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Physical Properties of the Function and Number of Empirical Macrohardness of the Material: Universal Physical Unit of Measurement of Macro Hardness (Part 2)

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Abstract: Results of analytical studies of the physical properties of the function and number of empirical macrohardness based on the standard experimental force diagram of kinetic macroindentation by a sphere. An analytical comparison method and a criterion for the similarity of the physical and empirical macrohardness of a material are proposed. The physical properties of the hardness measurement process using the Calvert-Johnson method are shown. The physical reasons for the size effect when measuring macrohardness are considered. The universal physical unit and standard of macrohardness of kinetic macroindentation by a sphere is substantiated.

Key words: Physical theory of kinetic indentation, method for determining the function and number of physical macrohardness, ratio of empirical and physical macrohardness of a material, universal physical unit of macrohardness.

1. Function and Number of Empirical Macrohardness, Physical Properties. Method for Comparing the Values of Physical and Empirical Macrohardness

Let us consider, using the example of indentation by a sphere, the properties of the function HI(h) and the number of empirical macrohardness of the material using physical analysis. Let us discuss the reason for the size effect. The results are presented in detail in Ref. [1]. In Fig. 1a, empirical hardness diagrams obtained using the formula HI(h)=F(h)/S(h). Fig. 1b is a characteristic view of the physical diagram of kinetic indentation (KI), constructed from the results of analytical processing of the function F(h), here there is a spherical indenter of different diameters, standard hardness measures HB103 and HB411. S(h) – conditional contact area and distribution of force F on the material. HI(h) –F(h)/S(h). Here S(h) is the conditional area of contact and distribution of the action of the force F on the material. HI(h)—empirical hardness according to Brinell, has a conditional dimension N/m3, this is a conditional pressure or conditional compressive stress on the contact surface. Usually, the area S(h) is determined by the formula proposed in the hardness measurement procedure. In hardness calculations, conditional geometric characteristics of the surface created in the process of pressing the indenter body into the material are used. The formulas determine the area of the conditional surface of a dent or imprint (a dent after the removal of the force F), the projection of the surface, etc. All formulas for calculating the area S are approximate, conditional, empirical, do not have a correct physical connection with internal irreversible processes of material structure transformations. Models of plastic deformations of a material belong to the theory of mechanics of a deformed solid body, they do not contain statistical physics. The task of physical theory is to establish the relationship between changes in the structural-energy parameters of the material and empirical parameters (F, S, V, h) of kinetic indentation. Using the example of the Brinell method, we will

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consider the relationship between the number and function of empirical and physical hardness.

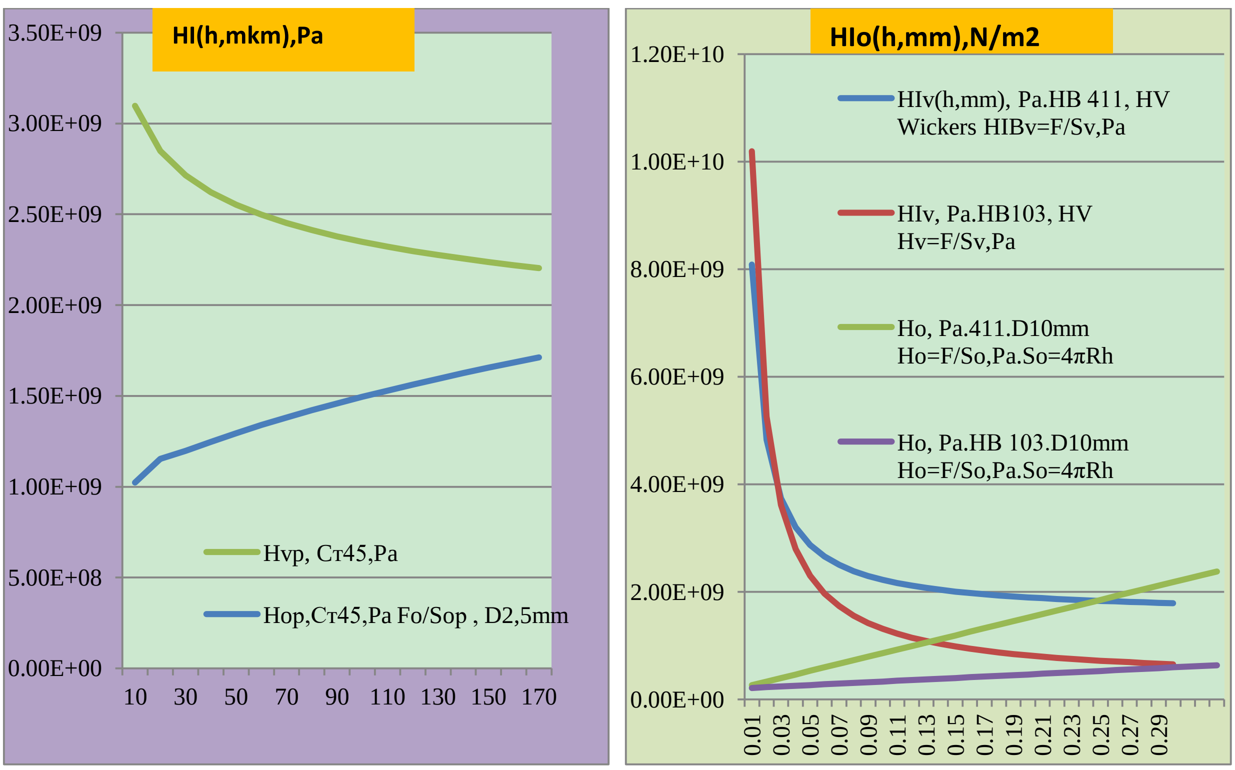
In [3], a formula (1) was obtained for the gradient of the energy field density of the thermomechanical indentation potential on the surface of the activated volume. We also indicated that the gradient of the indentation potential is the physical macrohardness of the laminar indentation of the material. In Cartesian coordinates, the total increment gradA of the activated volume is the sum of partial differentials, let us call them components of the density potential gradient:

$$\mathrm{grad}A = \frac{\partial A}{\partial V_x} + \frac{\partial A}{\partial V_y} + \frac{\partial A}{\partial V_z} = PHI_x(h) + 2PHI_z(h) \quad (1)$$

