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A Market Model of Risk Transfer and Insurance

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We introduce a model of a market where risk-averse consumers pay a fee to transfer their future losses to one or more firms. The future loss of each consumer is stochastic with a unique, known mean and variance. The law of large numbers allows the firms to know with certainty the expected aggregate loss of the consumers to whom they sell. The model could describe the behavior of agents in the market for property insurance where an insurance company sells a single type of policy to a specific group of consumers based upon the expected losses of those consumers and their willingness to pay for coverage. The model demonstrates how a single firm can choose the optimal segment of the market to which they sell a policy and how that choice might change when the distribution of consumers and their risk aversion changes. The model also demonstrates how two firms might engage in a cooperative strategy and share the market. The model shows how a firm entering the market will find it more advantageous to target a segment of the market with consumers that have a lower expected loss.

Keywords: insurance, quality competition

Introduction

There is an extensive literature on quality and price competition. We adapt an established model in this area so that it can describe a market for risk transfer from risk-averse individuals to one or more firms. It could apply to a market for property insurance contracts that are sold to consumers characterized by their unique distribution of losses. The foundation of the model is based on a model developed by Anderson, de Palma, and Thisse (1992), henceforth ADT. The ADT model is well established in the industrial organization literature, and it describes a duopoly market where the firms avoid direct competition by offering products with different qualities and prices. We build upon the ADT model by defining quality as the amount of insurance coverage. Risk averse consumers can choose to not insure, i.e., retain risk, or purchase an insurance contract that will cover 100 percent of the loss in an upcoming period. The loss is a random variable with a known mean and variance. In the model presented here, we add that firms can choose to not sell to consumers whose expected value of loss is too high.

The review of the literature in the next section provides the foundation of the theories and assumptions used in this model. After that section, we introduce the parameters for a market where there is only one company with the moniker Firm 1. In the initial analysis, Firm 1 does not have competition, but consumers can choose either to purchase insurance or to not purchase and retain their risk. After demonstrating the basic properties of the model with only Firm 1, we expand the model to include how a second firm might choose to enter and share the market with Firm 1. Additional analysis demonstrates how changes in market parameters can influence the results.

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Models of Quality Competition

Models analyzing the effects of quality competition are important because economic welfare is generally better when consumers and/or consumers have a wider variety of products and services from which to choose (see Mussa and Rosen, 1978; Anderson et al., 1992). Firms offering the products and services can benefit from being able to distinguish themselves and avoid direct competition. New entrants to an industry must usually try to find their own market niche, and that niche may be a quality that is different from the quality of existing products. Existing firms in the industry can benefit by allowing other firms to enter the industry with products that do not directly compete with the products already offered as opposed to entering with a product that directly competes.

Previous researchers have verified how it is often too costly for monopolists offering a given quality of goods to prevent entrants with a different quality product from entering the market; see Dixit (1980), Schwartz and Thompson (1986), Schwartz and Baumann (1987). This section summarizes the development of the literature that explores the dynamics and effects of this competitive process. Over the years, a large literature has built upon early works such as Leland (1977), Shaked and Sutton (1982) to show how quality differentiation relaxes price competition.

The reason for such differentiation is to avoid the Bertrand (1883) outcome associated with firms that compete with the same quality product. Researchers created the moniker “Bertrand death” to describe a competitive situation where prices are falling and production costs are increasing from direct competition. In the resulting equilibrium, the firms have non-positive profits (Anderson et al., 1992). According to Sutton (1997), “loss-making strategies will be avoided” by firms; also, “if a profitable opportunity exists in the market, there is ‘one smart agent’ who will fill it.” Rather than risking a competitive outcome where profits are non-positive, the entering firm can choose a quality that is different enough to allow it to not directly compete with the existing firm.

There is a lengthy literature on the entry of firms into industries where profit opportunities exist. Prescott and Visscher (1977), and Hay (1976) pioneered the modern sequential-entry models. Lane (1980) extended the model to allow for endogenous prices, and this development has characterized later models including the one in this paper. Subsequent works in this area include Shaked and Sutton (1982), Bernheim (1984), Harris (1985), Eaton and Ware (1987), Dewatripont (1987), Benoit and Krishna (1987), Tirole (1988), Vives (1988), Mclean and Riordan (1989), Donnenfeld and Weber (1992), and Anderson et al. (1992), Wauthy (1996).

