An Inelastic Material Model for Lateral Torsional Buckling and Biaxial Bending of Steel W-Shapes

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Abstract: A new material model for beam elements was developed for use as normalized tangent modulus expressions when performing 3-dimensional second-order inelastic analyses of steel I-section beams. The stiffness matrix of a 14 degree-of-freedom beam element was updated to include the effects of yielding on St. Venant’s torsion and bimoment stiffness at the initial and terminal nodes. A validation study compared the new model’s results with those from published detailed finite element analyses and was found to be in very close agreement. A biaxial end-moment study with two different depth-to-flange-width ratios provided expected and consistent results over a range of moment conditions.

Key words: Nonlinear analysis, lateral torsional buckling, biaxial bending, stiffness reduction, material model.

1. Introduction

Appendix 1 of the American Institute of Steel Construction Specification for Structural Steel Buildings [1] provides the designer with the option to use advanced analysis methods of structural analysis to directly model localized yielding and its effects on system behavior. The analysis requirements stipulate that the second-order inelastic analysis must account for geometric imperfections and the influence of residual stresses and partial yielding effects. Recent research has focused on developing design procedures and equations to account for the reduction in stiffness due to partial yielding effects [2-6]. The primary goal of this research is to provide designers with an accurate and efficient material model for a 14 degree-of-freedom beam element when performing a 3-dimensional second-order inelastic analysis to determine the lateral torsional buckling and biaxial moment capacities of rolled I-section beams.

2. Stiffness Reduction Model

The proposed stiffness reduction (\(\tau\)) model that accounts for partial yielding of the beam’s cross-section due to uniaxial or biaxial bending moments is given in Fig. 1 [7]. The triangular plateau at the top of the model represents the moment conditions for which yielding does not occur (\(\tau = 1\)). The coordinates \((m_x, m_y, \tau)\) on the red dashed curve represent the biaxial moments and corresponding stiffness reduction that is assumed to vary as an exponential function between 1 and 0. The blue curve at the bottom represents the biaxial moment conditions when \(\tau = 0\) and is assumed to vary as a second-order polynomial defined by the three coordinates \((1,0,0), (m_x^*, m_y^*, 0)\) and \((0,1,0)\). The model variables are a function of the W-shape’s cross-section dimensions, the residual stress ratio \(c_r\), and exponent constants \(n_x\) and \(n_y\).

The major-axis moment \(M_x\) and minor-axis moment \(M_y\) are normalized using the respective plastic moment capacities as \(m_x = M_x/M_{pz}\) and \(m_y = M_y/M_{py}\). For a given W-shape, and assuming an ECCS (European Convention for Constructional Steel) residual stress pattern [8], the maximum moment for which \(\tau = 1\) is maintained for uniaxial major-axis bending is

\[m_{1x} = \frac{S_z}{Z_z}(1 - c_r)\]  \(\text{(1)}\)

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where $S_z$ = major-axis elastic section modulus, $Z_z$ = major-axis plastic section modulus and $c_r$ = residual stress ratio $(\sigma_r/\sigma_y)$. The equation for uniaxial minor-axis bending is

$$m_{1y} = \frac{S_y}{Z_y}(1 - c_r)$$  \hspace{1cm} (2)

where $S_y$ = minor-axis elastic section modulus and $Z_y$ = minor-axis plastic section modulus. The $m_{zl}$ and $m_{yl}$ coordinates in Fig. 1 are found assuming a linear relationship between $m_{1z}$ and $m_{1y}$. The $m_i$ scalar magnitude in Fig. 1 is given as

$$m_i = \left( \frac{m_{1y}}{m_y} \right) \sqrt{1 + \left( \frac{m_y}{m_z} \right)^2}$$  \hspace{1cm} (3)

The $m^*_z$ and $m^*_y$ coordinates in Fig. 1 are determined from the dimensions of the W-shape and correspond to the moments for the plastic neutral axis to be in the position as given in Fig. 2. This coordinate is unique to every W-shape and helps to accurately define the $\tau = 0$ curve in Fig. 1.

