Journal of Environmental Science and Engineering B 12 (2023) 236-247
doi:10.17265/2162-5263/2023.05.004

Multi-objective Optimization of a Two-Stage Helical Gearbox to Improve Efficiency and Reduce Height

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Abstract: The goal of this research is to look at multi-target optimization of a two-stage helical gearbox in order to determine the best key design elements for reducing gearbox height and enhancing gearbox efficiency. To do this, the method known as Taguchi and GRA (Grey Relation Analysis) were used in two stages to address the problem. The single-objective optimization problem was addressed first to close the gap between variable levels, and then the multi-objective optimization problem was solved to determine the best primary design variables. The first and second stage CWFWs (Coefficients of Wheel Face Width), ACS (Permissible Contact Stresses), and first stage gear ratio were also calculated. The study's findings were utilized to identify the best values for five critical design aspects of a two-stage helical gearbox.

Key words: Helical gearbox, multi-objective optimization, gear ratio, gearbox height, gearbox efficiency.

1. Introduction

The gearbox is the most important component of a mechanical drive system. It is used to decrease motor speed and boost torque in order to satisfy the demands of the operating machines. As a result, the most effective gearbox design has been, is, and will continue to be an important topic.

A variety of studies have indicated that selecting the proper gear ratio is an important consideration when designing a gearbox, because the gear ratio is an important parameter that influences the size, volume, mass, and cost of the gearbox. As a result, extensive study is conducted to establish the optimal gear ratios.

Across the literature, there have been various studies on helical gearboxes. Pi et al. [1] present a novel study on optimizing partial transmission ratios in mechanical drive systems using a chain and a two-stage helical reducer. Vu et al. [2] aimed to find optimal partial gear ratios for cost reduction in a three-stage helical gearbox. Pi et al. [3] proposed models that offer accurate and simple calculation of optimal partial ratios for both a V-belt drive and a two-stage helical reducer. With an optimization problem focused on system cross-sectional area, Tuan et al. [4] present a study on optimizing gear ratios in mechanical systems with a chain drive and a two-step helical gearbox featuring double gear sets in the first step. Cam et al. [5] discuss optimal gear ratios in mechanical systems using a two-stage helical gearbox with double gear sets in the second stage and a chain drive. Using a simulation experiment to analyze the impact of different input factors on optimal transmission ratios, Tu et al. [6] suggested equations that can determine optimal gear ratios for two-stage helical gearbox featuring double gear sets in the second stage and a chain drive. Hung et al. [7] discuss optimal gear ratios in mechanical systems featuring a two-stage helical gearbox with double gear sets in the first stage and a chain drive. Linh et al. [8] employ simulation, Minitab R18, and Taguchi method to analyze different design parameters' impact on the first stage's optimal partial gear ratio of a two-stage helical gearbox with double gear-sets. Patil et al. [9] focus on an optimized 3-stage

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gearbox design for industrial applications like mining conveyer belts, aiming for enhanced safety, durability, and efficiency. Quang et al. [10] discuss optimizing partial gear ratios for a four-speed helical gearbox using cost optimization, with an analysis of 13 input parameters to find an easy-to-use regression formula.

Numerous studies on the best design of gearboxes have been conducted, according to the previously described study. However, no study on multi-objective optimization for a two-stage helical gearbox with the single target functions of maximum gearbox efficiency and lowest gearbox height has been implemented.

This research examines at multi-target optimization approaches for a two-stage helical gearbox. Two single aims were pursued in this effort: decreasing gear-box height and enhancing gearbox efficiency. In addition, the CWFW (Coefficients of Wheel Face Width) for both stages, the ACS for both stages, and the first stage gear ratio were also studied. Furthermore, by combining the Taguchi technique with the GRA (Grey Relation Analysis), the multi-objective optimization problem in gearbox design was handled in two stages. The best values for five critical design elements for producing a two-stage helical gearbox were also provided.

2. Optimization Problem

2.1 Gearbox Height Determination

The height H of a two-stage helical gearbox can be found in Fig. 1:

$$H = \max(d_{w21}, d_{w22}) + 8.5 \cdot S_G \tag{1}$$

where, d_{w21}, d_{w22} are pitch diameters of the gears of the first and second stages which are determined by Chat and Van Uyen [11]:

$$d_{w21} = 2\ a_{w1} \cdot u_1/(u_1 + 1) \tag{2}$$

$$d_{w22} = 2\ a_{w2} \cdot u_2\ /(u_2 + 1) \tag{3}$$

in which, a_{w1} and a_{w2} are the center distances of the first and the second stages which can be calculated by Chat and Van Uyen [11]:

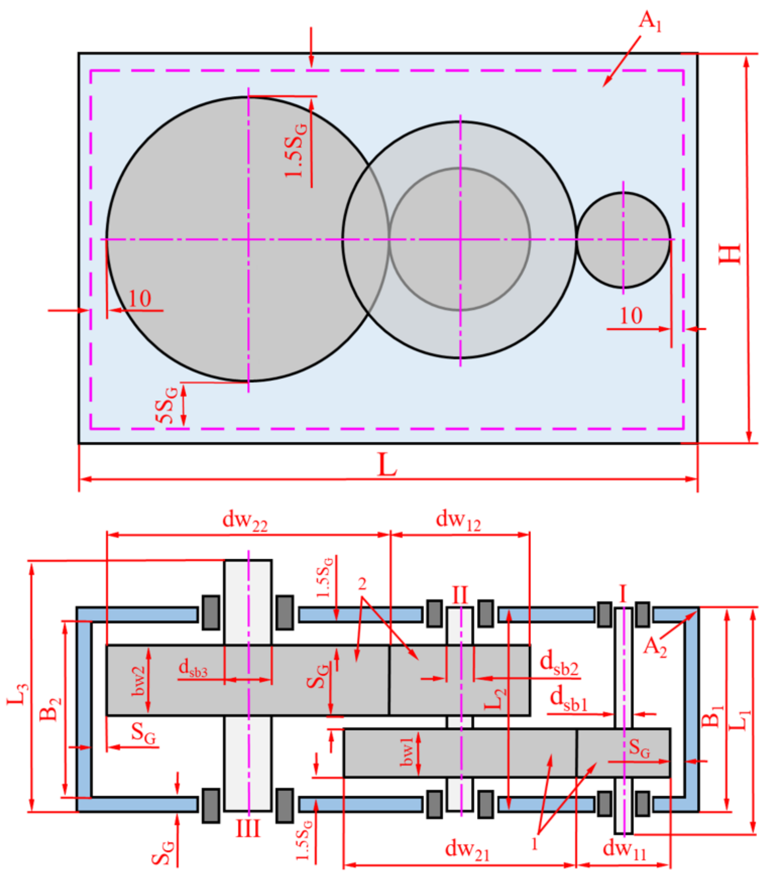

Fig. 1 Calculated schema [12].

$$a_{w1} = k_a(u_1+1)\sqrt[3]{T_{11}k_{H\beta 1}/([\sigma_{H1}]^2 u_1 X_{ba1})} \quad (4)$$

$$a_{w2} = k_a(u_2+1)\sqrt[3]{T_{12}k_{H\beta 2}/([\sigma_{H2}]^2 u_2 X_{ba2})} \quad (5)$$

In Eqs. (4) and (5), k_a is the material coefficient, while $k_{H\beta 1}$ and $k_{H\beta 2}$ are the contacting load ratios for the first and second stages of pitting resistance [11]. $[\sigma_{H1}]$ and $[\sigma_{H2}]$ are the ACS of the stage 1 and 2 (MPa), respectively; u_1 and u_2 are the first and second stage gear ratios. CWFWs of the stage 1 and 2 are X_{ba1} and X_{ba2}. T_{11} and T_{12} are the torques on the pinions of the first and second stages (in N mm):

$$T_{11} = T_{out}/\left(u_g \cdot \eta_{hg}^2 \cdot \eta_b^3\right) \quad (6)$$

$$T_{12} = T_{out}/\left(u_2 \cdot \eta_{hg} \cdot \eta_{be}^2\right) \quad (7)$$

in which, T_{out} is the output torque (N mm); η_{hg} is the helical gear efficiency ($\eta_{hg} = 0.96 - 0.98$); η_b is the rolling bearing efficiency (η_{be} = 0.99-0.995) [11].

In Eq. (1), S_G is determined by Römhild and Linke [13]:

$$S_G = 0.005 \cdot L + 4.5 \quad (8)$$

where, L is the length of the gearbox which can be found in Fig. 1:

$$L = d_{w11}/2 + a_{w1} + a_{w2} + d_{w22}/2 + 2 \cdot k \quad (9)$$

with $k = 8 - 12$ [11].

2.2 Gearbox Efficiency Determination

The gearbox efficiency is found by:

$$\eta_{gb} = \frac{100 \cdot P_l}{P_{in}} \quad (10)$$

in which, P_l is the total gearbox power loss [14]:

$$P_l = P_{lg} + P_{lb} + P_{ls} \quad (11)$$

In Eq. (11), P_{lg} indicates overall gear power loss, P_{lb} is bearing power loss, and P_{ls} is seal power loss. Those factors are determined by:

+) The gear power loss is found:

$$P_{lg} = \sum_{i=1}^{2} P_{lgi} \quad (12)$$

P_{lgi} is the gear power losses of i stage which can be determined by:

$$P_{lgi} = P_{gi} \cdot \left(1 - \eta_{gi}\right) \quad (13)$$

in which, η_{gi} is the expected efficiency of stage i [15]:

$$\eta_{gi} = 1 - \left(\frac{1 + 1/u_i}{\beta_{ai} + \beta_{ri}}\right) \cdot \frac{f_i}{2} \cdot \left(\beta_{ai}^2 + \beta_{ri}^2\right) \quad (14)$$

where, u_i is the gear ratio of i stage; β_{ai} and β_{ri} are the arcs of approach and retreat on the i stage which are found by Buckingham [15]:

$$\beta_{ai} = \frac{\left(R_{e2i}^2 - R_{02i}^2\right)^{1/2} - R_{2i} \cdot \sin\alpha}{R_{01i}} \quad (15)$$

$$\beta_{ri} = \frac{\left(R_{e1i}^2 - R_{01i}^2\right)^{1/2} - R_{1i} \cdot \sin\alpha}{R_{01i}} \quad (16)$$

in which, R_{e1i} and R_{e2i} are the pinion and gear's outer radiuses; R_{1i} and R_{2i} are the pinion and gear's pitch radiuses; R_{01i} and R_{02i} are the pinion and gear's base-circle radiuses; α is the pressure angle.

