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Risk Neutral Options Based on Maximum Profits

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This study uses risk-neutral-probability (RNP, thereafter) to forecast future excepted profits in copper, gold, and platinum group metals. The RNP measures are advantageous where ones want to compare values of the same asset in different states. At the heart of this study is using future profits based on a binomial tree, which has RNPs. Overall, RNP profits converge more to the future-normal profits. And, the robustness results based on the vector autoregressive (hereon, VAR) model reveal that the profits react differently to different shocks stages from revenues, risk-free interest rates and profits. The findings of this study apply to copper, gold, and platinum mining around the globe.

Keywords: option, profit, risk-neutral-probability

Introduction

RNP methods have been used in earlier studies because of their "ability to make sure that prices are exactly equal to the discounted expectations". Thus, in the context of that state, arbitrage opportunities are almost zero. Among advantages of RNP measures is their ability to transform parameters in between different states equitably (Chernov & Ghysells, 2000). The latter phenomenon is advantageous when pricing risk assets, in particular in incomplete setting. Moreover, uncertainty is usually appropriately captured in the model (Scott, Highfill, & Sattler, 1988).

In the mining industry, RNP measures have been used before (Moel & Tufano, 2002). However, the parameters of models have been fitted in with single-measured-parameters such as volatility. In a risky environment, volatility is huge factor and it is important that it should capture all correct and/or "real" stock market information. For example, in Moel and Tufano (2002), volatility is calculated only from single gold prices. The disadvantage of only prices volatility is that it is one of the few financial parameters in the mining industry. In the context of the mining industry, volatilities are sensitive to (i) financial, (ii) technical, and (iii) social parameters (Cortez, Saydam, Coulton, & Sammut, 2018). Then, the question is why researchers are not using incorporated volatilities.

This study uses incorporated volatilities to forecast future optimal profits in copper, gold, and platinum group metals industries using the future expected revenue model under RNP framework. Thereafter, the study uses VAR model to illustrate that future incorporated profits are slightly lower than future profits based on single-price-volatility. That might be due to the fact that clustering effect in incorporated parameters erodes some profits. Davis (1998) found similar findings in his study on real options in the mining industry. The VAR

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(1,1) results, when read in conjunction with Cholesky decomposition, illustrate that inter-causal relationship between revenues, risk-free interest rates and profits.

The remainder of the article proceeds as follows. Section 2 is on literature review. Section 3 is on modelling. Section 4 is on empirical application and the last section concludes the article.

Literature Review

McDonald and Siegel (1985) investigated the option to shut down, when valuing risky investment projects. That valuation is carried out under risk-neutral framework. The focus is on firms to maximise profits, when one takes into account utilities of firms. In building their model, they start from "problem under risk neutrality" (p. 322). Broadly, the profits are a convex function of prices. One of the parameters contributing to the latter shape is the technology. The cash flow is the discounted exercise price of an option. Overall, the value of the firm is a probability density function. The typical risk aversion is driven by risk premium based on Intertemporal Capital Asset Pricing Model (ICAPM). Interestingly, risk premium is collapsed to risk neutral state. In order to enhance their modelling, they incorporated the futures prices. The future prices formula is pretty much similar to Black-Scholes (1973) model.

The results of McDonald and Siegel (1985) illustrate that the option to shut-down can be extended top Cobb-Douglas case. Moreover, that option can be shown by hump-shaped curve, when value is mapped against time. The common real option parameters (spot price, time to expiration, risk-free interest rates, costs, volatility, etc.) reacted to changes in consistent manner to option pricing theory. Lastly, the model should be treated in a rudimentary way.

Moel and Tufano (2002) illustrated switching options (i.e., option to shut down and option to open) in the United States gold mining industry. The data were mainly from *Metallica* 2000 during 1998-1997 period. The independent variables are (i) gold prices, (ii) interest rates, (iii) volatility, and (iv) mine cost structure and prior state (open or closed). Prior studies synthesised by Moel and Tufano (2002) found that (i) fixed costs are function of remaining reserves, (ii) marginal costs are function of current level of production and cumulative amount of already extricated minerals, and (iii) mine technology. Preliminary costs of North American mines showed that fixed and marginal costs are mainly positive and statistically significant. And, the dummies (with and without mine effects) are statistically significant and mainly negative. When one compares empirical findings on known states of variables (prior state, metal prices, volatility, operating costs, shut down costs, reopening costs, maintenance costs, reserves and interest rates), one finds the results of parameters consistent with the predictions of the literature.

