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# An Example of Multiplane Balancing Techniques in Situ

Beatriz F. De Cal[1] and Pedro Fraga[2]

1. *Naval and Industrial Engineering Department, A Coruña University, Ferrol 15403, Spain*

2. *Higher Polytechnic School, A Coruña University, Ferrol 15403, Spain*

**Abstract:** This paper presents the dynamic motion response by rotor unbalance malfunctions and the restraints available to oppose these applied forces and corrective techniques that can be used to reduce the effects of mass unbalance. The mass unbalance is the most common and frequent anomaly in rotating machines, and therefore, although there are many computer programs that solve many cases, we believe it is important to remember his theory here. About this subject should techniques for correcting unbalance problems described in this document be applied. And, more importantly, a tape is made without disassembling the machine, if the transducers described in this work are installed.

**Key words:** Vibration, rotating machinery, mass unbalance, correction techniques.

## 1. Introduction

Many rotating machines have very serious breakdowns, due to their failure to perform adequate predictive maintenance that impair the success of their operation and cause negative performance, not so much because of their energy efficiency, but because of their feasibility and technical-economic performance results.

A strong imbalance, which is not corrected, ends up producing other problems such as the curvatures of the shaft, misalignments, rubs and cracks.

Rotor motion is an orbit, in most cases elliptical, with $X$ and $Y$ amplitude semi-axes and $\omega$ rotation speed, as indicated by Fig. 1 [1], and can be measured as displacements, according to the API's (American Petroleum Institute's) Standard 670 recommendation, using proximity or induction transducers in horizontal and vertical positions.

The measurements of the $X$ and $Y$ transducers are seen as sine waves and, and using the third axis of an oscilloscope, it allows visualizing the orbit as well as the phase angle of the maximum response (high spot) for the reference mark transducer (Kp). The measurement of the phase angle, on a polar diagram of the orbit is measured according to Fig. 2 [2], between the phase reference transducer and the direction of maximum response (high spot).

The equation to use in a simple equilibrium is:

$$F_u = m_* r_{e*} \omega^2 \tag{1}$$

caused because the center of gravity of the rotor does not coincide with the geometric axis of the rotation, where: $m$ is the rotor mass, $r_e$ is the distance from the geometry axis to the center of gravity, and $\omega$ is rotation speed, according to Fig. 3.

This CF (centrifugal force) has to be compensated by creating another (Fc), equally counter-direction, adding a mass $W_c$ at a distance $r_c$ [3]:

$$CF = \frac{W_c}{g} r_c \omega^2 \tag{2}$$

From both Eqs. (1) and (2), we obtain:

$$W_c = \frac{m r_c}{r_e}$$

Suppose that, the eccentricity $r_e$, at low revolutions, is equal to half of the amplitude reading of a displacement transducer, since this measures amplitude peak to peak, so that:

$$W_c = \frac{m * \frac{Amp_{pk-pk}}{2}}{r_c}$$

**Corresponding author:** Pedro Fraga, Ph.D., A Coruña University Professor (mechanical engineer).

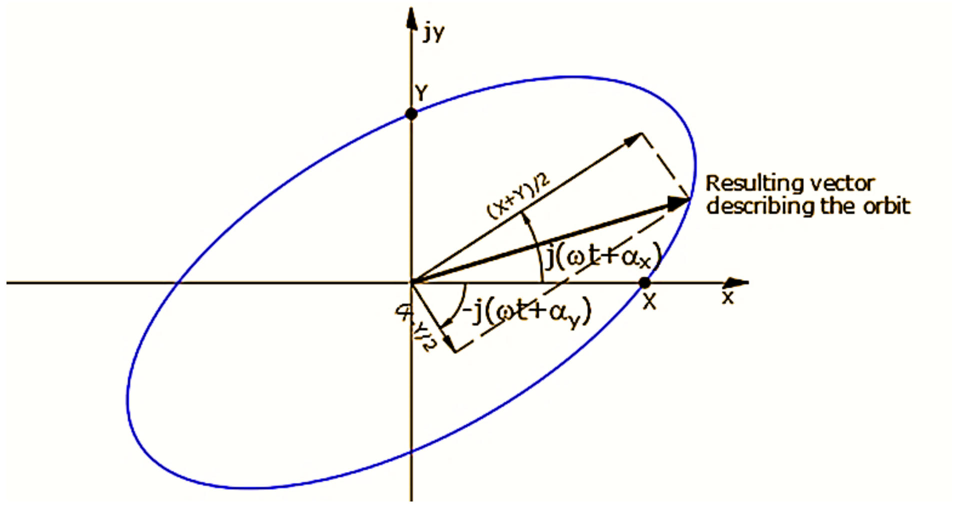


**Fig. 1 Orbit of the centre of the shaft.**

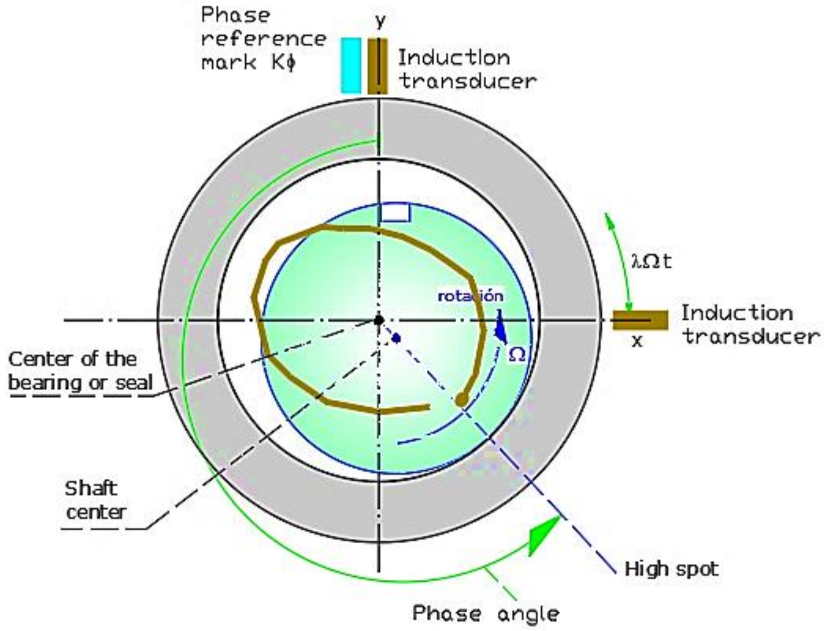


**Fig. 2 Phase angle of orbit.**

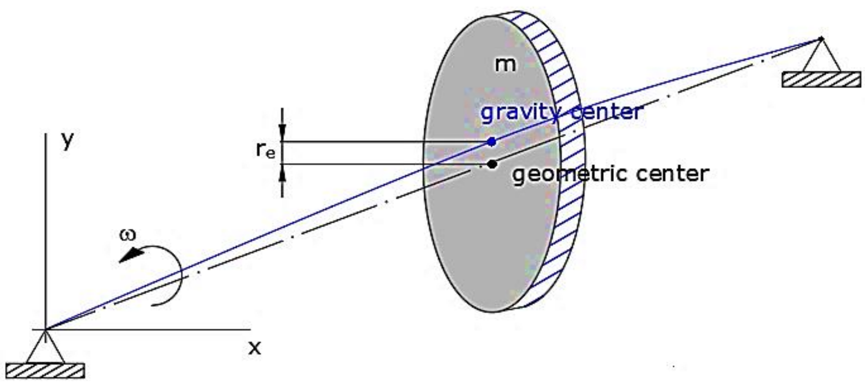


**Fig. 3 Rotor model.**

## 2. Method: Balancing Correction through Stiffness, Balancing "in Situ"

The first balance resonance is controlled primarily by the net spring constant of the rotor system, so that the spring of the rotor can be calculated and the mass unbalance force can also be calculated in this area. The frequency resonance is [4]:

$$\omega_{res}\left({rad}/{s}\right) = \sqrt{\frac{K}{M}}$$

where, $K$ = spring rate (N/m) and $M$ = rotor mass (kg).

The dynamic motion is indicated by:

$$Dynamic\ \ motion = \frac{Force}{Stiffness}$$

Consequently, in this area of first resonance, the motion is restrained by the spring of the rotor system, and the centrifugal force due to the mass unbalance, can be used to calculate the required corrective weight addition, whose expression was given by Eq. (2).

A rotor balanced at a certain low speed is not necessarily balanced at its operating speed, since it will be necessary to take into account different thermal expansions, curvatures of the shaft, modifications of its center of gravity, etc. when the rotor is operating at its nominal speed. Hence, there is the importance of balancing "in situ" in addition to the difficulty of often having to disassemble the machine and take it to the workshop, as it happens, for example, to a hydraulic turbine of a power plant (Fig. 4). For this reason, in many cases it is convenient to balance the rotors on their own bearings and working conditions.

In this method all parameters are considered vectors, with their amplitude and their angular position or phase angle.