The friction coefficient f in (14) is determined by [15]:

- When the sliding velocity $v \le 0.424$ m/s:

$$f = -0.0877 \cdot v + 0.0525 \quad (17)$$

- When the sliding velocity $v > 0.424$ m/s:

$$f = 0.0028 \cdot v + 0.0104 \quad (18)$$

+) The bearing power loss is found by Jelaska [14]:

$$P_{lb} = \sum_{i=1}^{6} f_b \cdot F_i \cdot v_i \quad (19)$$

where, f_b is the bearing friction coefficient; since radical ball bearings with angular contact were chosen $f_b = 0.0011$ [14]; F is the bearing load (N), v is the peripheral speed, and i is the bearing ordinal number (i = 1-6).

+) The overall seal power loss is calculated by Jelaska [14]:

$$P_s = \sum_{i=1}^{2} P_{si} \quad (20)$$

P_{si} is the power loss in a single seal (w), which can be found by:

$$P_{si} = [145 - 1.6 \cdot t_{oil} + 350 \cdot log\, log(VG_{40} + 0.8)] \cdot d_s^2 \cdot n \cdot 10^{-7} \quad (21)$$

where, VG_{40} is the ISO (International Organization for Standardization) Viscosity Grade number.

2.3 Objective Function and Constrains

2.3.1 Objectives Functions

The multi-objective optimization problem consists of two single objectives:

(1) Minimizing the gearbox height:

$$\min f_2(X) = \mathrm{H} \tag{22}$$

(2) Maximizing the gearbox efficiency:

$$\max f_1(X) = \eta_{gb} \tag{23}$$

where, X is the design variable vector of variables. Five main design factors were chosen as variables in this work: u_1, Xba_1, Xba_2, AS_1, and AS_2, resulting in:

$$X = \{u_1, Xba_1, Xba_2, AS_1, AS_2\ \} \tag{24}$$

2.3.2 Constrains

The multi-objective function must keep to the following constraints:

$$1 \le u_1 \le 9 \text{ and } 1 \le u_2 \le 9 \tag{25}$$

$$0.25 \le k_{be} \le 0.3 \text{ and } 0.25 \le X_{ba} \le 0.4 \tag{26}$$

$$350 \le AS_1 \le 420 \text{ and } 350 \le AS_2 \le 420 \tag{27}$$

3. Methodology

Five primary elements of design were chosen for consideration in this study. Table 1 displays the minimum and maximum values for different factors. To solve the optimization problem, the Taguchi approach and GRA were utilized. The L25 (5^5) design was used to optimize the number of levels for each variable. u_1, on the other hand, has a rather large range (u_1 runs from 1 to 9 in Table 1). Even with five levels, there was a considerable variation in the values of these traits (in this example, the difference is ((9-1)/4 = 2).

To assist to reduce the gap between variable values spread throughout a large range, the 2-stage multi-objective optimization problem solution approach (Fig. 2) [12] was applied. This technique's first stage addresses a single-objective optimization issue, while the second stage addresses a multi-objective optimization problem to determine the optimal primary design features.

Table 1 Main design factors and their maximum and minimum limits.

Factor	Notation	Lower limit	Upper limit
Gear ratio of stage 1	u_1	1	9
CWFW of stage 1	X_{ba1}	0.25	0.4
CWFW of stage 2	X_{ba2}	0.25	0.4
ACS of stage 1 (MPa)	AS_1	350	420
ACS of stage 2 (MPa)	AS_2	350	420

