

The Win-Win-Win Papakonstantinidis Model in the Ultimatum Game

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In this article it is investigated the case of a three-part ultimatum game, where the proposer makes repeated sharing suggestions to the responder and a “third”. This “third” is the regulatory factor of recurring bids, at a rate—at each time of the immediately preceding. This article answers to question like “what share would the proposer be willing to give to the responder in the case of three taking part in the ultimatum game?” Which would be moral and which fair? At which point of distribution would it be accepted by the responder? A Chi Square (χ^2) test is used. For this reason, after investigating the social integration factors through the literature, two small surveys trying to identify the distribution rate are conducted. We conclude that a share 0.40 could be accepted by the responder and the “third”.

Keywords: ultimatum game, the bargaining problem, win-win-win model sharing problem, sociological aspects

Introduction

The Ultimatum Game, introduced by Güth, Schmittberger, and Schwarze (1982), is a simple, take it-or-leave-it bargaining environment. In ultimatum experiments two people are randomly and anonymously matched, one as proposer and one as responder, and told they will play a game exactly one time. The proposer is endowed with an amount of money, and suggests a division of that amount between herself and her responder. The responder observes the suggestion and then decides whether to accept or reject. If the division is accepted then both earn the amount implied by the proposer’s suggestion. If rejected, then both the proposer and responder earn nothing for the experiment.

The key result of ultimatum experiments is that most proposers offer between 40% and 50% of the endowed amount, and that this split is almost always accepted by responders. When the proposal falls to 20% of the endowment it is rejected about half of the time, and rejection rates increase as the proposal falls to 10% and

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lower. As discussed by Camerer (2003), ultimatum game results are highly robust to a variety of natural design manipulations (e.g., repetition, stake size, degree of anonymity and a variety of demographic variables).

An important exception to the robustness results is reported by Hoffman and Spitzer (1985), who show that offers become significantly smaller, and rejections significantly less frequent, when participants compete for and earn the right to propose. An explanation is that this procedure changes the perception of “fair”, and draws attention to the importance of context in personal (as compared to market) exchange environments. Effects might also stem from varying the degree of anonymity among the subjects, or between the subjects and the experimenter (Hoffman, 1996).

A key focus of recent ultimatum game research has been to understand why responders reject low offers. Economic theory based on self-interested preferences suggests responders should accept any positive offer and consequently, proposers should offer the smallest possible positive amount (Daniel, & Kevin, 2014).

Literature Review

The Ultimatum game is a behavioral economics exchange game that is played over numerous trials. The situation places the monetary interests of two people into close association (Güth, Schmittberger, & Schwarze, 1982).

According to the classical theory, any strategy pair “offer $1-x$ ”, “accept $1-x$ ” corresponds to a Nash equilibrium, if the responder can convey a credible threat that she would reject any offer lower than $1-x$ or when this is common knowledge. The strategy profile can be written as pairs $(x, f(x))$ for all x between 0 and 1, where $f(x)$ is a bi-valued function expressing whether x is accepted or not (denoted by A and R). $(x, f(x))$ is a Nash equilibrium of the UG for some x , if $f(x) = \text{“accept”}$ and there is no $x' > x$, such that $f(x')$ would be “accept” (Schuster, 2017).

Simultaneous ultimata games compare social expectations and self-interests in a multi player bargaining experiment. Expanding the standard ultimatum game from two to more players has been done for the receiving side. The core bargaining relation between proposer and responder does not change fundamentally when a passive dummy player is added as an additional receiver who has no option to reject the overall allocation. Dummy players mostly receive comparably small amounts (Güth, & van Damme, 1998; Kagel, & Wolfe 2001), and responders frequently reject extreme offers (i.e., above 50% of the pie) to the dummy player when the responder’s payoff does not depend on this decision and is independent of the proposer’s allocation (Bosch-Rosa, 2015). Generally, differentiating roles and information in three-player bargaining can lead to systematic variations of inequity aversion (i.e. Bolton, & Ockenfels, 1998). Furthermore, when responders in the ultimatum game compete for the offer of one proposer, lower offers are less often rejected (Halko, & Seppala, 2006). Grosskopf (2003) experiments with one proposer bargaining at the same time with three responders, who have to individually agree to their division (or not), and where only one proposer-responder pie division out of the accepted proposals is randomly selected. The overall decreasing offers over repetitions are explained by reinforcement or directional learning. Güth, Ockenfels and Wendelb (1997) implement stronger competition with five responders with only the one with the lowest acceptance threshold (in a strategy method experiment) receiving their division. A trade-off between fairness and responder competition is proposed to account for overall lower (and further decreasing) offers.

