

6th Graders' Mathematics Communicating Reasoning Ability Development: Status Quo and Existing Problems

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The study explored the current situation and existing problems of primary school 6th graders' mathematical communicating reasoning ability development. In this study, the researchers adopted quantitative survey methods. A total of 306 6th graders from J School in Jinzhong City, China voluntarily participated in the study. The research was done qualitatively by using a test that comprised of six math problems which were adopted from SBAC. 306 6th grade students were administered with a 45-min test to find out their performance in mathematics communicating reasoning ability. They solved problems regarding their personal knowledge, skills, and ability in learning math. The results showed that the development level of mathematics communicating reasoning ability of 6th graders was low.

Keywords: mathematical communicating reasoning ability, current situation, problem solving

Introduction

With the rapid development of technology, human beings have entered the era of big data. To be able to express accurately and make others fully understand, which seems to be simple, however it's not easy, logical thinking and language skills play an impact role in accurate expression. In modern society, the ability to express accurately greatly affects individuals' development potential. Among oral language expression, written expression, or other forms of expression, it is a necessary skill to select the appropriate language and method to accurately convey. For example, in most of job interviews or entrance examinations at all levels, interviewers are required to present accurately and provide effective answers within limited time.

In order to adapt to the development trend of the new era, the core literacy models of major countries (or organizations) in the world, such as the European Union, the United States, New Zealand, Finland, the United Kingdom, Japan, Germany, France, Australia, and other countries (or organizations), have jointly put forward new requirements for talent cultivation, one of which is thinking ability and communication literacy.

For a long time, in the research field of mathematics education, mathematical reasoning and mathematical communication have been regarded as two independent concepts, which has achieved fruitful results and promoted the sustainable development of mathematics education. Some scholars have discussed the relationship between mathematical reasoning and mathematical communication (Yu, 2016; Lv, 2017; Chen, 2016; Ning,

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2019), but the premise of such research is still the independence of the two concepts. In view of this, the current academic research on mathematical reasoning and mathematical communication is in an independent stage.

The Smarter Balanced Assessment Consortium (SBAC) clarifies the math academic goals for three-eight grades and 11 grade. The connotation of the four sub-goals is further elaborated in detail: Concepts & Procedures, Problem Solving, Communicating Reasoning (CR), Modelling & Data Analysis. This system regards communicating reasoning as one of the goals of mathematics, and defines communicative reasoning as students can clearly and precisely construct viable arguments to support their own reasoning and to critique the reasoning of others (SBAC, 2015).

Therefore, the purpose of this study is to investigate the current situation and existing problems of the development of mathematical communication reasoning ability of primary school 6th grade students.

Methodology

Site and Subjects

All the participants are 6th graders of J primary school in Shanxi Province. The school was one of the national experimental advanced schools, advanced unit, and demonstration schools in its district. The level of teaching quality in all primary schools in the area was above average. However, compared with primary schools in Beijing, Shanghai, and Guangzhou, J school was at a medium level nationwide.

Table 1

Statistics Information of Subjects

Class #	Boys		Girls		Anonymous	
	#	%	#	%	#	%
A	28	51.85	25	46.30	1	1.85
B	23	46.94	24	48.98	2	4.08
C	27	51.92	25	48.08	0	0
D	26	50.98	25	49.02	0	0
E	22	44.90	26	53.06	1	2.04
F	26	50.98	25	49.02	0	0
Sum	152	49.67	150	49.02	4	1.31
	306					

J school had a total of eight 6th-grade classes (A, B, C, D, E, F, G, H) with 45-55 students in each class. There were four-five mathematics classes a week. A total of 306 6th graders voluntarily participated in the test.

Instrument and Procedure

According to the evaluation framework of SBAC, the mathematical communication reasoning ability was divided into A-F sub-abilities. A: Test propositions or conjectures with specific examples. B: Construct, autonomously, chains of reasoning that will justify or refute propositions or conjectures. C: State logical assumptions being used. D: Use the technique of breaking an argument into cases. E: Distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in the argument—explain. F: Base arguments on concrete referents such as objects, drawings, diagrams, and actions.

Cai and Xu (2016) pointed out that the first important thing in evaluating mathematical communication is to develop test tasks (p. 21). Similarly, in the evaluation of mathematical communication reasoning ability, the

development of corresponding test tasks is also an essential part. Based on the above-mentioned evaluation framework of mathematical communication and reasoning in the sixth grade of primary school, and based on the online items of SBAC test resources in the U.S., six items that meet the above conditions were selected to draft the test in total. Slight modifications were made to the items combined with the arrangement of the content in the mathematics textbooks of the People's Education Press in China. Each question respectively tested a sub-goal (A, B, C, D, E, F) of mathematical communicating reasoning.