The shape of the indenter (sphere, pyramid, etc.) affects the value of each component. The magnitude of the increment of the component depends on the coordinate axis and the shape of the surface of the activated volume, respectively, and the shape of the indenter. According to the gradient theory [4], each component characterizes the increment of potential energy density *A* on the surface *S* of the activated volume. Consequently, each component of the gradient in Eq. (1) characterizes the change in the energy density in the direction of the chosen axis. In the direction of each axis, we have a certain specific generalized power $PHM_{x,y,z}(h, HB)$ .The component of the physical hardness potential depends on the selected direction, depth, hardness of the material and the shape of the indenter. Let us denote $PHI_x(h, HB)$—the main component of the gradient of the generalized power of macroindentation of the material in the direction *h*. Let us study the main component of the potential gradient in the direction of indenter movement, the *X* axis, $h = x$. Volume partial differential in *h* (*X*-axis):

$$PHI_x(h, HB) = \frac{\partial A_x}{\partial Vx} = \frac{\partial A}{V'_p(h)\partial h} = \frac{A'(h)\partial h}{V'(h)\partial h} = \frac{A'(h)}{V'(h)} = \frac{F(h)}{V'_o(h)} \quad (2)$$

where $A' = \partial A / \partial h = F(h) \qquad \partial Vx / \partial h = V'(h) \qquad (3)$



(a) (b)

Fig. 1 Kinetic macro indentation with a sphere and a pyramid. Diagrams HI (*h*) of empirical macro-surface hardness: (a) Experimental. Upper curved Vickers pyramid, lower Brinell sphere *D*2.5 mm. Steel 45, data [2]; (b) Extended chart range HI=*F*/S_0, empirical hardness, (7) HB103/411, *D*10 mm, pyramid Wickers, built analytically, initial data [1].

For laminar macro indentation $A_x = \mu A$, μ is the parameter of the influence of the shape and direction of the axis on the function $S_{x,y,z}(X,Y,Z)$ of the indenter surface area component, for a sphere, *h* >> 0, we will approximately accept $\mu \approx 1$, we will obtain $A_x \cong A$. From the approximation of the function *F*(*h*) by a polynomial, for the sphere and pyramid [1] we have:

$$F(h) = ah^m + bh^{m-1} + c \qquad (4)$$

where, *a*, *b*, *c*, *m* are approximation constants. In Ref. [5] for macro KI with a Brinell ball and a Vickers pyramid, for standard hardness tests, it was established: *m* = 2, values, $a = a_o$, $b = b_o$, $c = 0$. From Eqs. (2), (3), (4) we obtain in general form:

$$PHI_x(h,HB) = \frac{\partial A_x}{\partial Vx} = \frac{F(h)}{V_o'(h)} = \frac{F(h)}{2\pi Rh} =$$

$$= \frac{F(h)}{S_{ax}} = \frac{a_o h^2 + b_o h}{2\pi Rh} = \frac{a_o h}{2\pi R} + \frac{b_o}{2\pi R} \qquad (5)$$

where, $V_o' = \partial V_o / \partial h = 2\pi Rh = S_{ax}$.

Substituting the partial derivative $V_o' = S_{ax}$ in Eq. (2) we get:

$$PHI_x(h,HB) = \frac{\partial A}{\partial Vx} = \frac{\partial A}{V_P'(h)\partial h} = \frac{\partial A}{S_{ax}\partial h} \quad N/m^2 \qquad (6)$$

From Eq. (6) it is obvious that the main component of physical hardness or the component of the generalized indentation power function $PHI_x(h,HB)$ is the specific amount of the energy flux gradient, therefore, the value is determined per unit area $S_a(h)$. From Eq. (6) it is obvious that the main component of the indentation gradient depends on the work done and the area of the contact surface (respectively, on the depth). Thus, there are two options for representing the power gradient component or the macro physical hardness component KI in Eqs. (2) and (5). Let us consider them in detail:

First option: Dimension of volume differential of physical hardness of indentation J/m^3, in Eq. (7)

For the sphere formulas of Eqs. (2) and (5), as a result of transformations, the physical dimension J/m^3 degenerates (reduces), we obtain the dimension of empirical hardness N/m^3. Therefore, in Eqs. (5) and (6) using the dimension of conditional stresses, Eqs. (2) and (5) are converted into the empirical hardness formula $HI(h)$ Eq. (7):

$$PHI_x(h,HB) = \frac{\partial A_x}{\partial Vx} = \frac{\partial A}{S_a \partial h} = \frac{F(h)}{2\pi Rh} = \frac{F(h)}{S_{ao}(h)} = HI(h), N/m^2, \quad (6)$$

where $S_a(h) = \delta\pi Rh$.

The final empirical hardness is the main component of the gradient of the generalized macroindentation volumetric power, divided by the area:

$$PHI_x(h,HB) = HI(h), N/m^2, \qquad (7)$$

Thus, empirical hardness is twice the specific indicator. The empirical hardness depends on the energy density A dissipated in the volume V_0 and the energy density gradient on the surface of the activated volume, moved in the direction of the h-X axis and changed in shape. This is the main component of the gradient Eq. (6) of physical hardness, it is equal to the empirical hardness Eq. (7), which characterizes the change in the energy density of indentation, when moving from the activated volume to the outer region of the indented material, in the direction of the axis of motion.

Second option: Expanded formula of the physical hardness component KI sphere

The relationship is shown with the main approximation parameters a_o, b_0 in the function *F*(*h*), in Eq. (4):

$$PHI_x(h,HB) = \frac{\partial A(h)}{\partial V_a(h)} = \frac{a_o h}{2\pi R} + \frac{b_o}{2\pi R} \qquad (8)$$

Summarize. The partial differential axis X(also h) this $PHI_x(h,HB)$, in Eqs. (6) and (7), is also a function of the empirical hardness of the material in the Brinell method. This function (8) contains the main component of the gradient in Eqs. (1). Empirical hardness is a specific characteristic of the stress work flow. As the depth *h* increases, the value of the empirical hardness, the main component of the gradient for the sphere, continuously grows linearly. In Fig.2a, an example $PHI(h,HB)$ for a material of different hardness is shown. Fig. 2b shows together $HI(h)$ the empirical hardness function and the gradient component function $gradA_x = PHI(h, HB)$, HB411 hardness block, sphere *D*5 mm. The functions of physical and empirical hardness of one material (standard measure) coincide. The slope of the linear function $PHI(h, HB)$ and $HI(h)$ depends on the hardness.