An example of expanding the ultimatum game without inducing direct competition between the responders is provided by Knez and Camerer (1995). Here a proposer simultaneously sends offers to two

formally independent responders. Expectations of half of the responders change with an increasing offer made to the other responder: some expect a larger and others a lower share of the total amount available. The authors assume that subjects dislike lower payoffs compared to the other players, however, responders were ambiguous with whom to compare, i.e. some compare with the other responder (decreasing willingness to accept a given offer depending on the size of the other responder's offer) and some compare with the proposer (increasing willingness to accept a given offer depending on the size of the other responder's offer). Thus, inequity aversion can be directed toward a person in the same role or toward a person in the opposing role. Also expanding the number of proposers appears to be a useful experimental expansion of the standard two-player ultimatum game, as often several offers are received in parallel in the real world. Roth et al. (1991) provide a study of ultimatum bargaining with multiple proposers, but here direct competition is induced as the responder is forced by the experimental design to accept only the highest offer. Under this specific bargaining structure results converge to the subgame perfect equilibrium of maximum offers. Responders with multiple ultimatums might follow strategic concerns of how to increase the (future) offers they (will) receive, or they might have social considerations regarding social norms, for example in the form of inequity aversion or pure altruism. Social considerations can also take place in the form of rivalry, where happiness results from the relative advantage over another player (Hopkins, 2008). On the one hand, this could be understood in evolutionary terms as a relative strategic advantage for obtaining resources (in evolutionary terms offspring). On the other hand, perception of relative success or failure can serve self-regulatory purposes and can provide help with setting reasonable goals (i.e. aspiration levels).

Especially in a behaviorally new domain, where information about the execution of a social norm is not readily available, offers could be evaluated in reference to each other. Many studies document influences of social comparisons on individual behavior. For example Cason and Mui (1998) study a sequential dictator game where dictators observe the offers made by other dictators and find that dictators maintain other regarding preferences when information about the behavior of another dictator is provided, which is usually not the case when this kind of information is lacking. Responders in the ultimatum game feel treated worse when another responder is treated more kindly, as more offers are rejected when a responder observes the average offers made to other responders (Bohnet, & Zeckhauser, 2004) -possibly enforcing a social norm, and outcomes generally further deviate from the subgame perfect equilibrium when providing the information about others' behavior in the same situation (Duffy, & Feltovich, 1999).

When structural information is not directly available, players could be more prone to a variety of social heuristics, while uncertainty could influence offer levels in the ultimatum game positively as well as negatively. If there is information asymmetry concerning the available pie to divide, this proposer's advantage is usually exploited by offering the equal split division of the minimum pie possible (Güth et al. 1996). With role uncertainty, when it is randomly decided who the receiving and who the providing side in a three-player dictator game is (i.e. "solidarity game" Ernst, & Klaus 1999), overall increased giving are observed.

Research Method

The paper used selected questionnaires from the urban centers of the Peloponnese district where questionnaires with closed questions were completed.

It is applied the χ^2 test on this data looking for correlations between acceptable offerings of the proposer in specific fields (eg loans, salaries, cakes) from the respondents.

Then the research tried to define an acceptable solution, by using the combination of the three mathematical constants i.e the φ, π, e .

Finally, it tries to find the approximate offer accepted by the respondent via the limit of a series of trial-error sequences, by also using the combination of φ, π, e .

Analysis

The Ultimatum game is identical to the Dictator game except that the recipient can reject the proposed allocation (Güth et al., 1982). If she rejects it, both players receive nothing. Rejections are evidence of negative reciprocity (Rabin, 1993), the motive to punish players who have treated you unfairly, or inequity aversion (Fehr, & Schmidt, 1999), which is a distaste for unfair outcomes. The amount a recipient loses by rejecting a proposed allocation serves as a measurement of the strength of these motives. Offers of less than 20% are rejected about half the time; proposers seem to anticipate these rejections and consequently offer on average approximately 40%. Cross-cultural studies, however, show that across small-scale societies, ultimatum offers are more generous when cooperative activity and market trade are more common (Ernst, & Ian (2014).