Table 2

Item Information of 6th Graders' Mathematics Communicating Reasoning Ability Test

Item	Sub-claim	Depth of knowledge	Grade	Mathematics content	Type of item
1.CR.A	A	2	5	Fractional Comparison Reduction	Blank
2.CR.B	B	2	6	Multiplication of Fractions	Short-answer
3.CR.C	C	3	4	Chicken with Rabbit Cage	Multiple-choice
4.CR.D	D	3	5	Fractional Addition & Subtraction	Blank
5.CR.E	E	2	5	Decimal Multiplication	Single-choice
6.CR.F	F	2	5	Meaning of Fraction	Multiple-choice

Table 3

Item for CR.B Information in the Test

2.CR.B
Xiaohua helps her mother to purchase 120 bottles of mineral water in the supermarket. The supplier Mr. Wang has 6 boxes of mineral water in total, and 18 bottles of mineral water in each box. Could Mr. Wang provide enough mineral water for Xiaohua? Please write your problem-solving steps and explain why.
Design Purpose: Construct, autonomously, chains of reasoning that will justify or refute propositions or conjectures. Content Purpose: (a) understand the meaning of integer addition, subtraction, and multiplication; (b) be able to calculate the two-digit integer multiplication accurately; (c) be able to calculate the three-digit integer multiplication accurately; (d) be able to compare integers within three digits.

Results and Discussion

Overall, there were 168 students, accounting for 54.9% of all subjects, their development of mathematical communicating reasoning ability was unqualified. That is, they were not able to clearly and accurately find arguments to support their own reasoning, of which 93 boys account for 60.78% of all boys, and 75 girls account for 49.02% of all girls. In the future, they need to make more efforts to improve the development of their mathematical communicating and reasoning ability.

There were 129 students, accounting for 42.16% of all subjects, their development of mathematical communicating reasoning ability was qualified, including 55 boys, accounting for 35.95% of the total number of 6th-grade boys, 74 girls, accounting for 48.37% of the total number of 6th-grade girls. That is, they were able to find some arguments to support their own reasoning, but there are errors.

The mathematical communicating reasoning ability development of only nine students, accounting for 2.94% of all participants, including five boys and four girls, is excellent. They were able to clearly and accurately find arguments to support their own reasoning. They had well prepared for the follow-up stage or future life and learning.

In general, the development level of mathematics communicating reasoning ability of 6th graders was low. Further teaching strategies are necessary to promote 6th graders' mathematics communicating reasoning ability.

Table 4

Evaluation Framework of Mathematical Communicative Reasoning Ability

First level index	Second level index	Level		
		Unqualified	Moderate	Excellent
Mathematical communicative reasoning ability: the ability to clearly and accurately find evidence to support their reasoning, and to use concise mathematical language, to accurately discuss their reasoning process and judge others.	Claim A: Test propositions or conjectures with specific examples	Not be able to use specific examples to test propositions or conjectures, and we can not use logical language to explain and discuss the above argument process.	Be able to test propositions or conjectures with some (or erroneous) specific examples, and can use mathematical language to explain and discuss errors in part of the argument process or process.	Be able to use, correct specific examples to test propositions or conjectures, and can use concise, correct, logical mathematical language to explain and discuss.
	Claim B: Construct, autonomously, chains of reasoning that will justify or refute propositions or conjectures	Not be able to independently construct the reasoning chain to prove or refute the proposition or conjecture, and it can not use logical language to explain and discuss the above argument process.	Be able to independently construct partial (or erroneous) reasoning chains to prove or refute propositions or conjectures, and can use mathematical language to explain and discuss errors in some arguments or processes.	Be able to independently construct a complete and correct reasoning chain to prove or refute a proposition or conjecture, and can use concise, correct and logical mathematical language to explain and discuss.
	Claim C: State logical assumptions being used	Not be able to state the logical assumptions used.	Be able to use, mathematical language, and logical assumptions used in discourse parts (or with errors).	Be able to use concise mathematical language, orderly, based, accurate discussion of the use of logical assumptions.
	Claim D: Use the technique of breaking an argument into cases	Not be able to a demonstration process into several argument segments, can not use logical language to explain and discuss the above argument process.	Be able to partially decompose an argument process into several argument segments, and can use mathematical language to explain and discuss some errors in the argument process or process.	Be able to correctly decompose a demonstration process into several argument paragraphs, and can use concise, correct and logical mathematical language to explain and discuss.
	Claim E: Distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in the argument—explain.	Not be able to distinguish between correct and wrong logic or reasoning, if there is a error in the argument, can not explain what the error is, and, can not use logical language to discuss the correct reasoning process.	Be able to roughly distinguish between correct and wrong logic or reasoning, if there is a mistake in the argument, and can use mathematical language to explain what some errors are and discuss the reasoning process, there are errors in the process.	Be able to accurately distinguish between correct and wrong logic or reasoning, if there is a mistake in the argument, and can use concise, correct, logical mathematical language to explain what the error is and the discussion, the correct reasoning process.
	Claim F: Base arguments on concrete referents such as objects, drawings, diagrams, and actions.	Not be able to demonstrate based on specific reference objects (such as objects, drawings, tables and behaviors), and it is unable to use logical language to explain and discuss the above argument process.	Be able to make partial (or erroneous) arguments based on specific references (e. g., objects, drawings, tables and behaviors), and can use mathematical language to explain and discuss part of the argument process or errors.	Be able to demonstrated based on specific references (e. g., objects, drawings, tables and behaviors), and can use concise, correct and logical mathematical language to explain and discuss.

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