2. Similarity Criterion for Measurements of Physical and Empirical Macrohardness: The Standard of Macrohardness of Indentation by a Sphere

Fig. 2a shows the characteristic functions $PHI(h, HB)$ for different HB176/411 hardness measures. For one measure of hardness, regardless of the diameter, the graphs of the functions almost coincided. For one measure HB_i, the function of empirical hardness or the component of the physical hardness gradient $PHI(h, HB)$ is invariant to the diameter of the sphere, the lines for three different diameters *D* almost coincided. Fig. 2b shows together the functions $PHI(h, HB_i)$ and the empirical hardness HB_i for the measure HB411, they are the same. For the value of the function or the hardness number, we calculate for a certain depth *h*. The value, the hardness number, depends on the chosen coordinate—the indentation depth *h*. It follows from the analysis that in order to unambiguously and correctly determine the number of empirical hardness of a material, it is necessary to ensure the same physical conditions or similarity of testing processes for materials of different hardness, taking into account the shape of the tool and the depth *h*. In this case, we have macro indentation by a sphere, the similarity of physical conditions is provided at one given depth h_{st}, regardless of the diameter of the sphere. For a sphere, the standard for measuring hardness is—a constant indentation depth. At the same time, this is the first condition for the similarity of the physical process of macroindentation. Fig. 2d shows the principle of determining the reference hardness values of different materials with a macro indenter. Provided that $h_{st} = h$ the physical similarity of processes is observed, for different materials, in this case, standard measures of hardness. Table 1 shows the results of calculating the physical hardness number for a conditional standard constant value of the indentation depth h_{st} = 0.25 mm.

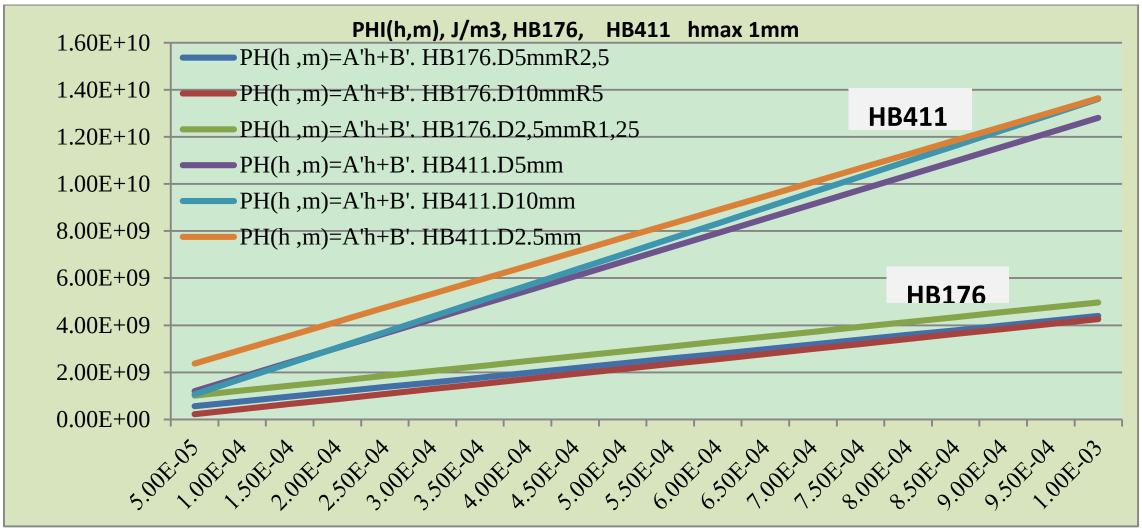
In Fig. 2c, the hardness functions $PHI(h)$, measure HB103, indenter sphere and Vickers pyramid. For a pyramid (also for a cone), the function has the form of a hyperbola; with a change in hardness, the graph shifts; the features of the KI process with a sharp indenter are considered in Ref. [5]. From the properties of the function and the definition of physical hardness, we see that the similarity is fulfilled if the indenter during the movement of KI generates the same specific value of the surface area $\Delta S_a / \Delta V_a, 1/m$, for different materials and different shapes of the indenter.

The calculation and comparison of the hardness number of different materials, indenters, should be performed under physical and mechanical similarity of conditions—the same increase in surface area per unit of activated (displaced) volume V_a. Such conditions for measuring the hardness of a material were first created by Calvert-Johnson (1859) [5,6]. In Ref. [5], in order to check the compliance with the condition of similarity of the macro process KI, a generalized physical and mechanical characteristic Eq. (9) of the material shape change function is proposed:

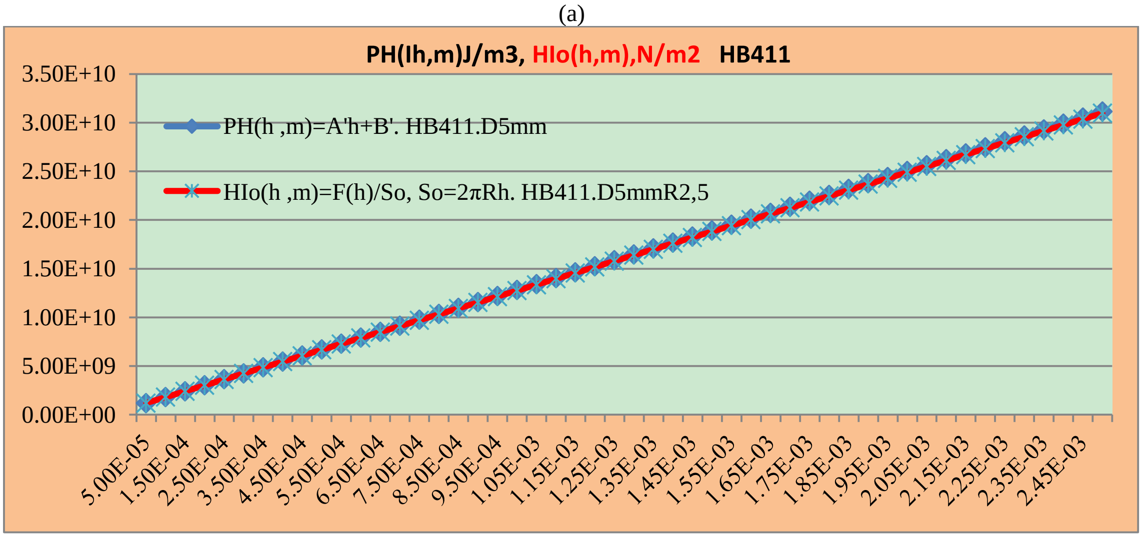
$$X_{SV}(h) = \frac{S_a(h)}{V_a(h)} \tag{9}$$

