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Teaching Math in Secondary (Middle and High) Schools: Complex Strategy and Its Successful Implementation

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This article deals with Math education in the Middle and High School in Kharkiv City (Ukraine) and in Academic Gymnasium No. 45 in particular. It shows the whole structure of education and the ways of motivation for learning Math at the high level by students. It also shows the obvious success of the strategy of complex Math teaching and analyzes its positive results for the last 25 years.

Keywords: math education, math competitions, Math Olympiads, development of critical thinking, students' scientific research

Introduction

This article deals with Math education in Secondary (Middle and High) Schools in Kharkiv City (Ukraine) and in the specialized school—Academic Gymnasium No. 45 in particular. It shows the whole structure of education in the chain country-city-school-class and ways to motivate students to learn higher level Math. It also shows the obvious success of the strategy of complex Math teaching and analyses its positive results for the last 25 years.

There are many countries with famous scientific schools and traditions in Mathematics. However, our world is changing rapidly. It takes teachers a long time to motivate their students to study science and its applications at universities. So, Math teachers should try to make efforts to encourage and motivate young people to get knowledge, the sooner the better. With the development of our society this is not an easy task, because of a lot of temptations far from science and learning.

The following information concerns the unique experience in Kharkiv city and at our school for a regular creation of student's motivation for deep Math learning.

The Unique Experience in Kharkiv City

Math Education in Kharkiv City

Kharkiv is one of the largest scientific centers of Ukraine (East Europe). There were three Nobel Prize winners in Kharkiv. It is the place where an atomic nucleus was split one of the first times in history. The scientists of Kharkiv University cooperated with students of city schools. But only at the beginning of 1980s we managed to create extra-curricular Math courses in the city. So, a great number of students were involved and they started learning additional Math topics. It was widespread in the 1990s and hundreds of students 5-17 years joined these

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courses. Today there are three big extra Math educational centers with about 5,000 students (there are about 114,000 students in Kharkiv schools).

As a result, a lot of students became the winners in Math Olympiads. It is also important, that learning Math is becoming more and more popular, prestigious, and trendy. Furthermore, our city council provides talented students with different grants and scholarships and reflects their success in Mass Media. Thus we have the basis for children learning Math in Elementary, Middle, and High Schools. Unfortunately, there is a lack of experienced Math teachers in Kharkiv to support students' interest in Math and to develop their abilities.

Math Education in Academic Gymnasium No. 45 and General Principles

Speaking about Academic Gymnasium No. 45 we have our own unique system of such complex Math development of students. We have been working on this task for about 25 years. The following information is about the special features of our teaching concept and how to apply this system successfully in other schools. The main idea is that successful Math teaching at school should be complex and includes the following:

(1) Teaching basic knowledge at lessons via heuristic methods;

(2) Preparing for Math competitions at lessons and extra-curricular lessons as well;

(3) Preparing science projects with students, under the leadership of Mathematicians in particular;

(4) Development of critical thinking skills and a scientific mindset, realization of the importance of Math and its connection to modern Computer Science.

Math teachers should not only give basic knowledge but also inspire students to solve problems, including playing and small competitions among themselves. So, I believe it is very important to recognize mathematically gifted children, pay them special attention, involve them into creative Math discoveries at lessons, additional lessons, tutorials, and organize their attendance in Math development centers. Due to participation in different competitions, scientific contests and conferences, a lot of students consider Math not only as a strict and boring school subject. It is also important to influence not only advanced students, but the whole class.

Unfortunately, the majority of schools are oriented on only one direction, such as: strictly following the course program, preparing for final and entrance exams, or working only with gifted students. But following this strategy we have a tendency of students' losing interest for Math, and Math teaching is becoming less effective. Let children do what they like, but under supervision. Most of them like playing so they can play Math games. If they like to compete, give them a chance to do it at lessons. If they are fond of gadgets, they can use them for Math modeling!

It is important for teachers to avoid putting certain boundaries on students' Math development. For example, we have to publish popular Math books as much as possible oriented not only for the top 5% students, but with a style such that it is available for most of students and their parents. Unfortunately, we have a gap in these types of Math literature. On the one hand, some of these books are for very low students, and on the other hand there are books for advanced students only. It looks like Math books are written in absolutely different styles, end even of different subjects. As a result, the majority of students can't find the appropriate books. It's important for children to know that lessons at school, popular science books, and additional Math literature for Olympiad participants are all aspects of the same science—Math!