The game was also played with chimpanzees, using raisins as the good to be shared. Responders among chimpanzees were satisfied with much lower offers. These deviating results are easy to explain. Due to the lower intellectual capacities of chimpanzees in comparison to humans, they cannot anticipate the effect of rejecting low offers and focus more on the instantaneous benefit.

The UG has been studied so extensively in experiment that Camerer wrote: “We should declare a moratorium on creating Ultimatum Game data and shift attention toward new games and new theories. Good new theories codify precisely what fairness is ...” In the present contribution, we address this question.

We denote the fraction the proposer wants to keep by x . Then the offer is $1-x$. Some authors distinguish between two parameters: p as the average offer (denoted here by $1-x$) and q as the minimum offer responders will accept, where clearly $q < p$ and also $q < 1-p$ because responders would not reject a fraction they would ask for when being a proposer. To avoid a proliferation of symbols, we do not introduce general symbols for the payoffs here. It is obvious that the payoffs of the proposer and responder are x and $1-x$, respectively, if the offer is accepted and both are zero if not.

According to the classical theory, any strategy pair “offer $1-x$ ”, “accept $1-x$ ” corresponds to a Nash equilibrium, if the responder can convey a credible threat that she would reject any offer lower than $1-x$ or when this is common knowledge. The strategy profile can be written as pairs $(x, f(x))$ for all x between 0 and 1, where $f(x)$ is a bi-valued function expressing whether x is accepted or not (denoted by A and R in Fig. 1). $(x, f(x))$ is a Nash equilibrium of the UG for some x if $f(x) = \text{“accept”}$ and there is no $x' > x$ such that $f(x')$ would be “accept”.

Somewhat paradoxically, the large number of Nash equilibria includes solutions in which the responder accepts $(1-x)$ but would reject some or all larger offers. It is very plausible to restrict the solutions such that accepting some offer implies that all larger offers would be accepted as well. Even then, however, the number of Nash equilibria is too large to be applicable. Thus, the Nash equilibrium is a “weak” concept in the context of the UG and related bargaining games. Thus, it is necessary to narrow down the solutions, in agreement with observations.

A frequently used way of restricting the set of solutions in sequential games is to only consider subgame-perfect Nash equilibria that concept can be explained as follows. Sequential games can be written as

decision trees (so-called extensive forms) (Fig. 1). A Nash equilibrium is subgame-perfect if at any node in the game tree, that equilibrium remains a Nash equilibrium of the subgame starting from that node 1. If we assume that the smallest unit of the used currency is such that the lowest non-zero offer is 1% (or 10% then the strategy pair “offer $1-x = 1\%$, accept this” (10% in the case) is the only subgame-perfect Nash equilibrium, the so-called rational solution. This is because the responder does not have an incentive to decline that offer, as it is still larger than 0, and she cannot convey any threat after the proposer has made an offer. The proposer would not deviate either from that strategy because it grants her the largest amount.

The subgame-perfect Nash equilibrium may, alternatively, be explained from a population perspective. In an initial population of responders where all of them decline any offer below 0.5, a subpopulation will start accepting

Lower and lower offers. This strategy would grant them an advantage, at least in the short-term. There are different opinions on whether or not games played in populations subject to infrequent mutations between strategies lead to subgame-perfect Nash equilibria. Gintis found, for the UG, that evolutionary dynamics with recurrent perturbations do not converge to the subgame-perfect equilibrium as the perturbation rate approaches zero.

As outlined above, the manifold experiments with the UG in diverse ethnical communities show that proposers offer a sizeable portion of the good and low offers are usually rejected. Thus, in the case of the UG, classical theory is at odds with observations. One way of deriving solutions other than the subgame-perfect Nash equilibrium in the UG is to consider perturbations in the choice of strategy here, we will derive an alternative solution concept without explicitly considering perturbations.

