

Time Series Forecasting of Hong Kong Inter-bank Offered Rate (HIBOR) using Exponential Smoothing State Space Model*

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This paper set out to analyze and forecast the Hong Kong Interbank Interest Rate (HIBOR) for a period 2006 to 2018. The main objective of this study is to propose an appropriate time series forecasting model for HIBOR. HIBOR conceptually captures the interaction between demand and supply of Hong Kong dollar in the interbank market. The volatility of HIBOR reflects market sentiment, changes in underlying macroeconomic environment, random events and even political climate. Thus, the time series data of HIBOR appears to have multiple seasonality during the aforesaid period. The TBATS model, the state space modeling framework developed by De Livera, Hyndman and Snyder (2010) is adopted for this study to improve the accuracy and efficiency of the time series modeling and forecasting of HIBOR. The TBATS model incorporates Box-Cox transformations, Fourier representations with time varying coefficients, and ARMA error correction. Likelihood evaluation and analytical expressions for point forecasts and interval predictions under the assumption of Gaussian errors are derived, leading to a simple, comprehensive approach to forecasting complex seasonal time series. In addition, the trigonometric formulation is used as a means of decomposing complex seasonal time series, which helps to identify and extract seasonal components which are otherwise not apparent in the time series plot itself. The performance of the TBATS model as evaluated by measures of forecast error are presented.

Keywords: HIBOR, forecast, multiple seasonal

Introduction

Traditionally, analysis and forecast of time series data take the view that the main patterns found in time series data are: trend, season, and cycle. Since each of these patterns might affect the time series in different ways, decomposition of time series is carried out to break the time series into many distinct components to understand how each one behaves in the series. The most frequently used forecasting models in literature are ARIMA and exponential smoothing. However, these models have a major disadvantage as they do not allow forecasting the series with double seasonality and periods of non-integer values. As it is known, seasonal variation of interest

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rates is closely related to seasonal Patterns in the Banking Industry from the demand and supply perspectives. In U.S., the basic seasonal pattern of the banking industry is a period of annual lows in late January and February, followed by a surge in loans that begins in March and rises sharply through May, usually peaking in early June. From there, demand for banking services typically remains relatively flat to slightly down through the summer months. This period extends to around the first of October. Then, from the first part of October through the first part of January, the financial services sector as a whole tends to experience a steady increase in business. Since movement of HIBOR is closely related to U.S. interest rate, it is worth exploring if a corresponding seasonality in HIBOR exists.

Algorithm

TBATS is a time series forecasting methods that are capable of modeling time series with multiple seasonality. The identifier TBATS is an acronym for key features of the model which include Trigonometric seasonality, Box-Cox transformation, ARMA errors, Trend and Seasonal Components. While taking its roots in exponential smoothing methods, TBATS model can be described by the following equations:

Model:

$$y_t^{(\omega)} = \begin{cases} \frac{y_t^\omega - 1}{\omega}; & \omega \neq 0 \\ \log y_t, & \omega = 0 \end{cases}$$

$$y_t^{(\omega)} = \ell_{t-1} + \phi b_{t-1} + \sum_{i=1}^M S_{t-m_i}^{(i)} + d_t$$

is the forecast equation

$$\ell_t = \ell_{t-1} + \phi b_{t-1} + a d_t$$

is the level equation

$$b_t = (1 - \phi)b + \phi b_{t-1} + \beta d_t$$

is the trend equation

$$S_t^{(i)} = S_{t-m_i}^{(i)} + \gamma_i d_t$$

$$d_t = \sum_{i=1}^p \varphi d_{t-i} + \sum_{i=1}^q \theta \varepsilon_{t-i} + \varepsilon_t \varepsilon_{t-i} + \varepsilon_t$$

Seasonal part:

$$S_t^{(i)} = \sum_{j=1}^{k-i} S_{j,t}^{(i)}$$

$$S_{j,t}^{(i)} = S_{j,t-1}^{(i)} \cos \lambda_j^{(i)} + S_{j,t-1}^{*(i)} \sin \lambda_j^{(i)} + \gamma_1^{(i)} dt$$

$$S_{j,t}^{*(i)} = -S_{j,t-1}^{(i)} \sin \lambda_j^{(i)} + S_{j,t-1}^{*(i)} \cos \lambda_j^{(i)} + \gamma_2^{(i)} dt$$

$$\lambda_j^{(i)} = 2\pi j/m_i$$

where

$y_t^{(\omega)}$ is the time series at moment t (Box-Cox transformed)

