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Minimum Number of Load Increments in Second-Order Elastic Analyses of Steel Frames

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Abstract: A relationship was discovered between the amplification factor and the number of load increments that are needed to limit the relative error to one percent in second-order elastic analyses with a predictor-corrector solution scheme. Previous research by the authors proposed a design equation to determine the required minimum number of load increments based on an evaluation of the elastic critical buckling load ratio. Further research has shown that an approximate amplification factor equation that is based on the B_2 multiplier equation produces similar results when the amplification factor is less than approximately four. Eleven moment frames are used to verify the use of the new approximate amplification factor in the proposed design equation.

Key words: Nonlinear geometric analysis, amplification factor, load increment, structural frame.

1. Introduction

The strength requirements of frames are often evaluated considering geometric nonlinear effects, which requires the engineer to make decisions about the required modeling effort and its associated computational time to achieve a desired level of accuracy. For steel frames modeled with beam elements, these nonlinear effects are accounted for using a solution scheme that incrementally applies the loads to approximate the "exact" equilibrium of the frame in the deformed configuration. The accuracy in modeling the frame in this configuration is dependent upon the number of load increments that are used to apply the external loads. Increasing the number of load increments to improve accuracy often comes at the cost of increased computational time since frame models often have a large number of degrees of freedom and multiple load combinations to consider. The effects of nonlinear material behavior may also need to be considered, but since the majority of routine building design considers only elastic material behavior [1], this other contributing influence on the number of load increments is ignored in the present study.

The number of load increments that are necessary to achieve a 1% relative error in a second-order elastic analysis was previously evaluated by the authors [2]. A total of 26 frames were modeled with an initial geometric imperfection of H/500 and increment size of 0.001 in a predictor-corrector solution scheme to obtain the "exact" lateral displacement results at the top of the frame. It was found that the amplification factor (AF) of the frame can be used to determine the minimum number of load increments. Using the frame's elastic buckling load factor α_{cr} to approximate the amplification factor $AF\alpha_{cr}$ [3-5], the displacement results using the number of increments equal to $5AF\alpha_{cr} - 2$ were compared with the "exact" displacement results and found to be within a 1% relative error.

This study explores the use of an alternative method to approximate the amplification factor using the B_2 multiplier equation in AISC 360, Appendix 8 [6]. The approximate amplification factor AF_{B2} is calculated by performing a first-order analysis, then the inter-story drift values are updated based on the appropriate B_2 multiplier at each level to approximate the second-order displacement at the top of the frame. It also

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investigates larger amplification factors than previously studied by increasing the gravity loads on 11 multi-story benchmark frames.

The frames were modeled in the MASTAN2 [7] analysis software, which accounts for second-order effects using an updated Lagrangian formulation, and for this study, the predictor-corrector solution scheme. The software is also capable of performing a linear buckling analysis using the inverse iteration method [8]. All members were modeled as planar 6-dof beam elements with elastic material behavior, and all frame models have out-of-plumb geometries.

2. Frame Amplification Factors

Numerous second-order elastic analyses were conducted to determine the minimum number of load increments that were needed to limit the relative error to 1% or less. Frames were modeled with an initial geometric imperfection of H/500 and an increment size of 0.001 in a predictor-corrector solution scheme to obtain the "exact" results. The amplification factor was evaluated for each analysis condition using Eq. (1), where δ_{2nd} is the lateral displacement of the top left node from a second-order elastic analysis, and δ_{1st} is the displacement at the same location from a first-order analysis.