Although the UG is a one-shot game, most people assume that it would be iterated. In experiments with a repeated UG, it was found that learning led to convergence to offers near even division (equipartition). It is generally assumed that people have learned rules of behavior from iterated games that they apply also to one-shot games. This may explain why very low offers are rejected. Responders tend to “punish” proposers who make low offers, hoping that they will make larger offers in the next round. Note that the proposer in the next round need not be the same person. In fact, the proposer may assume that in a future round of the game, she will be a responder and the proposer who was a responder before might retaliate. Since, usually, people in a population can communicate, it is common knowledge that selfishness would be punished and would imply a low reputation-see the scheme below.

In computer simulations based on an evolutionary approach, Nowak and coworkers observed that populations of players evolve their strategies so as to converge to equipartition, provided that a sufficiently large fraction of players is informed about any one offer accepted previously. The threshold of the offer below which all responders would reject converged to about 0.4. By computer simulations using adaptive dynamics and invoking a principle of empathy, similar results were found. Simulations indicate that taking into account stochastic effects in that players make mistakes when judging the payoffs and strategies of others favors fairness even more. All these results depend, of course, on the assumptions made in the simulations.

The proposer chooses her strategy first and has (in this special game) 11 options. Offers are given by the numbers at the upper branches. The responder chooses second, with A and R standing for “accept” and “reject”, respectively. Payoffs are indicated below the lower branches in the sequence proposer, responder. Underlined options “1, A” correspond to the subgame-perfect Nash equilibrium. Any strategy pair “offer p , accept p and

reject every lower offer” with $p > 10\%$ is not subgame-perfect because after the proposer had offered 1 (i.e. 10%), rejecting that would not be the best choice by the responder at that stage, that is, in the resulting subgame. Offers indicated in bold are empirically the most frequent ones and are usually accepted. The offer “4” is nearest to the GR in this special game. The offer “0” is a degenerate case because then the payoff for the responder is zero irrespective of her choice (Stefan, 2017).

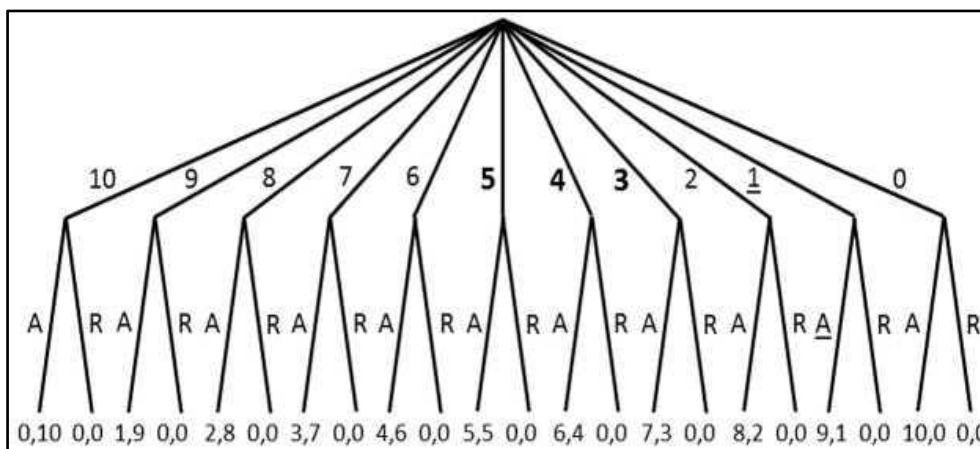


Figure 1. Game tree for an UG with 10 € being the total amount and 1 € being the smallest unit.

Proposer-Responder-the “Third”

The Golden Ratio ϕ Share. The Ultimatum Game is a paradigmatic two-player game. A proposer can offer a certain fraction of some valuable good. A responder can accept the offer or reject it, implying that the two players receive nothing. The only subgame-perfect Nash equilibrium is to only offer an infinitesimal amount and to accept this. However, this equilibrium is not in agreement with experimental observations, which show varying accepted offers around 40%. While some authors suggest that the fairest split of 50% vs. 50% would be explainable on theoretical grounds or by computer simulation, a few authors (including myself) have recently suggested that the Golden Ratio, about 0.618 vs. about 0.382, would be the solution, in striking agreement with observations. Here we propose a solution concept, based on an optimality approach and epistemic arguments, leading to that suggested solution. The optimality principle is explained both in an axiomatic way and by bargaining arguments, and the relation to Fibonacci numbers is outlined. Our presentation complements the Economic Harmony theory proposed by R. Suleiman and is based on infinite continued fractions. The results are likely to be important for the theory of fair salaries, justice theory and the predictive value of game theory.

