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Random Differential Settlement Effects on Reliability of Existing Bridges

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Abstract: This paper investigates the impact of differential foundation settlement on the reliability of bridge superstructure based on loads and resistances statistical properties in Missouri State. Maximum deterministic differential settlement is often used in current AASHTO LRFD (load and resistance factored design) specification. However, the expected foundation settlement is quite different from the actual settlement due to the soil's large variability. Therefore, it makes sense to consider settlement as a random variable. In this paper, a lognormal distribution with coefficient of variation of 0.25 of random settlement is considered in reliability analysis based on limited previous studies. Dead and live loads are modeled as random variables with normal and Gumbel Type I distributions, respectively. Considering the regional traffic condition on Missouri roadways, the live load effect on existing bridges based on weight-in-motion data is also investigated. The calibrated resistance statistical properties such as bias and COV (coefficient of variance) are used for reliability analysis. Total 14 existing bridges based on Strength I Limit State are analyzed. Since no differential settlement is considered in the past designed bridges in Missouri, small differential settlement can significantly reduce the reliability indices of the superstructure, depending upon the span length and rigidity of the girder. The analysis results also show that the reliability of existing steel-girder bridges is consistently higher than prestressed concrete and solid slab bridges; the shorter and stiffer the spans, the more significant the settlement's effect on the reliability of bridge superstructures; As the span length ratio becomes less than 0.75, the girder and solid slab bridges' reliability drops significantly at small settlements.

Key words: Bridges, superstructure, LRFD.

1. Introduction

Since October 2007, all state departments of transportation in the U.S. have been mandated to use the AASHTO LRFD (load and resistance factored design) specifications in their federally funded bridge projects. As a critical part of a bridge system, the foundation not only affects the safety and stability of the overall system, but also constitutes a significant portion of bridge construction costs.

In general, the vertical load capacity of foundations increases with the foundation settlements. Under a given design load, the more allowable settlement is, the smaller size or shorter length of foundations can be used. However, differential foundation settlement as one type of external loads as specified in the AASHTO design specifications [1] may induce additional responses in superstructure, such as deflection, moment, shear, and reaction. Researches done by Moulton [2] show that 76.2 mm differential settlement of continuous girder bridges can have very important effects upon the stresses, depending upon the span length and rigidity of the girder. To account the differential settlement effect on bridge superstructure, in the AASHTO specifications, the extreme differential settlement is considered as an external load with a load factor γ_{SE} = 1.0 when combined with other loads in strength limit states (I, II, III, and V) and service limit states (I and III). The extreme differential settlement used in AASHTO specification is obviously a maximum deterministic value. However, the calculated or expected differential settlement is quite different from the actual settlement due to the soil's large variability and uncertainty [3, 4]. More researches done by Zhang et al. [5] show that

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COV (coefficient of variance) of estimated pile settlement can be reached to 25%. Therefore, it is more reasonable to consider settlement as a random variable.

In current Missouri bridge design practice, support settlement is not considered because of the lack of well founded criteria for the tolerable support settlement of bridges. This design practice implies that all continuous bridges be supported on rock directly or on deep piles/shafts that are socketed into rock. However, the differential settlements of real bridges are often observed by engineers, therefore, it is quite necessary to study the differential settlement effect on the bridge safety. This paper investigates the impact of differential foundation settlement on the reliability of bridge superstructure based on the state-specific loads and resistance statistical properties. Differential settlement is modeled both deterministic value and probabilistic variable with lognormal distribution. Total 14 existing bridges (6 steel girder bridges, 5 pre-stressed concrete girder bridges, and 3 concrete slab bridges) based on Strength I Limit State are analyzed. The analysis results show that a small differential settlement has significant effect on the reliability indices of the superstructure, depending upon the span length and rigidity of the girder.