The "exact" amplification factor AF can be approximated by $AF_{\alpha_{cr}}$ using the elastic buckling load ratio of the frame α_{cr} as given in Eurocode EN 1993-1-1 [4] and AS 4100 [5]. The critical buckling load P_{cr} in Eq. (3) is obtained using closed-form equations for simple frames or from an eigenvalue analysis for more complex frames, and P is the applied load on the frame. When performing an elastic critical load analysis in MASTAN2, α_{cr} is the Mode #1 Applied Load Ratio.

$$\mathrm{AF} = \frac{\delta_{2nd}}{\delta_{1st}} \tag{1}$$

$$\mathrm{AF}_{\alpha_{cr}} = \frac{1}{1 - {}^{1}\!/_{\alpha_{cr}}} \tag{2}$$

$$\alpha_{cr} = \frac{\mathrm{P_{cr}}}{\mathrm{P}} \tag{3}$$

The B_2 multiplier in Eq. (4) is from Appendix 8 of AISC 360 [6] and is used to account for the P-Δ effect of each story. In this application, the B_2 multiplier is calculated for each story, where α is taken as 1, P_{story} is the total axial load supported by the story (ΣP_i), and $P_{e\,story}$ is the elastic critical buckling strength of the story in the direction of translation being considered.

$$B_2 = \frac{1}{1 - \frac{\alpha P_{story}}{P_{e\,story}}} \geq 1 \tag{4}$$

The elastic critical buckling load can be determined by an eigenvalue analysis or by using Eq. (5). In this equation, H represents the total story shear in the direction of translation ΣV_i, L is the height of the story, and Δ_H is the first-order inter-story drift. R_M is taken as 0.85 for moment frames. Fig. 1 illustrates the components of Eqs. (4) and (5).

$$P_{e\,story} = R_M \frac{HL}{\Delta_H} \tag{5}$$

Using first-order analysis results of the applied loads on the frame, Eq. (6) is used to convert the lateral displacement at each level δ_i to the corresponding inter-story drift values Δ_i.

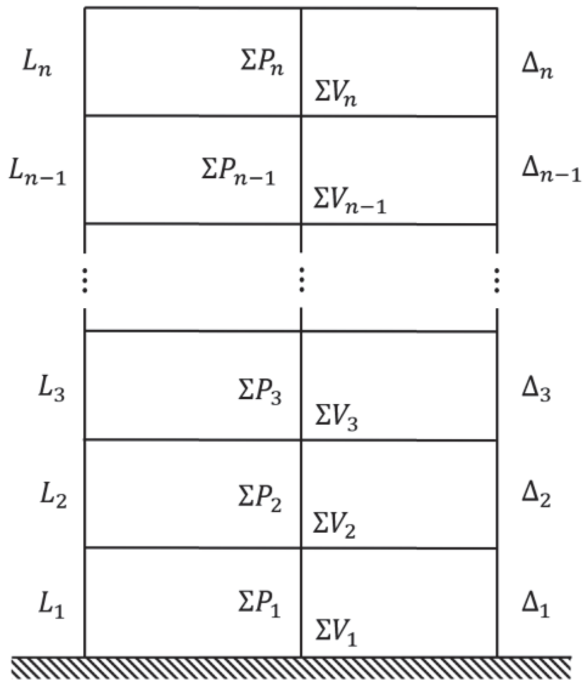


Fig. 1 Frame with B_2 equation components.

$$\begin{bmatrix} 1 & & & & & \\ -1 & 1 & & & & \\ & -1 & 1 & & & \\ & & \ddots & \ddots & & \\ & & & -1 & 1 & \\ & & & & -1 & 1 \end{bmatrix} \begin{Bmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \\ \vdots \\ \delta_{n-1} \\ \delta_n \end{Bmatrix} = \begin{Bmatrix} \Delta_1 \\ \Delta_2 \\ \Delta_3 \\ \vdots \\ \Delta_{n-1} \\ \Delta_n \end{Bmatrix} \quad (6)$$

The $C_{i,i}$ coefficients in Eq. (7) follow from the B_2 multiplier equation and are used to convert the first-order drift values to approximate second-order inter-story drift values. The variables in Eq. (7) are illustrated in Fig. 1.

$$C_{i,i} = \frac{1}{1 - \frac{\Sigma P_i \Delta_i}{0.85 \Sigma V_i L_i}} \quad (7)$$