The closed-form equations for $m^*_z$ and $m^*_y$ are

$$m^*_z = \frac{\lambda_1 + 6 \lambda + 5 \lambda_0}{\lambda_1 + 4 \lambda_1 + 4}$$  \hspace{1cm} (4)

$$m^*_y = \frac{1}{2 + \lambda \lambda_0} \times \left( 2 - \frac{[(4(\lambda_1 + 1) + \lambda \lambda_1)m^*_z - \lambda_0(\lambda_1 + 1)^2]^2}{8(\lambda_1 + 1)^2} \right)$$  \hspace{1cm} (5)

where $\lambda = A_w/A_f$, $\lambda_0 = t_w/b_f$ and $\lambda_1 = d_w/t_f$.

Using a W14×53 as an example, the red curve in Fig. 3 is the biaxial moment condition at $\tau = 0$ based on a detailed fiber element analysis. This curve is approximated using a second-order polynomial as given by the blue curve. In the assumed quadratic region, the $m_{2z}$ and $m_{2y}$ coordinate values are found using

$$m_{2z} = C_1 m^2_{2z} + C_2 m_{2z} + 1$$  \hspace{1cm} (6)

where the constants $C_1$ and $C_2$ are determined using the $m^*_z$ and $m^*_y$ values.
An Inelastic Material Model for Lateral Torsional Buckling and Biaxial Bending of Steel W-Shapes

\[ C_1 = \frac{1 - m_y^* - m_y}{m_x^* - m_y^2} \quad (7) \]

\[ C_2 = \frac{m_x^2 + m_y^* - 1}{m_y^2 - m_x^2} \quad (8) \]

The \( m_j \) scalar magnitude in Fig. 1 is given as

\[ m_j = \left( \frac{m_y}{m_x} - C_2 \right) - \sqrt{\left( \frac{m_y}{m_x} - C_2 \right)^2 - 4C_1} \times \frac{1 + \left( \frac{m_x}{m_y} \right)^2}{2C_1} \leq 1 \quad (9) \]

The \( m_{yz} \) scalar magnitude in Fig. 1 is determined using the given \( m_x \) and \( m_y \) moment condition.

\[ m_{yz} = \sqrt{m_x^2 + m_y^2} \quad (10) \]

Depending on the magnitude of \( m_{yz} \), the stiffness reduction can be determined using Eqs. (3), (9) and (10) such that

when

\[ m_{yz} \leq m_i \quad \tau = 1 \quad (11) \]

when

\[ m_i < m_{yz} \leq m_j \quad \tau = 1 - \left( \frac{m_{yz} - m_i}{m_j - m_i} \right)^{n_b} \quad (12) \]

where the exponent \( n_b \) is a function of the \( m_y/m_x \) ratio and the model input values \( n_x \) and \( n_y \).

\[ n_b = (n_x - n_y)e^{\left(-\frac{n_y}{m_x}\right)} + n_y \quad (13) \]

Eq. (13) is a nonlinear regression equation based on the assumption that \( n_b \) varies linearly between \( n_x \) and \( n_y \) with respect to the angle \( \phi \) in Fig. 1 for \( n_x > n_y \).

### 3. Inelastic Stiffness Matrix Terms

The stiffness reduction that occurs due to yielding over the member length is accounted for by using several beam elements per member and a stiffness matrix with coefficients that can vary depending on the severity of yielding at the initial and terminal nodes. Based on the assumption that the tangent modulus varies linearly over the length of the element, the coefficient terms provide a reasonable approximation of the reduced flexural and torsional stiffness of the element due to partial yielding. In practice, the error introduced by this assumption is reduced by using multiple elements along the length of the member. The stiffness matrix developed by Ziemian and McGuire [9] was used in this study because the \( \tau \) values from Eqs. (11) and (12) can be used directly as the \( a \) and \( b \) terms in the stiffness matrix coefficients. The reduced stiffness matrix coefficients for flexure about each axis are already in MASTAN2 [10], but the St. Venant's torsion and bimoment stiffness terms in Eqn. (14) were added to the source code. The same \( a \) and \( b \) terms based on flexural yielding are used because the stiffness reduction occurs primarily due to yielding of the cross-section from the bending moments \( m_x \) and \( m_y \).

