

Effect of the Earth Movement on the Measured Moon Trajectory

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All lunar theories within the framework of classical mechanics are based on the assumption of an inertial frame of reference, which is physically non-existent and has scarcely been examined in previous studies. This paper aims to examine the consequence of giving up the assumption of the existence of an inertial frame and the relevant influence on the measured moon trajectories. The general formulation of the lunar orbit is built from the perspective of the new general system theory (NGST), where the moon is modelled as a 6-DOF living body and active forces are introduced. This formulation can be retrograded to the framework of classical mechanics. Firstly, we calculate the active forces on the moon when a fixed earth movement and a fixed moon movement are assumed in the god-coordinate system. Then we investigate what will happen to the measured moon trajectory in the human-coordinate system when the earth movement and other parameters are perturbed in several cases. It is found that the results are sometimes inconsistent with the explanations using inertia forces in classical mechanics but can be well explained by NGST. We also discovered that the active forces on the moon should be adjusted constantly to maintain the stable measured moon trajectory.

Keywords: earth movement, measured moon trajectory, the new general system theory, active force

Introduction

The moon is the first object of pure science (Gutzwiller, 1998) and the relevant studies can be traced back to over 3,000 years ago when the motion of the moon was observed and documented in Mesopotamia (Ossendrijver, 2020). The early Greek astronomers and mathematicians advanced the development of lunar theory to the second stage when many fundamental theories were built, repudiated, rectified, and advanced (Gutzwiller, 1998). For instance, Ptolemy promoted the geocentric theory, advocating that the earth is motionless and is the center of the other celestial bodies' orbit. This theory dominated the human beings for more than 1,300 years and until the 16th century it was gradually superseded by the heliocentric theory advocated by Copernicus, where the sun is the static orbit center of the other celestial bodies (Gingerich, 1993). Tycho Brahe doubted both geocentric and heliocentric theories because the relevant computations were not in good accordance with the comet's path. He established the geoheliocentric model which assumed that the planets moved around the sun and the sun revolved around the earth in the hope of avoiding the drawbacks of both Ptolemy's and Copernicus's

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theories (Blair, 1990). Based on the invaluable data from Tycho Brahe, Kepler discovered the three fundamental laws (i.e., the elliptic law, the area law, and the harmonic law), of which an important discovery is that each planet orbits the sun along its own elliptical orbit and the sun is in a focal point of the ellipse (Wilson, 1972). It can be concluded that in this process, people's understanding of the world gradually developed from that the earth was the static orbit center of the other planets, to that the sun was the static orbit center of the other planets, to that the sun was the static focus of the other planets' orbits.

By the end of the 17th century, Issacs Newton promoted the lunar theory forward to a new stage of development. One of his most significant contributions is the relationship between the central force in a Kepler eclipse and the inverse square of the distance known as the gravitational law which is given in Equation (1) (Newton, 1726). Newton defined the main problem of the lunar theory as the solution of the motion of the moon relative to the earth, where the center of mass of the earth-moon system is assumed to move on a fixed Kepler ellipse around the sun (Newton, 1726), as is given in Equation (2) in modern mechanics language (Gutzwiller, 1998), where m_e , m_m , m_s , G , r , ρ , R , and \vec{x} denote the mass of the earth, the mass of the moon, the mass of the sun, the gravitational coefficient, the earth-moon distance, the sun-moon distance, the sun-earth distance, and the vector from the earth to the moon respectively.

$$F(r) = -\frac{Gm_em_m}{r^2} \quad (1)$$

$$\frac{d^2\vec{x}}{dt^2} = \frac{m_e + m_m}{m_em_m} \text{grad}_x \left(\frac{Gm_em_m}{r} + \frac{Gm_sm_m}{\rho} + \frac{Gm_sm_e}{R} \right) \quad (2)$$

However, scientists such as Leonard Euler, Alexis Clairaut, and Jean Le Rond d'Alembert doubted the inverse-square-of-the-distance in Newton's gravitational law because of its deviation from observation data, claiming that correction terms of higher orders should be added. However, all of them ultimately reached the conclusion that problems such as the perigee precession and the evection can be solved completely within the framework of Newton's gravitational theory without introducing correction terms (Airy, 1884), which further proved the greatness of Newton's gravitational formula. Hill proposed that the motion of the moon cannot be simply included in the main problem conforming to the Kepler equation and he used Hill's lunar equations as a solution, where an important modification is a rotating coordinate whose positive x-axis points to the sun, increasing the accuracy without arising many complicated expansion terms (Meyer & Schmidt, 1982). Ernest William Brown expanded the work of Hill and compiled the lunar movement table, which became the foundation of the international astronomical lunar calendar since 1923 (Gutzwiller, 1998). Lagrange studied the lunar orbit from the perspective of analytical mechanics using general equations of motion, where virtual displacements were introduced (Pepe, 2014). Although the formulations of Lagrange seemed very different from those of Newton at first glance, it had been confirmed that classical Newtonian mechanics can be derived from the framework of Lagrangian mechanics at the level of both individual model and global structure (Erik, 2013). It can be concluded that great achievements relevant to the main problem of lunar theory were made during this period, but all of the aforementioned work was based on Newton's assumption that the sun is static and that the orbit of the center of the earth-moon system is fixed.

Later, perturbation theories were developed which could obtain more accurate results by considering factors such as tides (Daher et al., 2021). E. Kant was the first researcher to attribute the lengthening of the day to the

tides and since then tidal interactions have been long recognized as the vital contribution to the variation in the motion parameters in the earth-moon system (Mignard, 1979). G. H. Darwin investigated the tidal effects with the conclusion of shorter lunar period, smaller eccentricity of the lunar orbit, and the shorter period of rotation of the earth in the past (Darwin, 1879). Those perturbation theories could explain numerous phenomena that solutions of the main problem are not capable of (i.e., the varying pattern of the earth's deformation (MacDonald, 1964)). However, most of them still rely on an inertial frame of reference assumption, which is physically non-existent based on the current knowledge. As one observes from a telescope, all stars observed are moving. Also from Gödel's incompleteness theorem (Gödel, 1931), one cannot measure the actual movement of his own platform. For example, the observers on the earth cannot measure the actual movement of the earth within the measurement system. If the observers moved to the sun, then he could see the earth's revolution around the sun as well as self-rotation along its own axis. So, the earth-fixed coordinate system is certainly not an inertial system. The same is true for any coordinate system where human observers are located. For instance, in the F. Mignard's calculation of the evolution of earth-moon system influenced by tides, he used a Laplacian plane as the inertial reference plane which is motionless if the total angular momentum is constant (Mignard, 1980). This condition is satisfied only when the environment outside the earth-moon system exerts no torque on the system according to the law of the conservation of momentum. However, the variation in the total angular momentum induced by the perturbation torque from the sun has long been recognized as the fundamental reason for the earth-moon system's precession and nutation (Urbassek, 2009). Therefore, in the investigation of the perturbation of the lunar orbit, changes in the system's angular momentum are by no means negligible and the assumption of the inertial plane of reference in F. Mignard's work cannot be justified.

From above discussions, it can be concluded that all the existing lunar theories regarding both the main problem and the perturbation within the framework of classical mechanics rely on an inertial frame of reference assumption. In addition to the previous argument based on Gödel's incompleteness theorem (Gödel, 1931), we can provide another two refutations.

Refutation 1: The Newton's second law of motion that the acceleration of the object is in proportional to the external force and inversely proportional to the mass of the object is only applicable in inertia frames (QS Study, n.d.). However, such a system based on the existence of absolute rest of a frame and no disturbance to the measurement results in the measuring process is physically impossible for human observers and thus we call it a God-coordinate system. From a historical point of view, it is obvious that this concept is from the geocentric theory and even up to Newton, God is a clear existence in his ontology. If we want to remove the concept of God from scientific ontology, we have to give up the assumption of the inertial frame assumption. We need to rely on the data measured by the realistic human observers. Indeed, all available data must be gained by the observer with the apparatus fixed on a massive celestial body, which inevitably exerts interactive force (i.e., disturbance) on the measured objects (Pan & Cui, 2021a). As a matter of fact, since geocentric theory was superseded by heliocentric theory, people have realized that the sun rather than the earth is the static orbit center. Furthermore, the sun is not absolutely unaccelerated either, as can be manifested in numerous modern investigations of the sun's radial migration in the milky way galaxy (Quillen, Minchev, Bland-Hawthorn, & Haywood, 2009; Sellwood & Binney, 2002; Martínez-Barbosa, Brown, & Zwart, 2015; Minchev, Chiappini, & Martig, 2013). Despite these knowledge, numerous contemporary studies are still conducted implicitly approximating the inertial coordinate system to be fixed on the sun center (Mignard, 1980; Mashhoon & Theiss, 1986), the earth center (Bellerose & Scheeres, 2008), or the barycenter of the earth-moon system (Breakwell &

Brown, 1979; Hou & Xin, 2018; Naidu & Margot, 2015) with scarcely any quantitative examination of such approximation.

Refutation 2: In order to apply Newton's second law of motion in non-inertia frames, one has to introduce inertial forces such as the centrifugal force and the Coriolis force. This has already been taken for granted in mainstream mechanics textbooks (Kibble & Berkshire, 2004). To be more specific, as is in Equation (3), \vec{a}_r is the object's acceleration in the non-inertial frame. \vec{a}_e (i.e., the entrainment acceleration) and \vec{a}_k (i.e., the Coriolis acceleration) are related to $\vec{\omega}$ (i.e., the angular velocity of the frame), $\vec{\epsilon}$ (i.e., the angular acceleration of the frame), \vec{r}' (i.e., the object's position in the frame), and \vec{v}_r' (i.e., the object's velocity in the frame). The inertial forces (i.e., $\vec{F}_{inertial} = -m(\vec{a}_e + \vec{a}_k)$) should be exerted on the object so that the Newton's second law of motion is applicable. However, the provider of those forces cannot be explained.

$$\vec{a}_a = \vec{a}_r + \vec{a}_e + \vec{a}_k = \vec{a}_r + \vec{\epsilon} \times (\vec{\epsilon} \times \vec{r}') + \vec{\omega} \times (\vec{\omega} \times \vec{r}') + 2\vec{\omega} \times \vec{v}_r' \quad (3)$$

$$m_m \frac{d^2 r}{dt^2} = -\frac{Gm_e m_m}{r^2} + m_m \omega^2 r \quad (4)$$

The introduction of these inertial forces can be traced to Newton's *Principia* (1726) as is shown in Equation (4) (i.e., $m_m \omega^2 r$), where ω is the parameter to keep the correct dimension of a force, with inadequate explanation from Newton why this term should be added (Gutzwiller, 1998). Newton claimed that inertia is an intrinsic property of matter independent of its environment, while Dennis Sciama et al. argued that inertia is due to the interaction of all the matter in the universe (Sciama, 1957). Dennis Sciama attributed the consistency between Newton's theory and observations to an arbitrary assumption (i.e., the choice of absolutely reference frames) and an arbitrary value of a constant (i.e., the value of the gravitational coefficient), and he interpreted the gravitational constant to be the mean density of matter in space (Sciama, 1957). In addition, some other scientists argue that the inertial force is essentially a derivation of coordinate transformation via kinematic technique or an effect of inertia rather than real forces (Persson, 1998; Tombe, 2022). In the appendix of this study, the inertial forces are derived as retrogradations of the general formulation.

Based on the above-mentioned arguments, this paper aims to study what are the consequences if one gives up the assumption of the existence of inertial frames and will this cause influence on the measured moon trajectory from the perspective of the new general system theory (NGST) proposed by Cui and his colleagues (Pan & Cui, 2021a; 2021b; Cui, 2021a; 2021b). According to the relativity of simultaneity axiom proposed in NGST, the essence of matter is ether and the essence of life is mind and they are the fundamental existences in the world we can observe which is a finite spacetime of the whole universe (Cui, 2021a; 2021b). According to NGST ontology (Cui, 2021a), human beings can never observe the whole universe. In addition to the commonly accepted four basic forces (i.e., gravitation, electromagnetism, strong interaction, and weak interaction) which are all passive, active forces are introduced as the fifth force caused by mind-body interactions and psychic field can be generated around the living body, which can explain various phenomena (e.g., the existence of centrifugal force) (Pan & Cui, 2021b). Pan and Cui (2021a) re-examined the two-body problem using NGST, where they pointed out the lack of consideration of the influence of the observer who must be fixed on a massive body and the impracticality of assuming an inertial frame. Pan and Cui (2021a) also proposed that any possible movements (e.g., the planets' spin movements) can be explained within the framework of classical mechanics by introducing an active force called psychic force there. As a matter of fact, studies treating celestial bodies as living bodies can be traced back to 1970s when Lovelock et al. proposed Gaia model to demonstrate that the earth is alive

because it can persistently maintain constant temperature and a compatible chemical composition in changing environment (Lovelock, 1983). Furthermore, one can find that in the early definition of heat by Joule, the father of heat, the force causing active moving of particles is called the living force which is similar as our active forces.

Heat must therefore consist of either living force or of attraction through space. In the former case we can conceive the constituent particles of heated bodies to be, either in whole or in part, in a state of motion. In the latter we may suppose the particles to be removed by the process of heating, so as to exert attraction through greater space. I am inclined to believe that both of these hypotheses will be found to hold good,—that in some instances, particularly in the case of sensible heat, or such as is indicated by the thermometer, heat will be found to consist in the living force of the particles of the bodies in which it is induced; whilst in others, particularly in the case of latent heat, the phenomena are produced by the separation of particle from particle, so as to cause them to attract one another through a greater space. (Joule, 1884, p. 274)

However, Cui (2021a) may be the first scientist to explicitly identify the missing of the active forces in classical mechanics based on his relativity of simultaneity axiom and to study the motion of celestial bodies with the existences of active forces (Pan & Cui, 2021a). This paper aims to further re-examine the two-body problem in the framework of NGST by investigating the measured moon trajectories under the influence of different earth movements.

The rest of the paper is structured as follows. In Section 2, the establishment of kinematic and dynamic equations is illustrated in detail. To begin with, basic assumptions are made and different coordinate systems and their transformation relationships are established. Next, an earth trajectory and a moon trajectory in the god-coordinate system are assumed and the active forces on the moon are derived in analytic form. Furthermore, the movement of the earth and other parameters are perturbed in several cases and the corresponding measured moon trajectories are calculated employing the moon's active forces which have been solved. In Section 3, the numerical results of the observed moon trajectories observed in the human-coordinate system under perturbation are shown and analyzed. In addition, the patterns of the active forces on the moon to maintain the previously observed trajectory are investigated. In the last section, some conclusions are drawn concerning the significance and limitations of this study.

General Formulation

Different Coordinate Systems and Problem Description

Under the framework of classical mechanics, two core coordinate systems (i.e., $O_g - X_g Y_g Z_g$ and $O_h - X_h Y_h Z_h$) are established. $O_g - X_g Y_g Z_g$ is absolutely fixed at a point in the universe, with its origin as the god observer who has zero mass and exerts no influence on the measurement. Since no human observer can make such an accurate observation, we call this coordinate system as the god-coordinate system. In comparison, $O_h - X_h Y_h Z_h$ is fixed on the surface of the earth, with its origin as the human observer. In this human-coordinate system, the masses of the earth, the observer, and the apparatus should all be considered. In addition, to better describe the motion of the moon as well as the relationship between the two core coordinates, four intermediate coordinates are built (i.e., $O_0 - X_0 Y_0 Z_0, O_1 - X_1 Y_1 Z_1, O_2 - X_2 Y_2 Z_2,$ and $O_e - X_e Y_e Z_e$). The detailed transformation relationships among the six coordinate systems are given in Section 2.3.

The effect of the earth movement on the measured moon trajectory is investigated by comparing the different moon trajectories observed in $O_h - X_h Y_h Z_h$ via the following steps. In Section 2.4, the dynamic equations governing the motion of the earth, the moon, and the human observer are given, where they are modelled as 6-degree of freedom (DOF) living bodies subjected to gravitational forces and drag forces and can exert active forces as well as influence the active forces of each other. In Section 2.5, a fixed earth trajectory and a fixed

moon trajectory in the god-coordinate system are assumed, and the active forces on the moon are calculated in analytic form. The moon trajectory observed in $O_h - X_h Y_h Z_h$ can also be derived. In Section 2.6, the active forces on the moon are assumed to be constant within the timespan discussed in this study. The movement of the earth and other parameters are perturbed in several cases, and the corresponding moon trajectories measured in $O_h - X_h Y_h Z_h$ are calculated and compared with the measured moon trajectory before perturbation obtained in Section 2.5.

Basic Assumptions

- (a) Force is the fundamental cause for changes in the motion of objects.
- (b) The units of time, space, and mass are the same in all different coordinate systems (this is called invariant physics by some scientists, e.g. Phipps, 2014; Sato, 2018).
- (c) The law of the conservation of mass, the law of the conservation of momentum, and the law of the conservation of energy hold in all the coordinates.
- (d) The moon can generate active forces. The environment of the earth-moon system is composed of the whole universe outside these two bodies and it exerts negligible influence on the system, which can be justified if the axiom of locality is adopted (Pan & Cui, 2021a). As a matter of the fact, a scientific theory is possible only under the condition of locality axiom.
- (e) The God observer in the coordinate system $O_g - X_g Y_g Z_g$ does not exert any influence on the measurement of object motion.
- (f) Human observers on the earth cannot measure how the earth moves by themselves. Only God can know the absolute movement of the earth.
- (g) Human observers must have mass which includes the mass of the human observer plus the mass of the apparatus for the measurement platform. Of course, this mass is much smaller than the mass of the earth and due to the fixity of the coordinate system on the earth surface, this influence is merged into the earth gravitational force. If the human observer is located in a spacecraft, the mass of the spacecraft must also be added to this total mass of the observation system.
- (h) The moon is subjected to the active forces, the gravitational force from the earth and the drag force during its movement.

The Transformation Relationships Among Different Coordinate Systems

The god-coordinate system $O_g - X_g Y_g Z_g$ is absolutely fixed at a position in the universe with its origin as the god observer. The human-coordinate system $O_h - X_h Y_h Z_h$ is fixed on the earth surface and has its origin as the human observer. Since the effect of the movement of the earth rather than that of the observer is the key topic to be investigated in this study, the observer is assumed to be stationary relative to the earth. The directions of $O_h Z_h$, $O_h Y_h$, and $O_h X_h$ axes are from the centroid of the earth to the observer, tangent to the longitude line and tangent to the latitude line respectively. The earth-coordinate system $O_e - X_e Y_e Z_e$ is fixed on the earth and has its origin at the barycenter of the earth. The plane $O_e - X_e Y_e$ and the plane $O_e - X_e Z_e$ coincide with the earth's equatorial plane and the prime meridian plane respectively. The detailed transformation relationships among the six coordinate systems are given in Equation (5). $O_e - X_e Y_e Z_e$ is obtained by the translation from $O_g - X_g Y_g Z_g$ to $O_0 - X_0 Y_0 Z_0$ and then the fixed-point rotation of $O_0 - X_0 Y_0 Z_0$ around O_0 . $O_h - X_h Y_h Z_h$ is obtained by the translation from $O_e - X_e Y_e Z_e$ to $O_1 - X_1 Y_1 Z_1$ and then the fixed-point rotation of $O_1 - X_1 Y_1 Z_1$ around O_1 . $O_2 - X_2 Y_2 Z_2$ is introduced to describe the nodal precession of the lunar orbit around the earth and is

obtained by the fixed-point rotation of $O_0 - X_0Y_0Z_0$ around O_0 . These coordinate systems together with the three celestial bodies are shown in Figure 1 and all the symbols used are explained in detail in Table 1.