Furthermore, the results illustrated that (i) mines that never closed had 50% more costs that mines that never closed before, (ii) mines close due high costs-breakeven point is at $27/ounce, (iii) probability of mine being open increased with increase in gold price, (iv) gold prices is positively correlated with interest rates, and (v) incorporated costs and mine technology explain the closing and opening of mines. The volatility used in this article is incorporated as well, while prior studies such as Model and Tufano (2002) used single-parameter-volatility. In order to strengthen their results, Moel and Tufano (2002) by exploring (i) managerial decisions, (ii) portfolio effects, (iii) effects of multiple owners, and (iv) stakeholders concerns. The results of robustness tests confirm that robustness tests lead to optionality.

Modelling

This article focuses on estimated maximum future profits. Given that the commodity environment is volatility, therefore, one proposes a model that will forecast future profits while taking into account the stochastic nature of commodities. One such model is the future forecasted expected profits:

$$\mathrm{E}_p(S_1) = S_0(p * u + q * d) \tag{1}$$

where $\mathrm{E}_p(S_1)$ can be future expected revenues or profits, S_0 current revenues or profits, u up movement, d down price. And, p and $1-p$ are $p = \frac{e^r - d}{u - d}$ and $q = 1 - p$. The other parameters of Eq. (1) are as follows: $u = e^{\sigma\sqrt{t}}$ and $d = e^{-\sigma\sqrt{t}} = \frac{1}{u}$. Eq. (1) is forecasted future profits or revenues under risk-neutral measure. In order to test the robustness of Eq. (1), the study uses the VAR model. The advantage of using VAR is that models multiple quantities into one portfolio. In the context, numerous parameters are from one commodity; therefore, it is of interest to understand if the relationship of those parameters of a single commodity explains the profitability of different firms.

$$y_t = c + A_1 y_{t-1} + A_2 y_{t-2} + \cdots + A_p y_{t-p} + \mathrm{e}_t \tag{2}$$

where the l-periods back observation, y_{t-1} is called the l-th lag of l-th lag of y, c is a $k * 1$ vector of constants (intercepts), A_j is the time-invariant $k * k$ matrix, and e_t is a $k * 1$ vector of error terms satisfying $\mathrm{E}(\mathrm{e}_t) = 0$, every error term has mean zero. $\mathrm{E}(\mathrm{e}_t \mathrm{e}_t') = \Omega$, the contemporaneous covariance matrix of error terms is Ω (a $k * k$ positive-semidefinite matrix). A simple switching model for the variable z_t involves two AR specifications:

$$z_t = \begin{cases} \alpha_0 + \beta z_t + \varepsilon_t, s_t = 0, \\ \alpha_0 + \alpha_1 + \beta z_t + \varepsilon_t, s_t = 1, \end{cases} \tag{3}$$

where $|\beta| < 1$ and ε_t are i.i.d. random variables with mean zero and variance σ_ε^2. For this study, there will be AR (1) to AR (4). Kola and Sebehela (2020) used VAR for the same reasons as ones that this thesis puts forward to substantiate for the usage of the VAR.

Empirical Application

Data

The weekly aggregated data will be supplied by mining companies (Palabora Copper Mining Limited, AngloGold Ashanti Limited, Gold Fields Limited, Anglo Platinum Limited, and Impala Platinum Limited); it will include economics, production, revenue, costs, including funding, tax, and hedging. The sample is obtained from 1 January 2004 to 31 December 2018. The aggregated data may be defined as quarterly ore production record which consist of five elements or components: (1) the aggregated copper production data, (2) the aggregated copper concentrate purchase data, (3) the aggregated process cost data, (4) the aggregated selling expense data, (5) the aggregated sales volumes data, (6) the aggregated capital cost data, and (7) the aggregated provisions and trust funds.

Firstly, the aggregated copper production data from internal will be derived from the aggregated feedstock production that copper production sources were delivered from mining production, underground production, and Foskor marginal ore production including the copper content and copper recovery in concentrator equipment data. Secondly, the aggregated copper concentrate data in tonnes were purchased from external mine

sources. Then, the copper grade in concentrate aggregation is recorded from the internal and external sources. From the mining database, the aggregated concentrate stockpile at end of period is reported per week and it is stated in tonnes. Thirdly, the aggregated smelter data will include total tonnage of input anode stock at end of period per three months. Next, the aggregated refinery data described the copper grade in anode and the refinery recovery from smelter process. In the end of quarter period, the input cathode stock was weighted in a tonne unit. Finally, the aggregated cathode was collected from the total tonnage purchase data per quarter.