2. Statistical Properties of Loads and Resistances

2.1 Statistic Parameters for Dead Load

Dead load mainly represents self-weights of structural and nonstructural elements that are permanently attached to bridges. It is often considered to be uniformly distributed along the length of each member. In this study, three components of bridge dead loads are considered: prefabricated members (steel and precast concrete), cast-in-place concrete members, and wearing surfaces [6]. The mean and standard deviation of a dead load variable were estimated from the bias factor and the COV listed in Table 1 [6]. The mean of the dead load is defined as the product of its nominal value and the bias factor. The standard deviation is defined as the product of the mean and COV values. They can be expressed into:

$$\mu_D = D_n \times \lambda_D \tag{1}$$

$$\sigma_D = \mu_D \times COV_D \tag{2}$$

in which μ_D and D_n represent the mean and nominal values, λ_D is the bias factor, σ_D is the standard deviation, and COV_D is the coefficient of variation of the dead load.

2.2 Statistic Parameters for Live Load

In bridge designs, live load basically means the weight of vehicles plus their impact effect. As an extreme value of the day, the daily maximum value of a live load can be assumed to follow the extreme value distribution. In the recent study by Kwon et al. [7, 8], the Gumbel Type I distribution was adopted to represent the maximum daily load effect. Due to limited weigh-in-motion data over a short period of time in comparison with a bridge design life of 75 years, it is necessary to project the short-term field observations for a long-term prediction of the 75-year maximum load effect using the extreme value theory. The Gumbel Type I probability distribution function, $F_{X\text{-1day}}(x)$, and the probability density function, $f_{X\text{-1day}}(x)$, of the daily maximum load effect can be expressed into [9, 10]:

Table 1 Statistical values of dead load [6].

Component	Bias factor	COV	Distribution
Prefabricated members	1.03	0.08	Normal distribution
Cast-in-place members	1.05	0.10	
Wearing surfaces	1.00	0.25	
Miscellaneous	1.03-1.05	0.08-0.10	

$$F_{X-1day}(x) = \exp(-\exp(-\frac{x-u}{\alpha})) \quad (3)$$

$$f_{X-1day}(x) = \frac{1}{\alpha}\exp(-\frac{x-u}{\alpha})F_{X-1day}(x) \quad (4)$$

in which the scale parameter (α) and the location parameter (u) can be determined by the maximum likelihood estimation to fit the distribution model into the available observed data. Assuming that the maximum daily load effects are independent in 75 years, the probability distribution function of the 75-year maximum load effect can be projected by,

$$\begin{aligned} F_{X-75years}(x) &= \left[\exp(-\exp(-\frac{x-u}{\alpha}))\right]^N \\ &= \exp\left\{-\exp(-\frac{x-u_n}{\alpha_n})\right\} \end{aligned} \quad (5)$$

where N is the number of days during 75 years. The scale parameter of $F_{X-75years}(x)$, α_n is the same as that of $F_{X-1day}(x)$, and the location parameter of $F_{X-75years}(x)$, u_n is equal to $u+\alpha\ln(N)$ [10]. Thus, the mean value and standard deviation of the maximum live load effect in 75 years can be estimated by:

$$\mu_L = u_n + \gamma\sigma_n \quad (6)$$

$$\sigma_L = \sqrt{\frac{\pi^2\alpha_n}{6}} \quad (7)$$

in which μ_L and σ_L represent the mean value and standard deviation of the live load, α_n and u_n are the scale and location parameters of Gumbel Type I distribution for the 75-year maximum live load, and γ = 0.577216 is the Euler number.

2.3 Statistic Parameters of Settlement

The current AASHTO bridge design specifications [1] recommend that the angular distortion between adjacent foundations be less than 0.008 rad in simple spans and 0.004 rad in continuous spans [11-13]. They correspond to the maximum recommended differential support settlements of L/125 and L/250 for simple and continuous spans, respectively. In this study, the support settlements are assumed to be either deterministic with extreme values or random with a lognormal distribution. The deterministic extreme values or mean values of support settlement selected in this study do not exceed the recommended maximum values of L/250 for continuous spans.

Another important parameter for the random variable of differential settlement is the COV. For granular soils, there are a wide variety of methods currently in use for settlement prediction. However, the settlement of granular soils occurs so rapidly that at each stage of loading during the construction process, the settlement is essentially completed before the next stage of loading is applied. Thus, adjustments can be made during construction to minimize the post-construction settlement of significance to this study. For cohesive soils, a few sophisticated methods are available for settlement prediction. Based on a comparative study by Moulton et al. [2], and Zhang et al. [5], the ultimate foundation settlement can be numerically estimated to within 25 percent of its measured value so long as reliable subsurface exploration and consolidation test data are available. Therefore, the coefficient of variation for the support settlement is limited to 0.25 in this study.