$X_{SV}(h)$ —specific area of the created (generated) surface per unit volume of material activated in the CI process. The value of the function, in general, depends on the shape of the indenter and displacement *h*. For a sphere, function Eq. (9) has a special property in Eq. (10), it does not depend on *R*, but depends on the indentation depth *h* and the parameter in the contact surface formula (determined by an empirical method):

$$X_{SVo}(h) = \frac{S_{ao}(h)}{V_{ao}(h)} = \frac{\delta \pi R h}{\pi R h^2} = \frac{\delta}{h}, \ \delta = 2\sim4 \tag{10}$$



(a)



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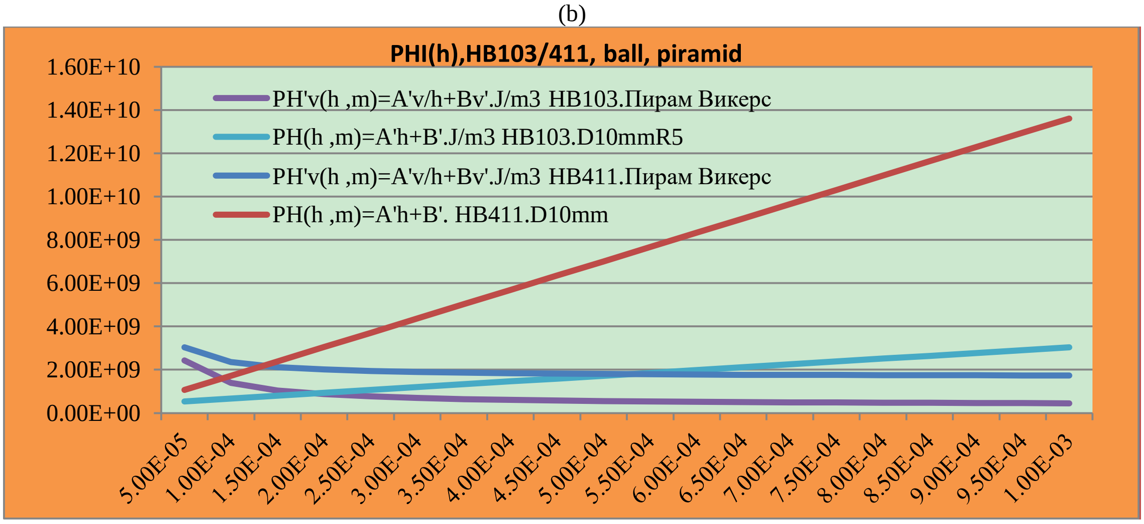


(c)

Fig. 2 Function of physical and empirical macrohardness CI: (a) physical hardness PHI(*h*, HB, D_n), standard measure of hardness HB176, HB411, diameter *D*10/5/2.5 mm, according to Ref. [1]; (b) HI(*h*) and PHI(*h*) are shown together, HB411, *D*5/2.5 mm; (c) Vickers sphere and pyramid diagrams.