$S_t^{(i)}$ is the i th seasonal component

ℓ_t is the local level

b_t is the trend with damping

d_t is ARMA (p, q) process for residuals

ε_t is Guassian white noise

Model Parameters:

T is amount of seasonality

m_i is the seasonal periods

k_i is the amount of harmonics for i th seasonal period

ω is Box-Cox transformation

α, β is smoothing

ϕ is trend damping

φ, θ_i is ARMA (p, q) coefficients

γ_1, γ_2 is seasonal smoothing (two for each period)

Each seasonality is modeled by a trigonometric representation based on Fourier series. The trigonometric function allows non-integer seasonal frequency since we define daily frequency for a year as 365.25 days in contemplating leap years. The smoothness of the seasonal pattern can be controlled by k_i , the number of Fourier sin and cos pairs, and the seasonal pattern is smoother for smaller values of k_i . TBATS choose The Final Model by considering various alternatives and fit a number of models which include models: with Box-Cox transformation and without it, with and without Trend, with and without Trend Damping, with and without ARMA (p, q) process used to model residuals, non-seasonal model, various amounts of harmonics used to model seasonal effects. The final model will be chosen using Akaike information criterion (AIC). To handle short term dynamic, ARMA is used to decide if residuals need modeling and what p and q values are suitable.

Decomposition of TBATS Model

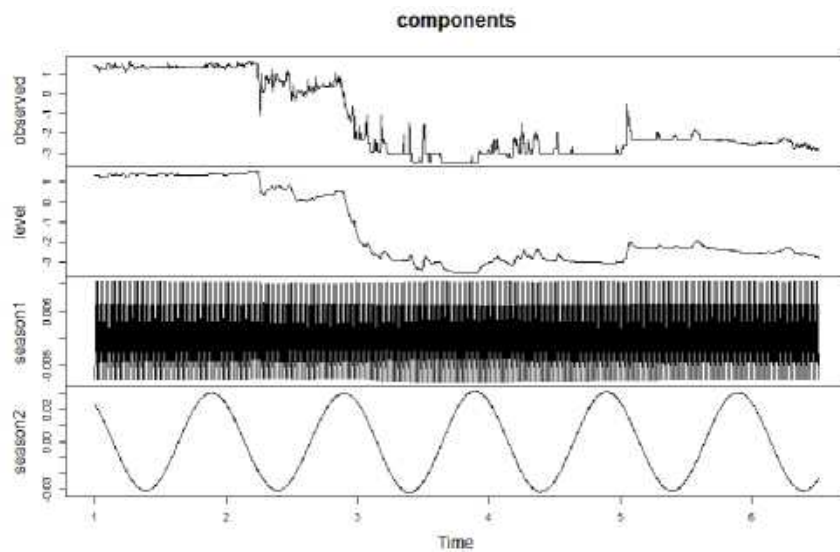


Figure 1. Decomposition of TBATS model.

A TBATS model allows seasonality to change slowly over time instead of forcing the seasonal patterns to repeat periodically without changing. The decomposition of the HIBOR series obtained by using the chosen TBATS model is shown in Figure 1. The vertical bars at the left side of each plot represent equal heights plotted on different scales, thus providing a comparison of the size of each component. The decomposition results shown in Figure 1 confirms both the weekly seasonality (season 1) and the yearly seasonality (season 2). The weekly seasonal pattern is relatively stable and the yearly seasonal pattern evolves considerably over time. From the time series plot, the level component is similar to the observed.