Analysis

Table 1

Amalgamated Forecasted Future Risk-Neutral Profits

Parameter	Historical profits	Future price profits	Future incorporated profits
Mean	0.912	0.888	0.789
Median	0.239	0.476	0.39
Min	0.1	0.162	0.16
Max	2.327	2.393	2.332
Std Dev	1.018	0.927	0.9
Kurt	-1.965	1.484	3.332
Skew	0.83	1.421	1.828

Notes. Mean is the average, Min is for minimum, Max is maximum, Std Dev is for standard deviation, Kurt is for kurtosis, and Skew is for Skewness. The future profits of AngloGold Ashanti, Gold Fields, and Impala Platinum are denominated U.S. dollar except for Palabora Copper Mining Ltd. (data are from 2004 to 2015) and Anglo Platinum (data are from 2004-2019). And profits are in billions. All dollar denominated values are converted into ZAR by multiplying by ZAR/U.S.$. Historical profits are from financial statements of the company. Future price profits are forecasted based on only price volatility and future incorporated profits are based on incorporated volatility.

Before outlining the results in Table 1, here is the salient information that is not included in Table 1. The salient point on the profits of the mining companies based on their income statements are as follows. The average profits for Palabora Copper Mining Ltd. are ZAR2.37 billion, AngloGold Ashanti are U.S.$0.029 billion, Gold Fields are U.S.$0.028 billion, Anglo Platinum are ZAR0.028 billion, and Impala Platinum are U.S.$0.201 billion.

Table 1 illustrates that historical profits are lower than both profits, when one uses future price profits and future incorporated profits. Overall, commodity prices were low in 2009 and highest in 2008-supercycle. The latter pattern is during the 2004-2019 period. The forecasted profits show that future price profits are higher than future incorporated profits, especially when using the mean parameter. One possible reason for that is the future incorporated profits are modelled against volatility, which comprises of prices, metal grades, production rates, and operating costs. That might lead to erosion of profits because of clustering effects of different inputted parameters. Studies such as Fessehaie (2012) illustrated when a number of variables are put together, there is usually clustering effect from the variables and that clustering leads to erosion of some margins in the copper industry. Interestingly, future price and future incorporated profits are positively skewed while historical profits are negatively skewed. The positive skewness in the two profits would imply that there is more information asymmetry embedded in those two profits. Although, the difference seems minute in Table 1 for future price and future incorporated profits; the mining is capital intensive and in terms of value, the difference might be huge.