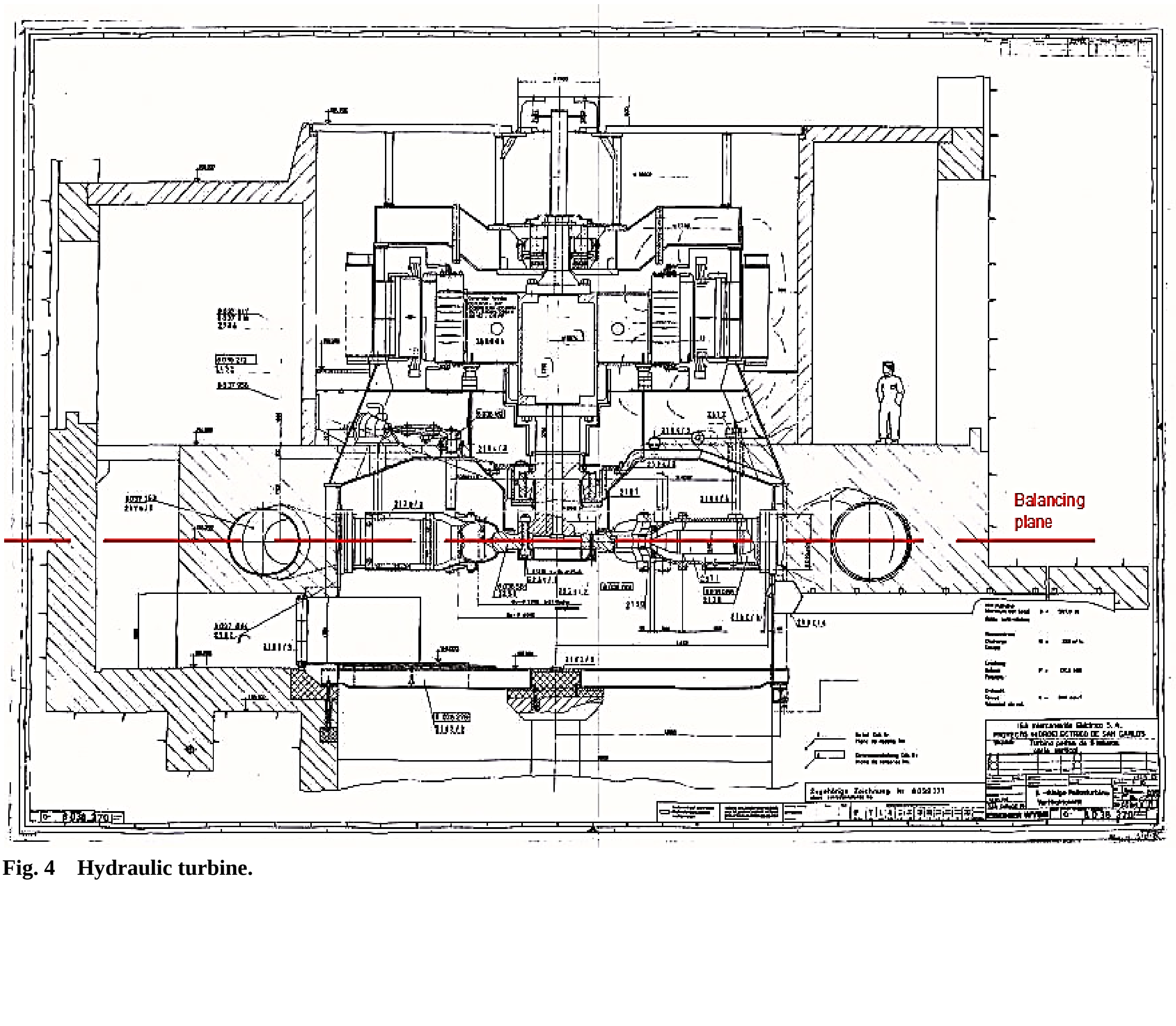


**Fig. 4 Hydraulic turbine.**

*2.1 For the Example of Hydraulic Turbine*

Rotor operating speed: $\omega$ = 2,200 rpm = 230.38 rad/s

Fist balance resonance: 1,600 rpm = 167.55 rad/s

Rotor mass = 650 kg

The procedure, for a plane located on the turbine impeller, consists of the following stages:

(1) The rotor is run at its rated speed to measure the amplitude and phase angle of the unbalance response 1X, vector $R_1$, as shown in Fig. 5, with an amplitude peak to peak of 3.5 mils (1 mil = 25.4 μm) and phase angle of 170°, measured counter clockwise:

Radius of correction: $r_c$ = 1,300 mm (turbine impeller radius)

Proximity transducer reading dynamic motion = 3,5 mils peak to peak = 88.90 μm

Phase angle = 170°

$K = \omega_{res}^2 * M =$ 230.38$^2$ *650/9.81 = 3,531,969.76 N/m

$$\text{Dynamic motion} = \frac{88.90}{10^6}\ \text{m}$$

$$= \frac{\text{force}}{3531969.76\ \text{N/m}} \Rightarrow \text{Force}$$

$$= 313.99\ \text{N}$$

From Eq. (2), the weight correction is:

$$W_c = \frac{CF * g}{r_c \omega^2} = \frac{313.99\ \text{N} * 9.81}{\left(\frac{1300}{1000}\right)\ m * 167.55^2 \text{rad/s}}$$

$$= 0.0843\ \text{kg} = 84.30\ \text{g}$$

This weight will be added to the rotor system with a phase angle 180° opposite the indicated or measured phase angle. For the previous example, the weight would add at an angle of 170°+ 180°= 350°, and if the corrective weight is subtracted, it must subtract at phase angle of 170°.