Selective Exams After the Elementary School

My work with the students of Academic Gymnasium No. 45 starts at their entrance exams after the 4th grade. These selective exams give us a chance to find out students with good mathematical abilities. But it doesn't mean that all gifted children can pass these exams, and all of the selected students will connect their life with Math in the future. Though it is very important to develop the students' personalities and to give them an opportunity for creative research in an appropriate surrounding. There are two steps in our entrance exams:

(1) A competition for students of 3rd-6th grade, which is called "The world of Math";

(2) Entrance tests "Student of Gymnasium".

The first step is oriented on finding out most of the gifted students with a strong and special mindset. As usual, these students have been attending different city Math courses for a while, but some of them are real prodigies. The second step is based on testing the learned basic knowledge of the elementary school and students' abilities of applying it in unusual situations. The winners of these competitions form a special class. In fact, most of these children are really good at Math, so my goal as a teacher is to develop their abilities based on certain topics in Math.

Three Parts of Math Education

Usual lessons and the development of students' critical thinking. The further organizational work is conducted in three directions. The first one is making lessons in which compulsory topics are combined with solving Olympiad problems and tasks for the development of thinking skills. It takes the same time as usual, because gifted students are very quick in standard methods of solving problems and need a challenge. It is very effective to organize a group work at lessons while solving multi-case problems especially in geometry. It influences the development of students' critical thinking and teamwork skills.

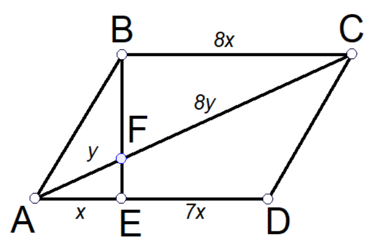

Figure 1. Multi-case parallelogram problem, first case.

For example, let us consider the following problem (Kryzhanovskiy, 2016). In the given parallelogram there is a height from the vertex of the obtuse angle. This height divides the opposite side by two segments with the ratio 1:7. Find the ratio of the two segments of the diagonal obtained by the intersection with the given height.

Consider two triangles in Figure 1: ΔAFE and ΔCFB : $\angle AFE = \angle BFC$ as vertical; $\angle FAE = \angle BCF$ as interior alternate angles for $\overleftrightarrow{BC} \parallel \overleftrightarrow{AD}$, AC transversal. So, we have a similarity of two triangles ΔAFE and ΔCFB. Hence, $\dfrac{AF}{FC} = \dfrac{AE}{BC} = \dfrac{1}{1+7} = \dfrac{1}{8}$.

Let's analyze how our figure corresponds to the given. In the given we have the ratio of two parts of the side of the parallelogram, but nothing about the order. So, we have another case in this problem. Let's look at Figure 2.

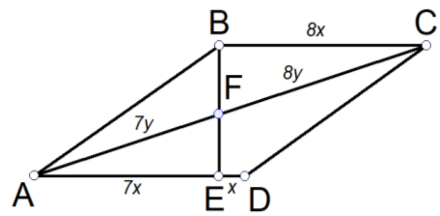

Figure 2. Multi-case parallelogram problem, second case.

In the same way, we have a similarity of two triangles ΔAFE and ΔCFB, and a corresponding proportion:

$$\frac{AF}{FC} = \frac{AE}{BC} = \frac{7}{1+7} = \frac{7}{8}.$$

Hence, in this case we have another answer.

It seems now, that we considered all possible cases. But no! We have two more cases for the location of the given height. Let's consider two new situations in Figures 3 and 4:

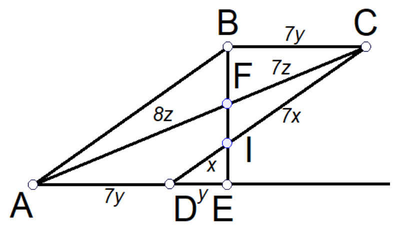

Figure 3. Multi-case parallelogram problem, third case.