The model in this paper builds upon the model established in Anderson et al. (1992), or ADT. The ADT model is a vertical differentiation model with a stable equilibrium where the quality levels could theoretically increase to infinity. In our model, the quality is an amount of coverage that will be sufficient to satisfy a consumer with a given loss distribution and risk aversion.

The next section summarizes the assumptions of the ADT model and modifies that model so that its inputs correspond to a market where an insurance company offers a contract with a given level of coverage for a premium. That section lays the groundwork for the section that describes the possible outcomes when a second firm enters the market.

A Single-Seller Market

This section serves two purposes. First, it introduces the assumptions of the model as well as the parameters that describe the characteristics of Firm 1 and the consumers in the model. Second, this section derives the equilibrium where Firm 1 only competes with the willingness of consumers to retain their risk of loss.

To describe how the consumers make their choices, we adapt the indirect utility function of consumers used by ADT. In the ADT model, indirect utility is a function of income, price, taste and quality, which are represented by y, P_i, θ, and q_i, respectively:

$$V_i(\theta) = y - P_i + \theta q_i \ (i = 1 \ldots n) \quad (1)$$

Each period, consumers purchase and consume a single unit of the product indexed i. The price and quality of the product are P_i and q_i, respectively. The increase in utility from consuming the product depends on each agent's unique preference factor denoted θ. Preferences follow a uniform distribution over an interval bounded by a lower and upper value of θ. The symbol q_H represents the quality level offered by the high-quality producer or the monopolist in the single-seller case. In the duopoly market, the low-quality producer offers q_L; therefore, $q_L < q_H$. The net benefit of consuming product variety i is given by the quantity θq_i-P_i.

Using the same basic concept, we propose the following utility function, which is a variation of the quadratic utility function used in the finance literature.[1] Utility is a function of the expected value and variance of loss:

$$V(\lambda) = (-\lambda - \rho\sigma^2/2)W \quad (2)$$

where

λ: the expected loss of the consumer expressed as percent;

ρ: the risk aversion of the consumer;

σ^2: the variance of the consumer's loss;

W: the nominal amount of wealth at risk.

To reduce the potential loss with the purchase of an insurance contract, a risk-averse consumer will pay a premium greater than that consumer's expected loss. The premium of that contract is P_i, which is the price charged by Firm "i". A consumer chooses to purchase the contract if $P_i < \lambda + \rho\sigma^2/2$. In other words, the consumer's utility from purchasing the contract from firm i is

$$V_i(\lambda) = (-P_i + \lambda + \rho\sigma^2/2)W \ (i = 1, \ldots n) \quad (3)$$

The consumer will purchase insurance if this value is positive. Going forward we employ the assumptions listed below.

The expected loss, λ, is unique to each consumer and the distribution of the values of λ can be approximated by a truncated exponential distribution that has a base with an upper limit of one, which represents an expected loss equal to 100 percent.

There is perfect information so that the consumers and the firms know the value of each λ.

The level of risk aversion, ρ, is the same for all consumers and ρ = 2 in this introductory study.

The distribution of a consumer's loss is normal and $\lambda = \sigma^2$.

All consumers have equal wealth at risk and W = 1.

The number of potential consumers is sufficient enough that the law of large numbers allows a precise estimate of the expected loss to the firm for the targeted market segment.

Given these demand-side assumptions, the utility function is

$$V_i(\lambda) = -P_i + 2\lambda.$$

On the supply side, for a given premium, the firm will only sell to consumers with values of λ below a

[1] A survey of the use of the quadratic utility model in game theory research can be found in Choné and Linnemer (2019).

specified level. In this model, that value is the premium, P_i. The profit function of Firm 1 is the number of buyers times P_i minus the aggregate expected loss of those buyers.

As mentioned, the consumers in the market are each identified by a unique λ. The distribution of λ is assumed to be

$$\lambda \sim 5e^{-5x} \; 0 < \lambda < 1 \text{ else zero.}$$

This has the desirable property of a declining density for higher level of losses. The parameter five was chosen because, for that value, the cumulative density over the range is equal to 0.99326, which is reasonably close to unity. As explained later, firms would generally not want to sell to consumers at the upper end of the distribution, so the truncation should not affect the outcome.