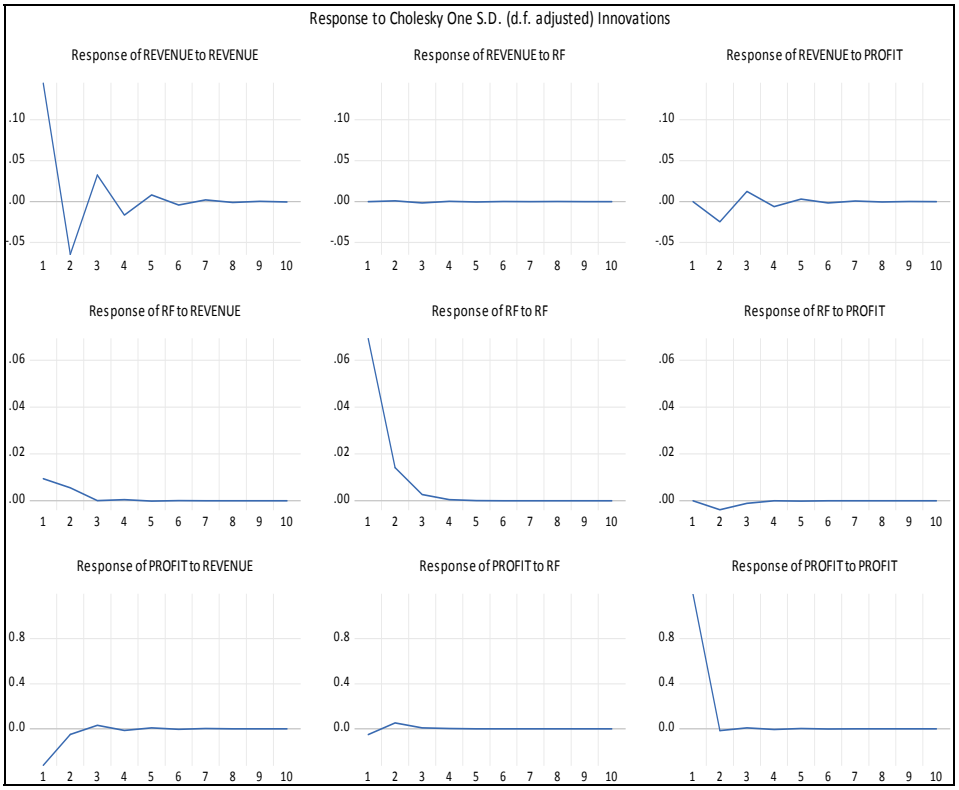
Robustness Test

In running the VAR model, Kola and Sebehela (2020) illustrated how one input parameter in terms of numbering order in the model is important in the type of results one will get. This study uses economic heuristics in assembling the pattern of the parameters. Firstly, the analysis is on commodities. And, the first thing that determines whether a commodity is affordable or not—it is its price based on demand and supply. Then, investors buy the commodity leading to mining companies generating revenues. In determining what price the mining company will sell at, they take into account interest rates. However, the interest rates effect is felt throughout the value change of each commodity. Similar, the operating costs of the mining company. The capital expenditures are funds used to maintain physical operations of the mines. Thus, capital expenditure is incurred before even the commodity is sold. Based on the economic heuristic, then capex will be the first variable, followed by price, then revenue; thereafter, operation costs. Then, interest rates and final profit as profit is the last parameter in the economic value chain. However, based on Eq. (1), the only variable appropriate for this hypothesis is revenue, interest rate and profit because of appropriate synchronisation with Eq. (1)—same and/or similar factors. Numerically, the appropriate VAR parameters can be represented as follows as an inequality;

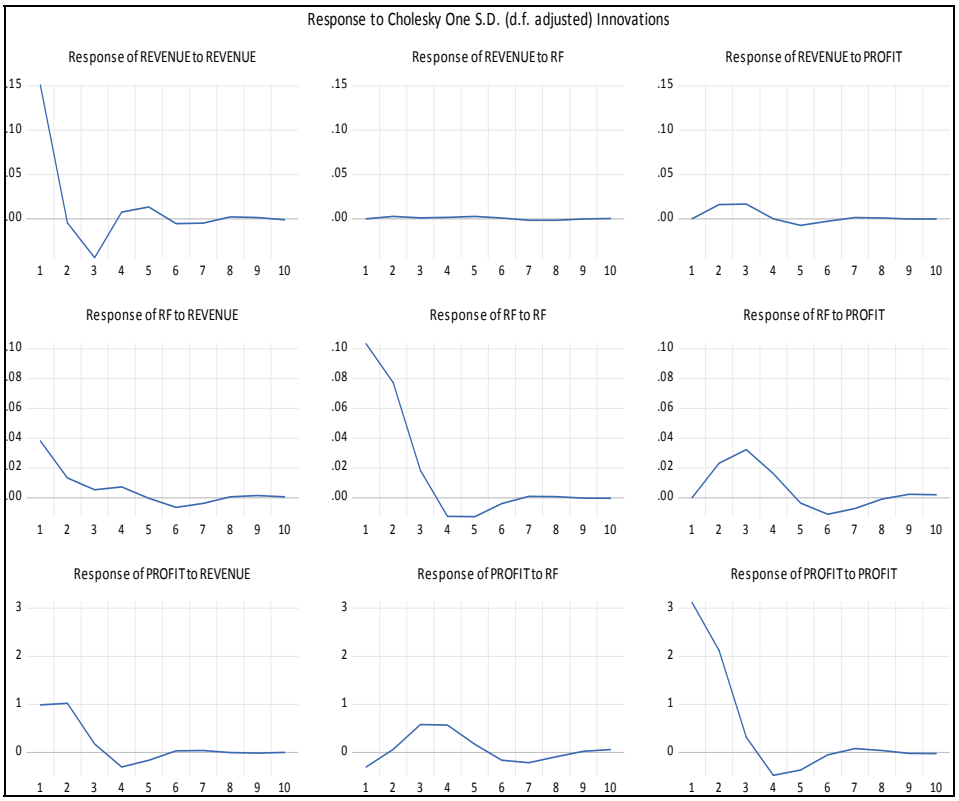
$$\begin{Bmatrix} revenue, (i) \\ interest\ rate, (ii) \\ profit, (iii) \end{Bmatrix} \tag{a}$$

In order to decide which lag is appropriate for our studies; firstly, one draws from prior studies—it can be inferred from Bakshi, Gao, and Rossi (2019) that one lag is appropriate for commodities markets when studying time series because commodities are very much sensitive to immediate changes in the markets. Therefore, "longer" lags will not account effectively for immediate shocks. Secondly, the first and second order tests, and lag-length criterion supporting using one lag for the time series. The graphs of Cholesky decomposition will be interpreted jointly with the VAR (1,1) results in Table 2.