For the convenience we locate the god O_g at the center of the sun, which is the left focus of the elliptic orbit of the earth around the sun. The relationship between R (i.e., the distance between the earth and the sun) and ϕ (i.e., the earth's revolution angle around the sun) is given in the polar coordinate. Likewise, the centroid of the earth is at $O_2(O_0)(O_e)$, which is the left focus of the elliptic orbit of the moon around the earth. The relationship between r (i.e., the distance between the moon and the earth) and θ (i.e., the moon's revolution angle around the earth) is given. The lunar nodal precession and the lunar perigee precession are both considered, the frequencies of which are referred to the data in Haigh, Eliot, and Pattiaratchi (2011). More specifically, the lunar nodal precession indicates that the intersection of the ecliptic and the lunar orbit has a retrograde motion (i.e., westward) of the period of roughly 18.61 years (i.e., O_2Z_2 is rotating around O_0Z_0), which is manifested in the rotational relationship between $O_0 - X_0Y_0Z_0$ and $O_2 - X_2Y_2Z_2$. The lunar perigee precession indicates that the long axis of the moon (i.e., the line connecting the perigee and apogee) takes roughly 8.85 years to make a complete revolution eastward. With such transformation relationship, \vec{R}_m^0 (i.e., the moon's position in $O_0 - X_0Y_0Z_0$) is derived in Equation (5), which is almost the same as the Equation (50) in Yoder (1995), where the author derived the spatial orientation of an orbit relative to the ecliptic and the equinox by three Euler angles. The only difference is the sign of z_m^0 , which is caused by the opposite choice of O_0Z_0 direction in this paper and in Yoder (1995).

$$\begin{aligned} \vec{R}_k^g &\xleftrightarrow{\text{translate}} \vec{R}_k^0 \xleftrightarrow{\text{rotate}} \vec{R}_k^e \xleftrightarrow{\text{translate}} \vec{R}_k^1 \xleftrightarrow{\text{rotate}} \vec{R}_k^h; \vec{R}_k^0 \xleftrightarrow{\text{rotate}} \vec{R}_k^2 \\ \left\{ \begin{array}{l} \vec{R}_k^0 = \vec{R}_k^g - \vec{R}_e^g \\ \vec{R}_k^g = \vec{R}_k^0 + \vec{R}_e^g \end{array} \right. & \left\{ \begin{array}{l} \vec{R}_k^e = [C_0^e] \vec{R}_k^0 \\ \vec{R}_k^0 = [C_e^0] \vec{R}_k^e \end{array} \right. & \left\{ \begin{array}{l} \vec{R}_k^1 = \vec{R}_k^e - \vec{R}_h^e \\ \vec{R}_k^e = \vec{R}_k^1 + \vec{R}_h^e \end{array} \right. & \left\{ \begin{array}{l} \vec{R}_k^h = [C_1^h] \vec{R}_k^1 \\ \vec{R}_k^1 = [C_h^1] \vec{R}_k^h \end{array} \right. & \left\{ \begin{array}{l} \vec{R}_k^2 = [C_0^2] \vec{R}_k^0 \\ \vec{R}_k^0 = [C_2^0] \vec{R}_k^2 \end{array} \right. \\ \vec{R}_e^g = \begin{bmatrix} R \cos \phi \\ R \sin \phi \\ 0 \end{bmatrix}; R = \frac{A(1-E^2)}{1-E \cos \phi}; \vec{R}_h^e = \begin{bmatrix} r_e \cos \xi \cos \lambda \\ r_e \cos \xi \sin \lambda \\ r_e \sin \xi \end{bmatrix}; \vec{R}_m^2 = \begin{bmatrix} r \cos(\theta + \zeta) \\ r \sin(\theta + \zeta) \\ 0 \end{bmatrix}; r = \frac{a(1-e^2)}{1-e \cos \theta} \\ [C_0^e] = \begin{bmatrix} \cos \phi & \sin \phi & 0 \\ -\sin \phi & \cos \phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \alpha & -\sin \alpha \\ 0 & \sin \alpha & \cos \alpha \end{bmatrix} = \begin{bmatrix} \cos \phi & \sin \phi \cos \alpha & -\sin \phi \sin \alpha \\ -\sin \phi & \cos \phi \cos \alpha & -\cos \phi \sin \alpha \\ 0 & \sin \alpha & \cos \alpha \end{bmatrix}; [C_e^0] = [C_0^e]^{-1} \quad (5) \\ [C_1^h] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \sin \xi & \cos \xi \\ 0 & -\cos \xi & \sin \xi \end{bmatrix} \begin{bmatrix} -\sin \lambda & \cos \lambda & 0 \\ -\cos \lambda & -\sin \lambda & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -\sin \lambda & \cos \lambda & 0 \\ -\cos \lambda \sin \xi & -\sin \lambda \sin \xi & \cos \xi \\ \cos \lambda \cos \xi & \sin \lambda \cos \xi & \sin \xi \end{bmatrix}; [C_h^1] = [C_1^h]^{-1} \\ [C_0^2] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \beta & -\sin \beta \\ 0 & \sin \beta & \cos \beta \end{bmatrix} \begin{bmatrix} \cos \chi & \sin \chi & 0 \\ -\sin \chi & \cos \chi & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos \chi & \sin \chi & 0 \\ -\sin \chi \cos \beta & \cos \chi \cos \beta & -\sin \beta \\ -\sin \chi \sin \beta & \cos \chi \sin \beta & \cos \beta \end{bmatrix}; [C_2^0] = [C_0^2]^{-1} \\ \vec{R}_m^0 = [C_2^0] \vec{R}_m^2 = \begin{bmatrix} r \cos(\theta + \zeta) \cos \chi - r \cos \beta \sin(\theta + \zeta) \sin \chi \\ r \cos(\theta + \zeta) \sin \chi + r \cos \beta \sin(\theta + \zeta) \cos \chi \\ -r \sin \beta \sin(\theta + \zeta) \end{bmatrix} \end{aligned}$$

Also, the transformation relationship between the moon trajectory in $O_g - X_gY_gZ_g$ and in $O_h - X_hY_hZ_h$ can be derived which is given in Equation (6).

$$\vec{R}_m^h = [C_1^h] \cdot \{ [C_0^e] \cdot (\vec{R}_m^g - \vec{R}_e^g) - \vec{R}_h^e \} \tag{6}$$

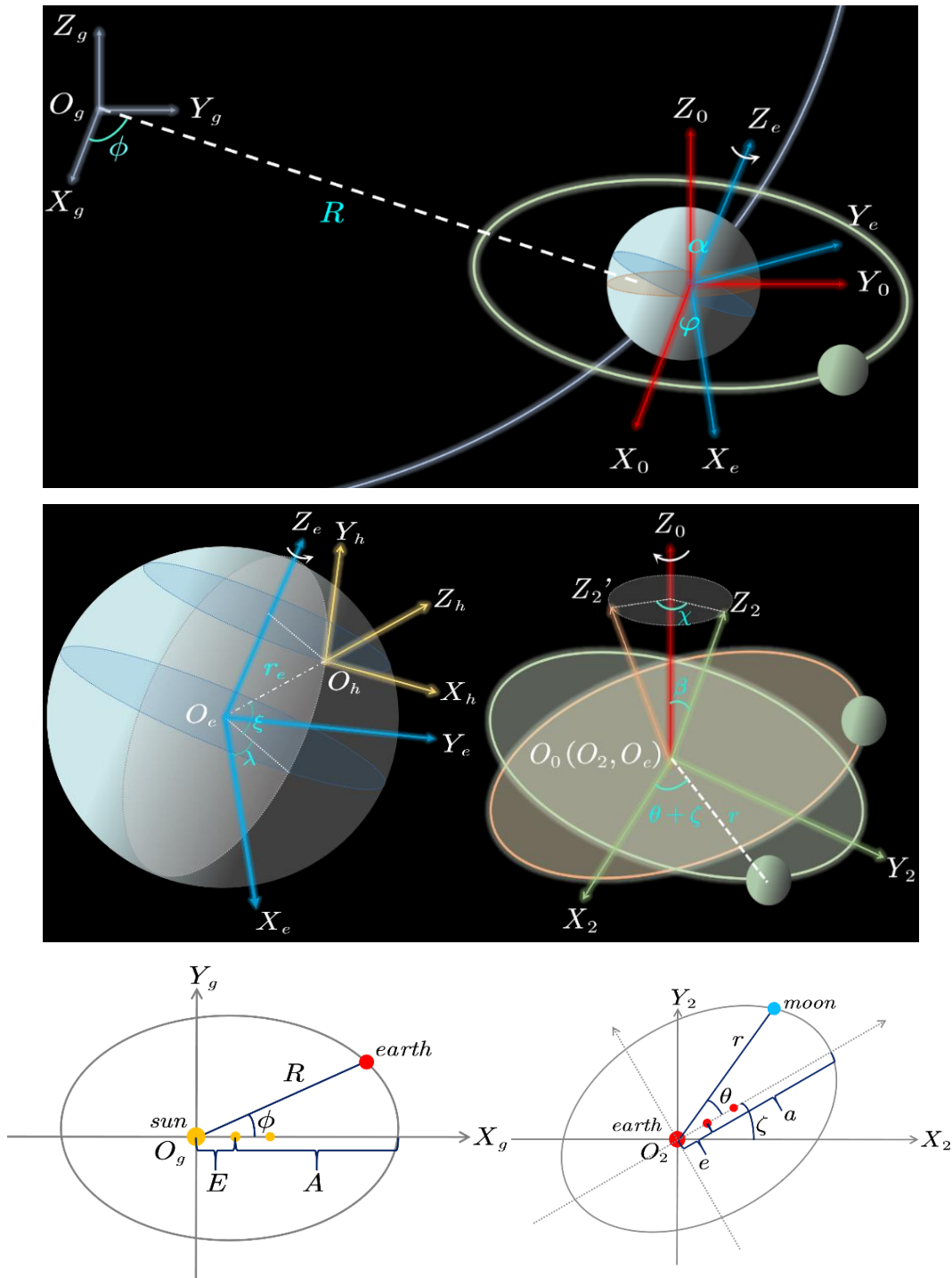


Figure 1. Different coordinate systems.

Table 1

Notations of Symbols in the Transformation Relationship Between Coordinates

Symbol	Meaning
$\phi(\omega_0)$	The earth's revolution angle around the sun (angular velocity)
$\varphi(\omega_1)$	The earth's rotation angle around its axis $O_e Z_e$ (angular velocity)
$\theta(\omega_2)$	The moon's revolution angle around the earth (angular velocity)
$\chi(\omega_3)$	The moon's nodal precession angle (angular velocity)
$\zeta(\omega_4)$	The moon's perigee precession angle (angular velocity)
α	The angle between the plane $O_e - X_e Y_e$ and the plane $O_g - X_g Y_g$
β	The angle between the plane $O_0 - X_0 Y_0$ and the plane $O_2 - X_2 Y_2$
$\lambda(\xi)$	The longitude (latitude) of the observer's location on the earth
$A(a)$	The length of the long semi-axis of the earth's orbit around the sun (the moon's orbit around the earth)
$E(e)$	The eccentricity of the earth's orbit around the sun (the moon's orbit around the earth)
$R(r)$	The distance from the sun to the earth (from the earth to the moon)
$r_e(r_m)$	The radius of the earth (the moon)
\vec{R}_e^g	The position vector of the earth in $O_g - X_g Y_g Z_g$
\vec{R}_m^2	The position vector of the moon in $O_2 - X_2 Y_2 Z_2$
\vec{R}_m^0	The position vector of the moon in $O_0 - X_0 Y_0 Z_0$
\vec{R}_m^e	The position vector of the moon in $O_e - X_e Y_e Z_e$
\vec{R}_m^h	The position vector of the moon in $O_h - X_h Y_h Z_h$
\vec{R}_m^g	The position vector of the moon in $O_g - X_g Y_g Z_g$
$[C_a^b]$	The transformation matrix from coordinate a to coordinate b

The Governing Equations for the Problem

In this section, the whole governing equations in the God-coordinate system are derived by assuming that Newton's second law is valid.

Forces on the earth. The earth is modeled as a 6-DOF rigid living body whose motion is governed by $\vec{F}_{D,e}$ (i.e., the drag force the earth receives during its movement), $\vec{F}_{G,e}^m$ (i.e., the gravitational force from the moon), $\vec{F}_{G,e}^h$ (i.e., the gravitational force from the observer including the measuring apparatus), $\vec{F}_{E,e}^e$ (i.e., the active force generated by the earth itself), $\vec{F}_{E,e}^m$ (i.e., the active force generated by the moon but acted on the earth), and $\vec{F}_{E,e}^h$ (i.e., the active force generated by the observer but acted on the earth). \vec{L}_e^g denotes the earth's moment of momentum in $O_g - X_g Y_g Z_g$. $\vec{M}_{G,e}^m$, $\vec{M}_{G,e}^h$, $\vec{M}_{D,e}$, $\vec{M}_{E,e}^e$, $\vec{M}_{E,e}^m$, and $\vec{M}_{E,e}^h$ denote the moments generated by $\vec{F}_{G,e}^m$, $\vec{F}_{G,e}^h$, $\vec{F}_{D,e}$, $\vec{F}_{E,e}^e$, $\vec{F}_{E,e}^m$, and $\vec{F}_{E,e}^h$ respectively.

$$\begin{cases} \frac{d^2(m_e \vec{R}_e^g)}{dt^2} = \vec{F}_{G,e}^m + \vec{F}_{G,e}^h + \vec{F}_{D,e} + \vec{F}_{E,e}^e + \vec{F}_{E,e}^m + \vec{F}_{E,e}^h \\ \frac{d^2 \vec{L}_e^g}{dt^2} = \vec{M}_{G,e}^m + \vec{M}_{G,e}^h + \vec{M}_{D,e} + \vec{M}_{E,e}^e + \vec{M}_{E,e}^m + \vec{M}_{E,e}^h \end{cases} \quad (7)$$

Forces on the moon. The moon is modeled as a 6-DOF rigid living body whose motion is governed by $\overrightarrow{F_{D,m}}$ (i.e., the drag force the moon receives during its movement), $\overrightarrow{F_{G,m}^e}$ (i.e., the gravitational force from the earth), $\overrightarrow{F_{G,m}^h}$ (i.e., the gravitational force from the observer including the measuring apparatus), $\overrightarrow{F_{E,m}^m}$ (i.e., the active force generated by the moon itself), $\overrightarrow{F_{E,m}^e}$ (i.e., the active force generated by the earth but acted on the moon), and $\overrightarrow{F_{E,m}^h}$ (i.e., the active force generated by the observer but acted on the moon). $\overrightarrow{L_m^g}$ denotes the moon's moment of momentum in $O_g - X_g Y_g Z_g$. $\overrightarrow{M_{G,m}^e}$, $\overrightarrow{M_{G,m}^h}$, $\overrightarrow{M_{D,m}}$, $\overrightarrow{M_{E,m}^e}$, $\overrightarrow{M_{E,m}^m}$, and $\overrightarrow{M_{E,m}^h}$ denote the moments generated by $\overrightarrow{F_{G,m}^e}$, $\overrightarrow{F_{G,m}^h}$, $\overrightarrow{F_{D,m}}$, $\overrightarrow{F_{E,m}^e}$, $\overrightarrow{F_{E,m}^m}$, and $\overrightarrow{F_{E,m}^h}$ respectively. In the calculation process, $\overrightarrow{F_{G,m}^h}$ is omitted because the mass of the observer plus the mass of the apparatus is much smaller than the mass of the earth. Also, $\overrightarrow{F_{E,m}^h}$ is omitted because it is highly unlikely that a human observer can noticeably alter the motion of the moon with his mind. Furthermore, since currently there is no method to distinguish between $\overrightarrow{F_{E,m}^m}$ and $\overrightarrow{F_{E,m}^e}$, they are added as $\overrightarrow{F_E}$ and calculated as one force.

$$\begin{cases} \frac{d^2(m_m \overrightarrow{R_m^g})}{dt^2} = \overrightarrow{F_{G,m}^e} + \overrightarrow{F_{G,m}^h} + \overrightarrow{F_{D,m}} + \overrightarrow{F_{E,m}^e} + \overrightarrow{F_{E,m}^m} + \overrightarrow{F_{E,m}^h} \\ \frac{d^2 \overrightarrow{L_m^g}}{dt^2} = \overrightarrow{M_{G,m}^e} + \overrightarrow{M_{G,m}^h} + \overrightarrow{M_{D,m}} + \overrightarrow{M_{E,m}^e} + \overrightarrow{M_{E,m}^m} + \overrightarrow{M_{E,m}^h} \end{cases} \quad (8)$$

Forces on the human observer. The human observer is modeled as a 6-DOF rigid living body whose motion is governed by $\overrightarrow{F_{D,h}}$ (i.e., the drag force he receives during his motion), $\overrightarrow{F_{G,h}^e}$ (i.e., the gravitational force from the earth), $\overrightarrow{F_{G,h}^m}$ (i.e., the gravitational force from the moon), $\overrightarrow{F_{E,h}^h}$ (i.e., the active force generated by the human observer himself), $\overrightarrow{F_{E,h}^e}$ (i.e., the active force generated by the earth but acted on the observer), $\overrightarrow{F_{E,h}^m}$ (i.e., the active force generated by the moon but acted on the observer), and $\overrightarrow{F_N}$ (i.e., the supporting force from the earth). $\overrightarrow{L_h^g}$ denotes the human observer's moment of momentum in $O_g - X_g Y_g Z_g$. $\overrightarrow{M_{G,h}^e}$, $\overrightarrow{M_{G,h}^m}$, $\overrightarrow{M_{D,h}}$, $\overrightarrow{M_{E,h}^e}$, $\overrightarrow{M_{E,h}^m}$, $\overrightarrow{M_{E,h}^h}$, and $\overrightarrow{M_N}$ denote the moments generated by $\overrightarrow{F_{G,h}^e}$, $\overrightarrow{F_{G,h}^m}$, $\overrightarrow{F_{D,h}}$, $\overrightarrow{F_{E,h}^e}$, $\overrightarrow{F_{E,h}^m}$, $\overrightarrow{F_{E,h}^h}$, and $\overrightarrow{F_N}$ respectively.

$$\begin{cases} \frac{d^2(m_h \overrightarrow{R_h^g})}{dt^2} = \overrightarrow{F_{G,h}^e} + \overrightarrow{F_{G,h}^m} + \overrightarrow{F_{D,h}} + \overrightarrow{F_{E,h}^e} + \overrightarrow{F_{E,h}^m} + \overrightarrow{F_{E,h}^h} + \overrightarrow{F_N} \\ \frac{d^2 \overrightarrow{L_h^g}}{dt^2} = \overrightarrow{M_{G,h}^e} + \overrightarrow{M_{G,h}^m} + \overrightarrow{M_{D,h}} + \overrightarrow{M_{E,h}^e} + \overrightarrow{M_{E,h}^m} + \overrightarrow{M_{E,h}^h} + \overrightarrow{M_N} \end{cases} \quad (9)$$

From our experience, we can move freely according to our free will and this is attributed to the role of $\overrightarrow{F_{E,h}^h}$. On the other hand, the active forces $\overrightarrow{F_{E,h}^e}$ and $\overrightarrow{F_{E,h}^m}$ generated by the earth and the moon are certainly not as changeable as human's free will. They can adapt to the environmental change in a certain range. In this paper, the human observer is assumed to be fixed at a certain point on the earth surface and this can also be attributed

to the role of $\overrightarrow{F_{E,h}^h}$. So no further discussion on the solution of Equation (9) will be carried out. For this analysis, we wish to help readers understand the existence of the active forces and their functions.