With these assumptions, Firm 1 would choose a market where it could charge $P_i = 2\lambda$ and sell to consumers where $P_i/2 < \lambda < P_i$. Consumers characterized by $\lambda < P_i/2$ would not purchase the contract, and the firm would not sell to consumers where $P_i < \lambda$. Given these assumptions, the profit function of Firm 1 is

$$\pi_1 = 2\lambda(e^{-5\lambda^*} - e^{-10\lambda^*}) - (\lambda e^{-5\lambda^*} + 0.2e^{-5\lambda^*} - 2\lambda e^{-10\lambda^*} - 0.2e^{-10\lambda^*})$$

where the λ^* in the expression represents the lower bound of the targeted market. The profit-maximizing value for this lower bound and the symbol going forward is in the following expression: $\lambda^* = \lambda_{L1,1} = 0.31872$.

If Firm 1 sells only a single contract, it will sell to consumers where

$$\lambda_{L1,1} = 0.31872 < \lambda < 0.63774 = \lambda_{U1,1}.$$

In the subscripts, L1 is the lower bound of the market segment for Firm 1. The second entry in the subscript, 1, identifies it as the value found in the single-seller market. The profit for Firm 1 in the single-seller market is $\pi_{1,1} = 0.03238$.

Table 1 displays results for variations from the assumed values for risk aversion and the scale parameter in the exponential distribution. Future research will attempt to generalize the model to find results that can be expressed as the inputs. The relationships in Table 1 are consistent with respect to changes in each input and each output. In the case of an increase in the scale parameter for the exponential distribution, this leads to a decrease in λ_{L1} and $\pi_{1,1}$. This is logical in that the higher scale factor means that there is a higher concentration of persons in the area of the distribution where there is a lower expected loss. When the scale factor goes up, the insurance company lowers $\lambda_{L1,1}$ and targets a market area where there is a higher concentration of consumers. The profit level is lower for a higher scale factor since the premium will be lower. When the scale factor increases from five to six but keeping $\rho = 2$, for example, the premium will decrease from 0.6368 to 0.5312.

Table 1

Results with Variations of the Risk Aversion of Consumers and the Scale Parameter of the Distribution

↓ρ Scale Parameter→	4	5	6
1.5	$\lambda_{L1,1} = 0.4602$ $\pi_{1,1} = 0.0114$	$\lambda_{L1,1} = 0.3326$ $\pi_{1,1} = 0.0203$	$\lambda_{L1,1} = 0.2772$ $\pi_{1,1} = 0.0169$
2	$\lambda_{L1,1} = 0.3984$ $\pi_{1,1} = 0.0405$	$\lambda_{L1,1} = 0.3187$ $\pi_{1,1} = 0.0324$	$\lambda_{L1,1} = 0.2656$ $\pi_{1,1} = 0.0270$
2.5	$\lambda_{L1,1} = 0.3840$ $\pi_{1,1} = 0.0574$	$\lambda_{L1,1} = 0.3072$ $\pi_{1,1} = 0.0459$	$\lambda_{L1,1} = 0.2560$ $\pi_{1,1} = 0.0383$

When the scale factor remains at five, an increase in risk aversion from $\rho = 2$ to $\rho = 2.5$ leads to a decline in $\lambda_{L1,1}$ to 0.3072 but the upper bound and premium increase to $2.25*\lambda_{L1,1} = \lambda_{U1,1} = 0.6912$, and this increases $\pi_{1,1}$.[2]

[2] $V_i(\lambda) = -P_i + \lambda + \rho\lambda/2$ become $V_i(\lambda) = -P_i + \lambda + 2.5\lambda/2$. The firm charges $P_{1,1} = 2.25\lambda_{L1,1}$.

Consumers are willing to pay more, and the firm can sell to a larger segment of the market. For the subsequent sections, the scale factor and coefficient of risk aversion will remain at five and two respectively.

Duopoly Market

The previous section finds a profit-maximizing solution when a single insurance company sells a single type of insurance contract in the market and charges the same premium to all buyers while promising to cover 100 percent of each consumer's loss. This section explores the possible outcomes when two firms compete in the market and each sells a single type of contract.