To fully describe the random variable of support settlement, its distribution function needs to be defined. Lognormal distribution has been widely used in various engineering applications based on observed histogram shapes [9, 14]. In this study, it is also considered for differential support settlement.

2.4 Statistic Parameters of Resistance

The statistical distribution of resistance is based on the uncertainties in material (strength, modulus of elasticity, etc.), fabrication (geometry), and analysis (accuracy of analysis equations). The resistance R can thus be expressed into its nominal value R_n multiplied by three random factors: M for material properties, F for fabrication outcomes, and P for professional analyses [6]. That is,

$$R = R_n MFP \tag{8}$$

Since the three factors are associated with three independent processes in the creation of a bridge structure, they can be assumed to be statistically independent. In this case, the COV of the overall resistance can be determined by the square root of the sum of the squared COV values of individual factors provided they are small. That is,

$$COV_R = \left(COV_M^2 + COV_F^2 + COV_P^2\right)^{1/2} \tag{9}$$

The statistical distribution of the resistance R can be characterized by a bias factor λ_R and the COV_R. The bias factor is the ratio of the mean to the nominal design value. The COV_R is the ratio of the standard deviation to the mean of resistance, giving an indication of uncertainty.

In order to determine the statistical distribution of resistance, Kwon et al. [7, 8] and Orton et al. [15] recently analyzed 100 sample bridges (14 reinforced concrete, 58 prestressed, and 28 steel) from MoDOT's bridge inventory to determine the strength of representative bridges according to the 2008 AASHTO bridge design specifications. Based on their research results, the moment statistical parameters were updated to reflect the bridge samples in MoDOT inventory. They are presented in Table 2.

The mean value and standard deviation of resistance can then be determined using the bias factor and COV value like Eqs. (1) and (2):

$$\mu_R = R_n \times \lambda_R \tag{10}$$

$$\sigma_R = \mu_R \times COV_R \tag{11}$$

in which μ_R and σ_R are the mean value and standard deviation of resistance, R_n and λ_R are the nominal value and bias factor of resistance, and COV_R is the coefficient of variation of resistance.

3. Reliability Analysis of Existing Bridges with Settlement Effects

The reliability indices of bridges will be evaluated based on the uncertainties in live load, dead load, resistance, and settlement effect. In this study, the reliability indices are calculated using the FORM (first order reliability method) [16, 17]. To investigate the impact of differential foundation settlement on the reliability of existing bridges, total 14 bridges were analyzed in this paper. The bridge information and allowable settlement are presented in Table 3. Settlement is modeled both probabilistically and deterministically. For the first case, settlement is defined as a random variable with a mean of nominal value and a COV of 0.25. The current MoDOT bridge design specifications require that all continuous bridges be founded on rock or piles/shafts and no settlement is permitted. However, foundation actually settles in applications. This case can shed light on how random settlement (mean value) affects the reliability of a bridge that is not designed for settlement under a combined dead, live, and settlement effect. Thus, the limit state function for the safety margin g is given by:

$$g = R - (DC + DW + LL_{75-year} + SE) \tag{12}$$

Table 2 Statistical parameters of resistance.

Type of structure	Moment [6]		Shear force [6]		Moment [7, 8]		Distribution
	Bias	COV	Bias	COV	Bias	COV	
Steel	1.12	0.100	1.14	0.105	1.23	0.081	Lognormal
Concrete slab	1.14	0.130	1.20	0.156	1.17	0.090	
Prestressed concrete girder	1.05	0.075	1.15	0.140	1.055	0.069	

Table 3 Selected bridges for analysis.