Determination of the gravitational force and moment. The gravitational force and moment between two bodies of random shape is given in Equation (10), where the mutual gravitational potential and torque is expanded by means of Legendre polynomials to the second order (Ashenberg, 2007). $\{l, m, n\}$ and $\{l', m', n'\}$ denote the directional cosines of $\overrightarrow{R_m^e}$ in the comoving coordinate fixed on the earth and fixed on the moon respectively. $\{I_{e,x}^g, I_{e,y}^g, I_{e,z}^g\}$ and $\{I_{m,x}^g, I_{m,y}^g, I_{m,z}^g\}$ denote the components of the moment of inertia of the earth and the moment of inertia of the moon respectively. $\overrightarrow{\rho_1}$ denotes the position vector of the microelement of the earth in the comoving coordinate fixed on the earth and $\overrightarrow{r_1}$ denotes the position vector from the microelement vector of the earth to the microelement of the moon. V denotes the gravitational potential. Considering the relative magnitude of r (i.e., the earth-moon distance) and r_e (i.e., the radius of the earth), $V^{(2)}$ is smaller than $V^{(0)}$ by at least four orders of magnitude and is reasonable to be omitted. $\overrightarrow{F_G}$ and $\overrightarrow{M_G}$ are both truncated to the second order in this study, which means that the three equations of momentum and the three equations of the moment of momentum are decoupled.

$$\begin{aligned} \overrightarrow{F_G} &= \nabla V = \nabla \left(-G \int_{m_e} \int_{m_m} \frac{1}{r} dm_e dm_m \right); & \overrightarrow{M_G} &= \overrightarrow{T} = G \int_{m_e} \int_{m_m} \frac{\overrightarrow{\rho_1} \times \overrightarrow{r_1}}{r^3} dm_e dm_m \\ \begin{cases} V^{(0)} = -\frac{Gm_em_m}{r} \\ V^{(1)} = 0 \\ V^{(2)} = \frac{-Gm_m[(1-3l^2)I_{e,x}^g + (1-3m^2)I_{e,y}^g + (1-3n^2)I_{e,z}^g]}{2r^3} \\ \quad + \frac{-Gm_e[(1-3l'^2)I_{m,x}^g + (1-3m'^2)I_{m,y}^g + (1-3n'^2)I_{m,z}^g]}{2r^3} \\ \dots \\ \dots \end{cases} & ; \begin{cases} \overrightarrow{T}^{(0)} = \vec{0} \\ \overrightarrow{T}^{(1)} = \vec{0} \\ \overrightarrow{T}^{(2)} = \frac{3m_m m_e}{r^3} \begin{bmatrix} mn(I_{e,z}^g - I_{e,y}^g) \\ ln(I_{e,x}^g - I_{e,z}^g) \\ lm(I_{e,y}^g - I_{e,x}^g) \end{bmatrix} \\ \dots \\ \dots \end{cases} \end{aligned} \quad (10)$$

Determination of the drag force and moment. The drag force in Equation (11) is based on the analogy to the aerodynamic force of sand grain discussed in Xie, Ling, and Zheng (2007), where ρ_a , Re , V_r , r_m , and μ denote the density of the air, the Reynolds number, the velocity of the moon relative to the space gas around it, the radius of the moon, and the viscosity of the air. C_D (i.e., the coefficient of drag for a sphere) is based on the empirical formula in Ungar and Haff (1987). $\overrightarrow{V_r}$ can be approximated as the moon's velocity in $O_g - X_g Y_g Z_g$ since the velocity of the space gas around the moon is much smaller than the velocity of the moon and is not the main topic to be investigated in this study. Since the space gas is much more rarefied than the air around the earth, we multiply the drag force by the coefficient k . The value of k is initially chosen as 0.001, which is later altered when we study the measured moon trajectory under perturbation.

$$\begin{aligned} \overrightarrow{F_D} &= -k \frac{1}{2} \pi r_m^2 \rho_a C_D V_r \overrightarrow{V_r} \\ C_D &= \frac{24}{Re} + \frac{6}{1 + \sqrt{Re}} + 0.4 \\ Re &= \frac{2\rho_a r_m V_r}{\mu} \end{aligned} \quad (11)$$

Determination of the active force and moment. While the gravitational forces and drag forces are

functions of the positions of the earth, the moon, and the observer, the active forces are unknown and must be derived by assuming a fixed earth movement and a fixed moon movement in the god-coordinate system, as is illustrated in detail in Section 2.5. The active force due to his mind-body interaction is dependent on his free will and this is very complicated. In this paper, the human observer is assumed to be fixed at a position on the earth surface and his active force on the earth and on the moon is very small and they are also neglected in the analyses.

Retrogradation of the general formulation to the solutions in classical mechanics. In the appendix, it is demonstrated that the inertial forces in classical mechanics (e.g., centrifugal force and Coriolis force) can be derived via coordinate transformation in the general formulation.

The Active Forces of the Moon in Analytic Form

In this section, a fixed earth movement and a fixed moon movement are assumed in the god-coordinate system. The motion parameters chosen are given in Table 2, which are all assumed to be constant. \overline{R}_m^g is obtained by the transformation relationships in Section 2.3 and the assumed movements of the earth and the moon are given in Equation (12). Combining Equation (8), Equation (10), Equation (11), and Equation (12), the active forces on the moon are derived in analytic form in Equation (13), which are initially assumed to be constant within the time span of perturbation discussed in this study. In addition, once $\{x_m^g, y_m^g, z_m^g\}$ is presumed, $\{x_m^h, y_m^h, z_m^h\}$ is obtained using the relationship between $O_g - X_g Y_g Z_g$ and $O_h - X_h Y_h Z_h$ in Equation (6) in Section 2.3.

$$\overline{R}_e^g = \begin{bmatrix} x_e^g \\ y_e^g \\ z_e^g \end{bmatrix} = \begin{bmatrix} \frac{A(1 - E^2)\cos\omega_0 t}{1 - E\cos\omega_0 t} \\ \frac{A(1 - E^2)\sin\omega_0 t}{1 - E\cos\omega_0 t} \\ 0 \end{bmatrix}; \quad \overline{R}_m^g = \begin{bmatrix} x_m^g \\ y_m^g \\ z_m^g \end{bmatrix}$$

$$= \begin{bmatrix} \frac{a(1 - e^2)[\cos(\omega_2 + \omega_4)t\cos\omega_3 t - \cos\beta\sin(\omega_2 + \omega_4)t\sin\omega_3 t]}{1 - e\cos\omega_2 t} + \frac{A(1 - E^2)\cos\omega_0 t}{1 - E\cos\omega_0 t} \\ \frac{a(1 - e^2)[\cos(\omega_2 + \omega_4)t\sin\omega_3 t + \cos\beta\sin(\omega_2 + \omega_4)t\cos\omega_3 t]}{1 - e\cos\omega_2 t} + \frac{A(1 - E^2)\sin\omega_0 t}{1 - E\cos\omega_0 t} \\ - \frac{a(1 - e^2)\sin\beta\sin(\omega_2 + \omega_4)t}{1 - e\cos\omega_2 t} \end{bmatrix} \quad (12)$$

$$\overline{F}_E \triangleq \overline{F}_{E,m}^e + \overline{F}_{E,m}^m + \overline{F}_{E,m}^h = m_m \frac{d^2 \overline{R}_m^g}{dt^2} - \overline{F}_{G,m}^e - \overline{F}_{D,m}$$

$$= \begin{bmatrix} m_m \ddot{x}_m^g + \frac{Gm_e m_m (x_m^g - x_e^g)}{[(x_m^g - x_e^g)^2 + (y_m^g - y_e^g)^2 + (z_m^g - z_e^g)^2]^{1.5}} + \frac{k\pi r_m^2 \rho_a C_D \dot{x}_m^g}{2} \sqrt{\dot{x}_m^{g2} + \dot{y}_m^{g2} + \dot{z}_m^{g2}} \\ m_m \ddot{y}_m^g + \frac{Gm_e m_m (y_m^g - y_e^g)}{[(x_m^g - x_e^g)^2 + (y_m^g - y_e^g)^2 + (z_m^g - z_e^g)^2]^{1.5}} + \frac{k\pi r_m^2 \rho_a C_D \dot{y}_m^g}{2} \sqrt{\dot{x}_m^{g2} + \dot{y}_m^{g2} + \dot{z}_m^{g2}} \\ m_m \ddot{z}_m^g + \frac{Gm_e m_m (z_m^g - z_e^g)}{[(x_m^g - x_e^g)^2 + (y_m^g - y_e^g)^2 + (z_m^g - z_e^g)^2]^{1.5}} + \frac{k\pi r_m^2 \rho_a C_D \dot{z}_m^g}{2} \sqrt{\dot{x}_m^{g2} + \dot{y}_m^{g2} + \dot{z}_m^{g2}} \end{bmatrix} \quad (13)$$

$$\overline{F}_E = \overline{F}_E(G, m_m, m_e, \beta, a, e, A, E, \omega_0, \omega_1, \omega_2, \omega_3, \omega_4, k, r_m, \mu, \rho_a) = [F_{E,x} \quad F_{E,y} \quad F_{E,z}]^T$$

Table 2

Parameters of the Assumed Earth Movement and Moon Movement in the God-Coordinate System

Symbol	Value	Unit	Symbol	Value	Unit
ω_0	$2\pi/365.256$	rad/day	A	$1.496 \cdot 10^8$	km
ω_1	$2\pi/0.997$	rad/day	E	0.0167	1
ω_2	$2\pi/27.322$	rad/day	a	$3.844 \cdot 10^5$	km
ω_3	$-\omega_0/18.61$	rad/day	e	0.0549	—
ω_4	$\omega_0/8.85$	rad/day	r_e	$6.371 \cdot 10^3$	km
α	$23.433/180 \cdot \pi$	rad	r_m	$1.737 \cdot 10^3$	km
β	$5.145/180 \cdot \pi$	rad	G	$6.672 \cdot 10^{-17}$	$N \cdot km^2/kg^2$
k	0.001	—	m_e	$5.965 \cdot 10^{24}$	kg
μ	$1.8 \cdot 10/24/3600$	$N \cdot day/km^2$	m_m	$7.342 \cdot 10^{22}$	kg
ρ_a	$1.29 \cdot 10^9$	kg/km^3			

The Moon Trajectories Observed From $O_h - X_h Y_h Z_h$ Under Perturbations

$\{x_m^{g'}, y_m^{g'}, z_m^{g'}\}$ are the unknowns to be solved in the ordinary differential Equation (14), where the other symbols marked with ‘’ are those perturbed (i.e., $x_e^{g'}$, $y_e^{g'}$, $z_e^{g'}$, k' , and m_m'). More specifically, the perturbations in $x_e^{g'}$, $y_e^{g'}$, and $z_e^{g'}$ are achieved by the perturbations in A' , E' , and ω_0' . $\{F_{E,x}, F_{E,y}, F_{E,z}\}$ are those solved in Section 2.5 based on the movement parameters before perturbation. To make sure that the divergence phenomena under perturbations are not caused by the ordinary differential equation's sensitivity to the initial values, the initial values use perturbed parameters (i.e., A' and E'). After solving $\{x_m^{g'}, y_m^{g'}, z_m^{g'}\}$, $\{x_m^{h'}, y_m^{h'}, z_m^{h'}\}$ is derived using the transformation relationship between $O_g - X_g Y_g Z_g$ and $O_h - X_h Y_h Z_h$ in Equation (6) in Section 2.3. In this process, the perturbation in ω_1' is also considered.

$$\begin{cases} m_m \ddot{x}_m^{g'} + \frac{Gm_e m_m' (x_m^{g'} - x_e^{g'})}{[(x_m^{g'} - x_e^{g'})^2 + (y_m^{g'} - y_e^{g'})^2 + (z_m^{g'} - z_e^{g'})^2]^{1.5}} + \frac{k' \pi r_m^2 \rho_a C_D \dot{x}_m^{g'}}{2} \sqrt{\dot{x}_m^{g'2} + \dot{y}_m^{g'2} + \dot{z}_m^{g'2}} = F_{E,x} \\ m_m \ddot{y}_m^{g'} + \frac{Gm_e m_m' (y_m^{g'} - y_e^{g'})}{[(x_m^{g'} - x_e^{g'})^2 + (y_m^{g'} - y_e^{g'})^2 + (z_m^{g'} - z_e^{g'})^2]^{1.5}} + \frac{k' \pi r_m^2 \rho_a C_D \dot{y}_m^{g'}}{2} \sqrt{\dot{x}_m^{g'2} + \dot{y}_m^{g'2} + \dot{z}_m^{g'2}} = F_{E,y} \\ m_m \ddot{z}_m^{g'} + \frac{Gm_e m_m' (z_m^{g'} - z_e^{g'})}{[(x_m^{g'} - x_e^{g'})^2 + (y_m^{g'} - y_e^{g'})^2 + (z_m^{g'} - z_e^{g'})^2]^{1.5}} + \frac{k' \pi r_m^2 \rho_a C_D \dot{z}_m^{g'}}{2} \sqrt{\dot{x}_m^{g'2} + \dot{y}_m^{g'2} + \dot{z}_m^{g'2}} = F_{E,z} \end{cases} \quad (14)$$

$$\begin{cases} x_m^{g'}|_{t=0} = a(1+e) + A'(1+E') \\ y_m^{g'}|_{t=0} = 0 \\ z_m^{g'}|_{t=0} = 0 \\ \dot{x}_m^{g'}|_{t=0} = 0 \\ \dot{y}_m^{g'}|_{t=0} = a(1+e)\omega_3 + \cos\beta a(1+e)(\omega_2 + \omega_4) + A'(1+E') \\ \dot{z}_m^{g'}|_{t=0} = -\sin\beta(\omega_2 + \omega_4)a(1+e) \end{cases}$$

Results and Discussion

In Section 3.1, the forces on the earth and the forces on the moon are plotted, where the active forces are observed to dominate compared with other forces in terms of the order of magnitude.

In Section 3.2, the active forces on the moon are initially assumed to be constant within the time span of perturbation. The influences of several other factors on the measured moon trajectory are discussed. If there is divergence in the observer-moon distance caused by perturbation, the active forces on the moon to maintain the previously observed trajectory are investigated. In addition, the results are analyzed from the perspective of both the inertia forces in classical mechanics and the active (or psychic) forces in NGST, where contradictions sometimes occur. NGST perspectives are found to better explain the phenomena.

Before Perturbation: Active Forces on the Earth and the Moon

As can be seen in Figure 2, the active forces on both the earth and the moon dominate in comparison with other forces in terms of the order of magnitude. Therefore, the active forces play the major role in governing the motion of the earth and the moon. Furthermore, as can be seen from Table 3, the gravitational forces from other planets in the solar system or the sun cannot explain the active forces on the moon because even the sum of their maximum values is smaller than the active forces on the moon by 10 orders of magnitude.

Table 3

The Gravitational Forces to the Moon From Other Celestial Bodies in the Solar System

Star (Planet)	Mass/kg	The nearest distance to the moon/km	Maximum gravitational force to the moon/N
Sun	1.99E+27	1.49E+08	4.38E+17
Mercury	3.30E+23	9.13E+07	1.94E+14
Venus	4.87E+24	4.10E+07	1.42E+16
Mars	6.42E+23	7.79E+07	5.18E+14
Jupiter	1.90E+27	6.28E+08	2.36E+16
Saturn	5.68E+26	1.28E+09	1.70E+15
Uranus	8.68E+25	2.72E+09	5.75E+13
Neptune	1.02E+26	4.35E+09	2.65E+13
Total			4.78E+17

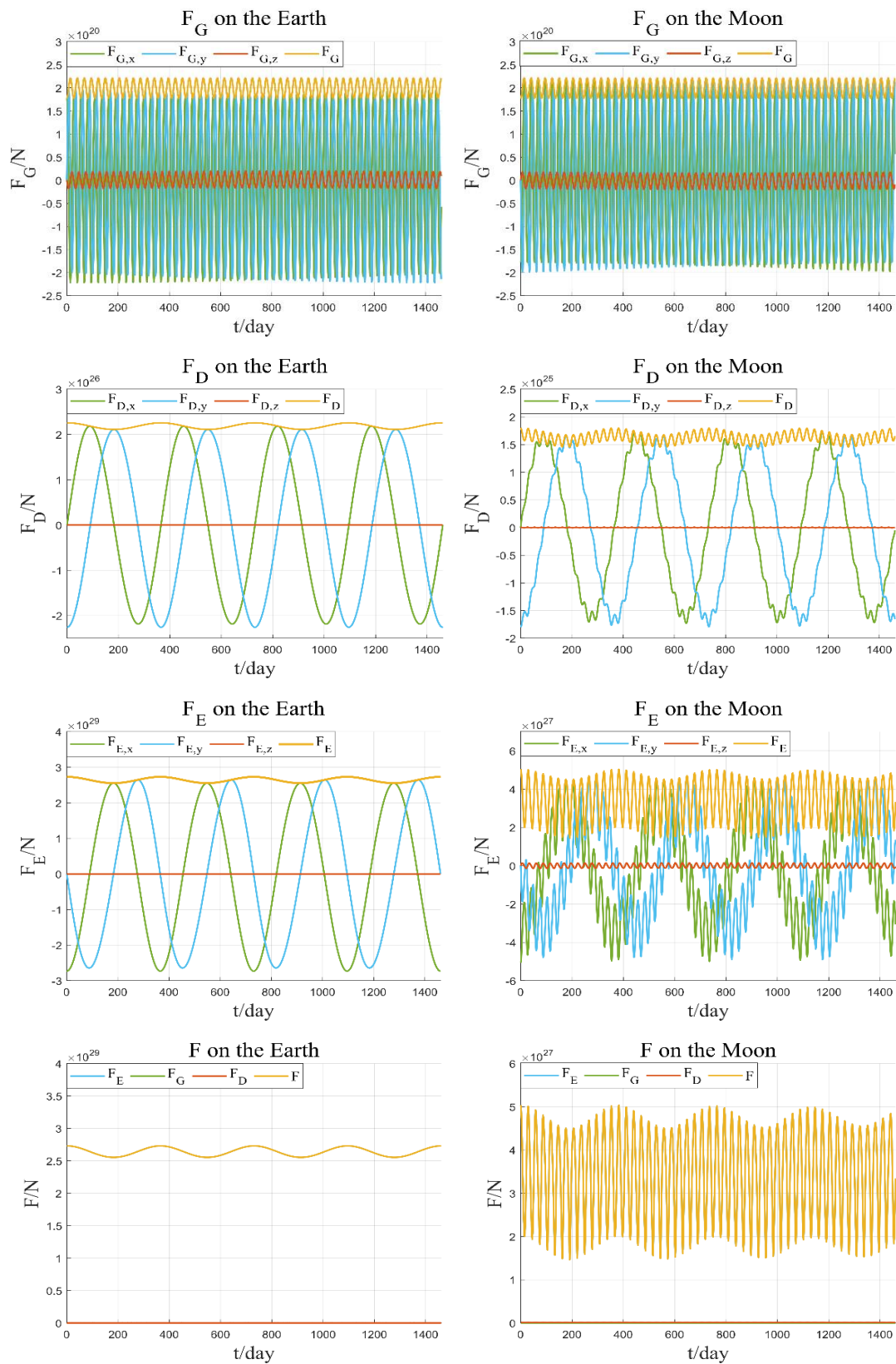


Figure 2. Forces on the earth and the moon in the case of the assumed earth movement and moon movement under no perturbation.