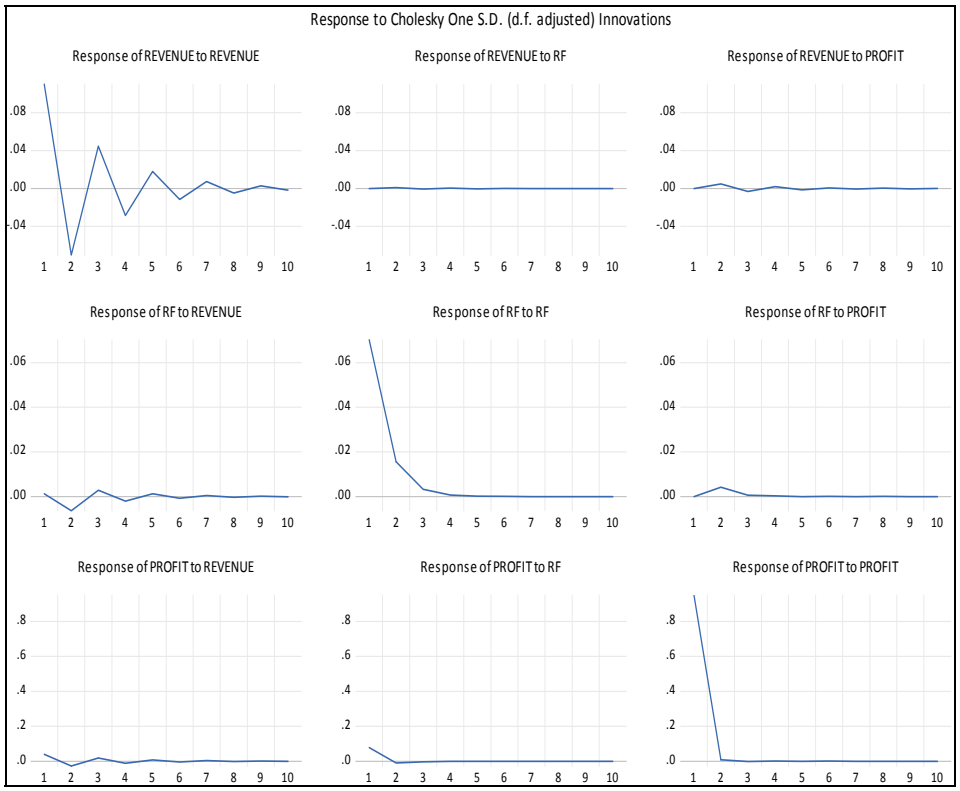
In order to interpret the VAR results one should read them in conjunction with the Cholesky decomposition (Figure 1). Based on Figure 1(e), start with the first curve at the top-left corner. Basically, it tells us changes in revenue reacting to first shocks coming from revenue, then second shocks from interest rates and thirdly, shocks coming from profit. That interpretation of Cholesky decomposition applies to all figures for hypothesis three—Figure 1. So, when one interprets the top-left corner graph of Figure 1(a), it tells one that revenue changes negatively to first shocks from revenue. Then, positively to second shocks from interest rate and negatively to shocks from profits. Note that all first shocks are statistically significant. Although, the second and third shocks are statistically insignificant, those shocks are in line with prior expectations and studies. For figures starting at zero, it means that they have been imposed by Cholesky decomposition. In terms of coefficient signs, interest rate is positive and profit negative—this is consistent with prior studies (Jarociński & Karadi, 2020). In the second row of Figure 1(a), interest rates react positively to the second shocks emanating from interest rate while interest rates do not react to first and third shocks from revenue and profit, respectively. Interestingly, in row 3 of Figure 1(a), profits react negatively to firs shocks from revenue. That would be expected as profits include revenues. According to Choi and Prasad (1995), in the U.S., revenues can be as high 50% of profits depending on exchange rates. Figure 1(a) relates to Palabora Copper Mining Ltd.



(a) Palabora Copper Mining Ltd.