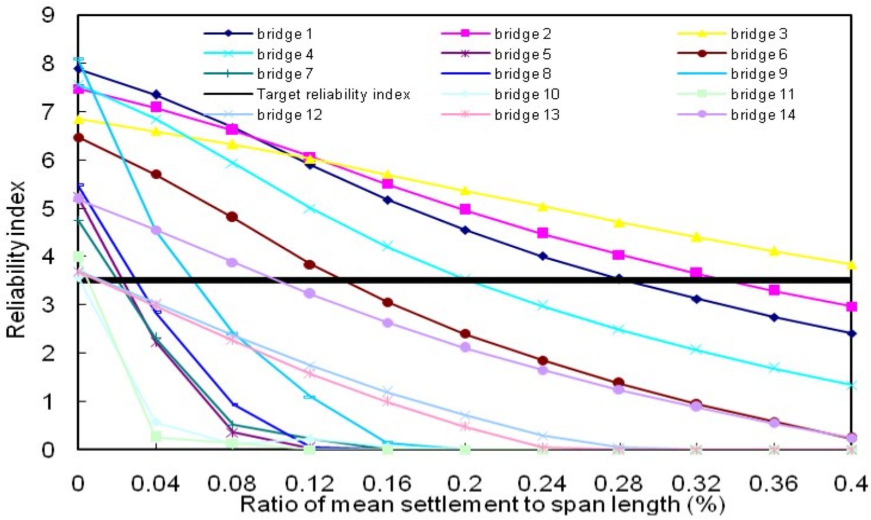
Bridge index	Bridge No.	Drawing No.	Bridge description	Minimum span (m)	Allowable settlement 0.004*L* (mm)
1	2664	A3101	36.6 m + 36.6 m steel girder	36.6	146
2	3945	A4840	42.1 m + 43.0 m steel girder	42.1	168
3	31500	A7300	19.7 m + 19.7 m steel girder	19.7	42
4	2852	A3386	22.9 m + 29.6 m + 22.9 m steel girder	22.9	91
5	3475	A4256	5.9 m + 7.9 m + 7.2 m steel girder	5.9	24
6	4043	A4999	17.7 m + 36.3 m + 16.5 m steel girder	16.5	66
7	3332	A4058	11.3 m + 19.8 m + 12.8 m prestressed concrete girder	11.3	45
8	11893	A5161	11.6 m + 19.8 m + 12.2 m prestressed concrete girder	11.6	46
9	29023	A6569	19.8 m + 30.5 m + 22.6 m prestressed concrete girder	19.8	79
10	3276	A3973	18.0 m + 18.0 m + 13.1 m + 13.1 m prestressed concrete girder	13.1	52
11	3753	A4582	11.6 m + 11.6 m + 19.8 m + 11.6 m prestressed concrete girder	11.6	46
12	2983	A3562	10.4 m + 14.0 m + 10.4 m slab bridge	10.4	41
13	28993	A6450	5.5 m + 7.0 m + 5.5 m slab bridge	5.5	22
14	2856	A3390	14.6 m + 18.3 m + 14.6 m + 16.8 m slab bridge	14.6	58

in which R, DC, DW, $LL_{75\text{-}year}$, and SE are random variables, DW is the weight of the wearing surface, DC is the dead load excluding the wearing surface (DW), $LL_{75-year}$ is the 75-year live load based on the weight-in-motion data.

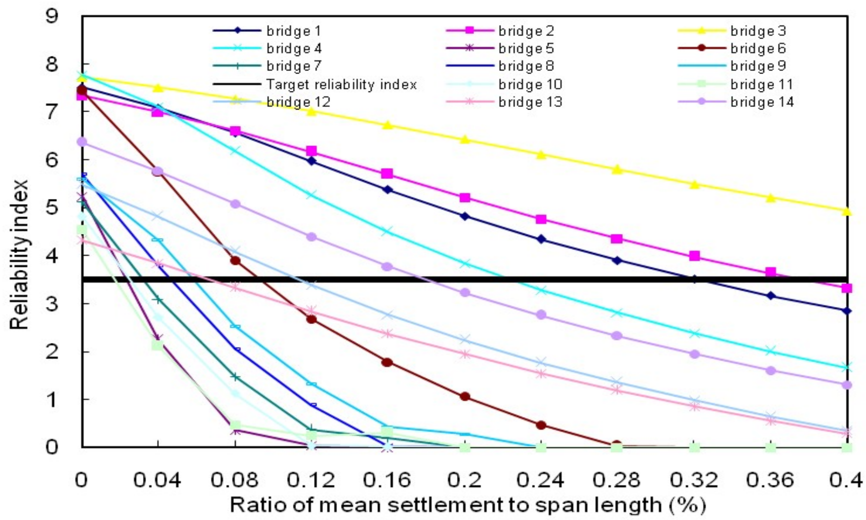
The second case models the settlement as deterministic value. This case represents the current MoDOT practice when settlement is defined as an extreme value. In this case, settlement is not treated as a random variable or its COV is equal to zero. The minimum resistance is the same as Eq. (12). The safety margin function is also the same as Eq. (13) except that SE is now a deterministic extreme value. This case can shed light on how extreme settlement (extreme value) affects the reliability of a bridge that is not designed for settlement under a combined dead, live, and settlement effect. The moments and shear forces due to dead load, live load, differential settlement are calculated based on general finite element method.