After Perturbation: The Measured Moon Trajectories

This section shows the results of the observed moon trajectories influenced by the following factors. Factors 1-2 are related to the coordinate properties. Factors 3-6 are related to the perturbations in the movement of the earth. Factors 7-8 are related to the perturbations in other parameters.

Factor 1: Coordinate;

Factor 2: (λ, ξ) (i.e., the human observer's location on the earth);

Factor 3: ω_1 (i.e., the angular velocity of the earth's rotation around its own axis);

Factor 4: ω_0 (i.e., the angular velocity of the earth's revolution around the sun);

Factor 5: A (i.e., the length of the long semi-axis of the earth's orbit around the sun);

Factor 6: E (i.e., the eccentricity of the earth's orbit around the sun);

Factor 7: m_m (i.e., the moon's mass);

Factor 8: k (i.e., the coefficient of the drag force).

The closed set of ordinary differential Equation (14) is solved by the ode45 function in MATLAB, which is a kind of Runge-Kutta method, where 4th and 5th order methods are used to provide candidate solutions and control errors respectively with the step size of roughly $1.0007 \cdot 10^{-4}$ sidereal day and within the time span of one sidereal year and four sidereal years. We discuss the measured moon trajectories under the disturbance of different parameters of the motion of the earth and other parameters. In each case, one parameter is altered while the others are the same with those relevant to the initially assumed movement in Section 2.5 in Table 2. To avoid too many overlapping lines interfering the observation of patterns, the results are plotted with the interval of 2911 trajectories.

We use d and d' to denote the observer-moon distance before perturbation and after perturbation respectively. We use the dimensionless quantity $C_1 = (d' - d)/d$ to describe the relative change in the observer-moon distance. We define the divergence in d as the large deviation of the observer-moon distance under perturbation from the observer-moon distance in the previously assumed trajectory. We define "radial" as the rectilinear direction from the observer to the moon. Also, if d is found to diverge because of perturbations, the patterns of the active forces on the moon to maintain the previously observed trajectory are investigated. We use \vec{F}_E' and \vec{F}_E to describe the active forces on the moon before perturbation and the active forces on the moon to maintain the previously observed trajectory after perturbation respectively. Since at this time there is no method to distinguish between $\vec{F}_{E,m}^m$ (i.e., the active force generated by the moon itself) and $\vec{F}_{E,m}^e$ (i.e., the active force generated by the earth), we call the vector sum of them \vec{F}_E (i.e., the active forces on the moon). We use the dimensionless quantity $C_2 = (F_E' - F_E)/F_E$ to describe the relative change in the active forces on the moon.

The descriptions and analyses of the results are mainly structured as follows:

- (a) The descriptions of the phenomena.
- (b) The explanations of the phenomena according to the calculation process in this study.
- (c) What conclusion will a human observer arrive at when he analyzes the motion of the moon in $O_h - X_h Y_h Z_h$ using classical mechanics theories and if such a conclusion is consistent with the results in this study.
- (d) If contradictions exist, where do they come from?
- (e) If d is found to diverge because of perturbation, how should the active forces on the moon be adjusted so that the moon can maintain the previously observed trajectory?

Factor 1: Coordinate.

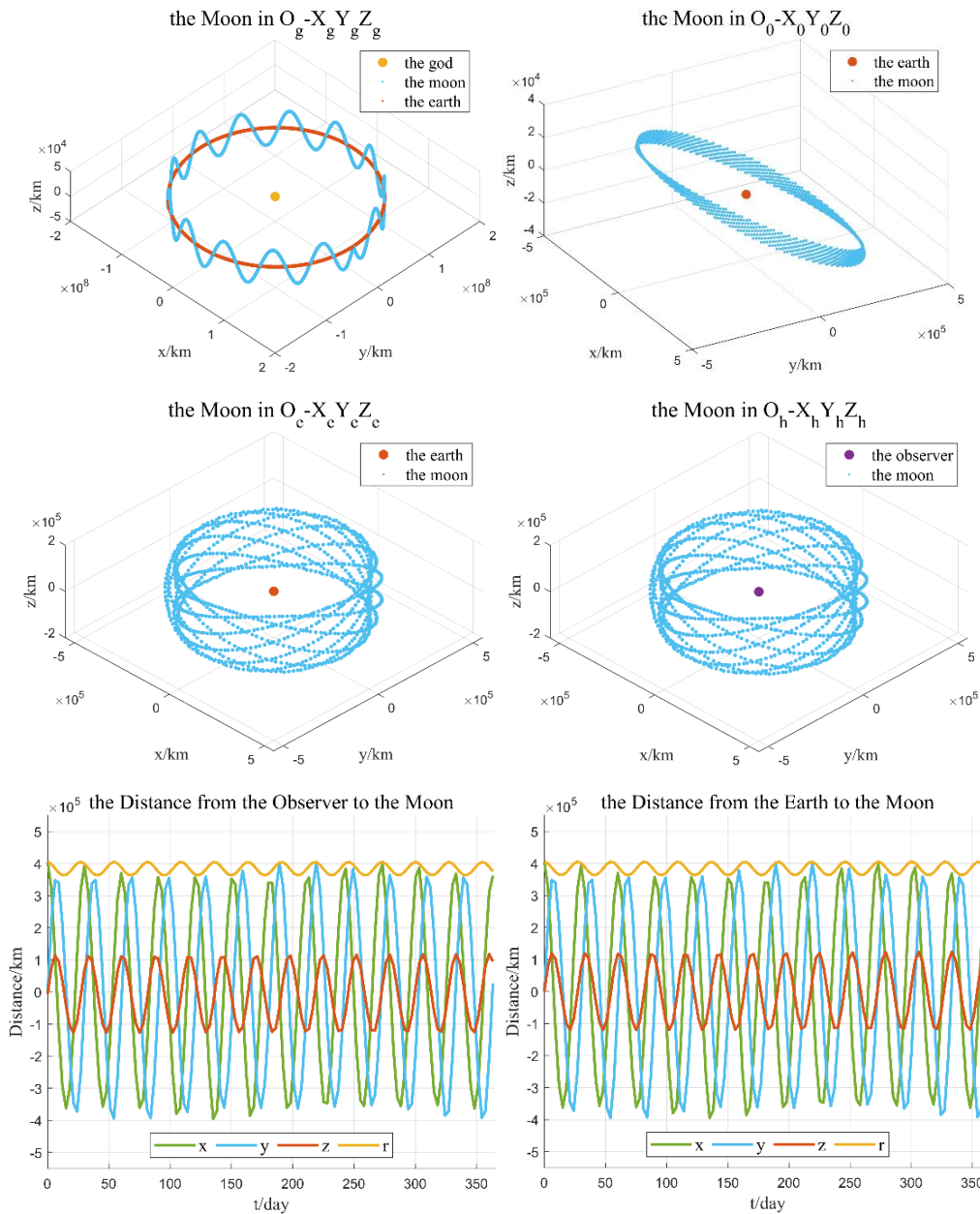


Figure 3. The moon trajectories in different coordinate systems under no perturbation.

Figure 3 shows the moon trajectories in $O_g - X_g Y_g Z_g$, $O_0 - X_0 Y_0 Z_0$, $O_e - X_e Y_e Z_e$, and $O_h - X_h Y_h Z_h$ when no parameter is perturbed. There is huge discrepancy between the moon trajectory observed from the god and the moon trajectory observed from the human observer fixed on the surface of the earth, and the measured moon trajectory under perturbation in $O_h - X_h Y_h Z_h$ is the main topic to be investigated. Also, the observer-moon distance and the earth-moon distance are plotted, which are very close because r_e is smaller than d by two orders of magnitude. Additionally, d fluctuates in the period a little shorter than one sidereal month because lunar perigee precession is considered. The local minimum and maximum indicate the moon's position at the perigee and the apogee respectively.

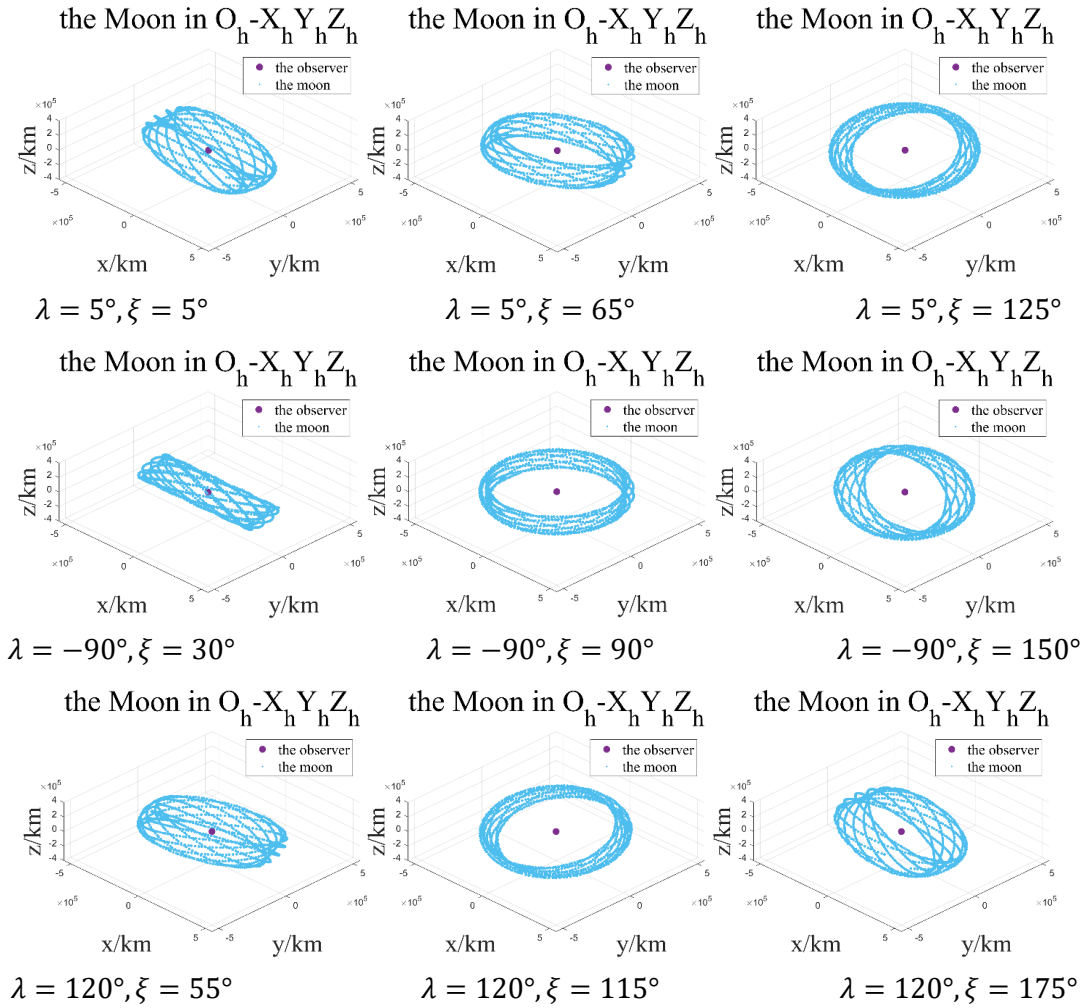
Factor 2: (λ, ξ) (i.e., the human observer's location on the earth).

Figure 4. The measured moon trajectories influenced by (λ, ξ) .

As can be seen from Figure 4, the moon trajectories in $O_h - X_h Y_h Z_h$ are different when the human observer's locations on the earth are different. The measured moon trajectories are observed to rotate as a whole, but there is no noticeable divergence in d or change in periodicity.

Analysis: In the whole calculation, (λ, ξ) exists in \overline{R}_h^e and $[C_1^h]$. In other words, it does not affect the moon trajectory in $O_e - X_e Y_e Z_e$ or $O_g - X_g Y_g Z_g$ and it only affects the transformation relationship between $O_h - X_h Y_h Z_h$ and $O_e - X_e Y_e Z_e$ (i.e., \overline{R}_h^e and $[C_1^h]$ indicate the transitional and rotational transformation respectively). Since r_e is smaller than d by two orders of magnitude, $[C_1^h]$ has a much more noticeable influence than \overline{R}_h^e . In addition, $[C_1^h]$ is independent of time because the human observer is assumed to be fixed on the earth surface. Therefore, the different moon trajectories observed from $O_h - X_h Y_h Z_h$ fixed on the human observers in different locations on the earth surface can be approximately transformed into each other by time-independent rotation. In the following we discuss cases where $O_h - X_h Y_h Z_h$ is fixed on the human observer located at $(\lambda = -0.5\pi, \xi = 0.5\pi)$ on the earth surface.

Factor 3: ω_1 (i.e., the angular velocity of the earth's rotation around its own axis).

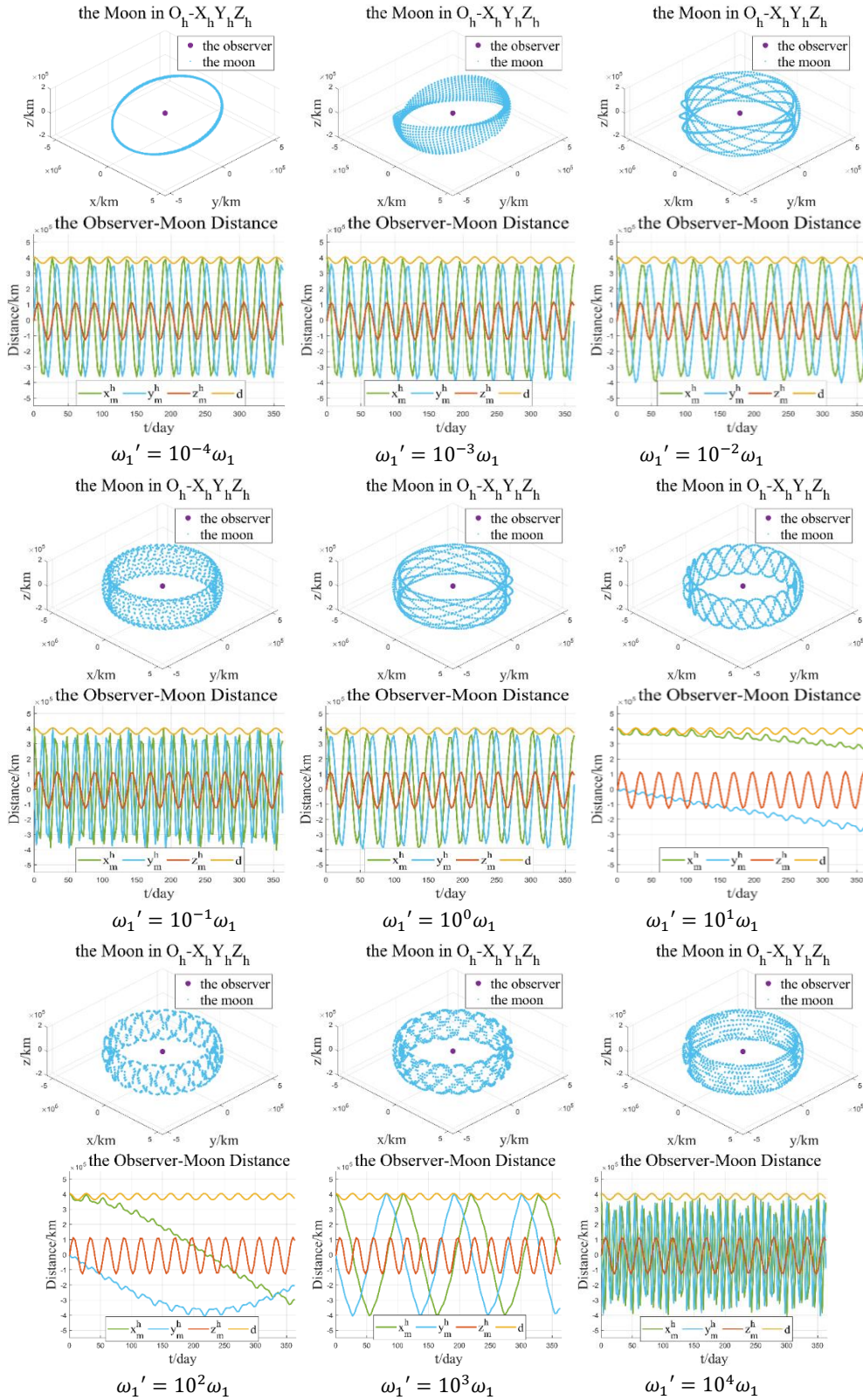


Figure 5. The measured moon trajectories under the perturbations in ω_1 (one year).

Figure 5 illustrates the measured moon trajectories in $O_h - X_h Y_h Z_h$ influenced by the perturbations in ω_1 (i.e., $\omega_1'/\omega_1 = 10^{-4}, 10^{-3}, 10^{-2}, 10^{-1}, 10^1, 10^2, 10^3, 10^4$) in one sidereal year, which show different patterns when ω_1 changes. However, there is no divergence in d even when ω_1 alters drastically.

Analysis: In the whole calculation, ω_1 only exists in $[C_0^e]$. The frequency of rotational transformation changes with ω_1 , which results in the different patterns of moon trajectories observed in $O_h - X_h Y_h Z_h$ when the moon positions are plotted with a fixed interval. However, d does not diverge no matter how drastic the change in ω_1 is. This is because ω_1 does not affect the solution of the moon trajectory in $O_g - X_g Y_g Z_g$ and only affects how this trajectory is transformed to $O_h - X_h Y_h Z_h$. Once the moon trajectory in $O_g - X_g Y_g Z_g$ is solved, considering that the earth's revolution around the sun is not perturbed, d is already determined (note that r_e is smaller than d by two orders of magnitude and can be neglected). Therefore, when ω_1 is perturbed, the divergence in d does not occur even if ω_1 largely alters.