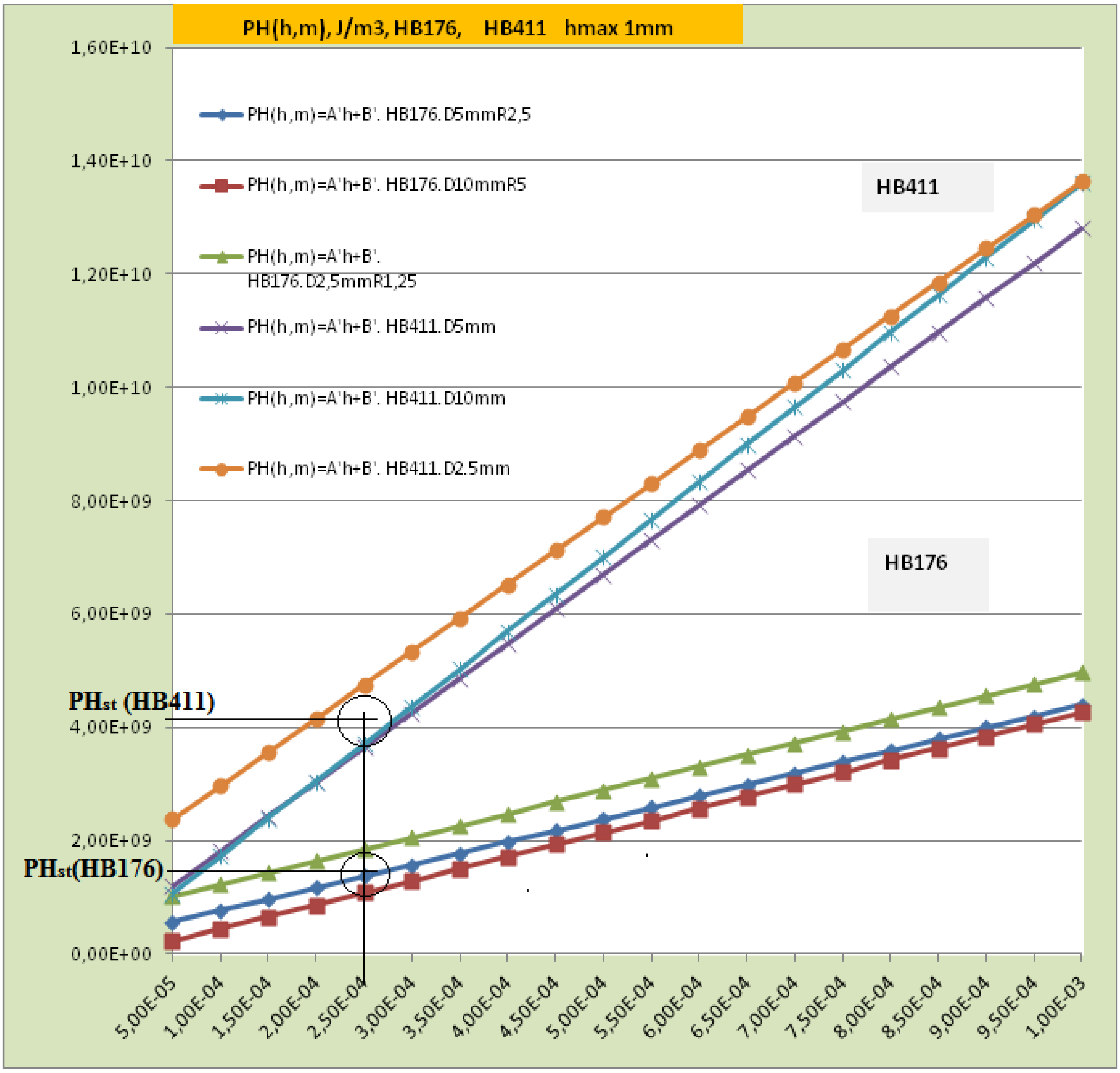


Fig. 2d Determination of the value of the standard of physical-empirical hardness, using the experimental charts of physical hardness PHI(*h*), for standard measures HB176, HB 411, indenters sphere *D* = 10.0/5.0/2.5 mm:. The value of the standard of physical hardness PH_{st} is approximately equal to the hardness of a standard measure. Reference depth h_{st} = 0.25 mm.

For the pyramid:

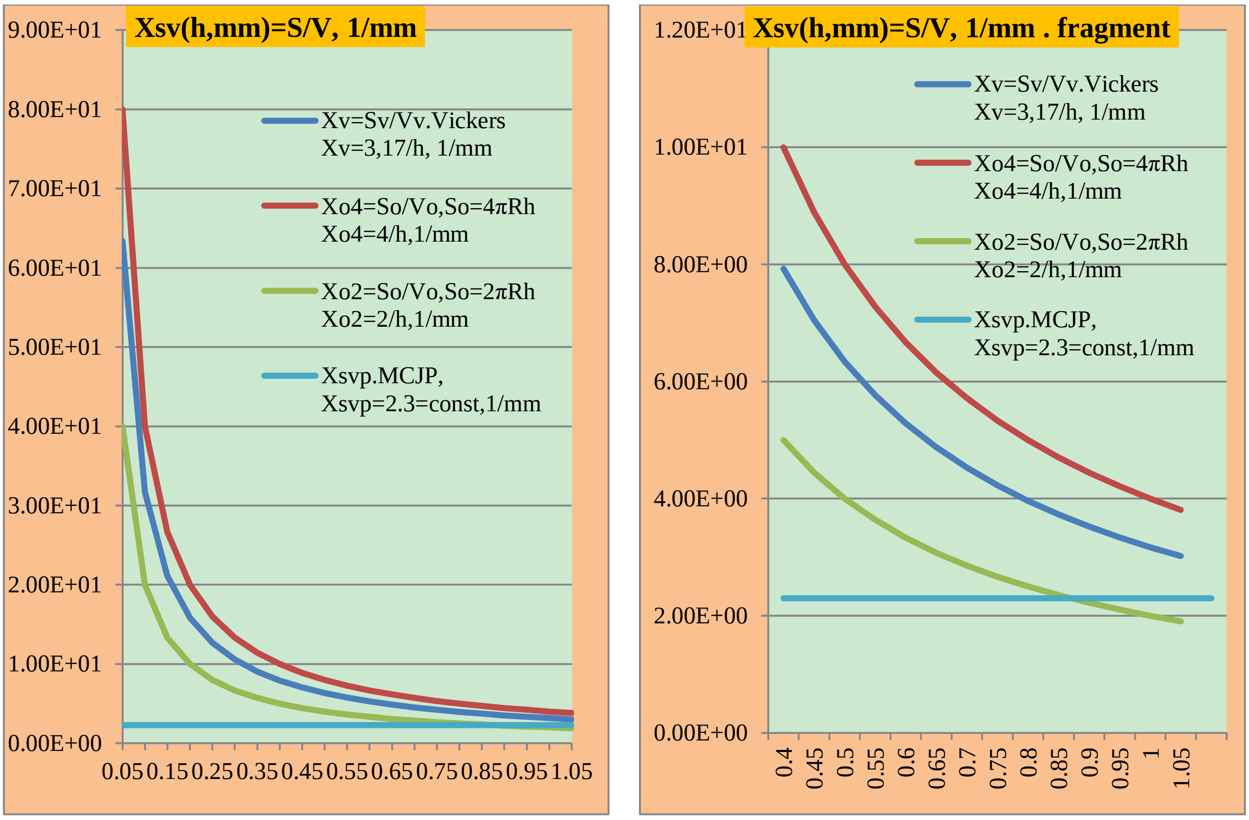
$$X_{SVv}(h) = \frac{\lambda}{h}$$

where, λ is a parameter that takes into account the influence of the shape of the pyramid $S_a = \lambda h^2$. For the Vickers pyramid λ = 3.17. In Fig. 3, the functions $X_{SV}(h)$ for the indenter are sphere, pyramid and truncated cone.

The value depends $X_{SV}(h)$ on the shape of the indenter and the depth *h*. The specific indicator is affected by the formula for calculating the contact surface $S_a(h)$, an example is shown for a sphere, two formulas $S_o(h)$. Similarity condition is listed below when measuring macrohardness:

$$X_{SV}(h) = \text{const} \qquad (11)$$

In the first physically correct method of hardness measurement, developed in 1859 [6] by Calvert-Johnson (denoted as MCJ), a truncated cone indenter was used, this shape has the property $X_{SV}MCJ \approx \text{const}$, in Fig. 3.



(a) (b)

Fig. 3 Volume shaping function $X_{SV}(h)$ for pyramid, sphere, cone MCJ. For sphere $S_{ao}(h)=\delta\pi Rh$, $\delta = 2$, $\delta = 4$. Truncated cone indenter, MCJ method, X_{SVP}MCJ ≈ 2,3.1/mm=const.

When measuring the hardness number in a laminar KI process, the specific surface area of the material must be the same. In this case, the similarity of the process of measuring the hardness number and the physically correct scale are preserved. In Fig.3a of the diagram $X_{SV}(h)$, we see that the similarity condition Eq. (11) is approximately satisfied for the sphere and the pyramid, if h > 1.0 mm. From this depth $X_{SV}(h) \approx \text{const}$, the physical hardness depends little on the depth h, the influence of the initial nonlinear section decreases, and conditions are created for correct measurements of the hardness number KI.

The values of the empirical standard hardness HB and the physical hardness potential in Eq. (7.3) PHM(HB) coincide (close) if the potential is determined in the interval that contains the value h, at which the hardness HB is determined according to the standard Table 1, at the same time, the values of the shape change parameter are $X_{SV}(h)$ close.