The Ultimatum Game is a famous asymmetric, sequential two-player game intensely studied in Game Theory. Game Theory has become a large and powerful theoretical framework, where entities such as countries, companies, people, animals, plants and living cells down to molecules are considered as players. The Ultimatum Game (abbreviated UG henceforth) had been devised by Güth and coworkers. It has the following minimalist rules: One player, called the “proposer”, is handed a valuable good, say 100 €. She is to offer any part of it to the second player, called the “responder”. The responder can choose between two strategies: to accept or to reject. If she accepts, the money is shared according to the offer. If she rejects, neither of the players receives anything. This game is both sequential and asymmetric because one player makes her move first and because the players have different sets of strategies.

There is a slightly older bargaining game proposed by Rubinstein. In that game, after every offer, the other player decides either to accept or to reject it, in which case she is to continue the bargaining. Costs of bargaining are considered, either in the form of fixed costs or of discounting factors. Note that it is not the iterated version of the UG because players swap roles after a rejection.

The standard UG is a model game for analyzing questions in economics such as fair salaries and fairness in economics in general. There is a case that at very large total amounts, say 10 million €, low fractional offers, say 10% might be accepted because the responder might not dare to decline 1 million €¹³. This point is still under debate in the literature. The experimental observations by Camerer and Thaler showed that the outcome changed only insignificantly if stakes were raised. Cameron performed experiments in Indonesia, which allowed her to raise stakes to three times the monthly expenditure of the average participant. The observed result was that offers were largely independent of the total amount while the responders' acceptance rates increased at higher amounts. Andersen reviewed the literature on this point and, by experiments in India, largely confirmed Cameron's results. In particular, they found that sufficiently high stakes led responder behavior to converge almost perfectly to full acceptance of low offers. In what follows, we consider so-called low-stake experiments, that is, we assume that the total amount is not very large (say, less than 10% of a monthly income), so that (in agreement with observations) it does not really matter for the decision, and we argue in terms of percentages (Daniel, 2018).

There is a huge body of experimental results on the UG available. Roth and coworkers run the UG in four different cities: Jerusalem, Ljubljana (Slovenia), Tokyo and Pittsburgh. The results were strikingly similar.



Figure 2. The golden ratio in the natural world.

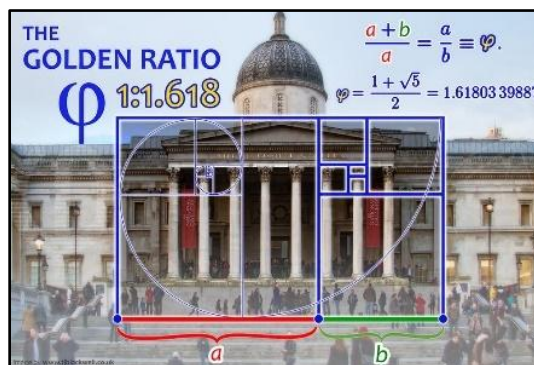


Figure 3. Golden Ratio in architecture.

In the last ten years, several authors started, first independently of each other, studying the Golden Ratio (GR) in the field of Justice Research and/or in relation to the UG. The GR, often denoted by Φ , is defined by the value $(\sqrt{5} + 1)/2$, which equals about 1.618, and is also known as the Golden Section or Golden Mean. The GR had been defined by Euclid in his “Elements” about 300 BC. Not mentioning the UG, Guillermina Jasso wrote that the GR has come to light in the context of the justice evaluation function (JEF), which compares the actual reward and the just reward. In that context, it is worth noting that Rubinstein distinguishes the positive question, “What is the agreement reached in practice” from the normative question, “What is the just agreement”? Jasso discusses the potential role of the GR in loss aversion. These ideas entered a monograph by Vermunt. He stated without proof that a fair division (the “justice zone”) should lie between the equal split and the inverse GR, which is about 0.618. This was, in turn, discussed by Jasso. She wrote: “it is possible that further theoretical work could yield a prediction for the golden ratio justice zone” (Stefan, 2017).

Manfred Langen suggested that the GR would be a plausible solution to the UG and published it in online magazines on economics. Without using mathematics, he argued that a ratio that occurs frequently in nature and is known to mankind since ancient times, is accepted most easily as well-proportionate and justified.