However, a human observer on the surface of the earth may well come to the opposite conclusion when analyzing the motion of the moon in $O_h - X_h Y_h Z_h$ based on certain classical mechanics explanations. To be more specific, since $O_h - X_h Y_h Z_h$ is a non-inertial frame of reference, inertial forces should be exerted on the moon. As is shown in Equation (15), both the centrifugal force and the Coriolis force are strongly related to ω_1 (note that ω_1 is about 365 times larger than ω_0). Also, the radial component of the inertia forces dominates. Therefore, the human observer will conclude that drastic changes in ω_1 inevitably bring divergence in d .

$$\begin{aligned} \overrightarrow{F_{centrifugal}} &= -m_m \overrightarrow{a_{entrainment}} = -m_m (\overrightarrow{\omega_0} + \overrightarrow{\omega_1}) \times [(\overrightarrow{\omega_0} + \overrightarrow{\omega_1}) \times \overrightarrow{R_m^h}] \\ \overrightarrow{F_{Coriolis}} &= -m_m \overrightarrow{a_{Coriolis}} = -2m_m (\overrightarrow{\omega_0} + \overrightarrow{\omega_1}) \times \frac{d\overrightarrow{R_m^h}}{dt} \end{aligned} \quad (15)$$

When considering the influence of the perturbation in ω_1 on d , why does the human observer on the earth come to the conclusion contradictory to that in Figure 5? The essential reason is the information gap between the god and the human observer. More specifically, the results in this study have certain traits of the god perspective by assuming a fixed earth movement and a fixed moon movement in $O_g - X_g Y_g Z_g$. From the perspective of the god, he knows that there is no perturbation in the earth's revolution around the sun and the moon's revolution around the earth, from which he already knows the divergence in d is completely independent of the perturbation in ω_1 . However, the human observer does not know the motion of the moon. What he knows is only the perturbation in ω_1 measured by the changing length in day and night. Thus, when the human observer analyzes the motion of the moon in $O_h - X_h Y_h Z_h$, changing ω_1 brings changing inertia forces on the moon and causes divergence in d .

While this study assumes certain god perspective, the information gap between the god and the human observer does exist, because a human is unable to measure the real movement of his platform (Gödel, 1931). Therefore, the motion of the moon explained by the introduction of inertia forces in classical mechanics may well be inconsistent with real lunar movements. It is noteworthy that we do not aim to exclude all the earth's rotation's influence on the earth-moon distance (e.g., the influence via tidal perturbation). However, in this paper, both the earth and the moon are modelled as rigid bodies and in that case, the above analyses are valid.

Factor 4: ω_0 (i.e., the angular velocity of the earth's revolution around the sun).

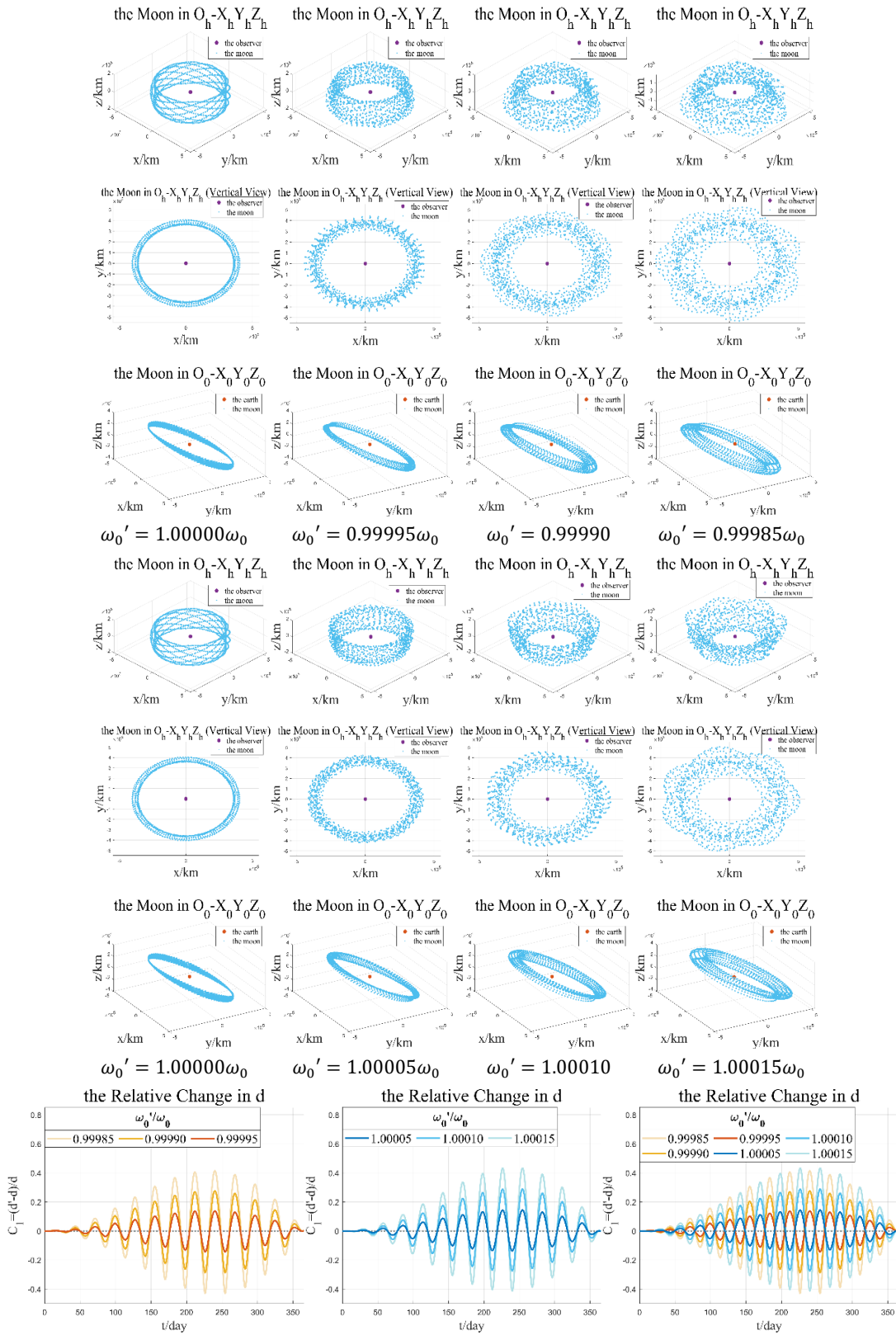


Figure 6. The measured moon trajectories under the perturbations in ω_0 (one year).

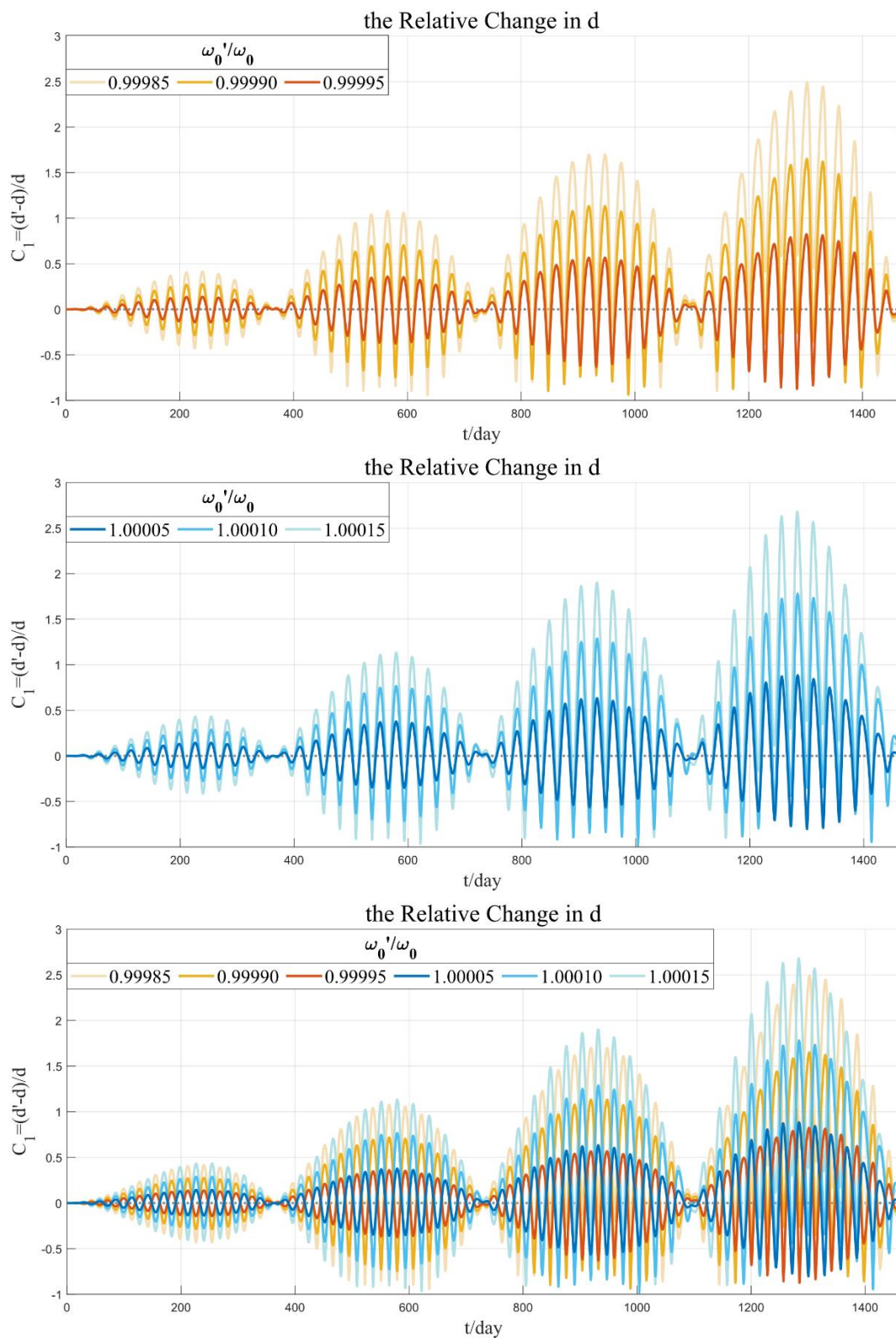


Figure 7. The measured moon trajectories under the perturbations in ω_0 (four years).

Factor 05: A (i.e., the major axis of the earth's elliptic orbit around the sun).

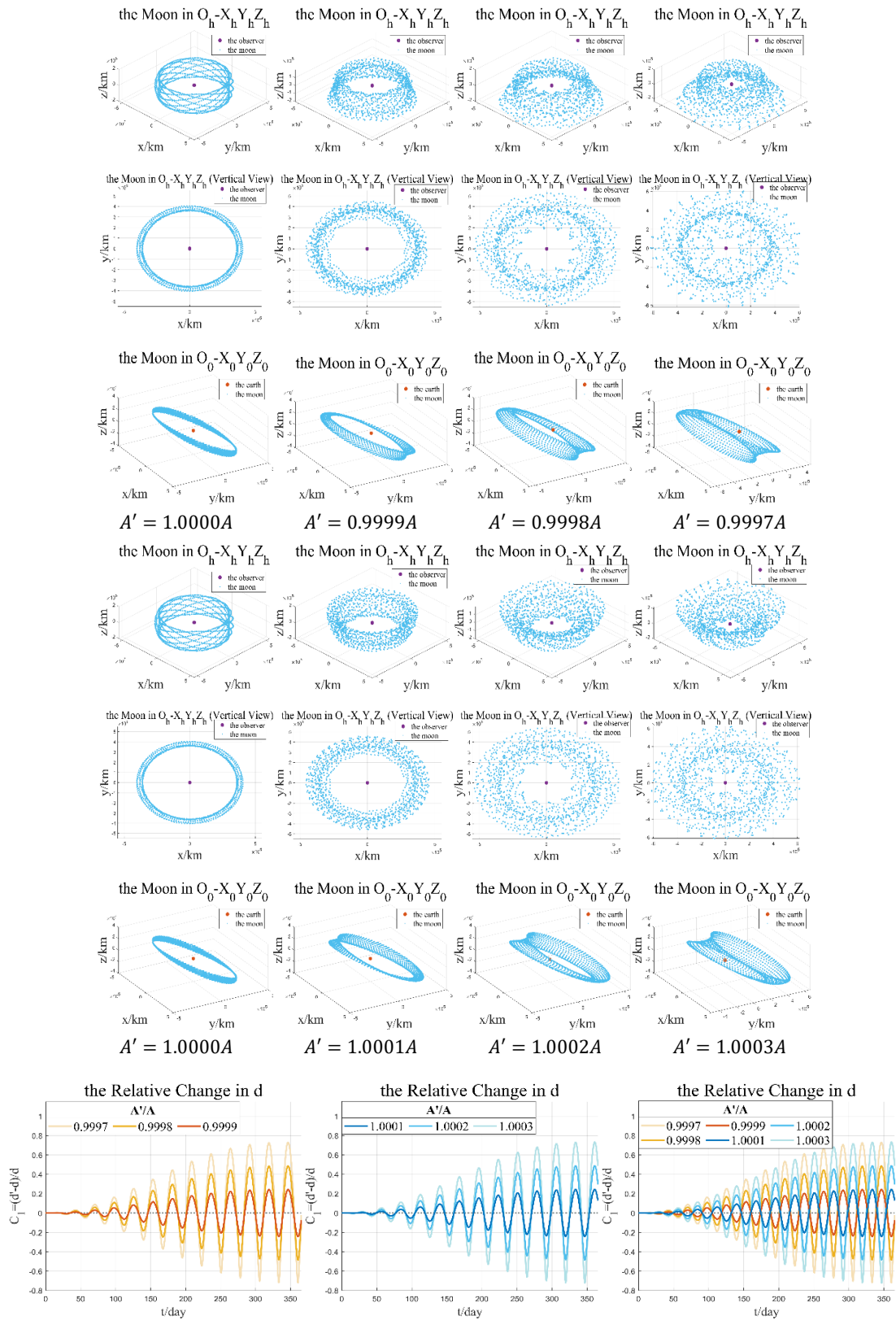


Figure 8. The measured moon trajectories under the perturbations in A (one year).

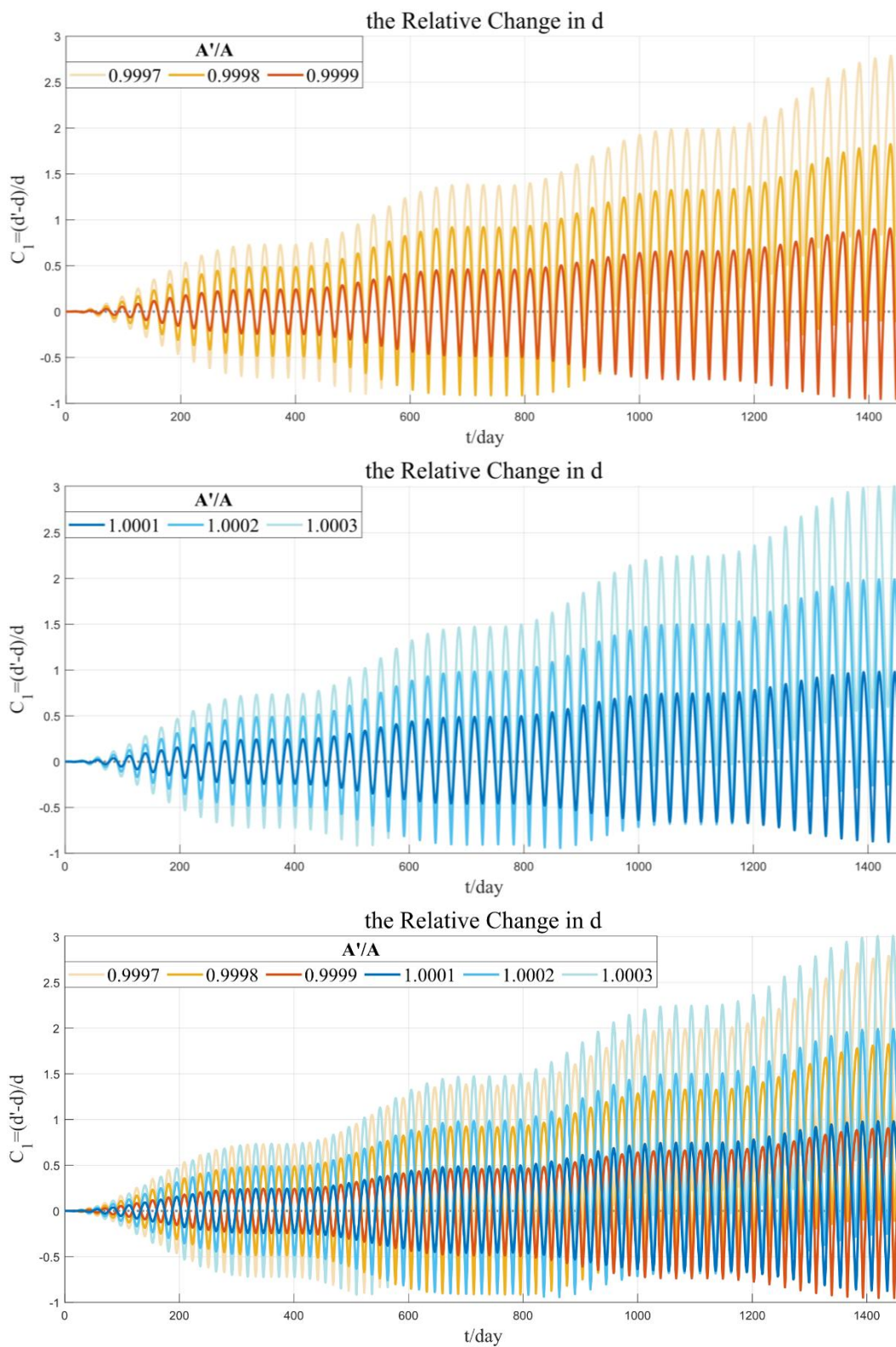


Figure 9. The measured moon trajectories under the perturbations in A (four years).

Factor 6: E (i.e., the eccentricity of the earth's orbit around the sun).

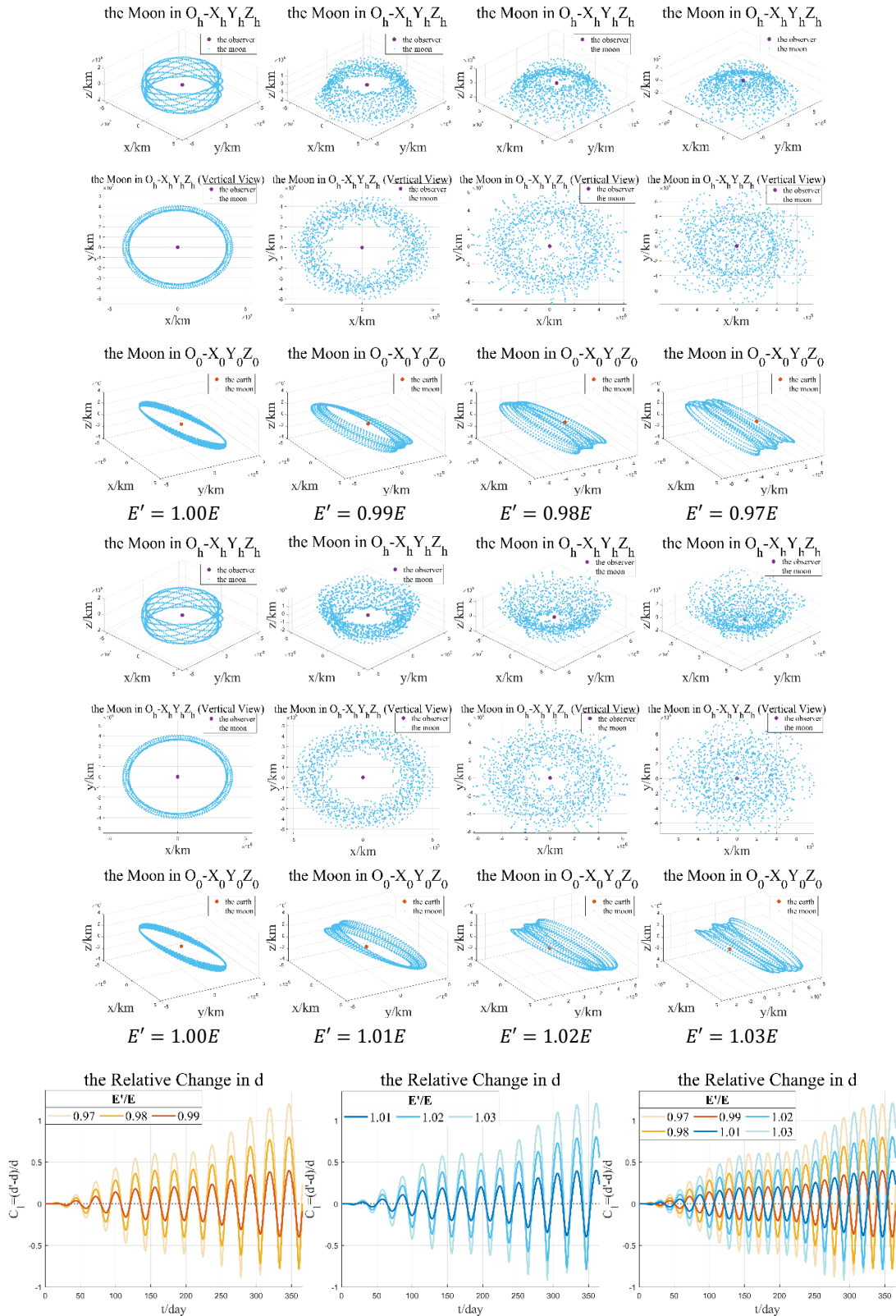


Figure 10. The measured moon trajectories under the perturbations in E (one year).