From the analysis of macrokinetic indentation functions in Figs. 1-3 it follows that a physically correct comparison of empirical hardness values in standard tests is possible if an indenter of the same shape is used (a sphere can have different D). When determining the hardness number, a constant depth is needed h_{st} (Fig. 2d).

For a different indenter shape, a physically correct comparison of the macrohardness number is possible only with the same value of the parameter $X_{SV}(h)$, the same depth does not provide similarity. Physical similarity of measurements of empirical hardness of different methods, tools and materials is performed under the condition in Eq. (11).

Table 1 gives the values of the standard reference physical differential hardness, defined by the formula Eqs. (3) and (4), for $h_{st} = 0.25\text{mm}$, D_i = 10/5/2.5 mm, three standard hardness measures HB103/176/411. The physical standard of standard depth h_{st} = 0.25 mm

Table 1. Physical hardness of standard measures for different sphere diameters.

Mechanical measure of hardness HBW	PH_{st} average physical hardness, J/m^3	Reference physical hardness value (9.4) PH_{st} J/m^3, different measure of hardness HB, diameter 10/5/2.5 (*D*, mm), h_{st} = 0.25 mm		
HB411	403×10^7	4.74×10^9 (*D* 2.5)	3.64×10^9 (*D* 5)	3.71×10^9 (*D* 10)
HB176	145×10^7	1.85×10^9 (*D* 2.5)	1.37×10^9 (*D* 5)	1.08×10^9 (*D* 10)
HB103	105×10^7	1.11×10^9 (*D* 2.5)	1.03×10^9 (*D* 5)	1.07×10^9 (*D* 01)

was adopted previously for comparative analysis. At the point h_{st}, the value PH_{st} and the empirical number of the standard measure HBW of Brinell hardness are approximately equal. The dimensions of physical and empirical hardness are formally reduced to the same value.

Physical hardness of standard measures for different sphere diameters is shown in Table 1.

To fulfill the similarity conditions in standard methods, the hardness number should be determined for the same tool shape and a constant value of the indentation depth h_{st} (Fig. 4b). If a different tool shape is used, then it is necessary to perform an appropriate corrective calculation, and satisfy condition Eq. (11) [4]. In this case, it is analytically possible to create a similarity of measurements with different tools, to perform a transition to another hardness scale, or to convert hardness number values using a universal physical unit of hardness. About the effect of different indenter shape, range on the hardness number, see Ref. [5] for more details. Due to the shape of the truncated cone, indenter in MCJ the similarity and condition Eq. (11) are met mechanically.

3. Physical Properties of the Hardness Measurement Process by the Calvert-Johnson Method

The paper [5] considers a physically correct method for measuring hardness, which was developed by Calvert-Johnson (1859) [6]. The authors of MCJ used a truncated cone indenter, initial contact diameter d = 1.25 mm, X_{SVP}MCJ ≈ 2.3 = const. This shape of the indenter ensures the fulfillment of condition Eq. (11), $X_{SV}(h) \approx$ const, see Figs.4a and 4b. This is a property of the truncated cone shape, not the material. Thus, when measured, the depth h in MCJ had almost no effect on the hardness number. Form change, surface formation, occurs approximately at $X_{SV}(h) \approx$ const. The MCJ experiment ends at point h_{st}, volume $V_a(h_{st})$. The MCJ did not measure the area and depth of indentation. We measured the required weight of the weights for the process of slow indentation, to a given depth h_{st}. The test time is always 30 min. The speed of movement of the indenter is approximately constant, conditions close to stationary creep are created. The value of the total weight of the weights, up to a constant factor, is equal to the work of indentation. The displaced volume and the contact surface area of the dent are the same for different materials. The total weight of weights for material of different hardness is different. But the root, physical indicator is the specific work of the weight of the weights. The weight is proportional to the specific work (J/m^3) of the indenter. Thus, the MCJ indirectly measures the indentation energy density ($\Delta A/\Delta V$) of the material. The first MCJ hardness scale uses a method of indirect comparison of physical hardness values. The specific indentation energy of each material under consideration is characterized by the individual weight of the load, which is necessary to form the same volume (depth) of the dent for all. As a result, we get the correct method. Different materials have different energy dissipation densities (different generalized specific power) for the same change in material shape, while at the same time this test shows different hardness of the material. In MCJ there was no size effect, since the physical process was the same for materials of different hardness, the shape of the indenter ensured the constancy of the specific power of the elastoplastic molding process. The original first scale was created in units of weights; a single “weight” (generalized specific energy) correct scale of hardness of materials from lead to cast iron arose [6]. The remaining indentation methods were then intuitively tuned to this scale. After that, the authors converted the energy (weight) hardness scale into a dimensionless one. There was no theoretical physical definition of hardness by the Calvert-Johnson method. An analysis of this method [5] showed that the basic physical principle of macro-instrumented indentation was intuitively created in it. The method has been used for over forty years.

In subsequent methods of measuring hardness, the MCJ scale was supported empirically, artificially. During this period of time, according to our assumption, an erroneous opinion was formed about the absence of influence on the hardness number by the ratio of the dimensions of the activated volume and the area of the contact surface, etc. The principle of similarity was lost in the new methods. The methods of data processing of a mechanical act, the shape of the tool, the algorithms for measuring the hardness number (different geometry parameters, forces, etc.) have changed. As a result, an incorrect empirical approach has been established, in which the original hardness scale is "artificially" maintained. The hardness (number) or the potential of the physical specific power of forming in MCJ does not depend on the depth of the tool movement *h* (in Fig. 4b, green lines). The shape of the indenter and the measurement rules in MCJ allowed the authors to form a basic correct physical scale of hardness. It became the basis for subsequent research, etc. The very physical principle of MCJ was subsequently unreasonably distorted.