Ramzi (2018) suggested the same ratio to be the solution. Suleiman had presented the idea in lectures in several talks back in 2013, e.g. at a conference in Israel. His idea is based on one out of several definitions for the GR:

$$(1-x)/x = x \quad (1)$$

That equation says that the ratio between the smaller and larger fractions is the same as the ratio between the larger fraction and the whole. This leads to the solution:

$$x^* = 5 - \sqrt{-12} \cong 0.618\dots, \quad x^* = 5 - 12 \cong 0.618\dots, \quad (2)$$

which is the reciprocal of the GR. Note that x^* is an irrational number. The GR is used frequently in architecture, painting and photography and occurs in phyllotaxis (leaf positions on plants). The evolution of plants has led to situations avoiding that leaves would be exactly above each other, that is, that integer numbers of leaves would fit into integer numbers of turns so that they minimize taking away sunlight from the others.

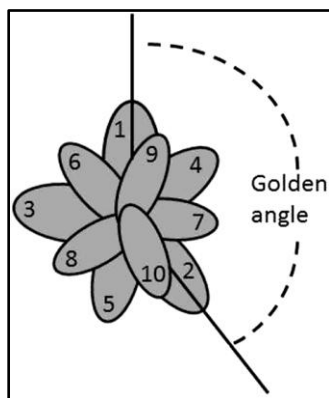


Figure 4. Schematic representation of plant leaves positioned according to the Golden Ratio.

The two solid lines make the Golden angle of $360^\circ(1 - 1/\Phi) \cong 137,5^\circ$. This arrangement implies that the overlap between leaves (numbered in ascending order) is minimized, since the GR can be approximated poorly by rational numbers t/n , where t and n are the number of turns and leaves, respectively. Analogously, a division of the “cake” in the UG following the GR can only be approximated poorly by ratios of small integers.

Research

Papakonstantinidis Proposals Scenarios

There is not a standard percentage for all the forms of the ultimatum games.

The answer is, it depends: According to Ramzi (2018), experiments on bargaining games have repeatedly shown that subjects fail to use backward induction, and that they only rarely make demands in accordance with the subgame perfect equilibrium. In a recent paper, he proposed an alternative model, termed “economic harmony” in which we modified the individual’s utility by defining it as a function of the ratio between the actual and aspired pay-offs. He also abandoned the notion of equilibrium, in favor of a new notion of “harmony”, defined as the intersection of strategies, at which all players are equally satisfied. He showed that the proposed model yields excellent predictions of offers in the ultimatum game, and requests in the sequential common pool resource dilemma game. Strikingly, the predicted demand in the ultimatum game is equal to the famous Golden Ratio (approx. 0.62 of the entire pie). The same prediction was recently derived independently by Schuster (2017).

A proposer can offer a certain fraction of some amount of a valuable good, for example, money. A responder can either accept, in which case the money is shared accordingly, or reject the offer, in which case the two players receive nothing. While most authors suggest that the fairest split of 50% vs. 50% would be the equilibrium solution, recently R. Suleiman (An aspirations-homeostasis theory of interactive decisions (2014)) suggested the Golden Ratio, 0.618, to be the solution and argued that such a partitioning would be considered fair by both sides. He provided a justification in terms of an approach termed aspirations-homeostasis theory. The main idea is that responders tend to accept the minor fraction of the Golden Ratio because they feel that this fraction equals, in comparison to the larger fraction obtained by the proposer, the ratio of the larger fraction and the whole amount.

Scenarios

Based on this-mainly-work, with backward inductions in our research we find two alternative cases:

In the first case, the recipient may accept an offer just above 0.4; in the second case the recipient, under pressure from the negotiating force of the proposer, accepts even 0.112.

The two scenarios proposed: The “good” and the “bad” between Allocators’ and Recipients’.

Scenario (A): The Chi Square Test

The sample: 862 persons from city centers of Peloponnese.