However, it would be very easy to think that the vibration response was due only to the unbalanced mass, so it is best to take this result as a guideline of the determined mass to install. It is not assumed that the mechanical system has a linear response to mass unbalance.

The presence of significant shaft preloads, thermal effects, fluidic forces, bearing instability and various other mechanisms will cause a non-linear response. So, a certain weight of 85 g shall be added, in principle, in the opposite direction to the $R_1$ obtained but in an arbitrary angular position and close, in any case to the opposite direction of that response $R_1$, at 30°, to measure their joint response and calculate what is due only to the determined added weight.

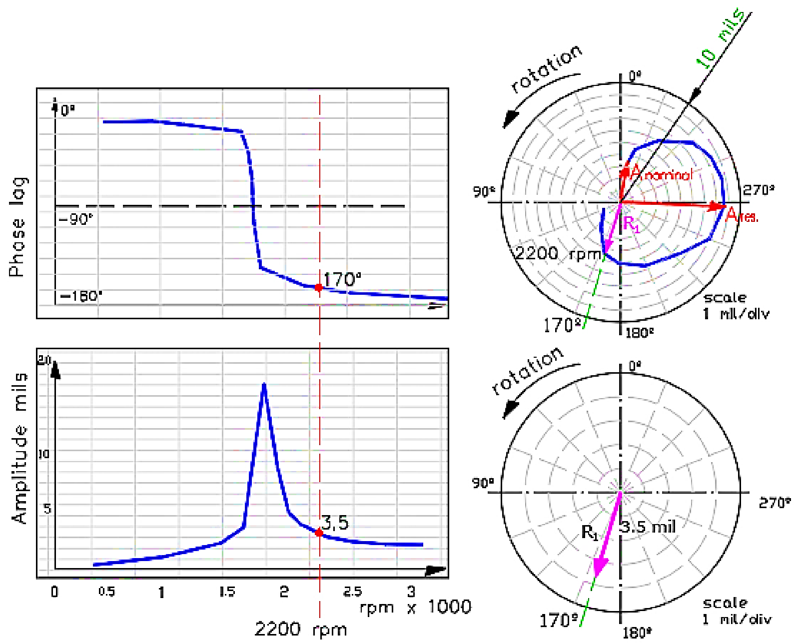


**Fig. 5 Original measurement.**

**Table 1 Resulting vector**

| | Polar | Rectangular |
|---|---|---|
| $R_1 + R_2 =$ | $4.5\angle 125°\Rightarrow$ | -4.34 – j 1.16 mils pp |
| $-R_1$ | $3.5\angle 170°\Rightarrow$ | -0.39 - j 3.48 mils pp |
| $R_3 =$ | $5.27\angle 64.65°$ | $\Leftarrow$ -4.73 + j 2.32 mils pp |

Consequently, we will place a certain weight approximate to the previous solution, 80 g, in the opposite quadrant in a somewhat arbitrary direction, 300°, for example. The response is then obtained $\overrightarrow{R_1 + R_2}$, 4.5mils∠125°, measured counterclockwise, corresponding to the signal that is produced now that added weight.

Subtracting now this last answer from the previous original $\overrightarrow{R_1}$ (Table 1), we obtain the unique answer corresponding to the added weight, vector $\overrightarrow{R_3} = 5.27\angle\ 64.5°$, counterclockwise. This situation is observed in Fig. 6.

To balance the rotor, we must find a vector operator $\overrightarrow{\Lambda}$ that when multiplied by the vibration vector produced by the determined added weight $R_3$ you get a vector equal to $-R_1$, that is [5]:

$$\overrightarrow{\Lambda} * \overrightarrow{R_3} = -\overrightarrow{R_1}$$

and to produce this change in the vector $\overrightarrow{R_3}$, the same change in the initial test weight must be made by applying the same vector operator $\overrightarrow{\Lambda}$, so that the correction weight that will produce a vibration vector equal to $-\overrightarrow{R_1}$, will be obtained:

$$\overrightarrow{W_c} = \overrightarrow{\Lambda} * \overrightarrow{W_p}$$

As the linear system has assumed, with respect to the mass disequilibrium response, any change in the added weight vector will give the same change in its vibration response vector. being then:

$$\overrightarrow{\Lambda} = \frac{\overrightarrow{R_3}}{\overline{-R_1}} = \frac{5.27\angle\ 64.65°}{3.5\ \angle\ 350°} = 1.50\ \angle\ 285.35$$

$$\overrightarrow{W_c} = 1.50\angle 285.35° * 85\ \text{g}\ \angle\ 30°$$
$$= 127.5\ \text{g}\ \angle 315.35°$$

A correction weight of 125 g must then be placed at 315.5° and at a distance of 1,300 mm from the centre of the impeller.

