Journal of Materials Science and Engineering A 12 (4-6) (2022) 55-66

doi: 10.17265/2161-6213/2022.4-6.002



Identification of Empirical Models for the Prediction of the Plasticity Index: Case of the Khô Depression in South Benin

Koffî Judicaël AGBELELE^{1,3}, Yvette Sèdjro TANKPINOU KIKI^{2,3}, Abalo P'KLA⁴, Gbênihon Céleste-Amour KENOUKON⁵

- 1. Normal Higher School of Technical Education (ENSET), National University of Science Technology Engineering and Mathematics (UNSTIM), BP 133, Abomey, Benin
- 2. Higher National Institute of Industrial Technology (INSTI), National University of Science Technology Engineering and Mathematics (UNSTIM), 01 BP 551, Abomey, Benin
- 3. Laboratory of Energy and Applied Mechanics (LEMA), EPAC, UAC, 01 BP 2009 Cotonou, Benin
- 4. Higher National Engineering School (ENSI) of the University of Lomé (UL), BP 1515 Lomé, Togo
- 5. National School of Public Works (ENSTP), National University of Science Technology Engineering and Mathematics (UNSTIM), BP 2282, Abomey, Benin

Abstract: The plasticity index is an essential design parameter used as a standard input in fine-grained soil investigation programs. It is used to estimate the plasticity and physical properties of soils, and indirectly their strength properties. This index is determined from the Atterberg limit tests, starting from the limits of liquidity and plasticity. However, the realization of the test considered as basic and simple, is not so much. The effects of the operator, the calibration of the apparatus and the environmental aspects during the tests affect the reliability and accuracy of the results. In this paper, the objective is to overcome these difficulties by evaluating the plasticity index of clay and loam soils by considering only the values of the liquid limit. Soil samples were collected from 0 to 5 m depth in the localities of the Khô depression in Benin. On these samples, Atterberg limit tests were performed in the laboratory. Using MATLAB's Curve Fitting Toolbox, linear, exponential and power prediction models were analyzed. The results showed that there is indeed a good correlation between the plasticity index and the liquid limit of the soils. For the linear model, it was observed R^2 equal to 0.9891. For the exponential model, R^2 is 0.98871 and for the power model 0.9802. A study of the residual plot validated the models found, as well as comparisons with well-known literature sources. Through the equations obtained, it is now possible to study the plasticity of soils in the Khô depression only from the liquid limit, without determining the plasticity limit.

 W_p

Key words: Plasticity index, liquidity limit, prediction, regression model, Lama depression, Khô depression.

List of Symbols

 W_L

		I_p	Plasticity index (-)
USCS	Unified soil classification system (-)	R^2	Correlation coefficient (-)
CH	High plastic clayey soil (-)	RMSE	Root mean square error (-)
CL	Low to medium plastic clayey soil (-)	TUVISE	reor mean square error ()
ML	Low to medium plastic silty soil (-)	1. Intro	duction
MH	High plastic silty soil (-)		

Corresponding author: Koffi Judicaël AGBELELE, Dr, research fields: geotechnical, materials science, composite materials, road.

Liquid limit (%)

Plasticity is an important characteristic of fine-grained soils. It is a measure of the soils ability to undergo irrecoverable deformation at constant volume without fracture. Plasticity is due to the presence of clay minerals or organic material [1]. The existence of

Plastic limit (%)

clay minerals in fine grained soil medium leads soil to be remolded in the presence of some moisture without crumbling. This nature of behavior is represented by cohesion phenomenon that is caused by the adsorbed water which surrounds the clay particles [2]. Depending on the moisture content, a soil may exist in a liquid, solid or plastic state [1]. The boundary water contents at which the soil undergoes a change from one state to another are called "Consistency Limits". The consistency limits (liquid limit, plastic limit) are related to the amount of water attracted to the surface of the soil particles and are predominant factors for identifying and classifying a soil [3]. If the water content is further reduced, the clay sample changes from the plastic state to the semi-solid state. If the soil does not have plasticity, it becomes brittle [4]. The mentioned natural behavior trend is initially revealed by Atterberg [5] with the use of a conservative method to describe the consistency of fine grained soils with various water contents [6]. The water content of the soil at the point of transition from solid to semisolid state is described as the shrinkage limit, and from semisolid to plastic state is identified as the plastic limit. In addition to these, the water content of the soil at the point of transition from plastic to liquid state is defined the liquid limit [2]. These three boundary water content values are named as Atterberg limits [6]. The Atterberg Limits remain the fundamental classification parameters in geotechnical engineering practice [7].