Multiplying the first-order drift values by a diagonal matrix of the $C_{i,i}$ coefficients results in the approximate second-order inter-story drift values Δ'_i .

$$\begin{bmatrix} C_{1,1} & & & & & \\ & C_{2,2} & & & & \\ & & C_{3,3} & & & \\ & & & \ddots & & \\ & & & & C_{n-1,n-1} & \\ & & & & & C_{n,n} \end{bmatrix} \begin{Bmatrix} \Delta_1 \\ \Delta_2 \\ \Delta_3 \\ \vdots \\ \Delta_{n-1} \\ \Delta_n \end{Bmatrix} = \begin{Bmatrix} \Delta'_1 \\ \Delta'_2 \\ \Delta'_3 \\ \vdots \\ \Delta'_{n-1} \\ \Delta'_n \end{Bmatrix} \quad (8)$$

The sum of the second-order drift values $\Sigma\Delta'_i$ gives an approximation for δ_{2nd} in Eq. (1), and since δ_{1st} is the same as δ_n, the approximate amplification factor AF_{B2} is given as

$$AF_{B2} = \frac{\Sigma \Delta'_i}{\delta_n} \quad (9)$$

3. Minimum Number of Increments

Eq. (10) was proposed for design purposes to determine the minimum number of load increments to use in a second-order elastic analysis with the predictor-corrector solution scheme [2].

$$\text{Number of Increments} = 5\text{AF} - 2 \quad (10)$$

Eleven moment frames that were developed by Lu et al. [9], Vogel [10] and Statler et al. [11] were used to test the validity of this expression to determine the minimum number of load increments in a second-order elastic analysis using the predictor-corrector solution scheme. An overview description of Frames 1 through 11 is given in Fig. 2. All frames were modeled with an initial geometric imperfection of H/500 and an increment size of 0.001 in a predictor-corrector solution scheme to obtain the “exact” results. The 11 frames were modeled with the original stiffness and loads as those given in the Benchmark Problems file of MASTAN2 [7], and they were also modeled with the same stiffness conditions but with increased gravity loads to produce larger amplification factors for all 11 frames. This was necessary because the original loads resulted in AF values between 1.153 and 1.686, but with the increased loads, the AF values ranged between 1.755 and 3.373.

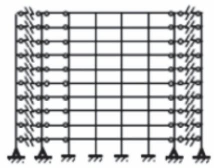
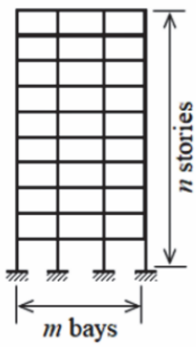
Frame No.	Description	Geometry
1	6 stories, 2 bays [Vogel 1985]	
2 - 5	10 stories, 3 bays; 0, 4, 8, 12 lean-ons [Lu et al. 1977; Statler et al. 2011]	
6	10 stories, 5 bays [Lu et al. 1977]	
7	20 stories, 1 bay [Lu et al. 1977]	
8	26 stories, 3 bays [Lu et al. 1977]	
9	30 stories, 2 bays [Lu et al. 1977]	
10	30 stories, 2 bays [Lu et al. 1977]	
11	40 stories, 2 bays [Lu et al. 1977]	

Fig. 2 Overview of benchmark frames 1 – 11.

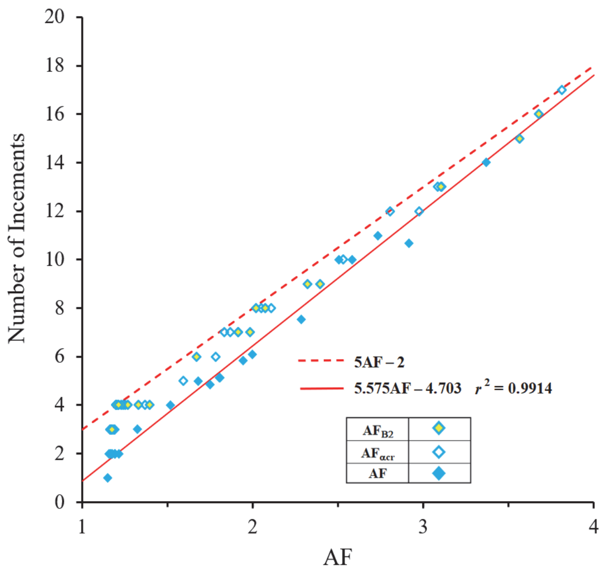


Fig. 3 Number of increments vs. amplification factor for Frames 5 − 15 (relative error ≤ 1%).