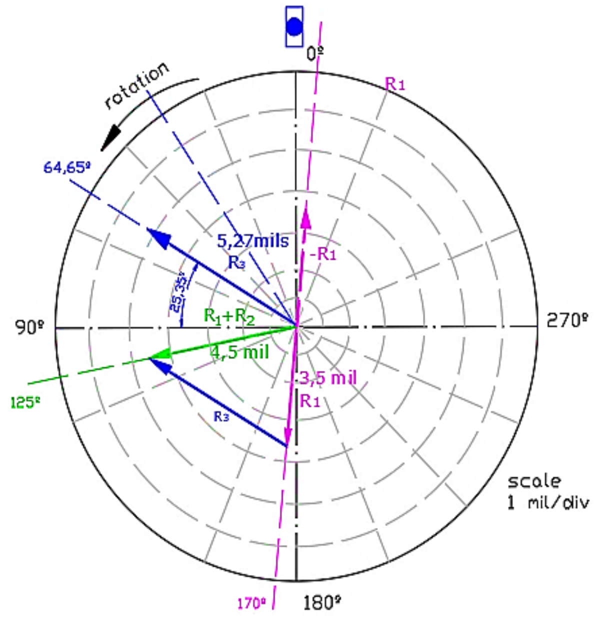


**Fig. 6 Polar diagram.**

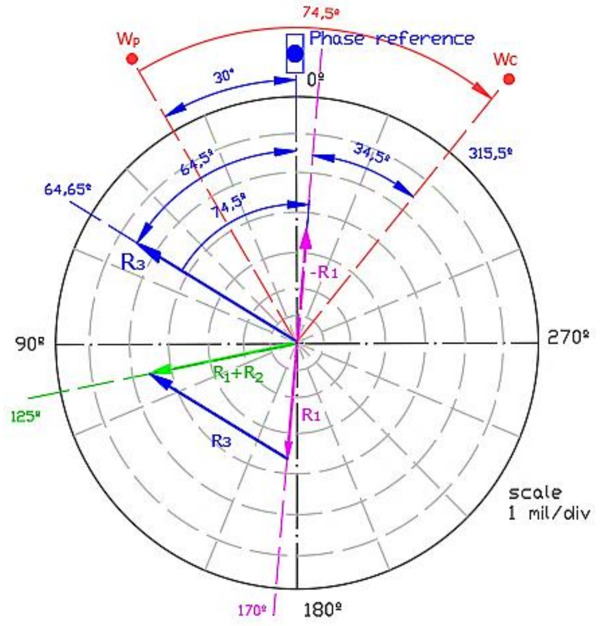


**Fig. 7 Correction weight turns.**

Next, we must make the vibration response vector $R_3$ to the added given weight *Wp* rotate 74.5° clockwise, to match the address of the original response vector $-R_1$, to counteract the original vibration of the turbine. Consequently, the determined test weight *Wp*, must also be rotated, at the same angle as $R_3$ to coincide with $-R_1$, another 74.5° clockwise, at *Wc* according to Fig. 7.

This *Wc* correction weight is placed on the turbine impeller in the above-mentioned position and the turbine is started again to verify the balance and check the need for new corrections, for which the previous steps indicated would have to be repeated.

## 3. Multiplane Balancing Techniques

Another approach to deriving a balance solution to a particular balance can be found in a discrete vector solution to the problem, being its method similar to the case of a plane, but considering, of course, the coupling and interconnection between planes. This technique is most useful on multiple casing machine trains or when the specific response and cross coupled effects of the rotor are unknown.

The study is carried out in a laboratory rotor, such as the one in Fig. 8, in which two balancing planes $V_1$ and $V_2$ are studied. Most rotor systems contain multiple mass elements supported between two bearings and a secondary effect of two plane balancing is the associated with cross coupling. That is, a balance correction at one of the rotor systems will be recognized through cross coupling at the other end of the rotor. Typically, this cross coupled action is 180 °opposite across the rotor and characteristic cross coupled force vector ratios are 4 to 1.

The discreet vector solution is:

$$\overrightarrow{V_0} = \vec{R}\ \vec{U} \tag{3}$$

where: $\overrightarrow{V_0}$ is the original measured vibration vector at balancing speed, amplitude (μm or mils) and angular location, no added determined weight.

*U* is the unbalance force vector (g. and angular location).

*R* is the machine response at unbalance (mils/g) and specific angle.