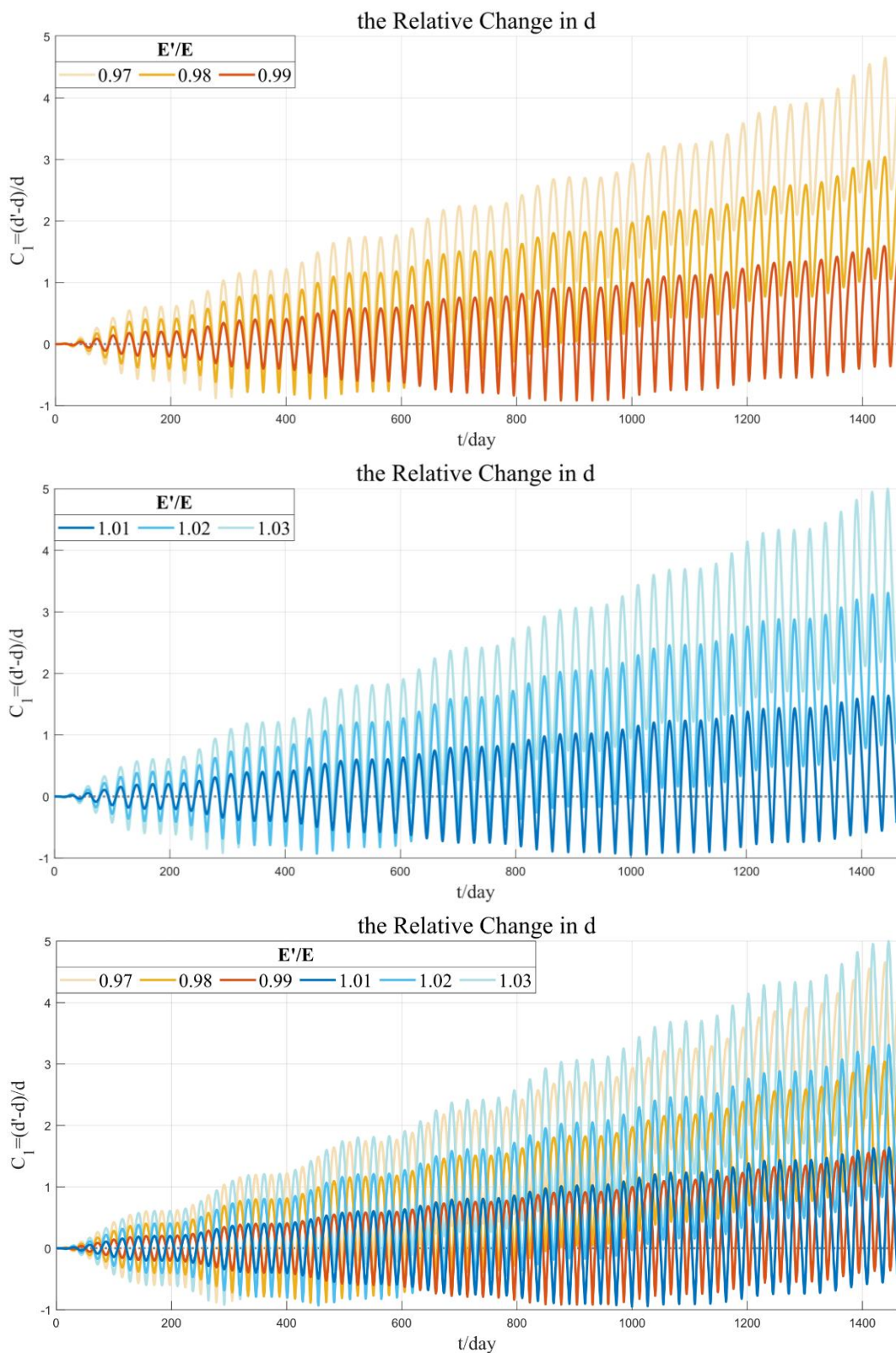


Figure 11. The measured moon trajectories under the perturbations in E (four years).

The moon trajectories influenced by the perturbations in ω_0 , A , and E have similar patterns. So, they are described and analyzed together.

Figure 6 and Figure 7 illustrate the measured moon trajectories in $O_h - X_h Y_h Z_h$ influenced by the perturbations in ω_0 (i.e., $\omega_0'/\omega_0 = 0.99985, 0.99990, 0.99995, 1.00005, 1.00010, \text{ and } 1.00015$) in one sidereal year and four sidereal years respectively. Figure 8 and Figure 9 illustrate the measured moon trajectories in $O_h - X_h Y_h Z_h$ influenced by the perturbations in A (i.e., $A'/A = 0.9997, 0.9998, 0.9999, 1.0001, 1.0002, \text{ and } 1.0003$) in one sidereal year and four sidereal years respectively. Figure 10 and Figure 11 illustrate the measured moon trajectories in $O_h - X_h Y_h Z_h$ influenced by the perturbations in E (i.e., $E'/E = 0.97, 0.98, 0.99, 1.01, 1.02, \text{ and } 1.03$) in one sidereal year and four sidereal years respectively.

When ω_0 is perturbed, C_1 fluctuates in constant frequency. In addition, there is no phase difference between the C_1 related to the perturbations of the same sign, while there is a phase difference of π between the C_1 related to positive and negative perturbations. Similar patterns of frequency and phase difference can be found when A or E is perturbed. Also, the perturbation of larger absolute value causes the C_1 of larger absolute value at the same time point.

To describe other fluctuation patterns of C_1 , we use D_1 to denote the amplitude of fluctuation and B_1 to denote the value around which C_1 fluctuates. In the first sidereal year, the B_1 related to the perturbations in A , E , or ω_0 remains zero. In four sidereal years, the B_1 related to the perturbations in A or E increases monotonically with time. In comparison, the B_1 related to the perturbations in ω_0 rises and falls with time and it turns back to zero after roughly each sidereal year. Such variation grows larger each year, as can be seen from the higher local maximum as time goes by in Figure 7. In the first sidereal year, the D_1 related to the perturbations in A or E increases monotonically. In comparison, the D_1 related to the perturbations in ω_0 rises and falls with time and it turns back to zero at the end of the first sidereal year. In four sidereal years, the D_1 related to the perturbations in A or E initially increases and then reaches a plateau in some cases (e.g., the amplitude of the fluctuation in C_1 becomes stable in the fourth sidereal year when $E'/E = 1.03$). In comparison, the D_1 related to the perturbations in ω_0 rises and falls with time and it turns back to zero after roughly each sidereal year. Such variation in D_1 grows larger each year.

As perturbation magnifies and as time passes, it is more and more prevalent that $D_1 - B_1$ has an increasing positive value, which means that even if the moon oscillates to the locally closest position to the observer (i.e., d reaches the locally minimum value), the observer-moon distance is still further and further away from the initial distance before perturbation. Therefore, it can be concluded that for both positive and negative perturbations in A , E , or ω_0 , if the active forces on the moon remain constant, eventually the moon will move further and further away from the earth, provided that it has never been absorbed into the earth when d fluctuates to very small values (under what circumstances the moon will be absorbed into the earth is not the topic to be investigated in this paper). This is qualitatively consistent with the Lunar Laser Ranging measurement that the moon is receding from the earth at a current rate of roughly 3.8 cm per year (Williams & Boggs, 2016).

Analysis: ω_0 , A , and E affect the position of the earth as well as the gravitational force on the moon. Therefore, their perturbations are relevant to both the solution of the moon trajectory in $O_g - X_g Y_g Z_g$ and how it is transformed to $O_h - X_h Y_h Z_h$. C_1 alternates between positive and negative values. This is because after perturbation, the relative position between the earth and the moon changes while the active forces on the moon remain the same as those before perturbation, which means that some prior attractive forces may well turn into

repulsive forces. For instance, at t_0 before perturbation, the position vector from the moon to earth is along the negative direction of $O_g X_g$, and the active force on the moon attracting it to the earth is also along the negative direction of $O_g X_g$. In contrast, at t_0 after perturbation, the position vector from the moon to the earth may be along the positive direction of $O_g X_g$. However, the constant active force on the moon is still along the negative direction of $O_g X_g$, and as a result it now plays the role of repelling the moon from the earth. Likewise, some prior repulsive forces may well turn into attractive forces; thus, we can observe the fluctuations of C_1 around 0. Therefore, it is incorrect to think the unadjusted active forces as the kind of interaction that always pull the moon back from deviations (i.e., unadjusted active forces can also magnify the deviations). In comparison, inertia force is defined as the force that keeps an object's original state of movement (Butto, 2021). Therefore, the active forces (even those unadjusted ones) have much more complicated impact than the inertia forces in classical mechanics and have more potential to explain the various phenomena of the motion of celestial bodies.

When certain parameter of the movement of the earth is perturbed (i.e., ω_0 , A , or E), the observed moon trajectory in $O_h - X_h Y_h Z_h$ should diverge, as can be seen in Figures 6-11. However, for a human observer on the earth, he may detect zero perturbation in the movement of the earth. As we emphasized before, the human observer is unable to know the real movement of his platform (Gödel, 1931), let alone the real perturbations of this platform. Therefore, when the human observer analyzes the motion of the moon in $O_h - X_h Y_h Z_h$ using the knowledge of classical mechanics, nothing changes (e.g., the inertial forces on the moon remain the same) and he is likely to conclude that d will not diverge. The contradictions between the conclusions of the human observer (i.e., the stable lunar trajectory) and the real phenomena (i.e., the divergence in d) also arise from information gap (i.e., the human observer's unawareness of the perturbations). This argument again demonstrates that the inertia forces in classical mechanics may explain the phenomena contradictory to the reality.

Even if the current measurements are well aligned with the conclusions derived by introducing inertia forces in classical mechanics (i.e., the measured moon trajectory is stable), this is not necessarily because of the correctness of the classical mechanics theory but because people have neglected two crucial aspects. Firstly, people have failed to detect certain perturbations of the earth's movement. Secondly, the moon has always been modelled as a lifeless object and the active forces generated by it have never been considered in previous studies. In fact, the reality may well be that the active forces on the moon are constantly adjusted to maintain a stable trajectory, so that the moon does not rapidly move away from the earth because of certain perturbations in the movement of the earth which people fail to detect. These active forces can easily explain the stability phenomena we have observed in the environmental changes and under the condition of energy dissipation due to drag. This self-adjusting or self-adapting ability is similar to the ability we human beings or any living creatures have. Similar as other living creatures, the moon's ability to maintain the stable state is limited and has upper and lower bounds. For example, a moon may change its trajectory if we let a satellite collide on it similar to the Double Asteroid Redirection Test (DART) carried out by NASA on 27th September, 2022.

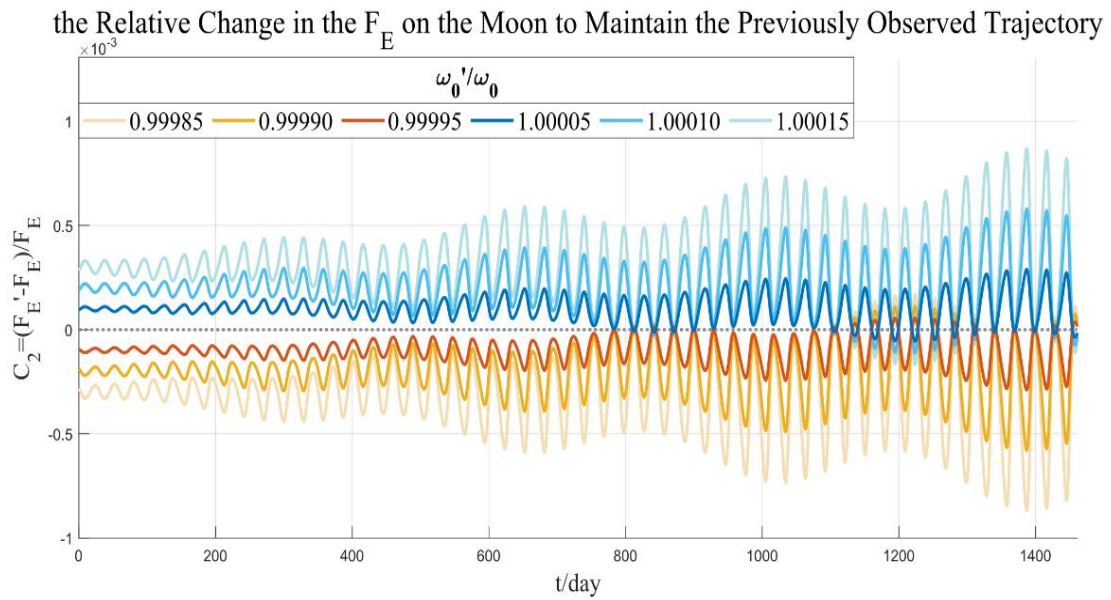


Figure 12. The relative change in the active forces on the moon to maintain the previously observed trajectory when ω_0 is perturbed (four years).

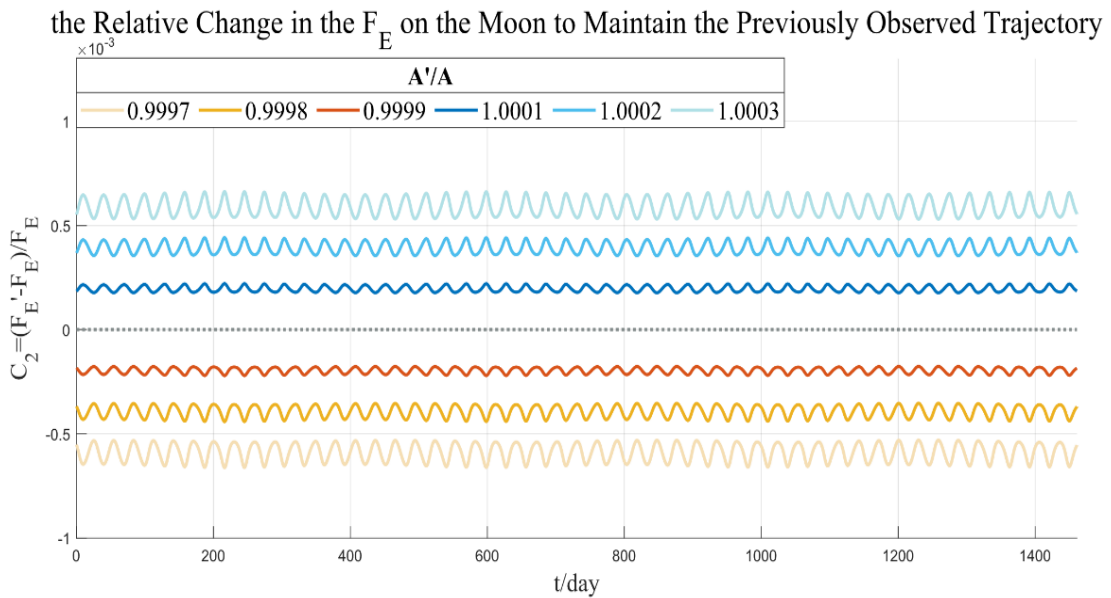


Figure 13. The relative change in the active forces on the moon to maintain the previously observed trajectory when A is perturbed (four years).

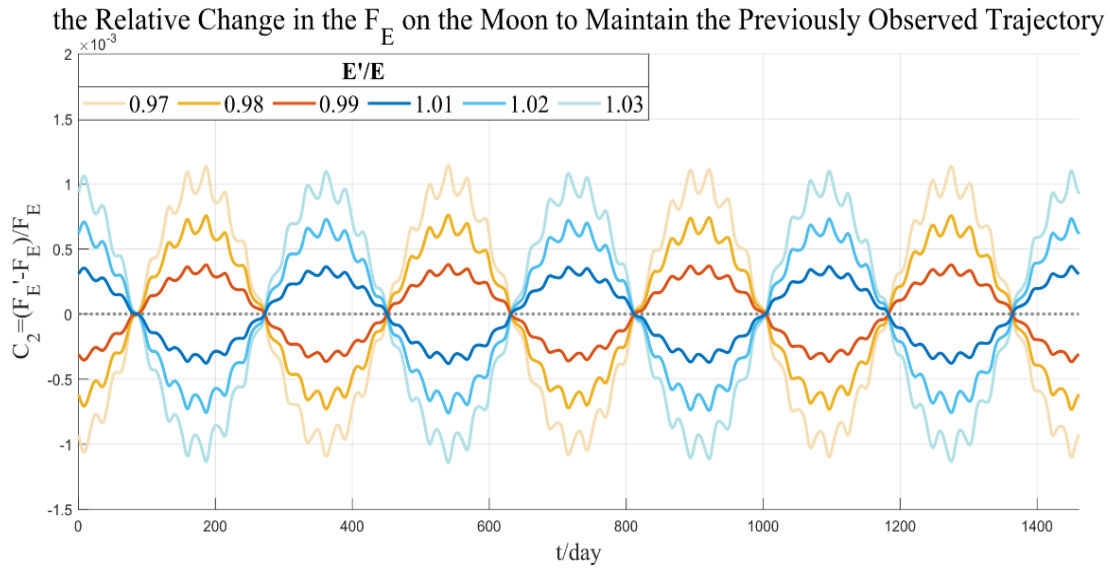


Figure 14. The relative change in the active forces on the moon to maintain the previously observed trajectory when E is perturbed (four years).