Relatively simple methods have been proposed to determine the liquid limit; the most important are the Atterberg percussion or Casagrande [8, 9] method and the fall cone penetrometer method. The classical method is described by American Society for Testing and Materials (ASTM) D-4318 [10] and the fall cone method by British Standards BS 1377 [11] and BS 1337:2001 [12]. The Casagrande method relies on the inducement of a slope failure as the cup is "tapped"—the water content when the "canal" fails after 25 blows is the liquid limit [7]. Wroth [13] and

Haigh [14] prove that percussion liquid limits correspond to a fixed ratio of strength to density. The fall cone test, developed in Sweden is a direct measurement of soil shear strength [15]. It is a mechanical test which removes the judgment that is required to determine failure when using the Casagrande cup but has been calibrated to give essentially the same results for soils with low plasticity indices [7]. Koumoto & Houlsby [16] give a detailed theoretical analysis of the mechanics of the fall cone test, showing the test's sensitivity to cone angle, cone bluntness, surface roughness of the cone and cone heave [7]. Several authors have studied correlation of W_L values estimated by the Casagrande and fall cone methods such as Rudolf [17], Sherwood and Ryley [18], Littleton and Farmilo [19], Wires [20], Belviso et al. [21], Sivapullaiah and Sridharan [22], Wasti [23], Leroueil and Le Bihan [24], in the 2000s and more recently, Dragoni et al. [25], Özer [26], Fojtová et al. [27], Grønbech et al. [28, 29], Di Matteo [30], Spagnoli [31], Bicalho et al. [32] and Kollaros and Boorman [33].

The plastic limit of the material is determined by rolling the soil into a thread, on a glass plate, using light finger pressure. The soil is said to have reached its plastic limit when it begins to crumble at a thread diameter of 3 mm [1]. The plasticity index (I_p) is a simple measure of the difference between the plastic limit (W_p) and the liquid limit (W_L) .

Plasticity index indicates the degree of plasticity of a soil [4]. It defines the range of moisture content where the soil exists in a plastic state, i.e. between the liquid limit (the moisture content at which the soil acts as a fluid) and the plastic limit (the moisture content at which the soil starts to behave as a brittle solid). By definition, the plasticity index of a soil is a measure of the affinity to water of its clay mineral constituents [1]. It is widely used to predict the engineering behaviour of a soil, as well as being a measure of the materials compressibility [33] and an input parameter in the determination of the activity index [1, 34].

The measured values for the liquid and plastic limits of soils are commonly used index parameters in geotechnical engineering. They are used to compute the plasticity index which is commonly used to empirically predict many soil properties. For instance, Casagrande's A-Line classifies soils into clays and silts based on a correlation between soil type and a combination of liquid limit and plasticity index [7]. The plasticity index has been used to predict several important parameters in soil mechanics as in road geotechnics. There are works of Skempton [35, 36] on the undrained strength ratios, Stroud [37] on the ratio of strength to SPT (Standard Penetration Test), Schofield & Wroth [38], Nakase et al. [39] on the critical state soil parameters, Black [40] on the CBR (California Bearing Ratio), and Heukelom & Klomp [41] to describe road subgrade quality.

In view of utility of the plasticity index, the acquirement process of liquid and plastic limit values gains importance. However, serious experience, attention and time are required to obtain valid and consistent results. Insufficient remolding effect applied during sample preparation in water contents other than the soils used in the experiments is one of the physical problems that may occur during the limit tests and this may trigger the interpretation errors and cause the wrong limit values to be obtained. Besides, the plastic limit test is shaped entirely based on the experience of the operator who is performing the experiment, and the main reason for the errors related to the experiment is the preparation and testing of the sample, which depends on the operator. In addition, the ambient temperature and remolding sensitivity of the operator while preparing the sample may cause the sample prepared in a certain water content to lose water and in fact the predicted content value cannot be obtained. This situation makes it difficult to interpret the average water content value and causes the data to be obtained in a wide range [6]. The plastic limit test is time-consuming, relying on hand-drying the sample, and highly subjective. Such subjectivity can introduce errors into the final calculation of the plasticity index of a soil [1]. Authors such as Nagaraj & Jayadeva [42], Giovanni et al. [43], Nihad [44], Al-Kahdaar et al. [45] and Arama et al. [6] have proposed models for predicting the plasticity index from the liquid limit.

On the other hand, some authors such as Arama et al. [6] and Dysli [46] have noted that the correlation models are only applicable to the soils that were used to develop them. These models are limited to the nature of the soil studied because not only are natural soils not stacked homogeneously as a single formation, but also these soils do not obey the same laws. The excessiveness and the actuality of the conducted studies about the evaluation and the derivation process of consistency limits show the importance and actuality of the subject. In addition, the developing information and computer technologies researchers to obtain the easy way to calculate all the soil engineering properties by the usage of advanced methods with prediction [6]. In this study, the values of the liquid limit test will be used to predict representative values of the plasticity index of clayey and silty soils that are selected in several localities of the central lama depression (Khô Depression) in Benin.