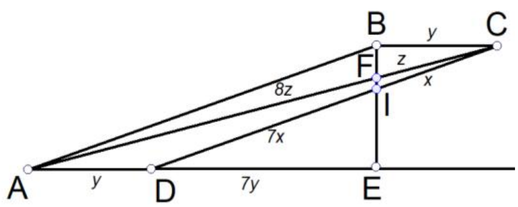

Figure 4. Multi-case parallelogram problem, fourth case.

These cases are interesting due to the fact, that the height is drawn to one side, but divides proportionally another side.

Let's consider these two situations. In both we have two similar triangles: ΔBIC and ΔEID. Indeed, $\angle BIC = \angle EID$ as vertical, $\angle BCI = \angle EDI$ as interior alternate angles for $\overleftrightarrow{BC} \parallel \overleftrightarrow{AD}$, CD transversal. So, $\frac{CI}{DI} = \frac{BC}{DE}$. With the same way we can obtain a similarity of two triangles ΔAFE and ΔCFB. Hence, $\frac{AF}{FC} = \frac{AE}{BC}$.

So, in the 3rd case $\frac{AF}{FC} = \frac{8}{7}$, and in the 4th case $\frac{AF}{FC} = \frac{8}{1}$.

As a result, we have four cases in this problem, and two answers only—8:7 and 8:1.

Now let's look at an example of another interesting side of complex Math teaching—connections with other subjects. The following shows a commonality between the AM-GM inequality and electric circuits.

Given two electric circuits, where both of them consist of one battery and two resistors (Figures 5 and 6).

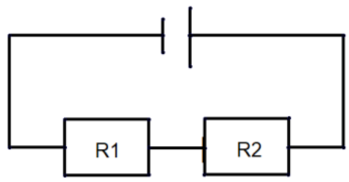

Figure 5. Electric circuit with two resistors that are connected in series with the battery.

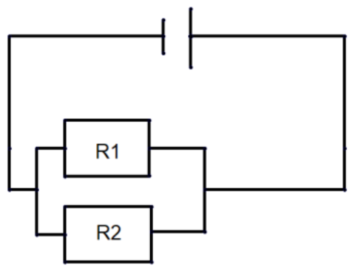

Figure 6. Electric circuit with two resistors that are connected in parallel with the battery.

In the first circuit two resistors are connected in series with the battery. In the second circuit the same resistors are connected in parallel with the battery. Find the minimum ratio between the total resistances in these two circuits.

Due to physical laws for total resistance, we have that in the first circuit: $R' = R_1 + R_2$, and in the second one $R'' = \dfrac{1}{\dfrac{1}{R_1} + \dfrac{1}{R_2}}$.

But how can we compare these results?

Let's use now the AM-GM inequality. With that we obtain the following:

$$\frac{R'}{R''} = \left(R_1 + R_2\right) \cdot \left(\frac{1}{R_1} + \frac{1}{R_2}\right) = \frac{\left(R_1 + R_2\right)^2}{R_1 R_2} = 4 \cdot \frac{\left(\dfrac{R_1 + R_2}{2}\right)^2}{R_1 R_2} \geq 4 \cdot \frac{R_1 R_2}{R_1 R_2} = 4.$$

With the smallest ratio of four, the resistance for the series circuit is at least four times the equivalent of the resistance of the parallel circuit.

Extra math classes "Solving of Math Olympiad Problems". The second direction is based on working with the children of the special class at our additional Math classes "Solving of Math Olympiad problems" systematically. At these lessons we study applying certain mathematical methods on appropriate examples adapted for children. As usual, the students enjoy this kind of activity, the content of which is related to serious mathematics, but it is unusual and often resembles as a game. These extra lessons exist in all grades from 5 to 11 (12), and are held once a week. The most important thing in this approach is adapting famous methods to develop the mathematical skills of children to school requirements (Kryzhanovskiy, 2015).

Individual and group lessons for the most gifted students. The third step is individual and group lessons prepare the most gifted students for Olympiads, competitions, and scientific conferences. Very often I'm not just a teacher, but also, I partner up with my students while solving actual problems. As usual, a good competition spirit and students' Math ambitions give a quite positive dynamic in the learning process. It is also important that all of the children are involved in creative work, in spite of changing forms and methods of learning.