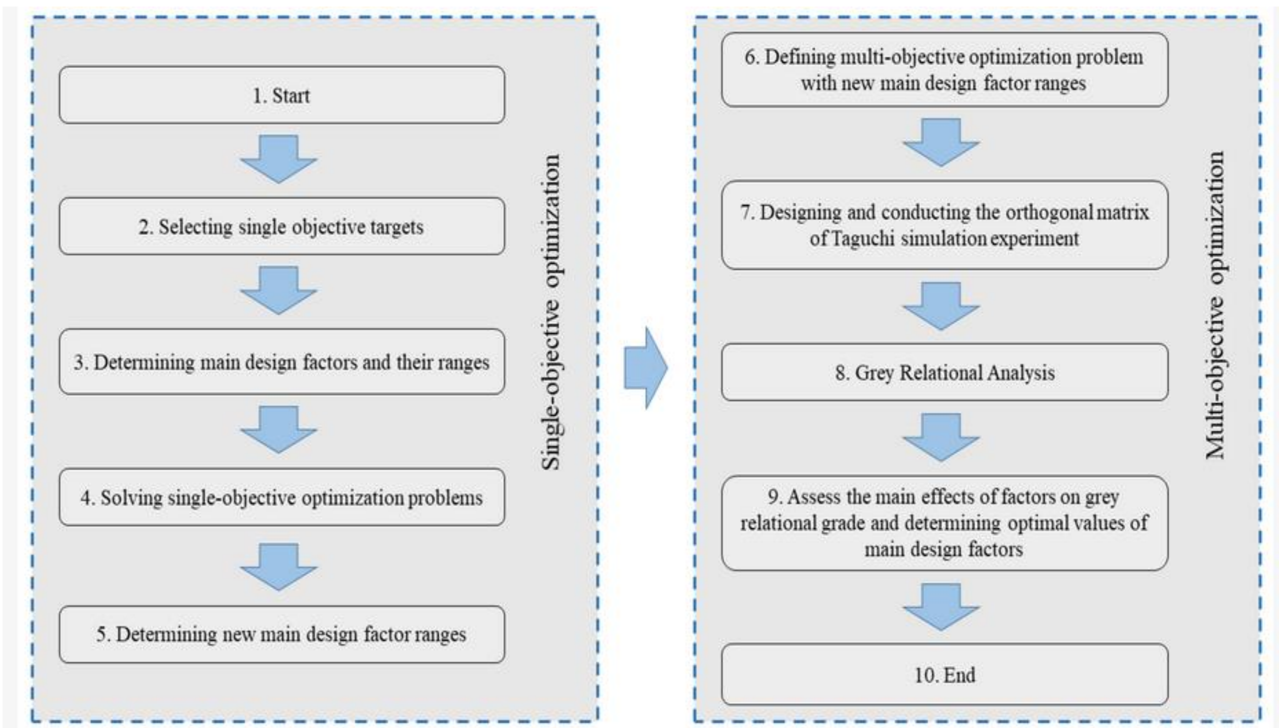

Fig. 2 Method to solve multi-objective problem [16].

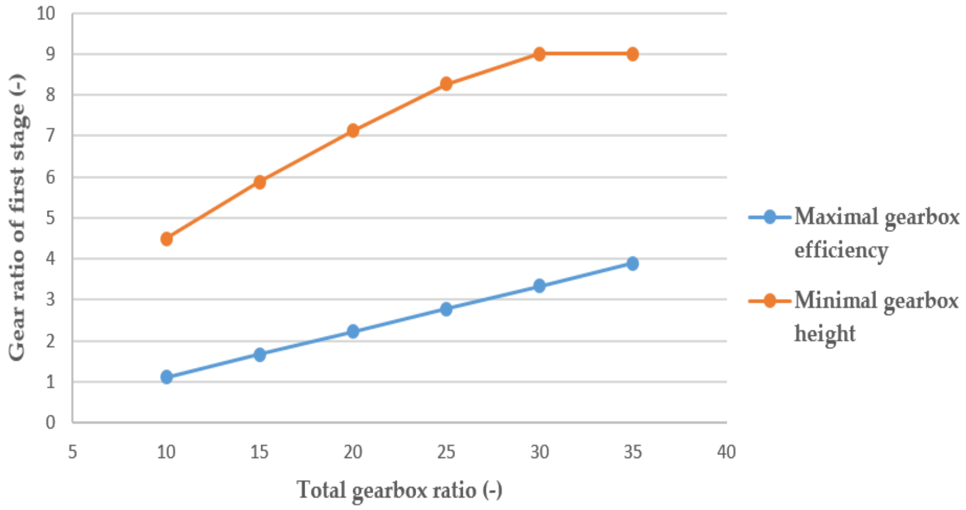

Fig. 3 **Relation between optimal gear ratio of first stage and total gearbox ratio.**

Table 2 **New constraints of u_1.**

u_t	u_1	
	Lower limit	Upper limit
10	1	4.59
15	1.57	5.98
20	2.12	7.22
25	2.68	8.36
30	3.23	9
35	3.79	9

4. Single-Objective Optimization

The direct search strategy is used in this inquiry to address the single-objective optimization problem. A Matlab-based computer program was also created to address two single-objective problems: reducing gearbox length and increasing gearbox efficiency. Fig. 3 depicts the link between the ideal gear ratio of the first stage u_1 and the total gear-box ratio u_t based on the program's findings. In addition, as shown in Table 2, additional restrictions for the variable u_1 have been created.

5. Multi-objective Optimization

The purpose of the multi-objective optimization problem for a two-stage helical gearbox in this study is to determine the optimal major design variables with a given total gear-box ratio that satisfy two single-objective functions: minimizing gearbox height and maximizing gearbox efficiency. A computer experiment was conducted out to accomplish this. The experimental design was built using the Taguchi technique using L25 (5^5) design, and the data were analyzed using the Minitab R18 application. Table 3 describes the survey input characteristics and levels when u_t = 15 that were chosen for inclusion. Table 4 describes the experimental matrix, as well as the results of the gearbox efficiency and gearbox height calculations.

When dealing with multi-objective optimization issues, the Taguchi and GRA techniques are applied. The key phases in this technique are as follows:

+) The S/N (Signal-To-Noise Ratio) is determined using the following equations:

The better the S/N, the shorter the gearbox length:

$$SN = -10log_{10}(\frac{1}{n}\sum_{i=1}^{m} y_i^2) \quad (28)$$

The higher the S/N, the more efficient the gearbox:

$$SN = -10log_{10}(\frac{1}{n}\sum_{i=1}^{m} \frac{1}{y_i^2}) \quad (29)$$

where y_i is the output result and m denotes the number of times the experiment is performed. Because this is a simulation, m = 1 and no repeats are predicted. Table 5 displays the estimated S/N indices for the two output objectives.

Table 3 Main design parameters and their levels for u_t = 15.

Factor	Notation	Level				
		1	2	3	4	5
Gear ratio of first stage	u_1	1.57	2.6725	3.775	4.8775	5.98
CWFW of stage 1	X_{ba1}	0.25	0.2875	0.325	0.3625	0.4
CWFW of stage 2	X_{ba2}	0.25	0.2875	0.325	0.3625	0.4
ACS of stage 1 (MPa)	AS_1	350	367.5	385	402.5	420
ACS of stage 2 (MPa)	AS_2	350	367.5	385	402.5	420

Table 4 Experimental matrix and output results for u_t = 15.