There are an equation and two unknowns: $\vec{R}$ *and* $\vec{U}$. To solve them, we add a known calibration weight ($\overrightarrow{W_p}$) at angle $\theta$ to the rotor and measure a new vector at operating speed, so we can provide a second equation:

$$\overrightarrow{V_1} \text{ (new measured vector at operating speed)} = \vec{R}\ \left(\vec{U} + \overrightarrow{W_p}\right)$$

$$\overrightarrow{V_1} = \vec{R}\ \vec{U} + \vec{R}\ \overrightarrow{W_p} = \overrightarrow{V_0} + \vec{R}\ \overrightarrow{W_p} \quad \text{and} \quad \vec{R} = \frac{\overrightarrow{V_1} - \overrightarrow{V_0}}{\overrightarrow{W_p}}$$

That provides a solution for the response $\vec{R}$ into Eq. (3) and solves the unbalance force vector:

$$\vec{U} = \frac{\overrightarrow{V_0}}{\vec{R}}$$

Taking into account that the balancing must be carried out in two planes, at least, by the different masses that are generally in a rotor, and by the effect of cross coupling that must be between one plane and another, applying this same tecnichue to a two plane balance ($V_1$ and $V_2$ in Fig. 9), we will have [6]:

$$\begin{aligned} \overrightarrow{V_{01}} &= \overrightarrow{R_{11}}\ \overrightarrow{U_1} + \overrightarrow{R_{12}}\ \overrightarrow{U_2} \\ \overrightarrow{V_{02}} &= \overrightarrow{R_{21}}\ \overrightarrow{U_1} + \overrightarrow{R_{22}}\ \overrightarrow{U_2} \end{aligned} \tag{4}$$

where:

$\overrightarrow{V_{01}} =$ original vector at balance plane 1, no added weight.

$\overrightarrow{V_{02}}$ = original vector at balance plane 2, no added weight.

$\overrightarrow{R_{11}}$ = response vector of rotor at plane 1 to unbalance at plane 1.

$\overrightarrow{R_{12}}$ = response vector of rotor at plane 1 to unbalance at plane 2.

$\overrightarrow{R_{21}}$ = response vector of rotor at plane 2 to unbalance at plane 1.

$\overrightarrow{R_{22}}$ = response vector of rotor at plane 2 to unbalance at plane 2.

$\overrightarrow{U_1}$ = unbalance vector at plane 1.

$\overrightarrow{U_2}$ = unbalance vector at plane 2.

If a calibration weight $\overrightarrow{W_{p1}}$ is added to plane 1, we will have:

$$\begin{aligned} \overrightarrow{V_{11}} &= \overrightarrow{R_{11}}\ \left(\overrightarrow{W_{p1}} + \overrightarrow{U_1}\right) + \overrightarrow{R_{12}}\ \overrightarrow{U_2} \\ \overrightarrow{V_{12}} &= \overrightarrow{R_{21}}\ \left(\overrightarrow{W_{p1}} + \overrightarrow{U_1}\right) + \overrightarrow{R_{22}}\ \overrightarrow{U_2} \end{aligned} \tag{5}$$

and on the other hand, also:

$$\begin{aligned} \overrightarrow{V_{11}} &= \overrightarrow{R_{11}}\ \overrightarrow{W_{p1}} + \underbrace{\overrightarrow{R_{11}}\ \overrightarrow{U_1} + \overrightarrow{R_{12}}\ \overrightarrow{U_2}}_{\overrightarrow{V_{01}}} \\ &= \overrightarrow{R_{11}}\ \overrightarrow{W_{p1}} + \overrightarrow{V_{01}} \\ \overrightarrow{V_{12}} &= \overrightarrow{R_{21}}\ \overrightarrow{W_{p1}} + \underbrace{\overrightarrow{R_{21}}\ \overrightarrow{U_1} + \overrightarrow{R_{22}}\ \overrightarrow{U_2}}_{\overrightarrow{V_{12}}} \\ &= \overrightarrow{R_{21}}\ \overrightarrow{W_{p1}} + \overrightarrow{V_{12}} \end{aligned}$$

being:

$\overrightarrow{V_{11}}$ = vibration vector at balance plane 1 with calibration weight at plane 1.

$\overrightarrow{V_{12}}$ = vibration vector at balance plane 2 with calibration weight at plane 1.

Substituing these equations at the original response Eq. (4), we will have the cross-effects:

$$\overrightarrow{R_{11}} = \frac{\overrightarrow{V_{11}} - \overrightarrow{V_{01}}}{\overrightarrow{W_{p1}}} \text{ and } \overrightarrow{R_{21}} = \frac{\overrightarrow{V_{12}} - \overrightarrow{V_{02}}}{\overrightarrow{W_{p1}}} \tag{6}$$

From Eq. (4) it is obtained:

$$\begin{aligned} \overrightarrow{U_1} &= \frac{\overrightarrow{R_{22}}\ \overrightarrow{V_{01}} - \overrightarrow{R_{12}}\ \overrightarrow{V_{02}}}{\overrightarrow{R_{11}}\ \overrightarrow{R_{22}} - \overrightarrow{R_{12}}\ \overrightarrow{R_{21}}} \\ \overrightarrow{U_2} &= \frac{\overrightarrow{R_{11}}\ \overrightarrow{V_{02}} - \overrightarrow{R_{21}}\ \overrightarrow{V_{01}}}{\overrightarrow{R_{22}}\ \overrightarrow{R_{11}} - \overrightarrow{R_{21}}\ \overrightarrow{R_{12}}} \end{aligned} \tag{7}$$