The moment and shear resistances of the all bridges are calculated based on the cross-section properties. The reliability indices of the 14 existing bridges based the statistical properties of external loads and resistance described in section 2 and FORM method are presented in Figs. 1 and 2.

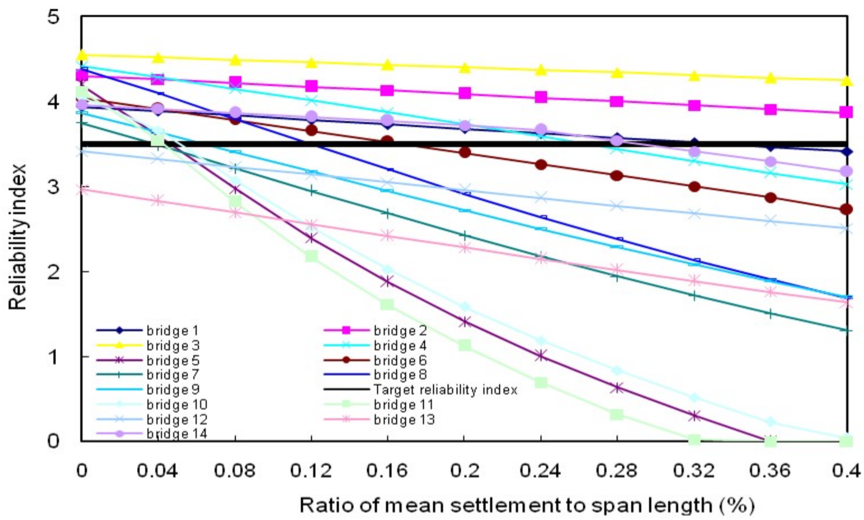
It can be seen from Figs. 1 and 2: (1) The reliability index for negative moment controls the minimum tolerable settlement of the existing bridges due to its additive combination with the maximum negative moments induced by dead and live loads. However, the overall distributions of the tolerable settlements for positive and negative moments of the 14 bridges are similar with slight differences. (2) The reliability index of prestressed or solid slab bridges is lower than that of steel-girder bridges. It is more sensitive to settlement effects due to their stiffer members. (3) Reliability index is sensitive to the ratio of span lengths. As the span length ratio becomes less than 0.75, the reliability index drops significantly at small settlements.