Exp. No.	Input factors					η_{gb}	H
	u_1	X_{ba1}	X_{ba2}	AS_1	AS_2	(%)	(cm)
1	1.5700	0.2500	0.2500	350.0	350.0	98.154	51.384
2	1.5700	0.2875	0.2875	367.5	367.5	98.189	47.699
3	1.5700	0.3250	0.3250	385.0	385.0	98.168	44.594
4	1.5700	0.3625	0.3625	402.5	402.5	98.123	41.932
5	1.5700	0.4000	0.4000	420.0	420.0	98.129	39.617
6	2.6725	0.2500	0.2875	385.0	402.5	97.775	38.239
7	2.6725	0.2875	0.3250	402.5	420.0	97.839	35.876
8	2.6725	0.3250	0.3625	420.0	350.0	97.793	38.797
9	2.6725	0.3625	0.4000	350.0	367.5	97.889	36.528
10	2.6725	0.4000	0.2500	367.5	385.0	97.781	41.025
11	3.7750	0.2500	0.3250	420.0	367.5	97.574	35.036
12	3.7750	0.2875	0.3625	350.0	385.0	97.569	32.951
13	3.7750	0.3250	0.4000	367.5	402.5	97.597	31.133
14	3.7750	0.3625	0.2500	385.0	420.0	97.561	34.983
15	3.7750	0.4000	0.2875	402.5	350.0	97.489	37.471
16	4.8775	0.2500	0.3625	367.5	420.0	97.289	29.635
17	4.8775	0.2875	0.4000	385.0	350.0	97.335	31.361
18	4.8775	0.3250	0.2500	402.5	367.5	97.181	35.107
19	4.8775	0.3625	0.2875	420.0	385.0	97.322	32.706
20	4.8775	0.4000	0.3250	350.0	402.5	97.332	30.687
21	5.9800	0.2500	0.4000	402.5	385.0	97.040	31.708
22	5.9800	0.2875	0.2500	420.0	402.5	97.026	31.240
23	5.9800	0.3250	0.2875	350.0	420.0	97.035	31.878
24	5.9800	0.3625	0.3250	367.5	350.0	97.106	31.403
25	5.9800	0.4000	0.3625	385.0	367.5	97.099	29.507

Table 5 S/N values for each test run when u_t = 15.

Exp. No.	Input factors					η_{gb}		H	
	u_1	X_{ba1}	X_{ba2}	AS_1	AS_2	(%)	S/N	(cm)	S/N
1	1.5700	0.2500	0.2500	350.0	350.0	98.154	39.8382	51.384	-34.2166
2	1.5700	0.2875	0.2875	367.5	367.5	98.189	39.8413	47.699	-33.5702
3	1.5700	0.3250	0.3250	385.0	385.0	98.168	39.8394	44.594	-32.9855
4	1.5700	0.3625	0.3625	402.5	402.5	98.123	39.8354	41.932	-32.4509
5	1.5700	0.4000	0.4000	420.0	420.0	98.129	39.8359	39.617	-31.9576
6	2.6725	0.2500	0.2875	385.0	402.5	97.775	39.8046	38.239	-31.6501
7	2.6725	0.2875	0.3250	402.5	420.0	97.839	39.8102	35.876	-31.0961
8	2.6725	0.3250	0.3625	420.0	350.0	97.793	39.8062	38.797	-31.7760

Table 5 to be continued

9	2.6725	0.3625	0.4000	350.0	367.5	97.889	39.8147	36.528	-31.2525
10	2.6725	0.4000	0.2500	367.5	385.0	97.781	39.8051	41.025	-32.2610
11	3.7750	0.2500	0.3250	420.0	367.5	97.574	39.7867	35.036	-30.8903
12	3.7750	0.2875	0.3625	350.0	385.0	97.569	39.7862	32.951	-30.3574
13	3.7750	0.3250	0.4000	367.5	402.5	97.597	39.7887	31.133	-29.8644
14	3.7750	0.3625	0.2500	385.0	420.0	97.561	39.7855	34.983	-30.8771
15	3.7750	0.4000	0.2875	402.5	350.0	97.489	39.7791	37.471	-31.4739
16	4.8775	0.2500	0.3625	367.5	420.0	97.289	39.7613	29.635	-29.4361
17	4.8775	0.2875	0.4000	385.0	350.0	97.335	39.7654	31.361	-29.9278
18	4.8775	0.3250	0.2500	402.5	367.5	97.181	39.7516	35.107	-30.9079
19	4.8775	0.3625	0.2875	420.0	385.0	97.322	39.7642	32.706	-30.2925
20	4.8775	0.4000	0.3250	350.0	402.5	97.332	39.7651	30.687	-29.7391
21	5.9800	0.2500	0.4000	402.5	385.0	97.040	39.7390	31.708	-30.0234
22	5.9800	0.2875	0.2500	420.0	402.5	97.026	39.7378	31.240	-29.8942
23	5.9800	0.3250	0.2875	350.0	420.0	97.035	39.7386	31.878	-30.0698
24	5.9800	0.3625	0.3250	367.5	350.0	97.106	39.7449	31.403	-29.9394
25	5.9800	0.4000	0.3625	385.0	367.5	97.099	39.7443	29.507	-29.3985

The data amounts are different for the two single-objective functions. The data must be normalized, or transformed to a standard scale, to guarantee comparability. To normalize the data, the normalization value Z_{ij}, which ranges from 0 to 1, is employed. The following formula is used to calculate this value:

$$Z_i = \frac{SN_i - min(SN_i, = 1,2,..n)}{max(SN_i, j = 1,2,..n) - min(SN_i, = 1,2,..n)} \quad (30)$$

where $n = 25$ is experimental runs.