2. Materials and Methods

2.1 Study Area

Benin is a sub-Saharan country located in West Africa and has a semi-arid climate. It is subdivided into 12 departments and has 77 communes. According to Baglo and Guedegbe [47] cited by Agbelele [48], the soils of the southern median zone are estimated at more than 3,000 km² and are located in the Lama Depression, which is divided into three depressions: Issaba in the east, Khô in the center, and Tchi in the west, Khô in the center, and Tchi in the west. The central depression constitutes our study area. This study focuses on the soils of the Khô depression. This depression crosses several localities such as

Koussoukpa, Zoukou, Massi center, Toffo-Kpamè, Massi, Lokounkpa, Sèhouè, Lonmè, and Adjikpokpa shown in Fig. 1, where the experimental study was conducted. To do this, Atterberg limit tests were performed on the soils of these localities which were sampled respectively of 0.40 to 1.00 m; 1.00 m to 2.00 m; 2.00 m to 3.00 m; 3.00 m to 4.00 m; and 4.00 m to 5.00 m relative to the ground.

2.2 Realization of the Test

The liquid limit (W_L) was determined by the Casagrande method as specified by NF P 94 051 [49].

This method consists in the determination of the water content for which a groove made in the soil sample placed in a cup of imposed characteristics closes when the cup and its contents are subjected to repeated shocks. The plasticity limit W_p determined according to the same standard NF P 94 051 [49], consists in determining the water content for which a roll of soil of a fixed size and made by hand will crack. The plasticity index I_p is nothing but the difference between the liquidity limit W_L and the plasticity limit W_p . Some materials used for the realization of the tests are illustrated at Fig.2.

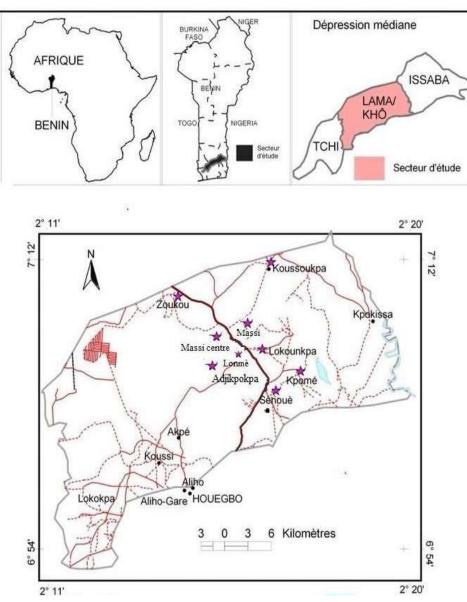
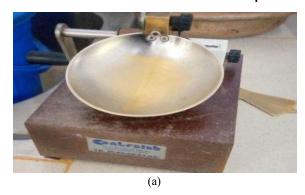


Fig. 1 Sample collection card.



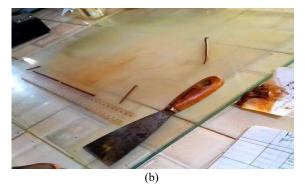


Fig. 2 Materials used for the realization of the test, (a) Apparatus of casagrande, (b) Thinned roller on the plate.

Table 1 Soil classification chart adapted from ASTM D-2487 [50].

	Criteria for Assigning Group Symbols and Group Names Using Laboratory Tests				
			Soil classification		
			Group symbol	Group name	
	Silts and clays liquid limit less than 50 Silts and clays liquid limit 50 or more	PI > 7 and plots on or above "A" line	CL	Lean clay	
Fine-grained soils 50% or more passes the No. 200 Sieve		PI < 4 or plots below "A" line	ML	Silt	
		PI plots on or above "A" line	CH	Fat clay	
		PI plots below "A" line	MH	Elastic silt	

Table 2 Symbol chart of USCS (unified soil classification system) [51].

Major divisions			Group symbol	Group name
		Inorganic	ML	Silt
			CL	Clay of low plasticity, lean clay
more passing the No. 200 (0.075 mm) sieve			MH	Silt of high plasticity, elastic silt
(0.0,0 11111) 510.0	Silt and clay liquid limit ≥ 50		СН	Clay of high plasticity, fat clay

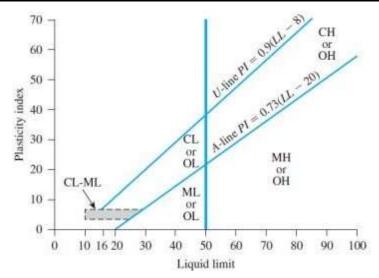


Fig. 3 Plasticity chart [2] cited by Arama et al. [6].