(a) Reliability indices for negative moment

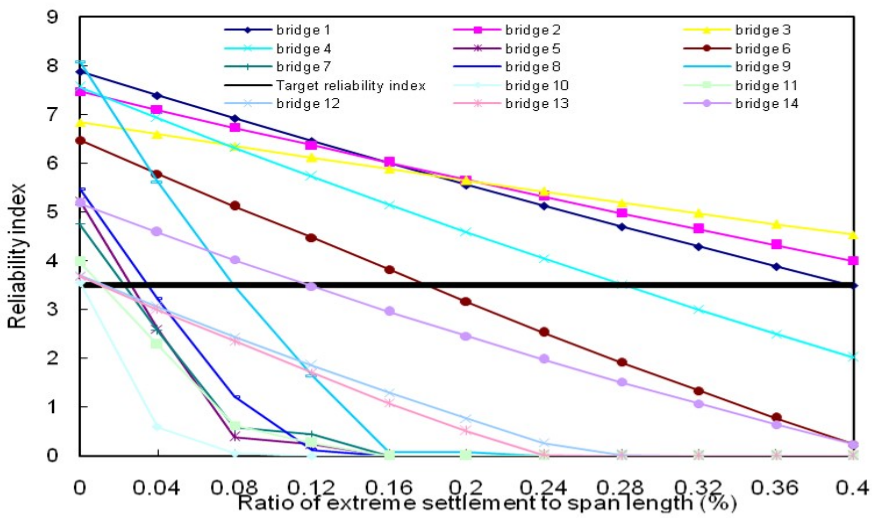


(b) Reliability indices for positive moment

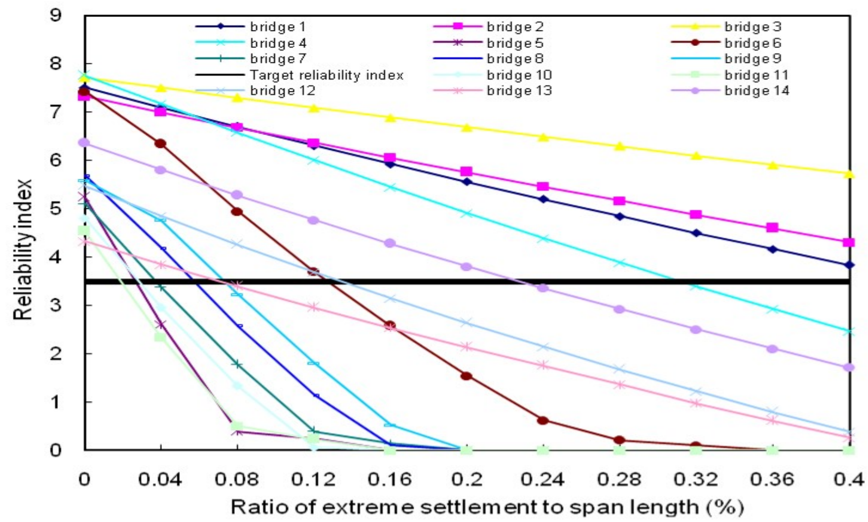


(c) Reliability indices for shear

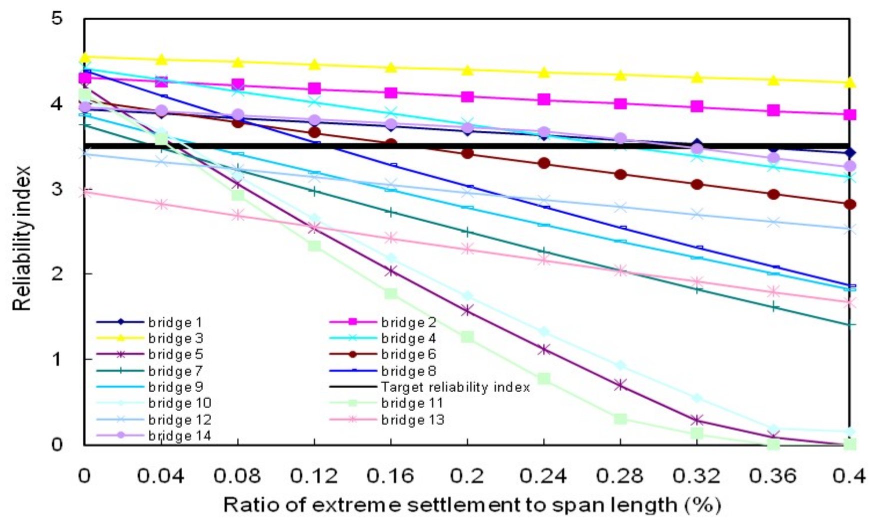
Fig. 1 Random settlement impact on reliability indices of 14 existing bridges.



(a) Reliability indices for negative moment



(b) Reliability indices for positive moment



(c) Reliability indices for shear

Fig. 2 Deterministic settlement impact on reliability indices of 14 existing bridges.

4. Conclusions

This study is focused on the effect of bridge settlement on the reliability of bridges under gravity loads. Dead and live loads were respectively modeled as random variables with a normal distribution and a Gumbel Type I distribution. Settlement was characterized by a random variable with lognormal distribution or a deterministic extreme value. The reliability indices for positive and negative moments as well as shear of 14 existing bridges were evaluated and compared. Based on the analysis results, the following collusions can be drawn:

(1) The settlement effect changes significantly, depending on the minimum span length, girder rigidity, and the span ratio of the bridges.