+) The grey relational factor is determined by:

$$\mathrm{y_i(k)} = \frac{\Delta_{\min} + \xi.\Delta_{\max}(k)}{\Delta_{\mathrm{i}}(\mathrm{k}) + \xi.\Delta_{\max}(k)} \quad (31)$$

in which, $i = 1, 2, ..., n$; $k = 2$ is the number of objectives; $\Delta_j(k) = \|Z_0(k) - Z_j(k)\|$ with $Z_0(k)$ and $Z_j(k)$ represent the reference and particular comparison sequences, respectively; Δ_{min} and Δ_{max} are the minimum and maximum values of $i(k)$, respectively; and $\zeta = 0.5$ is the characteristic coefficient.

+) Identifying the level of uncertainty in a relationship: It is determined by averaging the grey relational coefficients linked with the output objectives:

$$\overline{y_\iota} = \frac{1}{k}\sum_{j=0}^{k} y_{ij}\,(k) \quad (32)$$

where y_{ij} is the grey relation value of the i^{th} experiment's j^{th} output target. Table 6 shows the predicted grey relation number y_i as well as the average grey relation value $\overline{y_\iota}$ for each test.

To create harmony among the output elements, a higher average grey relation value is recommended. As a result, the objective function of a multi-objective issue may be reduced to a single-objective optimization problem, providing the mean grey relation value.

The results of an ANOVA (Analysis of Variance) test conducted to analyze the effect of the primary design elements on the average grey relation value $\overline{y_\iota}$ are shown in Table 7. Table 7 shows that u_1 has the most impact on $\overline{y_\iota}$ (39.28%), followed by X_{ba2} (27.78%), AS_1 (13.26%), AS_2 (8.72%), and X_{ba1} (2.24%). Using ANOVA analysis, Table 8 shows the order of the effect of the primary design components on $\overline{y_\iota}$.

+) Finding the best major design elements: In principle, the best major design factors would have the highest S/N values. As a consequence, the impact of major design elements on the S/N ratio (Fig. 4) was estimated. Furthermore, using Fig. 4 chart, the best set of multi-objective parameters (corresponding to the red points) may be easily determined. Table 9 shows the appropriate levels and values for the multi-objective function's essential design variables.

Table 6 Values of $\Delta_i(k)$ and $\bar{y}_i$.

	S/N		Z_i		$\Delta_i(k)$		Grey relation value y_i		$\bar{y}_i$
			η_{gb}	H					
	η_{gb}	H	Reference values		η_{gb}	H	η_{gb}	H	
			1.000	1.000					
1	39.8382	-34.2166	0.9701	0.0000	0.030	1.000	0.944	0.333	0.638
2	39.8413	-33.5702	1.0000	0.1342	0.000	0.866	1.000	0.366	0.683
3	39.8394	-32.9855	0.9820	0.2555	0.018	0.744	0.965	0.402	0.684
4	39.8354	-32.4509	0.9436	0.3665	0.056	0.634	0.899	0.441	0.670
5	39.8359	-31.9576	0.9487	0.4688	0.051	0.531	0.907	0.485	0.696
6	39.8046	-31.6501	0.6454	0.5327	0.355	0.467	0.585	0.517	0.551
7	39.8102	-31.0961	0.7003	0.6477	0.300	0.352	0.625	0.587	0.606
8	39.8062	-31.7760	0.6608	0.5066	0.339	0.493	0.596	0.503	0.550
9	39.8147	-31.2525	0.7432	0.6152	0.257	0.385	0.661	0.565	0.613
10	39.8051	-32.2610	0.6505	0.4059	0.349	0.594	0.589	0.457	0.523
11	39.7867	-30.8903	0.4727	0.6904	0.527	0.310	0.487	0.618	0.552
12	39.7862	-30.3574	0.4684	0.8010	0.532	0.199	0.485	0.715	0.600
13	39.7887	-29.8644	0.4925	0.9033	0.508	0.097	0.496	0.838	0.667
14	39.7855	-30.8771	0.4615	0.6931	0.539	0.307	0.481	0.620	0.551
15	39.7791	-31.4739	0.3995	0.5692	0.600	0.431	0.454	0.537	0.496
16	39.7613	-29.4361	0.2272	0.9922	0.773	0.008	0.393	0.985	0.689
17	39.7654	-29.9278	0.2669	0.8901	0.733	0.110	0.405	0.820	0.613
18	39.7516	-30.9079	0.1340	0.6867	0.866	0.313	0.366	0.615	0.490
19	39.7642	-30.2925	0.2556	0.8144	0.744	0.186	0.402	0.729	0.566
20	39.7651	-29.7391	0.2643	0.9293	0.736	0.071	0.405	0.876	0.640
21	39.7390	-30.0234	0.0121	0.8703	0.988	0.130	0.336	0.794	0.565
22	39.7378	-29.8942	0.0000	0.8971	1.000	0.103	0.333	0.829	0.581
23	39.7386	-30.0698	0.0078	0.8607	0.992	0.139	0.335	0.782	0.559
24	39.7449	-29.9394	0.0692	0.8877	0.931	0.112	0.349	0.817	0.583
25	39.7443	-29.3985	0.0631	1.0000	0.937	0.000	0.348	1.000	0.674

Table 7 Analysis of variance for means.