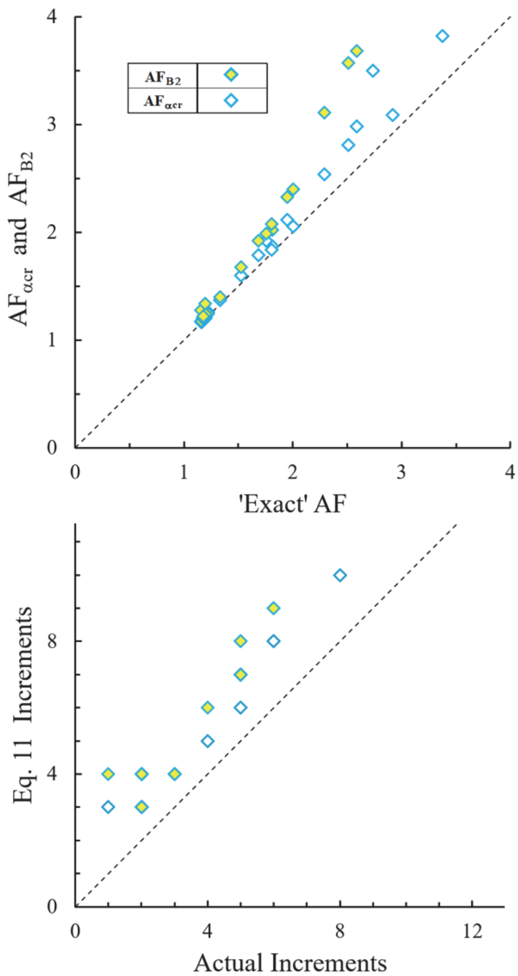


Fig. 4 Comparison of amplification factors and number of increments with "exact" results.

As indicated in Fig. 3, a linear relationship exists between the minimum number of load increments and the amplification factor (red line, r^2 = 0.9914). Eq. 10 was found to produce results for all 11 frames that were within 1% of the "exact" results. Fig. 4 illustrates the reason why this remains the case. The AFα_{cr} and AF$_{B2}$ values are always slightly larger than the actual AF values. This results in higher values for the minimum number of increments in Eq. 10 compared with the "exact" results; thus, they are conservative and always within 1% of the required minimum number of load increments. Fig. 4 also illustrates that using AF$_{B2}$ to determine the number of increments results in slightly higher values than those produced using AFα_{cr}, especially for amplification values greater than approximately 2

4. Conclusions

Based on the displacement results of 11 benchmark moment frames, a linear relationship was discovered between the amplification factor and the number of load increments that are needed to limit the relative error to 1% in a second-order elastic analysis with a predictor-corrector solution scheme. The integer result of 5AF − 2 is proposed for routine design purposes to determine the minimum number of load increments. Since a linear buckling analysis is required to calculate AFα_{cr}, an amplification factor based on the B_2 multiplier equation was investigated as an alternative. The use of AF$_{B2}$ was found to produce reliable and accurate results up to an amplification factor of approximately 4. The AF$_{B2}$ results were very comparable to those using AFα_{cr}, especially for amplification factors below approximately 2. It is recommended to limit the use of AF$_{B2}$ to amplification factors below 4; however, since most design conditions have amplification factors well below this threshold, using AF$_{B2}$ to determine the minimum number of load increments in a second-order elastic analysis can be confidently and widely used in practice.

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