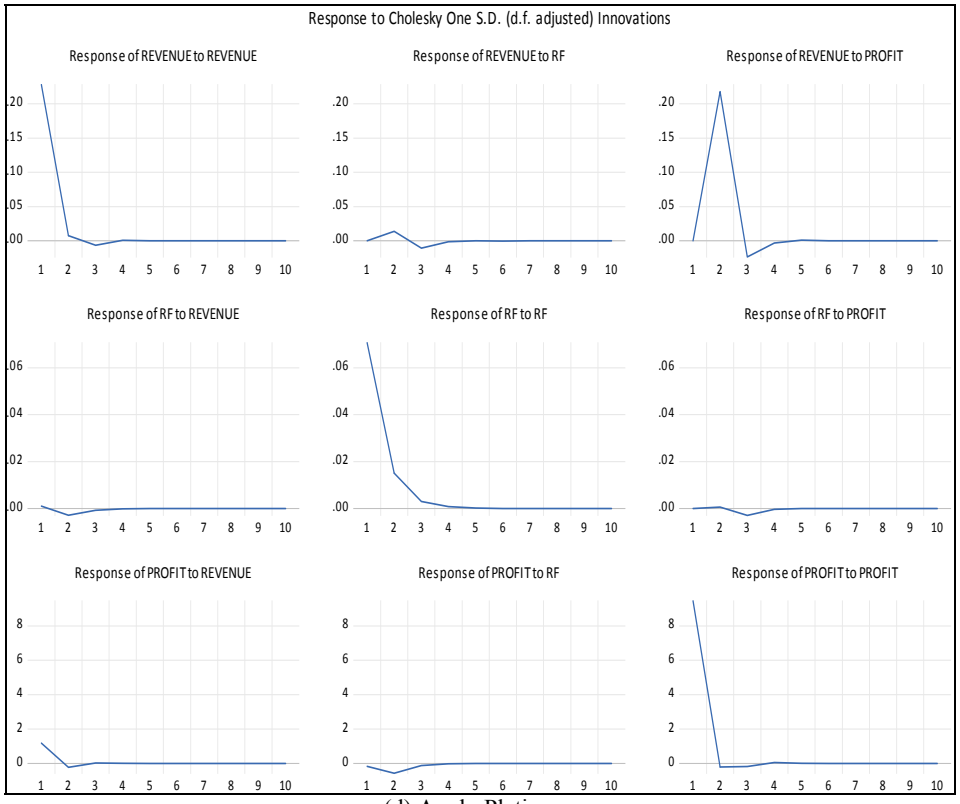


(b) AngloGold Ashanti.



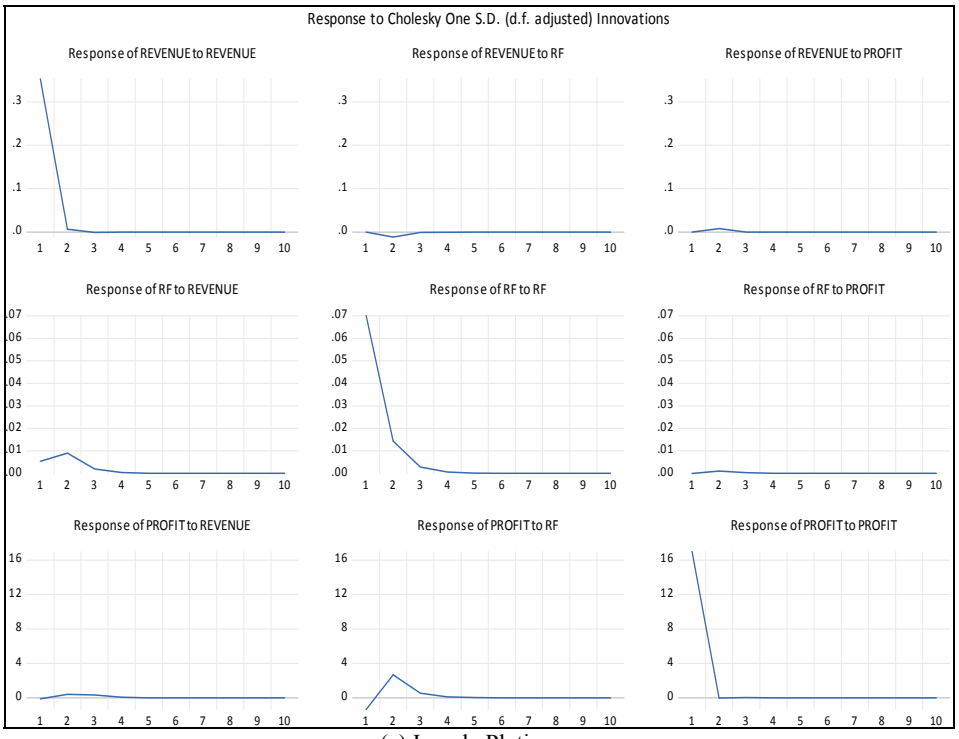
(c) Gold Fields Ltd.

Note. RF stands for interest rates.



(d) Anglo Platinum.

Note. RF stands for interest rates.



(e) Impala Platinum.

Note. RF stands for interest rates.

Figure 1. Cholesky Decomposition.