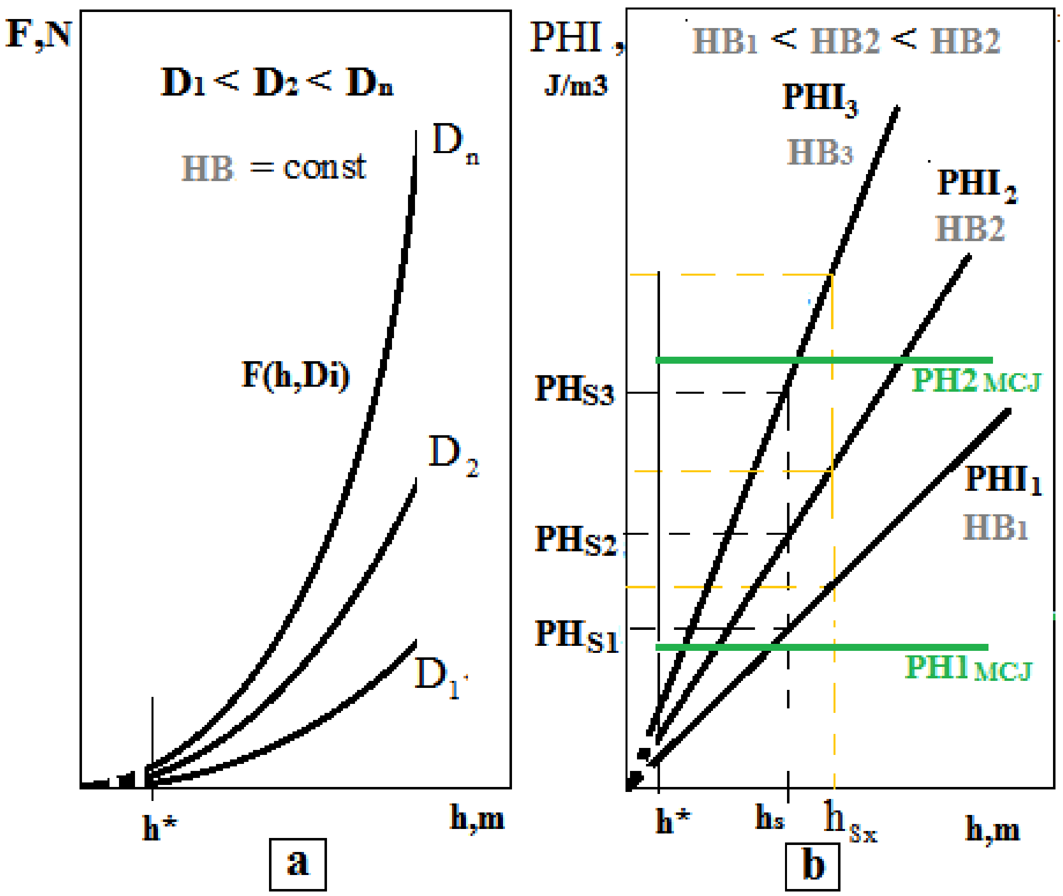


Fig. 4 Generalized diagrams of ideal laminar macro process KI by Brinell ball and cone MCJ: a) Function $F_n(h, D_n)$ diameter $D_1 < D_2 < D_n$, measure HB$_i$ =const; b) function PHI$_i$(h, HB$_i$) and standard PH$_{si}$ of physical hardness of each measure of hardness HB$_i$, D—arbitrary diameter, $D > D_{min}$, $h<D$. Physical diagrams of KI hardness PH1$_{MCJ}$, PH2$_{MCJ}$, Calvert and Johnson method, conditional soft and hard material, respectively, X$_{SV}$(h)=const.

4. The Discussion of the Results

As a result of an analytical study of the properties of the CI diagrams of various standard hardness measures, a stable characteristic of a solid material was found—physical hardness (Table 1). The main component of the gradient of physical hardness, under the condition of physical similarity of measurements, is equal to the value of the empirical hardness of this material on the Brinell scale. At the same time, the stable and fundamental nature of the relationship between the functions of physical hardness KI and empirical hardness is shown analytically.

From the analysis of the results of calculating the potential of physical hardness, it was found that this is a constant value, an objective physical characteristic for a given material. In a wide range of h values, the potential is numerically equal to the single value of the component of the physical hardness potential, at the same time it is numerically equal to the empirical hardness of HB of a given material (standard measure) under conditions of physical similarity. The value of the potential does not depend on the trajectory of the process $F(h)$. The property has been experimentally confirmed for different material hardness, different sphere sizes. These results confirmed the assumption that under macro KI by ISO 14577-1:2002 standard methods, there is a state function, the thermomechanical potential of the activated material volume in Eqs. (3) and (6.1). An objective characteristic of the physical-mechanical property of the hardness of the material for laminar CI is found.

In the works of researchers [7,8,9], the physical criterion of the specific energy of kinetic indentation was also used to assess the hardness of the material. Experimental results have been obtained in which there is practically no size effect at the same imprint size [9]. The authors of Ref. [7] suggested using the specific energy index KI to determine the hardness of the material. In these studies, a special case of KI is analyzed, there is no theoretical and physical generalization of the properties of this process, there is no physical analysis of the empirical method of measuring hardness. At the same time, these works experimentally confirm our theoretical assumptions and conclusions about the physical cause of the size effect.

4.1 Size Effect in the Measurement of Macrohardness

Fig. 4a shows in a generalized form the force diagrams $F_n(h, D_n)$ with different process trajectories, different diameters, constant hardness HB=const. Fig. 4b shows the corresponding functions of specific power $PHI_i(h, HB_i)$, three hardness values HB1 < HB2 < HB3, three sphere diameters, Eq. (7) is used. An analysis of the properties of functions PHI(h) in Eq. (5) and functions of empirical hardness HI(h) in Eq. (6) showed that the hardness number for macro indentation with standard indenters of different shapes should be found only for one established reference value h_{st}. With increasing depth $h_{st} \to h_{sx}$ (in Fig. 4b), there is a proportional increase in the values of the empirical hardness number, the scale of the hardness scale changes. A larger value of depth h_{st} corresponds to a “stretched” hardness scale. Thus, the number PHI_i of material formally increased. The scale with the new hardness scale (depth h_{sx}) is shown in yellow. The hardness for each HBi measure has increased on all lines of the diagrams PHI_i, and the scale of the empirical diagrams HI(h) will change similarly. As you decrease h_{sx}, the scale shrinks. At the same time, the value of the physical hardness potential PHM(V) of a given material does not depend on h in a sufficiently large KI range; this is a constant physical characteristic of the material, a given shape of the indenter, and a sufficiently large depth interval h. This hardness potential PHM(V) differs from the empirical or physical hardness number obtained by Eq. (5). For the KI diagram built by the MCJ indenter, the physical hardness $PH1_{MCJ}$, component Eq. (7), is practically independent of depth, it is equal to the physical hardness potential $PHM(V)=PH1_{MCJ} \approx const$. Green line in Fig.4b. In modern KI methods, there is no criterion for the similarity of physical processes when measuring the hardness number, this is the main reason for the appearance of the size effect (ISE). In empirical methods, the measurement of the hardness number of materials is performed at a different, uncontrolled value $X_{SV}(h)$, i.e. under different physical conditions. For the similarity of empirical tests, a sphere, a pyramid, a cone, it is necessary to assign and observe the standard of the physical and mechanical process. For a sphere in macro KI, this is a constant depth *h*, regardless of the diameter. The truncated cone indenter MCJ provided the same physical conditions in each act of indentation mechanically. If the Calvert and Johnson indenter is used in standard macro KI methods, it provides physical similarity conditions mechanically, over the entire macro depth range, and minimizes ISE (in Fig. 4). The MCJ indenter generates almost the same specific contact surface area $X_{SV}(h) \approx const$ (in Fig. 4). Under the condition *h* > *h**, the movement of the indenter in the MCJ has little effect on the macrohardness value. The effect of the relaxation region in this method is small. In MCJ, throughout the entire indentation process, the value of the specific energy expended on the formation of the material surface is approximately constant.