Forecast by TBATS model

The arguments of the model : TBATS (ω , { p , q }, ϕ , { $\langle m_1, k_1 \rangle$, $\langle m_2, k_2 \rangle$, ..., $\langle m_t, k_t \rangle$ } where: ω is a Box-Cox transformation (Box, & Cox, 1964), p , q are ARMA parameters (Whittle, 1951; Box, & Jenkins, 1976; Brockwell, & Davis, 1996), ϕ is a damping parameter (Gardner, & McKenzie, 1985; Snyder, 2006), m_1, \dots, m_t are seasonal periods, k_1, \dots, k_t are the number of Fourier series pairs (West, & Harrison, 1997; Harvey, 1989).

In Figure 2, the model is fitted using data from 2006-01-03 to 2016-01-04, and data from 2016-01-05 to 2018-10-16 is used to assess the out-of-sample forecasting performance of the models. TBATS model is fitted:

TBATS ($\omega = 0.002$, $p=1$, $q = 5$, $\phi = 0$, $\langle m_1, k_1 \rangle = \langle 7, 3 \rangle$, $\langle m_2, k_2 \rangle = \langle 365.25, 1 \rangle$) .

Since $\omega = 0.002$, the model imply that Box-Cox transformation is necessary for this time series. Decompositions between additive and multiplicative can be obtained using a Box-Cox transformation of the data with $0 < \omega < 1$. A value of $\omega = 0$ corresponds to the multiplicative decomposition while $\omega = 1$ is equivalent to an additive decomposition. A multiplicative decomposition is obtained by first taking logs of the data, then back-transforming the components. The estimated value of 0 for ϕ imply no damping on the trend. The arguments $k_1 = 3$ and $k_2 = 1$ indicate that the weekly seasonal component is more variable than the daily seasonal component.

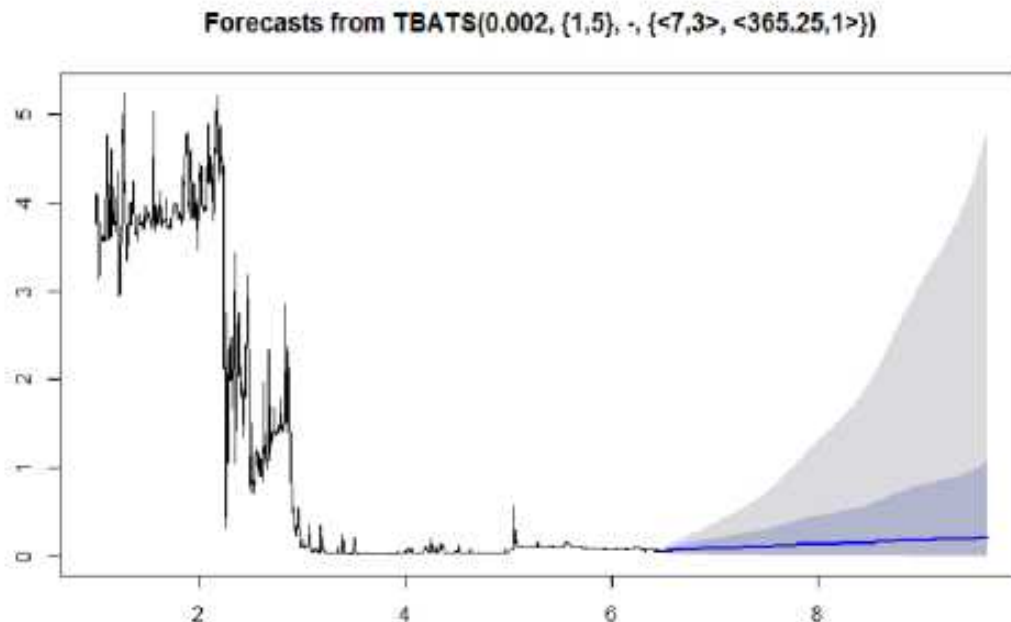


Figure 2. Forecast by TBATS model.