If the moon trajectory observed in $O_h - X_h Y_h Z_h$ is in fact stable, how the active forces on the moon should be adjusted so that the moon can maintain the previously observed trajectory when ω_0 , A , or E is perturbed is shown in Figure 12, Figure 13, and Figure 14 respectively. When ω_0 is positively or negatively perturbed, C_2 fluctuates around an accordingly positive or negative value in an amplitude increasing with time. When A is positively or negatively perturbed, C_2 fluctuates around an accordingly positive or negative value in a time-independent amplitude. When E is perturbed, C_2 fluctuates around 0 in a time-independent amplitude. In all cases, there is no phase difference between the C_2 related to the perturbations of the same sign, while there is a phase difference of π between the C_2 related to positive and negative perturbations. Larger perturbations cause larger fluctuation amplitudes.

While previously we do not know how to distinguish between $\overrightarrow{F_{E,m}^m}$ (i.e., the active force on the moon generated by the moon itself) and $\overrightarrow{F_{E,m}^e}$ (i.e., the active force on the moon generated by the earth), here they can be identified separately via the following analysis. As is in Equation (16), F_E' depends on both F_E and $C_2 \cdot F_E$. F_E depends on the moon's previous movement or what trajectory the moon wants to keep (which is calculated in analytic form in Section 2.5) and can be interpreted as $\overrightarrow{F_{E,m}^m}$. $C_2 \cdot F_E$ depends on F_E as well as the perturbation in the movement of the earth and can be interpreted as $\overrightarrow{F_{E,m}^e}$. It is noteworthy that $\overrightarrow{F_{E,m}^e}$ depends on both the previous motion of the moon and earth movement perturbations; therefore $\overrightarrow{F_{E,m}^e}$ is realized by the entanglement of the earth's mind and the moon's mind via psychic field according to NGST (Pan & Cui, 2021a; 2021b; Cui, 2021a; 2021b). The generation mechanism of the active forces provided by NGST can well explain all the phenomena we have observed about the moon's movement (i.e., the provider of $\overrightarrow{F_{E,m}^m}$ is the moon and the provider of $\overrightarrow{F_{E,m}^e}$ is the earth). In contrast, there exist no reasonable explanation of the providers of the centrifugal force and self-rotating moment along its own axis in classical mechanics.

$$F_E' = F_E + C_2 \cdot F_E \tag{16}$$

Factor 7: m_m (i.e., the moon's mass).

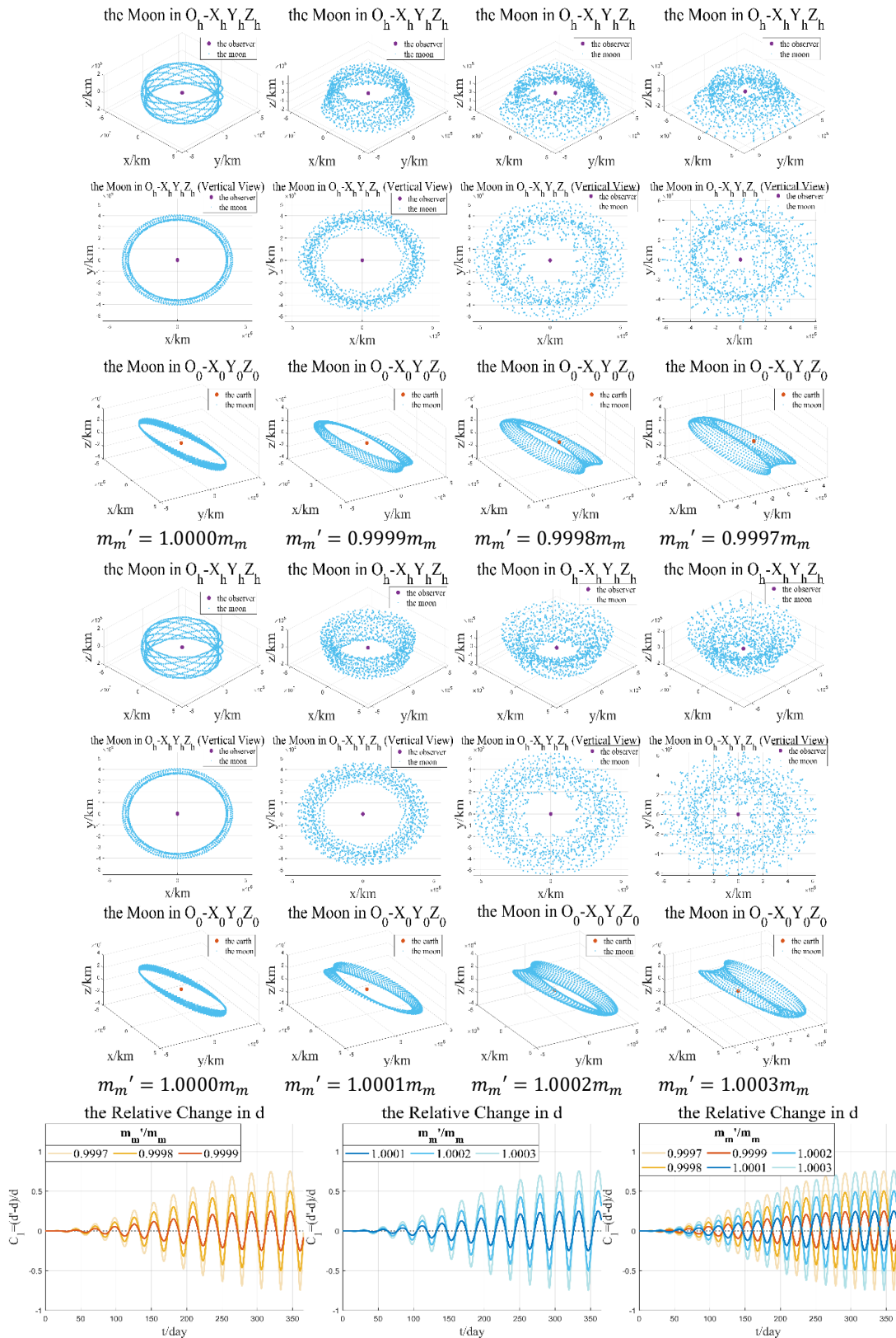


Figure 15. The measured moon trajectories under the perturbations in m_m (one year).

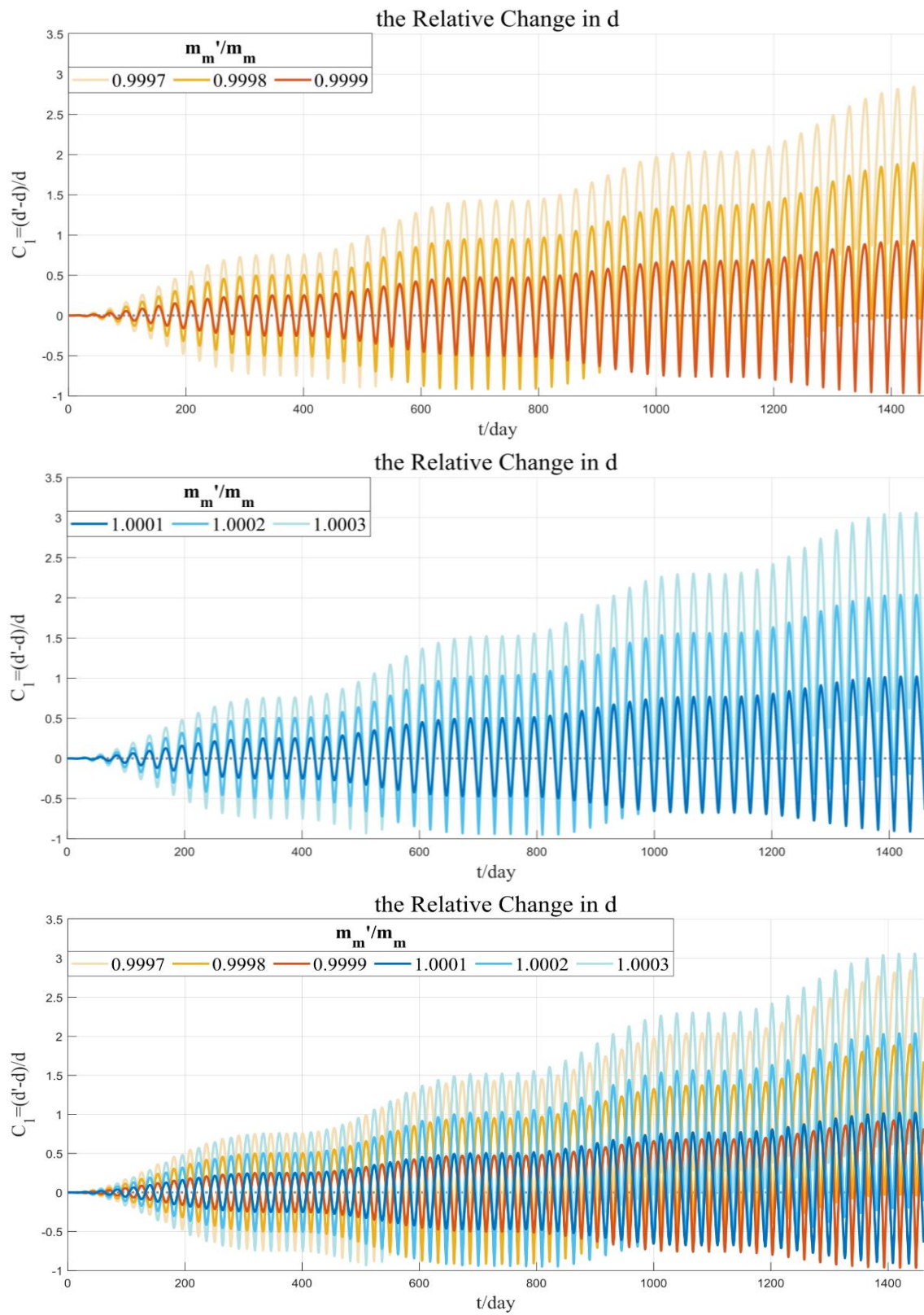


Figure 16. The measured moon trajectories under the perturbations in m_m (four years).

Factor 8: k (i.e., the coefficient of the drag force).

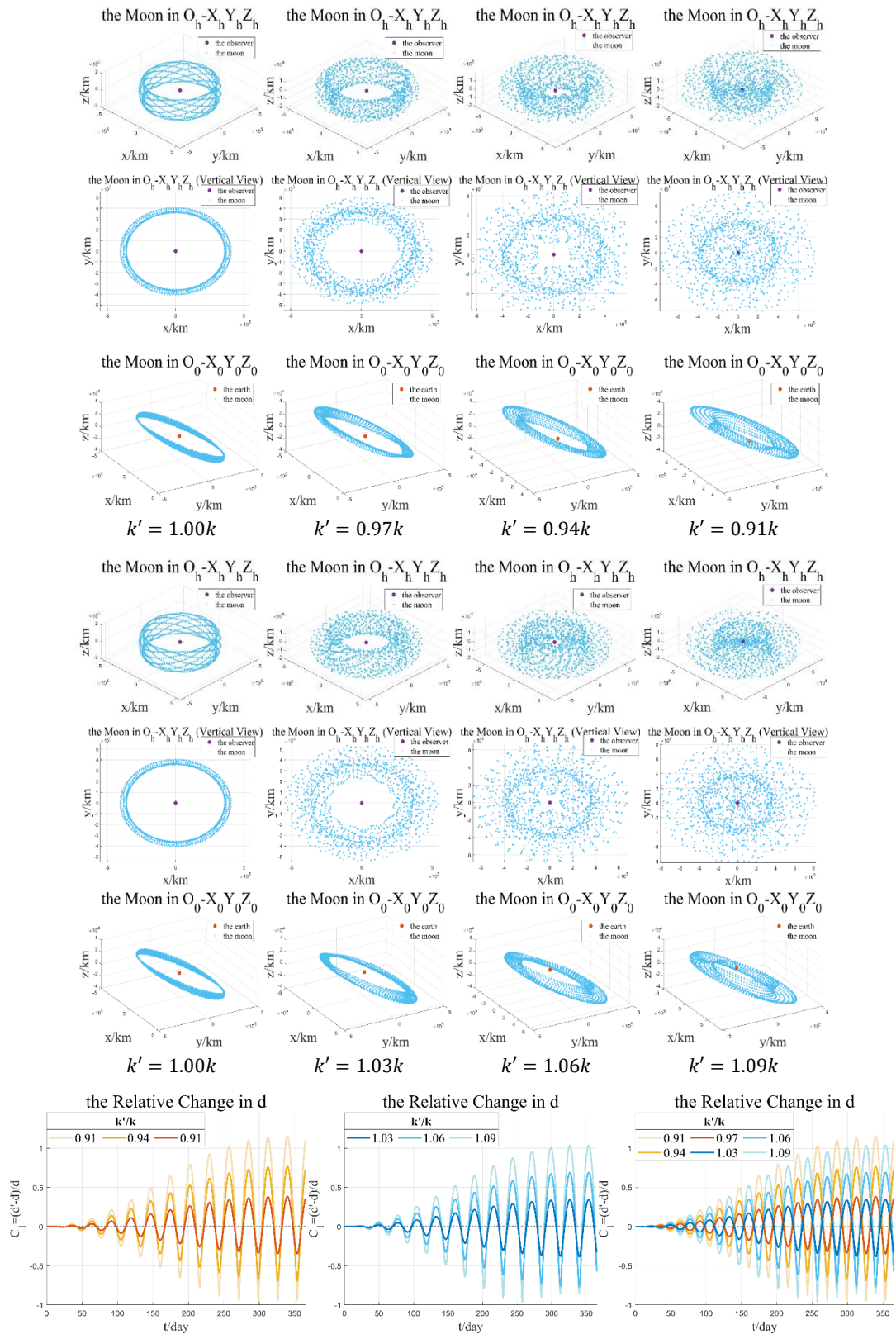


Figure 17. The measured moon trajectories under the perturbations in k (one year).

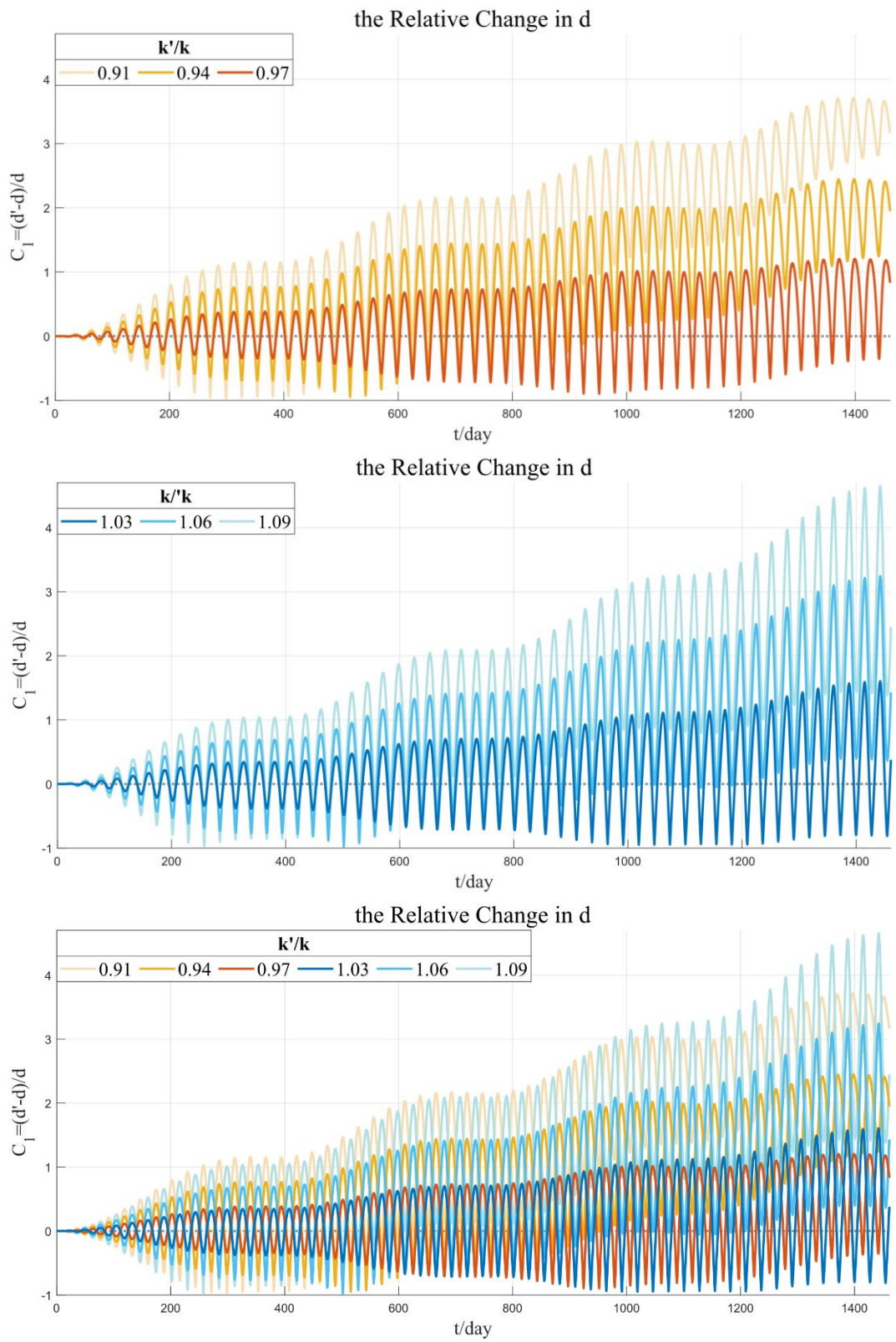


Figure 18. The measured moon trajectories under the perturbations in k (four years).

When parameters other than those related to the movement of the earth are perturbed (i.e., k and m_m), the measured moon trajectory undergoes changes as well. Figure 15 and Figure 16 illustrate the measured moon trajectories in $O_h - X_h Y_h Z_h$ influenced by the perturbations in m_m (i.e., $m_m'/m_m = 0.9997, 0.9998, 0.9999, 1.0001, 1.0002, 1.0003$) in one sidereal year and four sidereal years respectively. Figure 17 and Figure 18 illustrate the measured moon trajectories in $O_h - X_h Y_h Z_h$ influenced by the perturbations in k (i.e., $k'/k = 0.9997, 0.9998, 0.9999, 1.0001, 1.0002, 1.0003$) in one sidereal year and four sidereal years respectively.

When m_m or k is perturbed, C_1 fluctuates in constant frequency. In addition, there is no phase difference between the C_1 related to the perturbations of the same sign, while there is a phase difference of π between the C_1 related to positive and negative perturbations. Also, the perturbation of larger absolute value causes the C_1 of larger absolute value at the same time point.

To describe other fluctuation patterns of C_1 , we use D_1 to denote the amplitude of fluctuation and B_1 to denote the value around which C_1 fluctuates. In the first sidereal year, the B_1 related to the perturbations in m_m or k remains zero. In four sidereal years, the B_1 related to the perturbations in m_m or k monotonically increases with time. In the first sidereal year, the D_1 related to the perturbations in m_m or k monotonically increases with time. In comparison, the D_1 related to the perturbations in m_m initially increases and then reaches a plateau in some cases (e.g., the amplitude of the fluctuation in C_1 becomes stable in the fourth sidereal year when $m_m'/m_m = 1.0003$). The D_1 related to the perturbations in k initially increases and then decreases in some cases (e.g., the amplitude of the fluctuation in C_1 rises and falls in the four sidereal years when $k'/k = 0.91$).