When a second firm enters the market, it has five choices. After the introduction of those choices below, there is a discussion of the outcomes for each choice.

Choice A: Firm 2 targets the same market that Firm 1 chose in the single-seller market so that $\lambda_{L2,2} = \lambda_{L1,1}$ and $\lambda_{U2,2} = \lambda_{U1,1}$.

Choice B: Firm 2 enters and targets a market with a lower bound equal to the upper bound of the market targeted by Firm 1: $\lambda_{L2,2} = \lambda_{U1,1}$.

Choice C: Firm 2 targets a segment of the market where $\lambda_{L2,2}$ is low enough so there would not be direct competition with Firm 1 and its single-seller market choice: $2\lambda_{L2,2} = \lambda_{U2,2} = \lambda_{L1,1}$.

Choice D: Firm 2 targets a market with a value of $2\lambda_{L2,2} = \lambda_{U2,2} > \lambda_{U1,1}$, which would lead to direct competition.

Choice E: Firm 2 and Firm 1 can cooperate and each sells a contract to a unique segment and agree on an equitable share of the profits to be earned from that market.

Choice A would mean that Firm 2 sells a contract identical to that of Firm 1 and competes in the same market where $0.319 < \lambda < 0.638$. In that case, the two firms would have to agree to charge identical premiums and offer identical contracts. If not, the potential competition could lead to a Bertrand Death or similar outcome. If they cooperate, they would share in the profit that Firm 1 had to itself in the single-seller case so that each would earn $\pi_{2,2} = \pi_{1,2} = 0.0328/2 = 0.0164$. The two firms might try to cooperate and share the market to avoid competition and a Bertrand Death result; however, the total industry profit is lowest for this choice, and competitive behavior is likely to result.

Choices B and C are similar situations in that Firm 2 chooses a market share that would not interfere with the market established by Firm 1 in the single-seller market. For Choice B, Firm 2 chooses a market where the lower bound is $\lambda_{L2,2} = 0.638$, and it would sell to consumers where $0.638 < \lambda < 1 = \lambda_{U2,2}$. Firm 2's profit would be 0.0080. Comparing this outcome to Choice C illustrates how markets with higher values of λ are less desirable for several reasons. For Choice B, the potential market is limited to $\lambda < 1$ because that value represents consumers with an expected loss equal to 100 percent. Firms who compete from below have an advantage over those selling to a market defined by higher values of λ. Finally, there is a lower concentration of consumers for higher values of λ. Fewer consumers not only leads to lower revenue, but it also leads to less diversification of the losses of those customers. The model here assumes that the number of consumers in the chosen market is sufficient to provide a precise forecast of the loss for the group. Relaxing this assumption, so that there would be some uncertainty in the forecast based on the number of consumers in the target market, would mean that the firms would face more risk for intervals with higher values of λ than lower values of λ. For now, we continue with the given assumption that the number of consumers is large enough to allow for a precise forecast of the aggregate loss of the consumers in the target market. We will explore the implications of uncertainty of the forecasted aggregate loss for the target market segment in future research.

Choice C is where Firm 2 targets from below a market that would not overlap with the market to which Firm 1 sells in the single-seller case. Firm 2 would sell to the market defined by $\lambda_{L2,2} = 0.1595 < \lambda < 0.319 = \lambda_{U2,2}$ and earn a profit equal to $\pi_{2,2} = 0.02234$. This is a more desirable outcome than Choice A for both firms. Firm 1 would have no reason to engage in competition, but Firm 2 could be tempted to encroach upon Firm 1's target market, which is Choice D, and that could begin competitive behavior that could lead to an undesirable outcome.

Choice D leads to Firm 2 directly competing with Firm 1. In Choice C, Firm 2 selects a lower bound for its target market, $\lambda_{L2,2} = 0.1595$, such that twice that value equaled the lower bound of Firm 1's target market in the single-seller case, $2*0.1595 = 0.319 = \lambda_{L1,2}$. If Firm 2 increases the lower bound to a value $0.1595 + \varepsilon$ so that the upper bound of Firm 2's market is $0.319 + 2\varepsilon$, then consumers defined by $0.319 < \lambda < 0.319 + 2\varepsilon$ who were purchasing Firm 1's contract would now purchase Firm 2's contract with the lower premium charged by Firm 2. Without cooperation, the two firms would begin to compete, which could lead to a Bertrand Death result.