Source	DF	Seq SS	Adj SS	Adj MS	F	P	C (%)
u_1	4	0.036327	0.036327	0.009082	4.55	0.086	39.28
X_{ba1}	4	0.002071	0.002071	0.000518	0.26	0.890	2.24
X_{ba2}	4	0.025777	0.025777	0.006444	3.23	0.141	27.87
AS_1	4	0.012265	0.012265	0.003066	1.54	0.344	13.26
AS_2	4	0.008065	0.008065	0.002016	1.01	0.496	8.72
Residual error	4	0.007985	0.007985	0.001996			8.63
Total	24	0.092490					

Model summary

S	R-Sq	R-Sq(adj)
0.0447	91.37%	48.20%

Table 8 Response table for means.

Level	u_1	X_{ba1}	X_{ba2}	AS_1	AS_2
1	0.6742	0.5991	0.5567	0.61	0.5759
2	0.5684	0.6166	0.5708	0.6289	0.6025
3	0.5731	0.5898	0.613	0.6143	0.5874

Table 8 to be continued

4	0.5996	0.5964	0.6364	0.5654	0.6219
5	0.5924	0.6058	0.6307	0.5889	0.6199
Delta	0.1057	0.0268	0.0797	0.0635	0.046
Rank	1	5	2	3	4
Average of grey analysis value: 0.602					

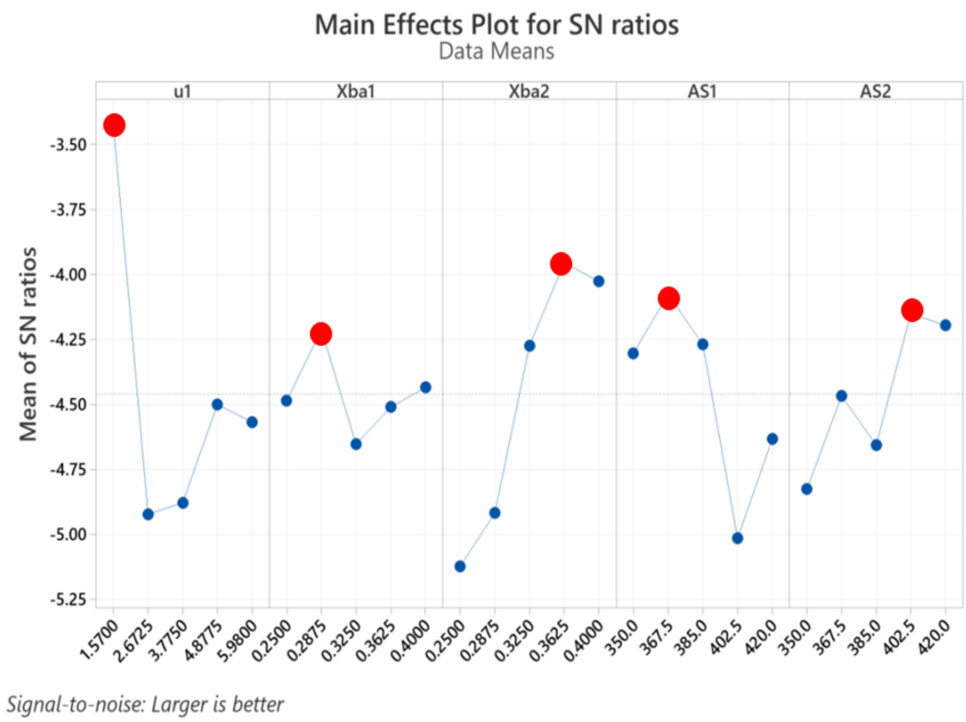

Fig. 4 Main effects plot for S/N ratios.

Table 9 Optimum values of main design factors.

No.	Input parameters	Code	Optimum level	Optimum value
1	Gear ratio of first stage	u_1	1	1.57
2	CWFW of stage 1	Xba1	2	0.2875
3	CWFW of stage 2	Xba2	4	0.3625
4	ACS of stage 1 (MPa)	AS1	2	367.5
5	ACS of stage 2 (MPa)	AS2	4	420

+) *Experimental modeling evaluation:* Fig. 5 depicts the Anderson-Darling method findings, which are used to assess the suitability of the proposed model. The data points corresponding to the experimental observations (shown as blue dots in the graph) fall within the 95% standard deviation zone defined by the top and bottom bounds. Furthermore, the p-value of 0.206 exceeds the significance level of α = 0.05. These data demonstrate that the empirical model utilized in this study is suitable for evaluation.

+) *Evaluating the experimental modeling:* Fig. 5 displays the Anderson-Darling approach findings, which are used to assess the suitability of the proposed model. The data points corresponding to the experimental observations (shown as blue dots in the graph) fall within the 95% standard deviation zone defined by the top and bottom bounds. Furthermore, the p-value of 0.206 exceeds the significance level of = 0.05. These data demonstrate that the empirical model utilized in this study is suitable for evaluation.