Evaluation of Forecast Accuracy

A forecast error is measured by the difference between an observed value and its forecast. However, as commented by Hyndman, & Koehler (2006), the term “error” in forecasting does not mean a mistake, it simply indicates the unpredictable part of an observation. A set of most commonly used measurements of forecast errors are suggested by Bratu (2012) as follows:

$$\text{Mean Error}(ME) = \frac{\sum_{t=1}^n e_t}{n}$$

$$\text{Mean Absolute Error}(MAE) = \frac{\sum_{t=1}^n |e_t|}{n}$$

$$\text{Root Mean Squared Error}(RMSE) = \sqrt{\frac{\sum_{t=1}^n e_t^2}{n-1}}$$

$$\text{Mean Absolute Percentage Error}(MAPE) = \frac{\sum_{t=1}^n \left| \frac{e_t}{y_t} \right|}{n} \cdot 100\%$$

$$\text{Mean Percentage Error}(MPE) = \frac{\sum_{t=1}^n \frac{e_t}{y_t}}{n} \cdot 100\%$$

$$\text{Mean Absolute Scaled Error}(MASE) = \frac{\sum_{t=1}^n |e_t|}{\frac{n}{n-m} \sum_{t=m+1}^n |y_t - y_{t-m}|}$$

$$\text{Autocorrelation Function of errors at lag 1}(ACF1) = \frac{\sum_{t=1}^{n-1} (e_t - ME) \cdot (e_{t+1} - ME)}{\sum_{t=1}^n (e_t - ME)^2}$$

where:

e_t is the error, $e_t = y_t - y_t^*$

y_t is the actual value,

y_t^* is the forecast value,

m is the seasonal period (Hyndman, & Koehler, 2006)

Measurements of forecast accuracy is presented in Table 1. Between MAE and RMSE errors there are no significant differences. Mean absolute scaled error (MASE) is a measure of forecast accuracy proposed by Koehler, & Hyndman (2006). Using the MASE=1.38 benchmark established by Hyndman (2006), the MASE is 1.078413 which is less than 1.38. Thus, the MASE error exhibits a greater forecasting accuracy in the training set than the average naive forecast. On the other hand, the MAPE which informs the percentage points that the forecasts are off, on average, is 6.12%. The mean error (ME) and mean percentage error (MPE) that are signed measures of error take the value of -0.0132 and -2.376 which indicate the forecasts tend to be biased negatively. In addition, the low autocorrelation (ACF1) criteria of 0.009423 between errors indicates that the model adjusted well to the data.

Table 1

Measurements of Forecast Error

ME	RMSE	MAE	MASE	MPE	MAPE
-0.0132439	0.155249	0.055361	1.078413	-2.375926	6.119754

Conclusion

The use of the trigonometric approach allows us to model the complex seasonality of HIBOR time series. Forecasting of such data would not be possible using traditional models such as ARIMA or exponential smoothing. The use of TBATS model helps to forecast data containing double seasonality and to specify the length of periods which took into account leap years. In this article we have not been able to check the model performance without the presence of other forecasting models. In subsequent studies we shall explore all possibilities in terms of data frequency and number of seasonality.

Reference

- Box, G. E. P., & Cox, D. R. (1964). An analysis of transformations. *Journal of the Royal Statistical Society, Series B*, 26(2), pp. 211-252.
- Box, G. E. P., & Jenkins, G. M. (1976). *Time series analysis: Forecasting and control*. San Francisco: Holden Day.
- Bratu, M. (2012). A comparison of two quantitative forecasting methods for macroeconomic. *Journal of Management Change*, 29(1), 104-124.
- Brockwell, P. J., & Davis, R. A. (1996). *Introduction to Time Series and Forecasting*. New York: Springer.
- Gardner, J. E. S., & McKenzie, E. (1985). Forecasting trends in time series. *Management Science*, 31(10), 1237-1246.
- Harvey, A. (1989). *Forecasting Structural Time Series Models and the Kalman Filter*. Cambridge Books: Cambridge University Press.
- Hyndman, R. J., & Koehler, A. B. (2006). Another look at measures of forecast accuracy. *International Journal of Forecasting*, 22(4), 679-688.
- Snyder, R. (2006). Discussion. *International Journal of Forecasting*, 22(4), 673-676.
- Whittle, P. (1951). *Hypothesis Testing in Time Series Analysis*. Uppsala: Almqvist and Wiksell.
- West, M., & Harrison, J. (1997). *Bayesian Forecasting and Dynamic Models*. 2nd ed. New York: Springer-Verlag.