Choice E is a potential, specific result of Choice D where the firms engage in a cooperative quality-differentiation strategy for the duopoly market. Firm 2 targets a market with a lower bound than that of Firm 1, but it will encroach on Firm 1's market only to the point where the profit of the two firms are equal. The target markets and profits for the firms are on Table 2. Although Firm 1 will have a lower profit, it is only 12 percent lower than what it was when it was a single seller. For both firms, it is clearly preferable to the cooperative result in Choice A. It is likely preferable to any result where the two firms engage in direct competition.

Table 2

Two-seller Market Cooperative Equilibrium Results Where Firms Agree to Equal Profits

	Market Share	Profit
Firm 2	$0.2115 < \lambda < 0.423$	0.0289
Firm 1	$0.4430 < \lambda < 0.886$	0.0289
Industry Profit =		0.0578

Interestingly enough, this sharing of the market is very close to the choice that would maximize industry profits. Table 3 provides the results where that is the goal. It suggests that a viable Choice F would be that the firms share the two market segments. They sell an identical contract to consumers where $0.2215 < \lambda < 0.433$ and a second identical contract to consumers where $0.4330 < \lambda < 0.866$. The possibility of firms selling more than one contract is discussed in the concluding section that follows.

Table 3

Two-seller Market That Maximizes Industry Profits

	Market Share	Profit
Firm 2	$0.2215 < \lambda < 0.433$	0.0281
Firm 1	$0.4330 < \lambda < 0.866$	0.0298
Industry Profit =		0.0579

Discussion

This paper introduces a model that can help analyze the behavior of firms who sell risk-transfer contracts, a.k.a., insurance contracts, to risk-averse consumers. When two firms compete, established research supports the proposition that an equilibrium is possible when the two firms lessen the intensity of the competition by differentiating themselves with respect to quality. This paper demonstrates that proposition in a case where two firms sell contracts that have differing amounts of insurance coverage and allows for an analysis of how the equilibrium changes when market parameter changes.

Consumer risk aversion is an important input, and for this introductory study, a quadratic utility function describes that risk aversion where the coefficient of risk aversion is two. It is found that if risk aversion increases, a firm can benefit by targeting consumers with a lower expected loss and charge a higher premium. The distribution of consumers' expected losses plays a role, too, and if that distribution's density increases for lower-risk consumers, then the firm will target more of those consumers but with a lower premium and earn a lower profit.

When two firms compete, they must share the potential profits to be earned. This paper explores the results of choices that a second firm might pursue when it enters a market where a single firm has been selling. A main goal would be to not initiate intense competition that would lead to an adverse outcome such as Bertrand Death. A viable solution would be for the entering firm, Firm 2, to target the market where expected losses are lower than that of the first firm, Firm 1, but to limit its encroachment on Firm 1's market to the point where they have equal profits. This is consistent with the findings in Lehmann-Grube (1997).

Future research will explore the implications of relaxing the current assumptions. One important assumption is that each firm sells the same contract to all consumers with the same premium, and each firm sells only one contract. Another assumption is that firms and consumers have perfect information concerning the distribution of the losses of each consumer in the upcoming period. Furthermore, the law of large numbers allows firms to forecast the loss of their target market precisely.

One possibility is to relax all of these assumptions and change the nature of the competition. If there is not perfect information, then firms would compete on their ability to estimate the loss characteristics of consumers. Also, relaxing the assumption concerning the Law of Large Numbers, would mean that firms would face uncertainty with respect to the expected loss, and that loss would be function of the number of consumers in the target market. There would more uncertainty for a firm targeting segments of the market where the expected losses are higher but the density of consumers is lower.

There is a lot to consider and do. The main point has been to introduce a model of a market for risk transfer from risk-averse consumers to one or more firms who can compose an accurate forecast of the aggregate losses for the consumers to whom they sell an insurance contract. In addition to proposing one cooperative equilibrium for a duopoly market, this study has shown how a firm will react to changes in inputs such as the distribution of risk among the consumers and their level of risk aversion.

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