2.3 Development of Correlations

The plasticity indices are correlated with the liquidity limits determined with the casagrande apparatus. To do this, three mathematical regression analysis models are tested for each tested in order to select the best ones:

• the linear model: $I_p = aW_L + b$ as determined by Nihad [44] and Al-Kahdaar et al. [45]

- the power model: $I_p = aW_L^b + c$ as shown by Arama et al. [6]
- the exponential model: $I_p = a \cdot e^{bW_L}$ shown by Spagnoli [31]

where:

 I_P is plasticity index;

 W_L is liquidity limit;

a, b and c are real constants to be determined. The existence of a correlation between the parameters will be assessed by the coefficient of determination R^2 which is an indicator giving the possibility of measuring the quality of the prediction of a linear regression [52]:

$$R^{2} = 1 - \frac{\sum_{i=1}^{n} y_{i} - y_{i}^{'}}{\sum_{i=1}^{n} y_{i} - y_{i}^{''}}$$
(1)

where:

n is the number of measurements;

 y_i is the value of the *i*-th measurement;

 y_i 'is the corresponding predicted value;

 y_i " is the average of the measurements.

The relationship between the parameters will be strong if the coefficient of determination R^2 is close to 1. The validity of the correlations will also be assessed on the basis of the RMSE (root mean square error). RMSE is the square root of the mean squared difference between the values calculated using a correlation and the corresponding observed values determined from laboratory tests. RMSE is most useful when large errors are particularly undesirable [53].

Regression analyses are conducted with Matlab 7.5.0 R2007b by the use of 2-dimensional graph systems to find the most proper expression of plasticity index. Different types of functions implemented in Matlab such as linear fitting, power and exponential are used for 2D analyses to search for the best fitting relationship between the variants in Matlab. Several analyses are performed for each existing method. Least squares method and Trust-region algorithm are used and the coefficients with 95% confidence bound.

3. Results and Discussion

3.1 Test Results

The tests carried out revealed that the soils tested are plastic and very plastic clays and silts. The liquidity limits vary from 26% to 184% and the plasticity indices are in the order of 5 to 131. The results are summarized in Fig. 3 and all the used soil types are classified according to the USCS. All the cohesive soil, according to USCS at the investigated area is clayey and silty soil with low to high plasticity (CL, CH and MH).

3.2 Correlations between Liquidity Limit and Plasticity Index

In order to establish these correlations, trend curves represented by the graphs below are drawn from the clouds of points whose coordinates correspond to the W_L and I_P values of Fig. 4.

A linear correlation presented in Fig. 5a shows that $I_p = 0.496W_L - 1.918$ with parameter a equal to 0.496 and parameter b equal to -1.918. Very good correlation is observed with correlation coefficient of $R^2 = 0.9891$ and R^2 adjusted equal to 0.9889. The RMSE is minimized until 2.476.

In case of exponential regression illustrated in Fig. 5b, the equation found is $I_p = 14.18 \cdot e^{0.012 \cdot W_L}$ and is characterized by a correlation coefficient of $R^2 = 0.98871$ and R^2 adjusted equal to 0.9885. The RMSE minimum is to 2.528.

From the power model (Fig. 5c), reliable correlation to predict the plasticity index (I_P) was proposed with R^2 equal to 0.9802 and R^2 adjusted is to 0.9794. RMSE is equal to 3.377 and confidence bound of parameters a, b and c is respectively [0.0035; 0.0242], [1.59-1.874] and [5.558; 11.97].

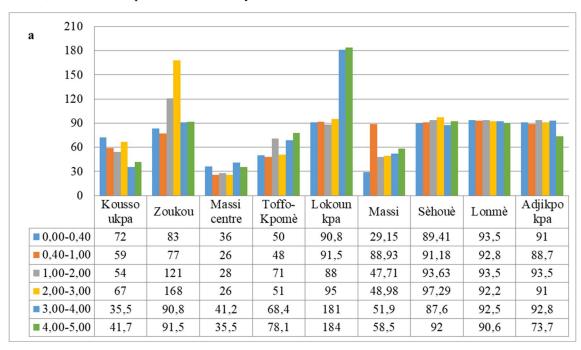
$$I_p = 0.0139W_L^{1.732} + 8.76$$

These different results are notified in Tables 3 and 4. For regression, there are numerous methods to evaluate the goodness of your fit i.e. how well the model fits the data. R^2 values are just one such

measure. A residual is a measure of how far away a point is vertical from the regression line. Simply, it is the error between a predicted value and the observed actual value [54].