It is very difficult to divide the topics which are learned at usual lessons and which are used only at Olympiads. Thus, I try to include Olympiad and research problems at our lessons as much as possible. Preparing for Math competitions we try to discover how usual methods of solving problems can be applied at Math Olympiads.

So, it is obvious that teachers, their students, and their parents should have close contact to each other. In fact, teachers and students communicate at lessons, preparing for Olympiads, visiting Math Festivals, at conferences, scientific competitions, and at summer Math schools. Thus, at Academic Gymnasium No. 45 in Kharkiv we are going to have the 15th year of our traditional summer school with the profile "Math and Computer Science". One of the main goals of this school is to stimulate the students' motivation in both Math and Computer Science studies, in the form of creative communication between students and their teachers.

Three types of Math competitions. To involve students in creative Math learning I try to organize their participation in competitions of three types. The first type consists of competitions, available for all students, such as "The Kangaroo", "The Championship of Math logical solving problems" (organized by France). The next one consists of competitions for mathematically gifted students, ready for intellectual and mental fighting. For instance, there is a system of national Olympiads in Ukraine, which includes four steps, with the final national Ukrainian Olympiad. Our students take also part in the "International Mathematical Tournament of Towns", "Math fest", and Olympiad named after Euler (organized by Russia, before 2.24.2022). The third type is the most difficult and consists of individual and group competitions for the most advanced and gifted students, such as IMO, EGMO, and Romanian Masters. Besides, there are some unique Ukrainian competitions, such as: "Young Math Tournament" (team research), "Champions Tournament" (including Math, Physics, and Computer Science), and Kiev international Math and Physics Fest (with the participation of scientists from the Math Institute of the National Academy of Science in Ukraine).

Students' scientific research. It is very important to mention that the problems of these competitions are often the beginning of scientific research. For instance, the problem, given by Prof. Valentine Leyfura, was the starting point for our cooperative research with my student Julia Fil. This project was presented at the National competition of students' scientific research.

Consider the triangle *ABC*, with the points *D*, *E*, *F* belonging to the sides *AB*, *BC*, *AC* respectively. Investigate the perimeter of the given triangle DEF using p, r, R—half of the perimeter, the inradius and the circumradius. In this project we found non-trivial lower and upper bounds for different cases, represented below.

(1) $\frac{2pr}{R} \le DE + EF + DF \le p$, given *D*, *E*, *F*—points of tangency of the circle, inscribed in the triangle *ABC*;

(2) $3\sqrt{3}r \le DE + EF + DF \le \dfrac{3\sqrt{3}R}{2}$, given *D*, *E*, *F*—foots of the angular bisectors of the triangle *ABC*;

(3) $DE + EF + DF = \dfrac{2S}{R}$ (for any right or acute triangles)

$\dfrac{2S}{R} < ED + DF + EF \le \dfrac{3\sqrt{3}}{2}R$ (for obtuse triangles), given *D*, *E*, *F*—foots of the altitudes of the triangle *ABC*;

(4) $\dfrac{2S}{R} \le DE + EF + DF < 3\sqrt{3}R$, given *D*, *E*, *F*—points of tangency of the excircles, points *D*, *E*, *F* belong to the sides *AB*, *BC*, *AC* respectively.

For example, let's find the upper bound in the inequality (4). First, find the sides and perimeter of the triangle *DEF* (Figure 7).

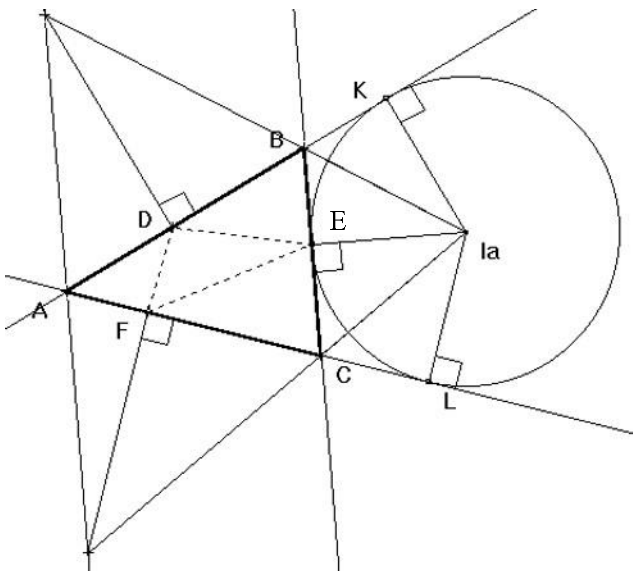

Figure 7. Estimating the perimeter of a triangle inscribed in a given one.