\[
\begin{bmatrix}
EC_L \\
\frac{6}{L} \left( \frac{a + b}{2} \right) - \frac{6}{L} \left( \frac{a + b}{3} \right) + \frac{6}{L} \left( \frac{a + 2b}{3} \right) & 12 \left( \frac{a + b}{2} \right) - \frac{12}{L} \left( \frac{a + b}{2} \right) & \frac{6}{L} \left( \frac{a + b}{4} \right) - \frac{6}{L} \left( \frac{a + 2b}{4} \right) \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
\frac{3}{5L} \left( \frac{a + b}{2} \right) - \frac{6}{5L} \left( \frac{a + b}{10} \right) + \frac{b}{10} & \frac{a}{10} \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
\frac{3}{5L} \left( \frac{a + b}{2} \right) - \frac{6}{5L} \left( \frac{a + b}{10} \right) - \frac{b}{10} & \frac{a}{10} \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
\frac{3}{5L} \left( \frac{a + b}{2} \right) - \frac{6}{5L} \left( \frac{a + b}{10} \right) - \frac{b}{10} & \frac{a}{10} \\
\end{bmatrix}
\]
4. Material Model Validation Study

The validity of the new inelastic material model was verified by comparing its results with published finite element results by Subramanian and White [4, 5]. Their finite element model used the full geometric and material nonlinear analysis capabilities of ABAQUS [11]. The cross-section was modeled using S4R shell elements with 20 elements through the web depth and 12 elements across the flange width. The element aspect ratio over the beam length was held to approximately 1.0 with the web elements. As part of their study, a simply-supported W21×44 W-shape was modeled with $C_b = 1$ and $C_b = 1.3$ concentrated end-moment conditions. To match their analysis conditions, $L_b/2,000$ was used at mid-span, bracing only at the supports, $E = 200$ GPa and $\sigma_y = 345$ MPa. The MASTAN2 model had 16 beam elements with continuous warping restraint over the entire length, except for warping free conditions at the ends. Material constants were $n_z = 1.5$, $n_y = 1.2$ and $c_r = 0.3$. The $M_z$ moments were incrementally applied using a second-order inelastic analysis up to the limit load condition. Comparison results are given in Fig. 4, and the new model results for both $C_b$ conditions agree very closely with their results.

5. Biaxial Beam Bending Study

Following the validation study, analyses were conducted on simply-supported beams using the new material model with uniaxial and biaxial bending end-moment conditions. Two W-shapes with different depth-to-flange-width ratios were used to demonstrate the differences in the beam capacity results for the biaxial bending conditions when applying the same percentage of $M_y/M_z$ concentrated end-moments. The analyses considered a W21×93 ($d/b_f = 2.6$) and W12×72 ($d/b_f = 1.0$) with imperfection $L_b/2,000$ at mid-span, bracing only at the supports, $E = 200$ GPa and $\sigma_y = 345$ MPa. The MASTAN2 model had 8 beam elements with the same warping restraint and material model constants as in the verification study.

The biaxial bending conditions were modeled with both the $M_z$ and $M_y$ moments applied simultaneously and incrementally up to the limit load. The results in Fig. 5 indicate a significant difference in the biaxial bending moment responses for the two W-shapes. There is a much larger reduction in the capacity for the W21×93 compared with the W12×72 for the same percentage of $M_y/M_z$ moments. As expected, the W12×72 with its smaller depth-to-flange-width ratio maintained higher biaxial bending capacities. The new material model also demonstrated its ability to provide consistent results over a range of $M_y/M_z$ conditions.
An Inelastic Material Model for Lateral Torsional Buckling and Biaxial Bending of Steel W-Shapes

6. Conclusions

A new inelastic material model for W-shapes under uniaxial and biaxial beam bending conditions is presented and validated. The material model was developed for use with a 14 degree-of-freedom beam element in a second-order inelastic analysis. The model is based on the actual dimensions of a given W-shape and allows the user to specify the residual stress ratio $c_r$ to control the initial yield condition and the values $n_x$ and $n_y$ to control the rate of stiffness reduction between the uniaxial major-axis moment and uniaxial minor-axis moment conditions, respectively. A validation study compared the new model’s results with those from detailed finite element analyses, and it was found that the new model provided very comparable limit load results. The biaxial end-moment study using two W-shapes with different depth-to-flange-width ratios provided expected and consistent results over a range of moment conditions.

References