We observe on the residual plot of the models in Fig. 6 below, that the data points are randomly and

symmetrically located around zero and that in general there is no clear pattern or trend. There is a high density of points near the origin. Based on these characteristics [55], it can be stated that they have a good residual plot. Therefore, the regression models obtained are valid.



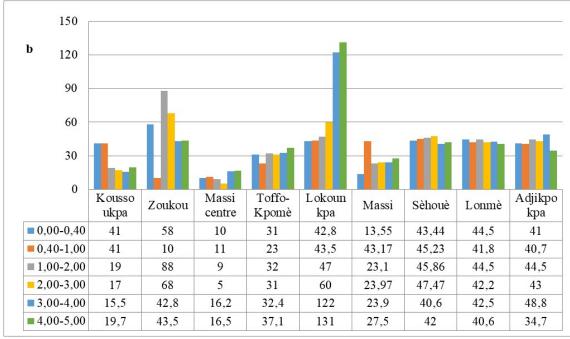


Fig. 4 Test results. (a) liquidity limits of soils. (b) plasticity index.

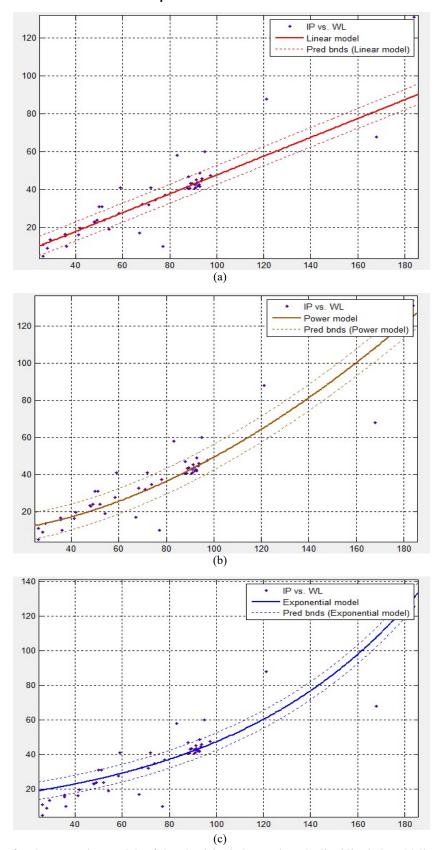


Fig. 5 Trend curves for the regression models of the plasticity index against the liquidity index, (a) linear model, (b) power model, (c) exponential model.

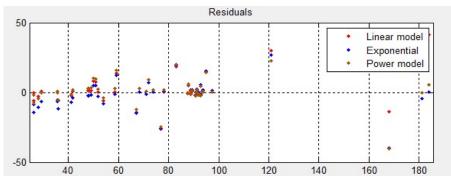


Fig. 6 Residuals plot.

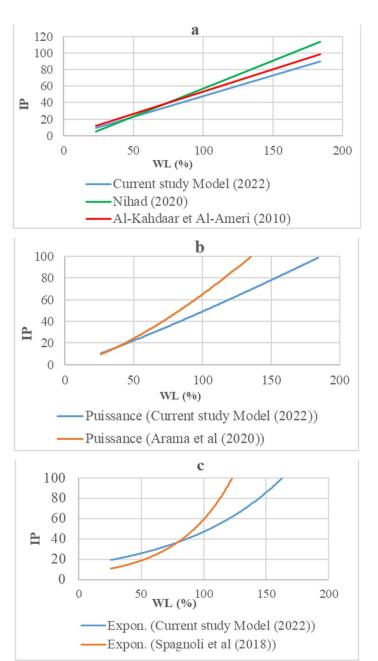


Fig. 7 Comparison of the suggested model with literature sources, (a) linear model, (b) power model, (c) exponential model.

Table 3 Summary of models parameters and confidence bound.

Model	Parameter a	Parameter b	Parameter c	Confidence bound of	a Confidence bound of	b Confidence bound of c
Linear	0.4969	-1.918		0.477-0.517	-3.624 to -0.213	
Exponential	14.18	0.012		13.6-4.75	0.011 - 0.012	
Power	0.0139	1.732	8.762	0.004-0.024	1.59-1.87	5.56-1.97

Table 4 Results of regression analyses.

Model	Correlation equation	R^2	R ² adjusted	RMSE
Linear	$I_p = 0.497 \text{ W}_{\text{L}} - 1.918$	0.9891	0.9889	2.476
Power	$I_p = 0.0139 \text{ W}_{\text{L}}^{1.732} + 8.762$	0.9802	0.9794	3.377
Exponential	$I_p = 14.18 \mathrm{e}^{0.012 \mathrm{W_L}}$	0.9887	0.9885	2.528

Table 5 Empirical model of the current study and those in the literature.