(2) The overall reliability of the bridges without differential settlement effects meets the 2008 AASHTO requirements except for one solid slab bridge for shear reliability. The reliability of the steel-girder bridges for moment is higher than that of prestressed and solid slab bridges.

(3) The reliability index for negative moment controls the minimum tolerable settlement of the existing bridges due to its additive combination with the maximum negative moments induced by dead and live loads. However, the overall distributions of the tolerable settlements for positive and negative moments of the bridges are similar.

(4) The reliability index of prestressed or solid slab bridges is lower than that of steel-girder bridges. It is more sensitive to settlement effects due to their stiffer members.

(5) Reliability index is sensitive to the ratio of span lengths. As the span length ratio becomes less than 0.75 (bridge No. 6 to No. 10), the reliability index drops significantly at small settlements.

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References

[1] AASHTO. 2008. *LRFD Bridge Design Specifications* (4th ed.). Washington, DC: AASHTO.

[2] Moulton, L. K. 1986. *Tolerable Movement Criteria for Highway Bridges, FHWA-86/228.* Washington, DC: Federal Highway Administration, U.S. Department of Transportation.

[3] Phoon, K. K., and Kulhawy, F. H. 1999. "Characterization of Geotechnical Variability." *Canadian Geotechnical Journal* 36: 612-24.

[4] Baecher, G. B., and Christian, J. T. 2003. *Reliability and Statistics in Geotechnical Engineering.* New York: John Wiley & Sons.

[5] Zhang, L. M., Xu, Y., and Tang, W. H. 2008. "Calibration of Models for Pile Settlement Analysis Using 64 Field Load Tests." *Canadian Geotechnical Journal* 45: 59-73.

[6] Nowak, A. S. 1999. *Calibration of LEFD Bridge Design Code, NCHRP Report 368.* Washington, DC: Transportation Research Board, National Research Council.

[7] Kwon, O., Orton, S., Kim, E., Hazlett, T., and Salim, H. 2011. *Calibration of Live Load Factor in LRFD Design Guidelines. Final Report No. TRyy0913.* Jefferson City, MO: Missouri Department of Transportation.

[8] Kwon, O., Kim, E., and Orton, S. 2011. "Calibration of the Live Load Factor in LRFD Bridge Design Specifications Based on State-Specific Traffic Environments." *Journal of Bridge Engineering* 16 (6): 812-9.

[9] Ang, A. H. S., and Tang, W. H. 1975. *Probability Concepts in Engineering Planning and Design, Volume I: Basic Principles.* New York: John Wiley & Sons, Inc.

[10] Ang, A. H. S., and Tang, W. H. 1984. *Probability Concepts in Engineering Planning and Design, Volume II: Decision, Risk and Reliability.* New York: John Wiley & Sons, Inc.

[11] Moulton, L. K., GangaRao, H. V. S., and Halverson, G. T. 1985. *Tolerable Movement Criteria for Highway Bridges, FHWA/RD-85/107.* Washington, DC: Federal Highway Administration, U.S. Department of Transportation.

[12] DiMillio, A. F., and Robinson, K. E. 1982. "Performance of Highway Bridge Abutments Supported by Spread Footing on Compacted Fill." United States: Federal Highway Administration.

[13] Barker, R. M., Duncan, J. M., Rojiani, K. B., Ooi, P. S. K., Tan, C. K., and Kim, S. G. 1991. *Manuals for the Design of Bridge Foundations, NCHRP Report 343.*

[14] Nour, A., Slimani, A., and Laouami, N. 2002. "Foundation Settlement Statistics via Finite Element Analysis." *Computers and Geotechnics* 29 (8): 641-72.

[15] Orton, S., Kwon, O., and Hazlett, T. 2012. "Statistical Distribution of Bridge Resistance Using Updated Material Parameters." *Journal of Bridge Engineering* 17: 462-9.

[16] Der Kiureghian, A. 2005. "First- and Second-Order Reliability Method." In *Engineering Design Reliability Handbook*. Boca Raton, FL: CRC.

[17] Choi, S. K., Grandhi, R. V., and Canfiedl, R. A. 2006. *Reliability-Based Structural Design.* London: Springer-Verlag.