BI_a is the angular bisector of $\angle KBC$ and CI_a is the angular bisector of $\angle LCB$, since I_a is the excenter relative to the vertex *A*.

So, $\angle ECI_a = = \dfrac{1}{2}\angle ECL = 90^\circ - \gamma/2 \Rightarrow \angle EI_aC = \gamma/2$. The same way, $\angle EI_aB = \beta/2$.

Hence, from the right triangles ΔBI_aE and ΔCI_aE: $EC = r_a tg\,\gamma/2, BE = r_a tg\,\beta/2$.

Notice, that $r_a tg\,\beta/2\; tg\,\gamma/2 = r$. Indeed, $BC = r_a\left(tg\,\beta/2 + tg\,\gamma/2\right)$; but, obviously

$$BC = r\left(ctg\,\beta/2 + ctg\,\gamma/2\right) = r\cdot\frac{tg\,\beta/2 + tg\,\gamma/2}{tg\,\beta/2\; tg\,\gamma/2}$$, so $r_a tg\,\beta/2\; tg\,\gamma/2 = r$.

That's why, $EC = \frac{r}{tg\,\beta/2}, BE = \frac{r}{tg\,\gamma/2}$.

The same way, $BD = \frac{r}{tg\,\alpha/2}, AD = \frac{r}{tg\,\beta/2}, CF = \frac{r}{tg\,\alpha/2}, AF = \frac{r}{tg\,\gamma/2}$.

According to the cosine theorem:

$$DE^2 = DB^2 + BE^2 - 2BD\cdot BE\cdot\cos\beta = \frac{r^2}{tg^2\,\alpha/2} + \frac{r^2}{tg^2\,\gamma/2} - 2\cdot\frac{r^2}{tg\,\alpha/2\, tg\,\gamma/2}\cdot\cos\beta =$$

$$= \frac{r^2}{tg^2\,\alpha/2} + \frac{r^2}{tg^2\,\gamma/2} + \frac{2r^2}{tg\,\alpha/2\, tg\,\gamma/2} - \frac{2r^2}{tg\,\alpha/2\, tg\,\gamma/2} - 2\cdot\frac{r^2}{tg\,\alpha/2\, tg\,\gamma/2}\cdot\cos\beta =$$

$$= \left(\frac{r}{tg\,\alpha/2} + \frac{r}{tg\,\gamma/2}\right)^2 - 2\cdot\frac{r^2}{tg\,\alpha/2\, tg\,\gamma/2}\cdot 2\cos^2\frac{\beta}{2} = b^2 - \frac{4r^2\cos^2\beta/2\,\cos\alpha/2\,\cos\gamma/2}{\sin\alpha/2\,\sin\gamma/2} =$$

$$= b^2 - \frac{2r^2\sin\beta\cos\beta/2\,\cos\alpha/2\,\cos\gamma/2}{\sin\alpha/2\,\sin\beta/2\,\sin\gamma/2} = b^2 - \frac{2r^2\sin\beta\cdot\frac{p}{4R}}{\frac{r}{4R}} = b^2 - 2pr\sin\beta.$$

We used the following formulas: $\sin\alpha/2\,\sin\beta/2\,\sin\gamma/2 = \frac{r}{4R}$ and $\frac{p}{4R} = \cos\alpha/2\,\cos\beta/2\,\cos\gamma/2$ (Prasolov, 2001).

Note that we have a useful corollary with the product of the two formulas above: $\frac{1}{8}\sin\alpha\sin\beta\sin\gamma = \frac{pr}{16R^2}$, than $\sin\alpha\sin\beta\sin\gamma = \frac{S}{2R^2}$.