Table 2

VAR (1,1)

Panel A: Copper			
A1: Palabora Copper Mining Ltd.			
Parameter	Revenue	Interest rate	Profit
Revenue (-1)	-0.489 (-6.414)	0.018 (0.486)	-0.424 (-0.651)
Interest rate (-1)	-0.002 (-0.013)	0.202 (2.429)	0.732 (0.495)
Profit (-1)	-0.021 (-2.031)	-0.003 (-0.645)	-0.015 (-0.167)
F-statistic	13.99	2.409	0.209
Akaike	-0.998	-2.459	3.293
Schwarz	-0.915	-2.375	3.376
B: Gold			
B1: AngloGold Ashanti			
Parameter	Revenue	Interest rate	Profit
Revenue (-1)	-0.074 (-0.875)	-0.155 (-2.504)	1.661 (0.902)
Interest rate (-1)	0.041 (0.327)	0.770 (8.345)	2.621 (0.952)
Profit (-1)	0.005 (1.569)	0.007 (3.114)	0.679 (9.516)
F-statistic	3.049	16.290	18.492
Akaike	-0.905	-1.535	5.256
Schwarz	-0.785	1.415	5.376
B2: Gold Fields			
Parameter	Revenue	Interest rate	Profit
Revenue (-1)	-0.639 (11.311)	-0.0061 (-1.698)	-0.256 (-0.524)
Interest rate (-1)	0.008 (0.068)	0.217 (3.030)	-0.156 (-0.161)

Table 2 to be continued

Profit (-1)	005 (0.619)	0.004 (0.819)	0.009 (0.127)
F-statistic	42.849	4.103	0.107
Akaike	-1.549	-2.453	2.764
Schwarz	-1.481	-2.385	2.832
C: Platinum			
C1: Anglo Platinum			
Parameter	Revenue	Interest rate	Profit
Revenue (-1)	-0.086 (-1.613)	-0.014 (-0.829)	-0.825 (-0.368)
Interest rate (-1)	0.256 (1.106)	0.213 (2.982)	-8.208 (0.850)
Profit (-1)	0.023 (12.727)	6.4e-05 (0.114)	-0.022 (-0.296)
F-statistic	54.327	3.149	0.323
Akaike	-0.095	-2.438	7.369
Schwarz	-0.027	2.369	7.438
C2: Impala Platinum			
Parameter	Revenue	Interest rate	Profit
Revenue (-1)	0.021 (0.288)	0.022 (1.526)	0.604 (0.170)
interest rate(-1)	-0.150 (-0.419)	0.205 (2.864)	38.047 (2.199)
Profit (-1)	0.001 (0.309)	6.01E-05 (0.201)	-0.001 (-0.019)
F-statistic	0.116	3.734	1.652
Akaike	0.774	-2.447	8.531
Schwarz	0.842	-2.379	8.599

Notes. In each cell, the first number is a coefficient and the number in the bracket is a t-test. The t-test is a standard student which is calculated in model from Eviews. All variables highlighted in grey are statistically significant for VAR values as they are at least two irrespective of being negative or positive.

Figure 1(b) is on AngloGold Ashanti. The panel B1 illustrates that all three parameters (revenue, interest rate, and profit) react to only to second shocks emanating from revenue, interest rate and profit, respectively. This is probably because AngloGold Ashanti is the third largest gold producer in the world. Therefore, any variable that affects gold directly and/or indirectly affects profitability of AngloGold Ashanti. Figure 1(d) is on Anglo Platinum and its VAR (1,1) is presented in panel C1. Panel C1 shows that interest rate reacts positively to second shocks coming from interest rate. Similarly, profits react positively to first shocks coming from revenue. The broader context of those findings is similar to ones from Palabora Copper Mining Ltd in Figure 1(e). Figure 1(c) is on Gold Fields. Panel B2 illustrates that revenues of Gold Fields react negative to shocks; firstly, shocks from revenues. For mining industry that would make sense as gold is regarded both as currency and "safe-haven", especially during turbulent times. And, some years were turbulent during the 2004-2019 period. Similarly, interest rate reacts positively to second shocks coming from interest rates. These are similar findings to ones based in Figure 1(e). Generally, copper and gold "control" commodities prices as they are widely used by different stakeholders throughout the world. Figure 1(e) is for Impala Platinum. They only variable that statistically significant is interest rate-react positively to second shocks coming from interest rates. This finding has been synthesised earlier. It seems that for platinum, interest rate is the cost of owning physical platinum. Some asset managers who hold platinum funds use interest rate when deciding to either increase or decrease their exposure to platinum stocks. Finally, statistical inference measures (i.e., Akaike and Schwarz) show that some VAR results are not normally distributed. That might have the data set for each parameter is 180 data points instead of at least 500 data points.