	References	Empirical models	R^2	RMSE	Type of soil
	Nihad [44]	$I_p = 0.6729 \text{ W}_L - 10.036$	0.87	2.18	CH, CL, MH, ML
Linear model	Al-Kahdaar and Al-Ameri [45]	$I_p = 0.536 \text{ (W}_L - 0.71)$	0.64		CH, CL
	Current study (2022)	$I_p = 0.497 \text{ W}_L - 1.918$	0.99	2.47	CL, CH, MH
Power model	Arama et al. [6]	$I_p = 1.96 \text{ W}_{L}^{0.82} - 21.6$	0.90		CH, CL
	Current study (2022)	$I_p = 0.0139 \text{ W}_{\text{L}}^{1.732} + 8.76$	0.98	3.37	CH, CL, MH
Exponential model	Spagnoli et al. [31]	$I_p = 5.94.e^{0.023W}$ L	0.80		CH, CL
	Current study (2022)	$I_p = 14.18.e^{0.012W}$ L	0.99	2.53	CH, CL, MH

A comparison of empirical equations from the study and those obtained from the literature has been made. The empirical model from literature are registered in table 5. After the comparisons of the different models found, a significant difference is noted as shown in Fig. 7. The conclusion is that the correlation models are only valid in the studied environment as already underlined by Dysli [46] in 2011.

4. Conclusion

This study is conducted to examine the relationships between plasticity index and liquid limit. Clays and silts with high and very high plasticity were collected in several localities of the Khô depression located in the center of the Lama depression in Benin within the framework of this study. Regression analyses were performed using linear, exponential and power models. Correlation equations are obtained for each model to calculate the I_p value. The residual plot of the models and a comparison with other models in the literature was done to verify the equations obtained. Therefore, in this study, these

results are obtained:

- (1) Appropriate equations were acquired to determine the value of I_p as a function of W_L , based on the actual experimental tests performed for the soils of Khô depression.
- (2) The value of the correlation coefficients and the plot of the residuals showed the applicability of the proposed relationships.
- (3) A significant result of this study is to draw attention to the relationships that are selected to represent geotechnical properties of sites in the design stages. This is because the expressions that allow the prediction of the parameters are a function of the dominant soil type used to establish the relationship.
- (4) It is in this context that the present study presents an applicable acquisition process of the I_p value for the soil types CH, CL of clays and MH of silts existing in the center of the Lama depression in Benin (Khô depression).

This situation also opens the way for different parameter estimates that can be made based on the value of the plasticity index.

References

- [1] Fall, D. 2000. "A Numerical Model for Rapid Determination of Plasticity of Fine Grained Soils." Engineering Geology Research Group, School of Earth, Environmental and Physical Sciences, University of Portsmouth, pp. 43-5.
- [2] Das, B. M., and Sobhan, K. 2017. *Principles of Geotechnical Engineering* (8th ed.). USA: C. Learning.
- [3] Kollaros, G. 2016. "Liquid Limit Values Obtained by Different Testing Methods." *Bulletin of the Geological Society of Greece* 50 (12): 778-87.
- [4] Naveena, S., and Chandanshree. 2018. "Establishing Relationship between Plasticity Index and Liquid Limit by Simple Linear Regression Analysis." *International Journal for Research in Applied Science & Engineering Technology* 6: 1975-8.
- [5] Atterberg, A. 1911. "Über die physikalische Bodenuntersuchung and und über dile Plastizität de Tone." *International Mitteilungen für Bodenkunde* 1: 10-40.
- [6] Arama, Z. A., Akın, M. S., Nuray, S. E., and Dalyan, İ. 2020. "Estimation of Consistency Limits of Fine-Grained Soils via Regression Analysis: A Special Case for High and Very High Plastic Clayey Soils in Istanbul." International Advanced Researches and Engineering Journal 4 (13): 255-66.
- [7] Paul, V. J., Asce, M., and Stuart, K. H. 2014. Some Recent Developments in the Determination of the Atterberg Limits. Davis, CA: Geotechnical Special Publication, pp. 1-8.
- [8] Casagrande, A. 1932. "Research on the Atterberg Limits of Soils." *Public Roads* 13 (18): 121-36.
- [9] Casagrande, A. 1958. "Notes on the Design of the Liquid Limit Device." *Géotechnique* 8 (12): 84-91.
- [10] American Society for Testing and Materials. 2010. "Standard Test Methods for Liquid Limit, Plastic Limit, and Plasticity Index of Soils." ASTM-D4318.
- [11] British Standards BS 1377. 1990. Part 2: Methods of Test for Soils for Civil Engineering Purposes—Classification Tests. London: BSI, p. 42.
- [12] British Standards BS EN 1337-2:2001. 2001. Structural Bearings. Part 2: Sliding Elements. London: BSI.
- [13] Wroth, C. P. 1979. "Correlations of Some Engineering Properties of Soils." In *Proc. 2nd International* Conference on the Behaviour of Offshore Structures, vol. 1, pp. 121-3.
- [14] Haigh, S. K. 2012. "Mechanics of the Casagrande Liquid Limit Test." *Canadian Geotech. J.* 49 (19): 1015-23.
- [15] Hansbo, S. 1957. "A New Approach to the Determination of the Shear Strength of Clay by the Fall Cone Test." Swedish Geotechnical Institute Proceedings 14: 5-47.
- [16] Koumoto, T., and Houlsby, G. T. 2001. "Theory and