The use of different indenter displacement depth *h* in one-stage empirical standard methods for measuring the macrohardness number by a sphere and a pyramid leads to a violation of the physical conditions of similarity. In this case, the measurement of hardness is accompanied by an uncontrolled transition to another scale or to another physical measure of the process. The hardness number of the empirical method depends on *h* on the trajectory of the physical process KI, that is, the hardness number depends on the value of the function PHI(h) in Eq. (7). Empirical hardness is the value of the function of the component of the specific generalized indentation power. The size effect arises when the condition of similarity of shape change is violated $X_{SV}(h) = const$.

4.2 Definitions of Physical Macrohardness of Kinetic Indentation

The thermomechanical potential of the indented material $U_p = A(h)$ is a function of the state of the activated volume. Physical macrohardness of the material— $PHM(V, HB)$, dimension J/m^3, in Eqs. (1), (2), and (5) different form of representation, specific potential of the generalized power of kinetic indentation, shape change of the activated volume of the material. $PHI_i(h, HB_i)$ —function of the potential component of the generalized specific power of indentation by the sphere, the potential gradient component is also a function of empirical hardness (in Figs. 3 and 4). The meaning and dimension of empirical and physical hardness are different. Formal translation of dimensions: $[J/m^3 = N/m^2 \times \frac{m}{m}]$. In Fig. 3b, there are two diagrams, for empirical $HI(h)$ and physical $PHI(h, HB, D_n)$ macro hardness, HB411, *D*5 mm. Using the Brinell indenter as an example, the physical meaning and properties of the macro empirical hardness function KI, in Eq. (7) and the essence of the surface macro indentation hardness numbers according to the ISO14577 standard are shown. Studies have shown that the methods for measuring the empirical macrohardness number of Brinell, Rockwell, Vickers, etc. according to the ISO14577 standard are incorrect from the standpoint of the physical theory of hardness. In empirical methods, there is no condition for the physical similarity of measuring the hardness number. The result of violation of the physical similarity of processes is the size effect. In Ref. [5], based on the physical approach, methods for comparing hardness values from different methods are considered. Using the dependences of physical methods for analyzing indentation, a virtual curve *F*(*h*) for the Vickers pyramid was analytically constructed, etc. Indentation is modeled as a process performed by a sphere with a variable diameter, a special function *D*(*h*) (dynamic Brinell ball) is set for this, etc.

The methods of mathematical modeling of the KI process, the function of the indenter shape from *h* is analytically given, showing the possibility of using physical theory to develop a universal program for comparing, converting hardness values of different standards and indentation methods.

Nano and micro indentation differs in its physical nature from the macro KI process [5]. Activation and shape change of the material occur in a very small volume of the body, the nano energy density is higher by an order of magnitude and more than in the macro range. The nano process has its own physical function of the state of the activated volume [5]. With an extended KI range, for example, for indentation with a Vickers pyramid, two mechanisms of shape change and transformation arise in succession. The contribution of each of the body shaping mechanisms changes during the KI process. The basis of the theory of the indentation process in any range remains the physical concept—the specific power, the energy of the process of formation of the activated volume and the contact surface KI [5]. The assessment of physical nano-micro hardness by KI analysis for a sharp tool cone, pyramid, micro and nano sphere, is a separate method. The physical hardness diagram of indentation by a pyramid, a cone is performed using its own physical state function, on this basis a universal indentation equation was obtained [3, 5], this is the topic of the next article.

4.3 Universal Physical Unit of Macro Hardness

Taking into account the perfect theoretical foundation laid down in the Calvert-Johnson method, the prospect of applying the physical analysis of the CI results, we propose to use the universal physical unit of macrohardness in the standard:

$$1 \text{ CJ} = 1\times10^7 \text{ J/m}^3, 1 \text{ CJ: one cal.}$$

The hardness of a 103HB standard measure is approximately equal to 100 CJ physical hardness. Physical macrohardness of structural materials is in the range of 1-1,000 CJ, and does not depend on the shape of the indenter. The function, scale and values of physical hardness are analytically related to the function and number of empirical hardness, for different indentation methods. Physical and empirical methods can operate in the new standard in parallel, until the abolition of empirical methods.

5. Conclusions

(1) The physical characteristics of the experimental process of kinetic macroindentation of a material are theoretically substantiated: the function and number of physical hardness, the physical meaning and dimension of hardness are determined.

(2) The function of the state of the activated volume of material for kinetic (instrumental) macro indentation by a sphere is determined.

(3) Based on the analytical analysis of the standard kinetic force diagram, methods have been developed for determining the value of the physical potential of the macrohardness of a material, the function of the physical kinetic hardness of macroindentation. There is no size effect in the method. Universal physical hardness specifications have a number of important advantages and can replace empirical standard methods for measuring hardness.

(4) The physical meaning of the standard empirical number of material macrohardness and the reason for the size effect in empirical macroindentation methods are shown.

(5) An analytical relationship between the values of empirical and physical hardness of the kinetic macro indentation of the material has been established. Principles, similarity criteria and an analytical method for comparing the hardness numbers of materials for different sizes, tool shapes, in the range of macro indenation are formulated.

(6) The results obtained form the basis for the development of an addition to the current ISO 14577 standard and the creation of a general physical theory of the hardness of structural materials in different ranges.

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