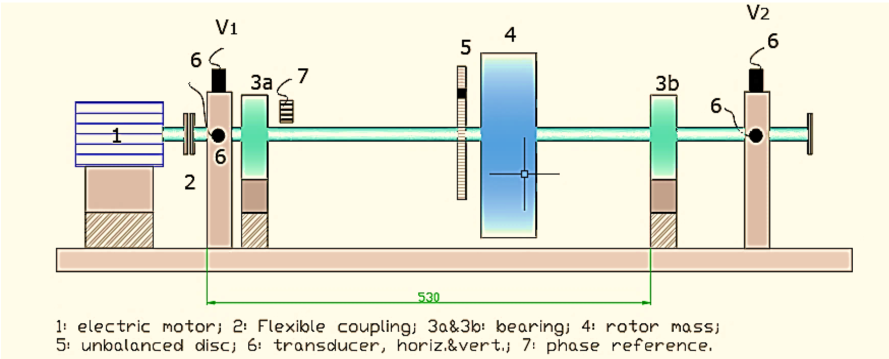


**Fig. 8 Laboratory rotor.**

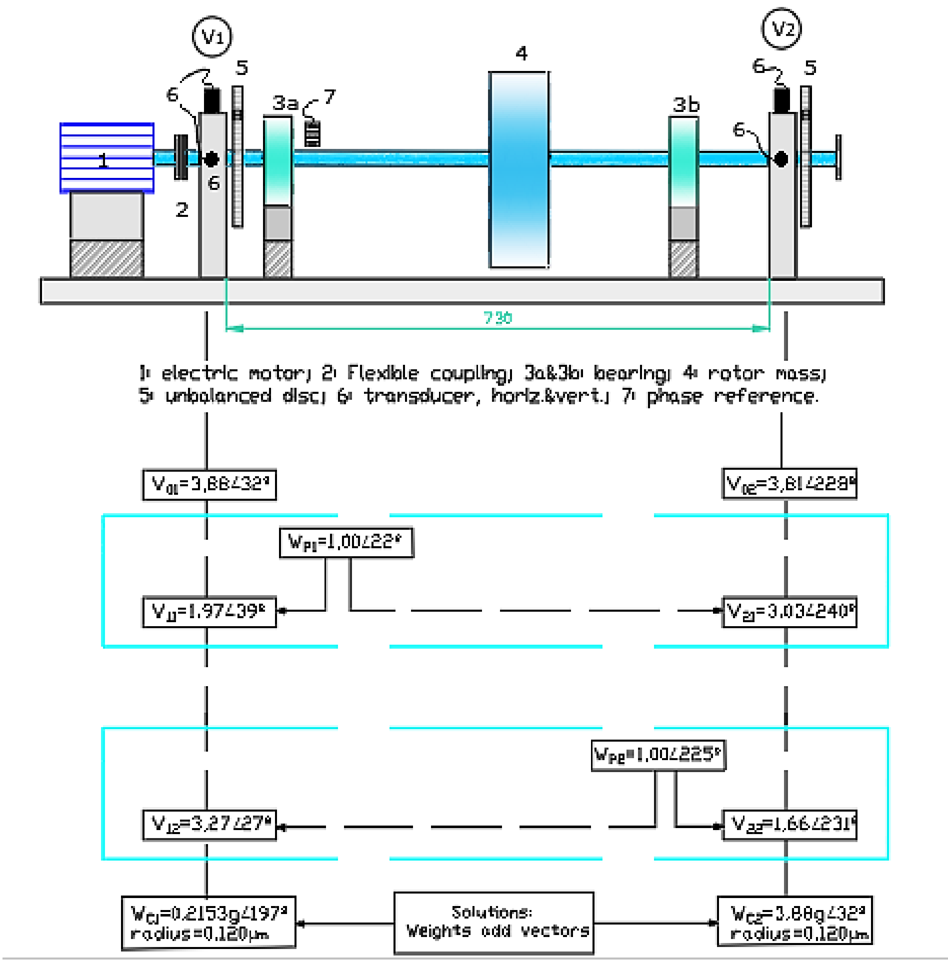


**Fig. 9 Results.**

Similarly, with a calibration weight $\overrightarrow{W_{p2}}$ now in plane 2, we calculate cross-coupling effects:

$$\overrightarrow{V_{21}} = \overrightarrow{R_{12}}\left(\overrightarrow{W_{p2}} + \overrightarrow{U_2}\right) + \overrightarrow{R_{11}}\ \overrightarrow{U_1}$$
$$\overrightarrow{V_{22}} = \overrightarrow{R_{22}}\left(\overrightarrow{W_{p2}} + \overrightarrow{U_2}\right) + \overrightarrow{R_{21}}\ \overrightarrow{U_1}$$

being:

$\overrightarrow{V_{21}}$ = vibration vector at balance plane 1 with calibration weight at plane 2.