- Practice of the Fall Cone Test." *Géotechnique* 51 (18): 701-12.
- [17] Rudolf, K. 1977. "Consistency Limits: A Manual for the Performance and Interpretation of Laboratory Investigations." Vols. 1 sur. 2 Part 6, Document D6, p. 40.
- [18] Sherwood, P., and Riley, M. 1970. "An Investigation of a Cone-Penetrometer Method for the Determination of the Liquid Limit." *Géotechnique* 20 (12): 203-8.
- [19] Littleton, I., and Farmilo, M. 1977. "Some Observations on Liquid Limit Values with Reference to Penetration and Casagrande Tests." *Ground Eng.* 10 (14): 39-40.
- [20] Wires, K. 1984. "The Casagrande Method versus the Drop-Cone Penetrometer Method for the Determination of Liquid Limit." *Canadian Journal of Soil Science* 64 (12): 297-300.
- [21] Belviso, R., Ciampoli, S., Cotecchia, V., and Federico, A. 1985. "Use of the Cone Penetrometer to Determine Consistency Limits." *Ground Engineering* 18 (15): 21-2.
- [22] Sivapullaiah, P., and Sridharan, A. 1985. "Liquid Limit of Soil Mixtures." *Geotechnical Testing Journal* 8 (13): 111-6.
- [23] Wasti, Y. 1987. "Liquid and Plastic Limits as Determined from the Fall Cone and the Casagrande Methods." Geotechnical Testing Journal 10 (11): 26-30.
- [24] Leroueil, S., and Le Bihan, J.-P. 1996. "Liquid Limits and Fall Cones." *Canadian Geotechnical Journal* 33 (15): 793-8.
- [25] Dragoni, W., Prosperini, N., and Vinti, G. 2008. "Some Observations on the Procedures for the Determination of the Liquid Limit: An Application on Plio-Pleistocenic Clayey Soils from Umbria Region (Italy)." *Italy Journal* engineering Geology 1: 185-97.
- [26] Özer, M. 2009. "Comparison of Liquid Limit Values Determined Using the Hard and Soft Base Casagrande Apparatus and the Cone Penetrometer." Bulletin of Engineering Geology and the Environment 68 (13): 289-96.
- [27] Fojtová, L., Marschalko, M., Franeková, R., and Kovàř, L. 2009. "Study of Compatibility of Methods for Liquid Limit Measurement according to Czech State Standard and Newly Adopted European Standard." GeoScience Engineering 11: 55-68.
- [28] Grønbech, G., Nielsen, B., and Ibsen, L. 2010. "Comparison of Plasticity Index of Søvind Marl Found by Use of Casagrande Cup, Fall Cone Apparatus and Loss on Ignition." Scientific Publications at the Department of Civil Engineering, Aalborg University, DCE Technical Reports, p. 14.
- [29] Grønbech, G., Nielsen, B., and Ibsen, L. 2011. "Comparison of Liquid Limit of Highly Plastic Clay by Means of Casagrande and Fall Cone Apparatus." In