Using the sine formula for the area of the triangle, we obtain:

$$b^2 - 2pr\sin\beta = b^2 - 2S\sin\beta = b^2 - ac\sin^2\beta = 4R^2\sin^2\beta - ac\sin^2\beta =$$

$$= 4R^2\sin^2\beta - 4R^2\sin\alpha\sin\gamma\sin^2\beta = 4R^2\sin^2\beta(1-\sin\alpha\sin\gamma)$$

Therefore, $DE = 2R\sin\beta\sqrt{1-\sin\alpha\sin\gamma}$.

With the same way we get,

$$EF = 2R\sin\gamma\sqrt{1-\sin\alpha\sin\beta}\ ;\ DF = 2R\sin\alpha\sqrt{1-\sin\beta\sin\gamma}\ .$$

To get the formula for the perimeter of the given triangle *DEF* we make an estimation using the Cauchy-Schwarz inequality:

$$DE + EF + DF = 2R(\sin\beta\sqrt{1-\sin\alpha\sin\gamma} + \sin\gamma\sqrt{1-\sin\alpha\sin\beta}) +$$

$$+\sin\alpha\sqrt{1-\sin\beta\sin\gamma}) \le$$

$$\le 2R\sqrt{\sin^2\alpha + \sin^2\beta + \sin^2\gamma}\cdot\sqrt{3-\left(\sin\alpha\sin\beta + \sin\beta\sin\gamma + \sin\alpha\sin\gamma\right)}\ .$$

Notice that from the famous Leibniz formula $MO^2 = R^2 - \dfrac{a^2+b^2+c^2}{9}$ we obtain the following Leibniz inequality: $a^2 + b^2 + c^2 \le 9R^2$, and therefore

$$\sin^2\alpha + \sin^2\beta + \sin^2\gamma = \frac{a^2}{4R^2} + \frac{b^2}{4R^2} + \frac{c^2}{4R^2} \le \frac{9R^2}{4R^2} = \frac{9}{4}.$$

Now, using the AM-GM inequality we get:

$$\sin\alpha\sin\beta + \sin\beta\sin\gamma + \sin\alpha\sin\gamma \ge 3\cdot\sqrt[3]{\left(\sin\alpha\sin\beta\sin\gamma\right)^2} = 3\cdot\sqrt[3]{\left(\frac{S}{2R^2}\right)^2}\ .$$

Thereby: $DF + DE + EF \le 2R\cdot\dfrac{3}{2}\cdot\sqrt{3-\sqrt[3]{\left(\dfrac{S}{2R^2}\right)^2}} < 3\sqrt{3}R$, Q.E.D. □

Notice that the first expression for the upper bound is exact, but really long. Another expression is $3\sqrt{3}R$ and is a good estimation too—not really exact, but shorter.

Math Olympiads and scientific ideas. It is necessary to say that Olympiads and other Math competitions are important in some aspects. As it was already said, competitive and game forms of learning encourage students' motivation at lessons. Besides that, scientific tournaments are a perfect starting point for the first scientific research. But there are also deep problems that show the connection of solving Math problems very quick and serious scientific ideas. It is obvious that the difficulties of these problems depend on the difficulty level of the

Olympiad. But involving these scientific ideas is one of the most essential parts of Math competitions. Let's have a look at applying these ideas of work with space basis and with functional equations. This unusual problem (author: Oleg F. Kryzhanovskiy, NYC) was given at Kharkiv Region Math Olympiad (Kryzhanovskiy, 2012).

Let's name the sum of triangles with sides $a_1 \le b_1 \le c_1$ and $a_2 \le b_2 \le c_2$ the triangle with sides $a_1 + a_2, b_1 + b_2, c_1 + c_2$. Name the product of the real number $x > 0$ and the triangle with sides a, b, c the triangle with sides xa, xb, xc. Find all functions with a set of triangles as the domain, and a set of real numbers as the range, with following properties:

(1) for any triangles T_1, T_2: $f(T_1 + T_2) = f(T_1) + f(T_2)$ ("additive property")

(2) for any triangle T and any real number $x > 0$: $f(xT) = xf(T)$ ("homogeneous property").