$\overrightarrow{V_{22}}$ = vibration vector at balance plane 2 with calibration weight at plane 2.

And, similary:

$$\overrightarrow{\boldsymbol{R_{22}}} = \frac{\overrightarrow{\boldsymbol{V_{22}}} - \overrightarrow{\boldsymbol{V_{02}}}}{\overrightarrow{\boldsymbol{W_{p2}}}} \text{ and } \overrightarrow{\boldsymbol{R_{21}}} = \frac{\overrightarrow{\boldsymbol{V_{21}}} - \overrightarrow{\boldsymbol{V_{01}}}}{\overrightarrow{\boldsymbol{W_{p2}}}}$$

This technique can be used for more than two planes. There will be *n* equations of the form:

$$\overrightarrow{V_{01}} = \overrightarrow{R_{11}}\ \overrightarrow{U_1} + \overrightarrow{R_{12}}\ \overrightarrow{U_2} + \overrightarrow{R_{13}}\ \overrightarrow{U_3} + \cdots \ldots + \overrightarrow{R_{1n}}\ \overrightarrow{U_n}$$
$$\overrightarrow{V_{0n}} = \overrightarrow{R_{n1}}\ \overrightarrow{U_n} + \overrightarrow{R_{n2}}\ \overrightarrow{U_n} + \overrightarrow{R_{n3}}\ \overrightarrow{U_3} + \cdots \ldots + \overrightarrow{R_{nn}}\ \overrightarrow{U_n}$$

This multiplane problem is best treated with a computer aided matrix solution [7].

*3.1 Example: Laboratory Rotor Kit*

Rotor mass = 0.550 kg

Rotor operating speed: $\omega$ = 4,000 rpm = 418.88 rad/s

Fist balance resonance: 1,800 rpm = 188.50 rad/s

Unbalance radius = 120 mm

Steps to follow:

(1) Vibration response (Fig. 9): Original response vector to correct, no weights added:

$$\overrightarrow{V_{01}} = 3.88\measuredangle 32°;\ \overrightarrow{V_{02}} = 3.81\measuredangle 228°$$

(2) A certain weight is added in plane 1 and the response vector is measured in planes $V_1$ and $V_2$.

Calibrations weight = 1g∠22 °and radius 0.12 µm.

With vibration response (Fig. 9), the result is obtained:

$$\overrightarrow{V_{11}} = 1.97\measuredangle 39°;\ \overrightarrow{V_{21}} = 3.03\measuredangle 240°$$

(3) Other certain weight is added in plane 2 and the response vector is measured in planes $V_1$ and $V_2$, vibration response (Fig. 9):

Calibrations weight = 1g∠225° and radius 0.12 µm

$$\overrightarrow{V_{12}} = 3.27\measuredangle 27°;\ \overrightarrow{V_{22}} = 1.66\measuredangle 231°$$

(4) The cross-effects of vibration between one plane and another are now calculated, according to Eq. (6). Continued subsitutions will permit solution of all the direct and cross-response coefficients:

$R_{11}$ = 3.24∠183 °; $R_{12}$ = 1.13∠10 °; $R_{21}$ = 1.74∠350 °; $R_{22}$ = 3.58∠180 °.

(5) The mass disequilibrium vector is then calculated, from Eq. (5):

$$\overrightarrow{U_1} = \frac{3.58\measuredangle 180° * 3.88\measuredangle 32° - 1.13\measuredangle 10° * 3.81\measuredangle 314°}{3.24\measuredangle 183° * 3.58\measuredangle 180° - 1.13\measuredangle 10° * 1.74\measuredangle 350°}$$
$$= \frac{18.18\measuredangle 33°}{9.64\measuredangle 3.63°} = 0.215\ \text{g}\ \measuredangle 17.28°$$

The mass to be added in plane 1 will be the one that has been obtained, but in the opposite direction, that is, rotated 180 °.

Plane 1: $\overrightarrow{U_1} + 180° = = 1.88\ g\ \measuredangle\ 29.37° + 180° = 1.88\ \text{g}\measuredangle 209.37°$

Similarly it will be done for plane 2. [8]

## 4. Conclusions

Two simple rotor cases have been analysed, a single plane and two planes, but the overall response of the rotating system to synchronous forcing functions includes observation of the specific bending modes or analysis of shaft modes.

However, after examining the total modal response of the rotor system, if the rotor operates in a second vibration mode, it may be that the weight placement location to correct the first bending mode, as was done in the example, implies that a weight placed at the plane 1 to correct the first bending mode will cause the second bending mode response to be considerably worse.

In the example on two-plane balancing, it was concluded that major weight correction must be applied at the plane 2. This weight correction also benefits the response at the plane 1 of the rotor system; so much less weight need be selected for the plane 1. The lighter weight is modally beneficial because it will not adversely affect the first bending mode response at the plane 2 of the rotor.

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