- Proceedings of the 14th Pan-American Conference on Soil Mechanics and Geotechnical Engineering, October 2-6, 2011, Toronto, Ontario, Canada.
- [30] Di Matteo, L. 2012. "Liquid Limit of Low- to Medium-Plasticity Soils: Comparison between Casagrande Cup and Cone Penetrometer Test." *Bulletin of Engineering Geology and the Environment* 71 (11): 79-85.
- [31] Spagnoli, G. 2012. "Comparison between Casagrande and Drop-Cone Methods to Calculate Liquid Limit for Pure Clay." *Canadian Journal of Soil Science* 92 (16): 859-64.
- [32] Bicalho, K., Gramelich, J., and dos Santos Cunha, C. 2014. "Comparação entre os valores do limite de liquidez obtidos pelos métodos de Casagrande e cone para solos argilosos Brasileiros." Comunicações Geológicas 101: 1097-9. (in Portuguese)
- [33] Tomlinson, M., and Boorman, R. 1995. *Foundation Design and Construction* (6th ed.). New York, NY: Longman Pub Group, p. 579.
- [34] Skempton, A. 1953. "The Colloidal Activities of Clays." In *Proceedings of the Third International Conference on Soil Mechanics and Foundation Engineering*, pp. 57-61.
- [35] Skempton, A. W. 1954. "Discussion: Sensitivity of Clays and C/P Ratio in Normally Consolidated Clays." In *Proceedings of the American Society of Civil Engineers*, pp. 19-22.
- [36] Skempton, A. W. 1957. "Discussion: Further Data on the C/P Ratio in Normally Consolidated Clays." In *Proceedings of the Institution of Civil Engineers*, pp. 305-7.
- [37] Stroud, M. A. 1974. "The Standard Penetration Test in Sensitive Clays and Rocks." *The European Seminar on Penetration Testing* 2 (12): 366-75.
- [38] Schofield, A. N., and Wroth, C. P. 1968. *Critical State Soil Mechanics*. London, UK: McGraw-Hill.
- [39] Nakase, A., Kamei, T., and Kusakabe, O. 1988. "Constitutive Parameters Estimated by Plasticity Index." *Journal of Geotechnical Engineering* 114 (17): 844-58.
- [40] Black, W. P. M. 1962. "A Method of Estimating the California Bearing Ratio of Cohesive Soils from Plasticity Data." *Géotechnique* 12 (14): 271-82.
- [41] Heukelom, W., and Klomp, A. J. G. 1962. "Dynamic Testing as a Means of Controlling Pavement during and after Construction." In Proceedings of the 1st International Conference on the Structural Design of Asphalt Pavements, University of Michigan, July 13-17, 1962, Ann Arbor, MI, USA.

- [42] Nagaraj, and Jayadeva 1984. "Critical Reappraisal of Plasticity Index of Soils." *Journal of Geotechnical Engineering* 110 (19): 1369-70.
- [43] Giovanni, S., Asuri, S., Pierpaolo, O., Diego, B., and Lucio, D. M. 2018. "Statistical Variability of the Correlation Plasticity Index versus Liquid Limit for Smectite and Kaolinite." Applied Clay Science 156: 152-9.
- [44] Nihad, B. S. 2020. "Corrélations des caractéristiques géotechniques pour les sols à grains fins." Série de conférences IOP, Science et génie des matériaux. (in French)
- [45] Al-Kahdaar, R. M., and Al-Ameri, A. F. I. 2010. "Correlations between Physical and Mechanical Properties of Al-Ammarah Soil in Messan Governorate." *Journal of Engineering* 16 (14): 5946-57.
- [46] DYSLI. 2001. Recherche bibliographique et synthèse des correlations entre les caracteristiques des sols. Suisse: Laboratoire de mécanique des sols de l'EPFL. (in French)
- [47] Baglo, M., and Guedegbe, B. 1995. Environnement naturel et socio-économique de la forêt classée de la LAMA, Document de travail No. 12, edited by UNESCO. Paris: Division des Sciences écologique Programme de Coopération Sud-Sud, pp. 1-28. (in French)
- [48] Agbelele, K. 2017. "Etude intégrée des sols instables en vue de la maîtrise du phenomene de gonflement: cas de la depression de la lama au Bénin." Ecole doctorale des sciences de l'ingénieur: Université d'Abomey-Calavi. (in French)
- [49] Association Française de Normalisation. 1993. NF P 94-051, Sols: reconnaissance et essais. Détermination des limites d'Atterberg. Limite de liquidité à la coupelle-Limite de plasticité au rouleau. (in French)
- [50] ASTM international D-2487-06. 2006. "Standard Practice for Classification of Soils for Engineering Purposes."(Unified Soil Classification System.
- [51] Wikipedia. 2022. The Free Encyclopedia. Accessed on 20 March 2022. https://en.wikipedia.org.
- [52] Zhang, D. 2016. "A Coefficient of Determination for Generalized Linear Models." *The American Statistician* 71 (14): 310-6.
- [53] Onyejekwe, S., Kang, X., and Ge, L. 2015. "Assessment of Empirical Equations for the Compression Index of Fine-Grained Soils in Missouri." *Bulletin of Engineering Geology and the Environment* 74 (13): 705-16.
- [54] Usman, G. 2020. "Towards Data Science." Accessed on 23 March 2022. Towardsdatascience.com.
- [55] Frost, J. 2017. statisticsbyjim.com.