Conclusion

Metal price plays a critical role in the maintenance of profits since the performance of production rates and operating costs are limited by constant values (Dooley & Lanihan, 2005). However, it is almost certain that production rates, operating costs, and other parameters in mining projects are generally associated with the future uncertainty contributing to the profits. The research to date has tended to focus on metal prices forecasting rather than future maximum profits forecasting. The synthesis of maximum future profits probabilities was done according to RNPs. A major advantage of RNP is that it has flexibility to be adjusted with actual data (Le Courtois & Quittard-Pinon, 2006). Investigating RNP is a continuing concern within volatility that traditionally has been assessed by measuring a single volatility. Such approach, however, has failed to address the future uncertainty that is suggested to develop an advanced volatility by incorporating technical and financial uncertainties in mining projects. The results of this study were approved by comparison between historical profits to a single volatility and an incorporated volatility. The average scores of different between historical profits and price volatility was -65% while an incorporated volatility was at -42%. The data reported here appear to support the assumption that an incorporated volatility has a correlation to identify the future uncertainty to forecast the future profits probability.

Many researchers have utilised the RNP approaches using a price volatility to measure the future probability of profits (Trigeorgis, 1993; Rubinstein, 1994; Dooley & Lanihan, 2005). However, these results therefore need to be interpreted with caution since a number of important variables in mining projects are not taken into account. Initial observations suggest that there may be a link between these variables and future uncertainty to influence accuracy in forecasting the future profits. A reasonable approach to tackle this issue could be to propose an incorporate volatility in the RNP approaches in calculating the future profits. Comparing two results between single volatility and an incorporated volatility, it can be seen that the RNP approaches using an incorporate volatility can optimise the future profits since it has a smaller difference mean to the historical profits than a single volatility.

Declarations

The authors have nothing to declare including any conflict of interest.

References

Bakshi, G., Gao, X., & Rossi, A. G. (2019). Understanding the sources of risk underlying the cross section of commodity returns. *Management Science, 65*(2), 619-641.

Chernov, M., & Ghysels, E. (2000). A study towards a unified approach to the joint estimation of objective and risk neutral measures for the purpose of options valuation. *Journal of Financial Economics, 56*(3), 407-458.

Choi, J. J., & Prasad, A. M., (1995). Exchange risk sensitivity and its determinants: a firm and industry analysis of US multinationals. *Financial Management*, *24*(3), 77-88.

Cortez, C. T., Saydam, S., Coulton, J., & Sammut, C. (2018). Alternative techniques for forecasting mineral commodity prices. *International Journal of Mining Science & Technology, 28*(2), 309-322.

Davis, G. A. (1998). Estimating volatility and dividend yield when valuing real options to invest or abandon. *Quarterly Review of Economics & Finance, 38*(3), 725-754.

Dooley, G., & Lenihan, H. (2005). An assessment of time series methods in metal price forecasting. *Resources Policy, 30*, 208-217.

Fessehaie, J. (2012). What determines the breadth and depth of Zambia's backward linkages to copper mining? The role of public policy and value chain dynamics. *Resources Policy, 37*(4), 443-451.

Jarociński, M., & Karadi, P. (2020). Deconstructing monetary policy surprises—the role of information shocks. *American Economic Journal: Macroeconomics, 12*(2), 1-43.

Kola, K. V., & Sebehela, T. (2020). The interdependence of indexed volatilities. In Mehmet K Terzioğlu (Ed.), *Handbook in linear and non-linear financial econometrics-theory & practice* (Chapter 7, pp. 127-154). London: IntechOpen.

Le Courtois, O., & Quittard-Pinon, F. (2006). Risk-neutral and actual default probabilities with an endogenous bankruptcy jump-diffusion model. *Asia-Pacific Financial Markets, 13*(11), 11-39.

McDonald, R. L., & Siegel, D. R. (1985). Investment and the valuation of firms when there is an option to shut down. *International Economic Review, 26*(2), 331-349.

Moel, A., & Tufano, P. (2002). When are real options exercised? An empirical study of mine closings. *Review of Financial Studies, 15*(1), 35-64.

Rubinstein, M. (1994). Implied binomial trees. *Journal of Finance, 49*(3), 771-818.

Scott, R. C., Highfill, J. K., & Sattler, E. L. (1988). Advantage to a risk neutral firm of flexible resources under demand uncertainty. *Southern Economic Journal, 54*(40), 934-949.

Trigeorgis, L. (1993). Real options and interactions with financial flexibility. *Financial Management, 22*(3), 202-204.