As perturbation magnifies and as time passes, it is more and more prevalent that $D_1 - B_1$ has an increasing positive value, which means that even if the moon oscillates to the locally closest position to the observer (i.e., d reaches the locally minimum value), the observer-moon distance is still further and further away from the initial distance before perturbation. Therefore, it can be concluded that for both positive and negative perturbations in m_m or k , if the active forces on the moon remain constant, eventually the moon will move further and further away from the earth, provided that it has never been absorbed into the earth when d fluctuates to very small values (under what circumstances will the moon be absorbed into the earth is not the topic to be investigated in this paper). This is qualitatively consistent with the Lunar Laser Ranging measurement that the moon is receding from the earth at a current rate of roughly 3.8 cm per year (Williams & Boggs, 2016).

Analysis: Similar to the analyses of Factor 4, Factor 5, and Factor 6, the human observer may well be unable to detect the perturbation in m_m or k and is likely to conclude that no divergence in d occur, while the fact is that such perturbation does cause large divergence and the observed stable lunar trajectory is actually achieved by the constantly adjusted active forces on the moon.

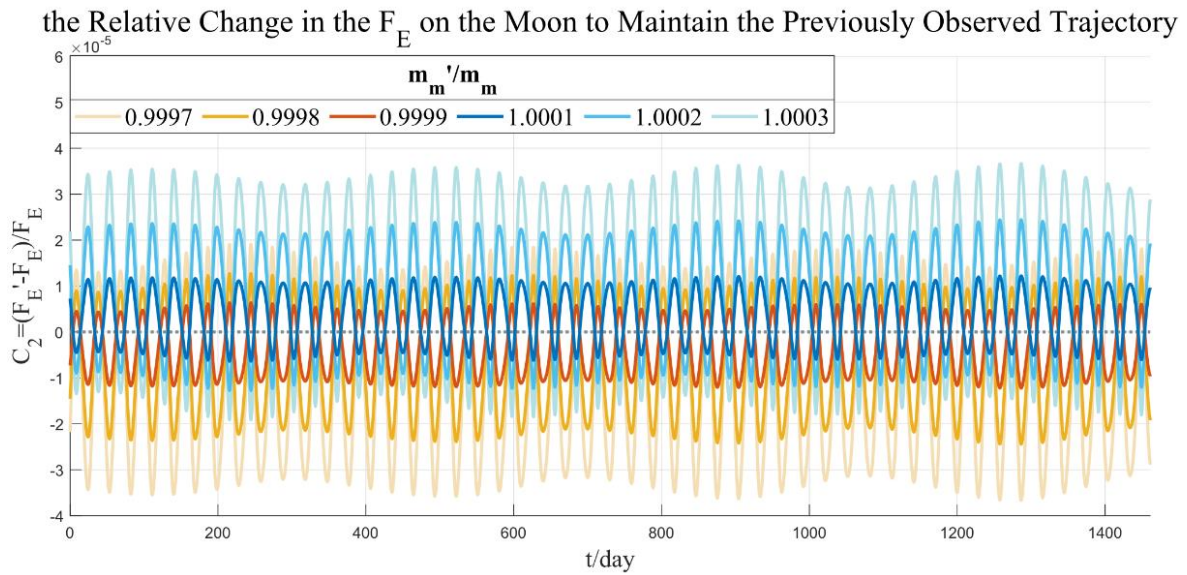


Figure 19. The changes in the active forces on the moon to maintain the previously observed trajectory when m_m is perturbed (four years).

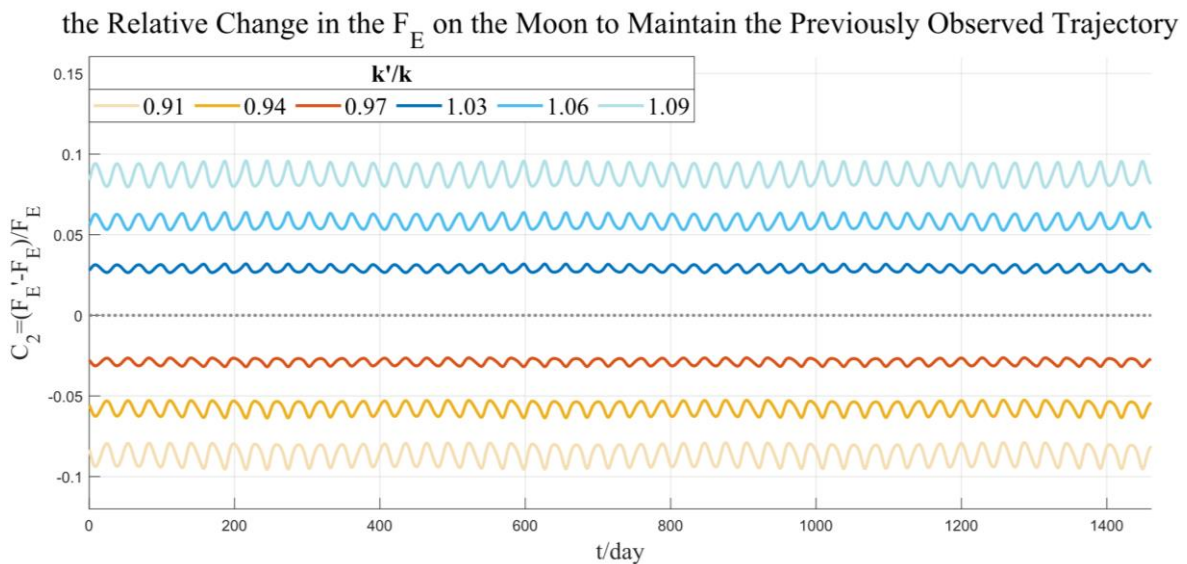


Figure 20. The changes in the active forces on the moon to maintain the previously observed trajectory when k is perturbed (four years).

If the moon trajectory observed in $O_h - X_h Y_h Z_h$ is in fact stable, how the active forces on the moon should be adjusted so that the moon can maintain the previously observed trajectory when m_m or k is perturbed is shown in Figure 19 and Figure 20 respectively.

When m_m is positively or negatively perturbed, C_2 fluctuates around an accordingly positive or negative value in the amplitude that rises and falls with time. When k is positively or negatively perturbed, C_2 fluctuates around an accordingly positive or negative value in a time-independent amplitude. In all cases, there is no phase difference between the C_2 related to the perturbations of the same sign, while there is a phase difference of π between the C_2 related to positive and negative perturbations. Larger perturbations cause larger fluctuation amplitudes.

Analysis: If the observed moon trajectory is stable and there are perturbations in m_m or k , the moon should constantly adjust the active forces so that d will not diverge. In fact, the perturbations in m_m and k happen all the time, as can be seen from the research about the lunar surface's exposure to the interplanetary dust particle streams, which causes dust influx to the lunar surface and lunar eject a cloud (Horányi et al., 2015). Since most biologists agree that a creature is alive if it can maintain a stable state in a perturbed environment (Lovelock, 1983), the moon should be modelled as a living body.

This study attributes the moon's ability to maintain observed stable trajectory under perturbation to the moon's ability in adjusting the active forces. More specifically, we use \vec{a}_G , \vec{a}_D , and \vec{a}_E to denote the moon's acceleration caused by the gravitational forces, the drag forces, and the active forces respectively. As can be analyzed from Equation (17), if m_m is negatively perturbed, \vec{a}_D and \vec{a}_G increase, which means that to keep the total acceleration stable, \vec{a}_E should include smaller components in the direction of \vec{a}_D and of \vec{a}_G , and since m_m is smaller, \vec{F}_E should include smaller components in the direction of \vec{a}_D and of \vec{a}_G . Likewise, if m_m is positively perturbed, \vec{F}_E should include larger components in the direction of \vec{a}_D and of \vec{a}_G . In addition, if k is negatively perturbed, \vec{a}_D decreases, which means that to keep the total acceleration stable, \vec{a}_E should include larger component in the direction of \vec{a}_D , and \vec{F}_E should include larger component in the direction of \vec{a}_D . Similarly, if k is positively perturbed, \vec{F}_E should include smaller component in the direction of \vec{a}_D .

$$\frac{d^2 \vec{R}_m^g}{dt^2} = \frac{\vec{F}_G}{m_m} + \frac{\vec{F}_D}{m_m} + \frac{\vec{F}_E}{m_m} = \vec{a}_G + \vec{a}_D + \vec{a}_E \quad (17)$$

Summary and Conclusions

Significance

This study investigates the consequence of giving up the assumption of the existence of an inertial frame and the relevant influence on the measured moon trajectories. The general formulations governing the motion of the moon and the earth are established from the perspective of NGST, where they are modelled as 6-DOF rigid living bodies who can generate active forces and these active forces can act on other objects. The varying patterns of the measured moon trajectories in the human-coordinate system are calculated when the earth movement or other parameters are perturbed, which are well explained by NGST. However, the human observer may come to contradictory conclusions due to unavoidable information gap and the introduction of inertia forces in classical mechanics with hidden flaws. This indicates that some current solutions within the framework of classical mechanics may not reflect the reality. Furthermore, it is demonstrated that to maintain stable measured trajectory, the active forces on the moon should be adjusted constantly so that the divergence in the observer-moon distance caused by perturbation will not happen. The active force generated by the moon and the active force generated by the earth via the entanglement of minds through psychic field can be distinguished. Overall, it is concluded that by introducing the active forces suggested in NGST and giving up the unrealistic inertial frame assumption, classical mechanics has adequate room to explain the intricate phenomena of the motion of the moon.

Limitations

(a) The numerical method to solve the ordinary differential equation requires that the time interval should be no less than approximately 0.0001 day to deliver stable results in a few years. However, the limited storage capability of MATLAB requires that the total number of computation steps should not be very large. This means

that the length of time span is limited to a few years. This also results in the fact that this study cannot provide solutions more accurate than all previous solutions. While the optimization of numerical computation methods is not the objective of this study, it is undoubtedly indispensable to the accurate predictions of the long-time evolution of the earth-moon system.

(b) In Section 2.5, parameters of the earth movement and the moon movement assumed in the god-coordinate system are time-independent, which may not reflect the reality. For instance, the precession of the equinoxes is not considered in this study. It is reasonable to be omitted since the precession period of the equinoxes (i.e., roughly 21,000 years as is in Hays, Imbrie, and Shackleton (1976)) is much longer than the investigated time span (i.e., one sidereal year or four sidereal years). However, if the results of longer time span are to be studied in the future, these factors should be considered.

(c) In Section 2.4, the truncation of the gravitational potential to the second order is reasonable because the observer-moon distance is much larger than the radius of the earth. However, it is shown in Section 3 that the observer-moon distance sometimes decreases under perturbation. In those cases, whether the gravitational potential should be expanded to higher-order terms will be investigated in future work.

(d) The perturbations in different parameters are investigated independently in this study. Future work will investigate the coupled influence of those parameters on the measured moon trajectory (e.g., under what circumstances the influence of the perturbation in ω_0 and the influence of the perturbation in A on the measured moon trajectory can be offset).

Appendix

In the appendix, how the terms of inertia forces in classical mechanics can be derived by certain coordinate transformation in the general formulation is deduced. In this part, for simplicity we neglect the lunar precession as well as the eccentricity of orbits. $\overline{\omega}_0$ is also neglected since its norm is much smaller than that of $\overline{\omega}_1$.

Firstly, we define the new operation \otimes as the operation that transforms a vector in $O_g - X_g Y_g Z_g$ from $O_g - X_g Y_g Z_g$ to $O_e - X_e Y_e Z_e$, as is in Equation (18).

$$\overline{R}_k^g \Big|_{\otimes} \triangleq [C_0^e](\overline{R}_k^g - \overline{R}_e^g) \quad (18)$$

Equation (19) is what we want to demonstrate, that is, the inertia forces can be derived by transforming the moon's acceleration in $O_g - X_g Y_g Z_g$ to $O_e - X_e Y_e Z_e$, subtracting the result by the moon's acceleration in $O_e - X_e Y_e Z_e$, and finally multiplying the result by the mass of the moon. And it can be deduced via the following derivation procedures.

$$\begin{aligned} m_m[(\overline{R}_m^g)'' \Big|_{\otimes} - (\overline{R}_m^e)'] &= m_m(\overline{a}_{entrainment} + \overline{a}_{Crocotus}) = \overline{F}_{Centrifugal} + \overline{F}_{Crocotus} \\ (\overline{R}_m^g)'' \Big|_{\otimes} &= ([C_2^0] \overline{R}_m^g + \overline{R}_e^g)'' \Big|_{\otimes} = [C_0^e] \{ ([C_2^0] \overline{R}_m^g)'' + (\overline{R}_e^g)'' - \overline{R}_e^g \} \\ &= [C_0^e] ([C_2^0] \overline{R}_m^g)'' + [C_0^e] \{ (\overline{R}_e^g)'' - \overline{R}_e^g \} = [C_0^e] ([C_2^0] \overline{R}_m^g)'' + \overline{R}_e^g \Big|_{\otimes} \quad (19) \\ (\overline{R}_m^e)'' &= ([C_0^e] [C_2^0] \overline{R}_m^g)'' \\ (\overline{R}_m^g)'' \Big|_{\otimes} - (\overline{R}_m^e)'' &= \overline{R}_e^g \Big|_{\otimes} + [C_0^e] ([C_2^0] \overline{R}_m^g)'' - ([C_0^e] [C_2^0] \overline{R}_m^g)'' \end{aligned}$$

$$\begin{aligned}
& [C_0^e]([C_2^0]\overline{R_m^2})'' - ([C_0^e][C_2^0]\overline{R_m^2})'' \\
&= \begin{bmatrix} \cos\varphi & \sin\varphi\cos\alpha & -\sin\varphi\sin\alpha \\ -\sin\varphi & \cos\varphi\cos\alpha & -\cos\varphi\sin\alpha \\ 0 & \sin\alpha & \cos\alpha \end{bmatrix} \frac{d^2}{dt^2} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\beta & \sin\beta \\ 0 & -\sin\beta & \cos\beta \end{bmatrix} \begin{bmatrix} r\cos\theta \\ r\sin\theta \\ 0 \end{bmatrix} \\
&- \frac{d^2}{dt^2} \begin{bmatrix} \cos\varphi & \sin\varphi\cos\alpha & -\sin\varphi\sin\alpha \\ -\sin\varphi & \cos\varphi\cos\alpha & -\cos\varphi\sin\alpha \\ 0 & \sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\beta & \sin\beta \\ 0 & -\sin\beta & \cos\beta \end{bmatrix} \begin{bmatrix} r\cos\theta \\ r\sin\theta \\ 0 \end{bmatrix} \\
&= \begin{bmatrix} \varepsilon_1[\sin\varphi\cos\theta - \cos\varphi\sin\theta\cos(\alpha - \beta)] + \omega_1^2[\cos\varphi\cos\theta + \sin\varphi\sin\theta\cos(\alpha - \beta)] + 2\omega_1\omega_2[-\sin\varphi\sin\theta - \cos\varphi\cos\theta\cos(\alpha - \beta)] \\ \varepsilon_1[\cos\varphi\cos\theta + \sin\varphi\sin\theta\cos(\alpha - \beta)] + \omega_1^2[-\sin\varphi\cos\theta + \cos\varphi\sin\theta\cos(\alpha - \beta)] + 2\omega_1\omega_2[-\cos\varphi\sin\theta + \sin\varphi\cos\theta\cos(\alpha - \beta)] \\ 0 \end{bmatrix} \\
&= r \begin{bmatrix} \varepsilon_1[\sin\varphi\cos\theta - \cos\varphi\sin\theta\cos(\alpha - \beta)] \\ \varepsilon_1[\cos\varphi\cos\theta + \sin\varphi\sin\theta\cos(\alpha - \beta)] \\ 0 \end{bmatrix} + r \begin{bmatrix} -\omega_1^2[\cos\varphi\cos\theta + \sin\varphi\sin\theta\cos(\alpha - \beta)] \\ -\omega_1^2[-\sin\varphi\cos\theta + \cos\varphi\sin\theta\cos(\alpha - \beta)] \\ 0 \end{bmatrix} \\
&+ 2r \begin{bmatrix} \omega_1^2[\cos\varphi\cos\theta + \sin\varphi\sin\theta\cos(\alpha - \beta)] - \omega_1\omega_2[\sin\varphi\sin\theta + \cos\varphi\cos\theta\cos(\alpha - \beta)] \\ \omega_1^2[-\sin\varphi\cos\theta + \cos\varphi\sin\theta\cos(\alpha - \beta)] + \omega_1\omega_2[-\cos\varphi\sin\theta + \sin\varphi\cos\theta\cos(\alpha - \beta)] \\ 0 \end{bmatrix} \\
&= \begin{bmatrix} 0 \\ 0 \\ \varepsilon_1 \end{bmatrix} \times r \begin{bmatrix} \cos\varphi\cos\theta + \sin\varphi\sin\theta\cos(\alpha - \beta) \\ -\sin\varphi\cos\theta + \cos\varphi\sin\theta\cos(\alpha - \beta) \\ \sin\theta\cos(\alpha - \beta) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \omega_1 \end{bmatrix} \times \left(\begin{bmatrix} 0 \\ 0 \\ \omega_1 \end{bmatrix} \times r \begin{bmatrix} \cos\varphi\cos\theta + \sin\varphi\sin\theta\cos(\alpha - \beta) \\ -\sin\varphi\cos\theta + \cos\varphi\sin\theta\cos(\alpha - \beta) \\ \sin\theta\cos(\alpha - \beta) \end{bmatrix} \right) \\
&+ 2 \begin{bmatrix} 0 \\ 0 \\ \omega_1 \end{bmatrix} \times r \begin{bmatrix} \omega_1[-\sin\varphi\cos\theta + \cos\varphi\sin\theta\cos(\alpha - \beta)] + \omega_2[-\cos\varphi\sin\theta + \sin\varphi\cos\theta\cos(\alpha - \beta)] \\ \omega_1[-\cos\varphi\cos\theta + \sin\varphi\sin\theta\cos(\alpha - \beta)] + \omega_2[\sin\varphi\sin\theta + \cos\varphi\cos\theta\cos(\alpha - \beta)] \\ 0 \end{bmatrix} \\
&= \overline{\varepsilon_1} \times \overline{R_m^e} + \overline{\omega_1} \times (\overline{\omega_1} \times \overline{R_m^e}) + 2\overline{\omega_1} \times \frac{d\overline{R_m^e}}{dt} \\
\therefore (\overline{R_m^g})'' \Big|_{\odot} - (\overline{R_m^e})'' &= \overline{R_e^g} \Big|_{\odot} + \overline{\varepsilon_1} \times \overline{R_m^e} + \overline{\omega_1} \times (\overline{\omega_1} \times \overline{R_m^e}) + 2\overline{\omega_1} \times \frac{d\overline{R_m^e}}{dt} = \overline{a_{\text{entrainment}}} + \overline{a_{\text{Crocolis}}} \\
\therefore m_m [(\overline{R_m^g})'' \Big|_{\odot} - (\overline{R_m^e})''] &= m_m (\overline{a_{\text{entrainment}}} + \overline{a_{\text{Crocolis}}}) = \overline{F_{\text{Centrifugal}}} + \overline{F_{\text{Crocolis}}}
\end{aligned}$$

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