Justify your answer.

Each triangle is defined by ordered triple of positive numbers (a,b,c), where $a \le b \le c$, which represent the triangle's sides. Try to guess the answer, using an analogy of vector components notation: $(a,b,c) = a(1,0,0) + b(0,1,0) + c(0,0,1)$. Thereby,

$$f(a,b,c) = f(a(1,0,0) + b(0,1,0) + c(0,0,1)) = f(a(1,0,0)) + f(b(0,1,0)) + f(c(0,0,1)) =$$
$$= af(1,0,0) + bf(0,1,0) + cf(0,0,1) = xa + yb + zc$$

However, the "basis" consists of "degenerated" triangles.

It would be easily improved by an operation, inverse to triangle addition—"subtraction of triangles". Indeed, if we have the equalities:

$$(1,0,0) = (3,3,3) - (2,3,3),\ (0,1,0) = (2,3,3) - (2,2,3),\ (0,0,1) = (2,2,3) - (2,2,2),$$

then: $(a,b,c) = a((3,3,3) - (2,3,3)) + b((2,3,3) - (2,2,3)) + c((2,2,3) - (2,2,2))$.

Since "subtraction of triangles" is not defined, amend last equality by shifting "subtraction" with addition:

$$(a,b,c) + a(2,3,3) + b(2,2,3) + c(2,2,2) = a(3,3,3) + b(2,3,3) + c(2,2,3),$$ or

$$(a,b,c) + a(2,3,3) + b(2,2,3) + 2c(1,1,1) = 3a(1,1,1) + b(2,3,3) + c(2,2,3).$$

Use the function f to both parts of the last equality and apply its "homogeneous" and "additive" properties:

$$f(a,b,c) + af(2,3,3) + bf(2,2,3) + 2cf(1,1,1) = 3af(1,1,1) + bf(2,3,3) + cf(2,2,3)$$

,

so

$$f(a,b,c) = a(3f(1,1,1) - f(2,3,3)) + b(f(2,3,3) - f(2,2,3)) + c(f(2,2,3) - 2f(1,1,1))$$

Thus, $f(a,b,c) = xa + yb + zc$, where x, y, z are any real numbers.

Checking by substitution of the given type function shows that they satisfy the given.

Finally, we obtained the answer: $f(a,b,c) = xa + yb + zc$, where x, y, z are any real numbers.

Summary

So, the main idea of the given experience is a selection of gifted students, complex development of their mathematical abilities, and encouraging students' motivation to study Math Science at school and university. The following results show the obvious success of the given method for 25 years in Academic Gymnasium No. 45, Kharkiv, Ukraine:

- More than 300 winners of Kharkiv Region Math Olympiad;
- More than 50 winners of the final level of the National Ukrainian Math Olympiad;
- Four winners of IMO (2003, Tokyo, Japan; 2011, Amsterdam, the Netherlands; 2018, Kluzh-Napoka Romania; 2023, Chiba, Japan);

For instance, the silver medal winner of IMO—2011 Olexii Kislinskij also won the International Mathematics Competition for University Students with a Gold medal in 2012. Recently he graduated from Yale University (the USA) with a Ph.D. in Math in 2021.

- One winner (Gold Medal) of EGMO (2017, Zurich, Switzerland);
- More than 80% of school graduates enter Ukrainian and foreign universities on specialties connected with Math and Computer Science.

The students of Kharkiv schools—the population of the city is 1,500,000—have even more impressive results. For instance, every year some students from Kharkiv become winners of the IMO.

But the main result of my work is the creation of the gifted student's mindset, involvement of them in the world of scientific research, Computer Science, and IT, and forming them as integrated personalities.

Certainly, it is possible to use my experience at Academic Gymnasium No. 45, Kharkiv, Ukraine in other schools. What we need is a cooperation between students and their parents, teachers, and school's senior management, city authorities, and different additional mathematical educational centers. Following this scheme of Complex Math education this experience might be useful for teachers of other schools. But it doesn't mean that this scheme should be followed and absolutely accurate. Of course, it should be adapted to the actual teachers' approaches. Thus, it helps them to succeed in their work and gets great results from their students in Math education.

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