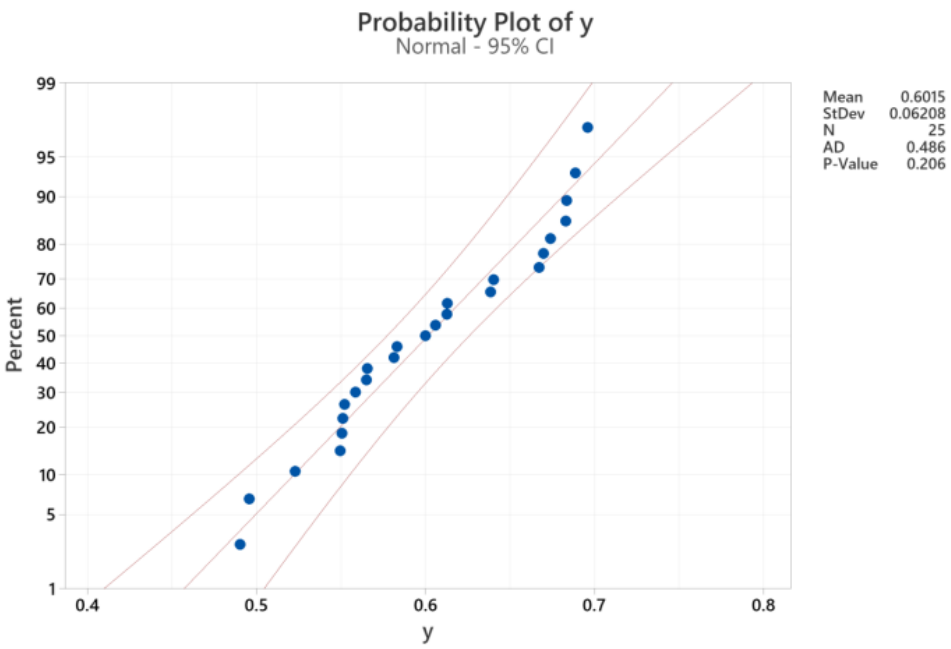

Fig. 5 Probability plot of $\bar{y}$.

Table 10 Optimal values of main design factors.

No.	u_t					
	10	15	20	25	30	35
u_1	1.0	1.57	2.12	2.68	3.23	3.79
X_{ba1}	0.2875	0.2875	0.2875	0.2875	0.25	0.25
X_{ba2}	0.3625	0.3625	0.3625	0.3625	0.3625	0.4
AS_1	367.5	367.5	367.5	367.5	367.5	350
AS_2	402.5	402.5	402.5	402.5	420	420

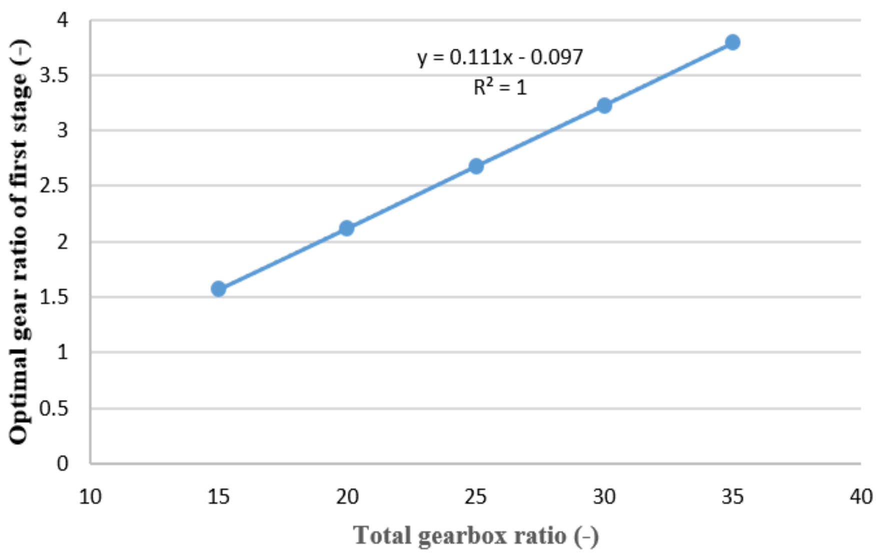

Fig. 6 Optimal first-stage gear ratio versus total gearbox ratio.

Proceed as with u_t = 15, but with the remaining u_t values of 10, 20, 25, 30, and 35. Table 10 illustrates the ideal values for each of the five primary design parameters at various u_t. The link between the appropriate first stage gear ratio and the total gearbox ratio is depicted in Fig. 6. The following regression formula (with R^2 = 1) is provided to obtain the best values of u_1.

$$u_1 = 0.111 \cdot u_t - 0.097 \quad (33)$$

After determining u_1, the optimal value of u_2 may be calculated using $u_2 = u_t / u_1$.

7. Conclusions

The results of a multi-objective optimization study on upgrading a two-stage helical gearbox to reduce gearbox height and increase gearbox efficiency are presented in this paper. This research optimized the first stage's gear ratio, the efficiency of the wheel face width in stages 1 and 2, and the permitted contact stress in stages 1 and 2. A simulation experiment based on the Taguchi L25 type was devised and carried out to address this issue. The impact of key design features on the multi-objective goal was also investigated. It was reported that u_1 has the most impact on $\overline{y_i}$ (39.28%), followed by X_{ba2} (27.78%), AS_1 (13.26%), AS_2 (8.72%), and X_{ba1} (2.24%). Furthermore, the optimal values for the critical gearbox characteristics have been advised. A regression approach (Eq. (33)) was also presented for determining the optimal first stage u_1 gear ratio.

Acknowledgment

Thai Nguyen University of Technology supported for this work.

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