Table 1

Sex Frequencies

Males	Females
502	360

Table 2

Educational Level

Educational level	Frequencies
Primary school	217
High School	399
University level	220

Table 2 to be continued

M. Sc.	25
Ph.D.	1
Total	862

Table 3

The Data

Frequencies categories	Salaries	Loan	Cake	Total
Recipients 1	80	18	40	138
Recipients 2	42	24	32	98
Recipients 3	58	43	45	146
Recipients 4	43	15	50	108
Recipients 5	57	32	68	157
Recipients 6	41	28	41	110
Recipients 7	40	21	44	105
total	361	181	320	862

Expected frequencies:

$$E_{cal} = \frac{\Sigma r \Sigma c}{total}$$

Table 4

Expected Frequencies

Frequencies categories	Salaries	Loan	Cake
Recipients 1	80-57, 7	18-26	40-46
Recipients 2	42-41, 04	24-20, 57	32-36, 38
Recipients 3	58-61, 14	43-30, 65	45-54, 19
Recipients 4	43-45, 2	15-22, 7	50-93, 1
Recipients 5	57-65, 37	32-32, 96	68-58, 28
Recipients 6	41-46, 06	28-23, 09	41-40, 83
Recipients 7	40-43, 97	21-22, 04	44-38, 97

The Formula for chi-square is:

$$\chi_c^2 = \Sigma \frac{(O - E)^2}{E}$$

where:

C = Degrees of freedom;

O = Observed value(s);

E = Expected value(s);

Degrees of freedom (df):

$$(r - 1)(c - 1) = (7 - 1) * (3 - 1) = 12$$

$$\alpha = 0.05$$

Table 5
Critical Values

Critical values of the Chi-square distribution with d degrees of freedom							
Probability of exceeding the critical value							
d	0.05	0.01	0.001	d	0.05	0.01	0.001
1	3.841	6.635	10.828	11	19.675	24.725	31.264
2	5.991	9.210	13.816	12	21.026	26.217	32.910
3	7.815	11.345	16.266	13	22.362	27.688	34.528
4	9.488	13.277	18.467	14	23.685	29.141	36.123
5	11.070	15.086	20.515	15	24.996	30.578	37.697
6	12.592	16.812	22.458	16	26.296	32.000	39.252
7	14.067	18.475	24.322	17	27.587	33.409	40.790
8	15.507	20.090	26.125	18	28.869	34.805	42.312

For $\alpha = 0.05$ and degrees of freedom (df) = 12, the critical value, $\chi^2_{critical} = 21.026$.

Table 6
Chi Square Test: Comparing $\chi^2_{calculated}$ With $\chi^2_{critical}$

Nr	$\chi^2_{calculated}$	$\chi^2_{critical}$	Notes
1	$8.618 + 2.461 + 0.782 = \mathbf{11.861} < ..$	21.026	H ₀ rejected
2<	21.026	H ₀ rejected
3<	21.026	H ₀ rejected
4	$0.100 + 2.610 + 19.90 = \mathbf{22.610} >$	21.026	H ₀ accepted
5<	21.026	H ₀ rejected
6<	21.026	H ₀ rejected
7<	21.026	H ₀ rejected

The Hypothesis that people are-about-willing to accept a salary of just over 40% or a loan of around 15% but a share of cake at 50%, has been accepted. It seems, therefore, that the bargaining power at this particular moment of the ultimatum is crucial. However, they are willing to accept a salary just above that 40% of those who ask. It is generally argued here that 60-40 is the most likely and not the 61.8-38.2 suggested by Suleiman.

We observe that the number. 40 results from the combination use of the three mathematical constants, ϕ , π , e which seems to be closer to the rule of law and justice.

$$\begin{aligned} \phi &= 1.618 \\ \pi &= 3.14 \\ e &= 2.718 \end{aligned}$$

indeed,

$$\frac{\phi}{\pi} \times e = \frac{1.618}{3.14} \times 2.718 = 1.4005$$

A three-player three-option ultimatum game. Hans-Rudiger Pfister and Gisela Bohm assumed that general feelings of satisfaction and dissatisfaction as well as specific emotions can play a role when responders decide to accept or reject the offer of the proposer. While this idea has been expressed before in the game theory literature relatively little direct experimental work has been done along these lines. A mounting literature

confirms the importance of general affect as well as of specific emotions in individual decision-making but only few studies has examined the preconditions and psychological mechanisms of affect and emotions in strategic games. With respect to general affect, Kirchsteiger, Rigotti, and Rustichini (2006) induced states of good or bad mood and found that bad mood leads to relatively more reciprocity, whereas good mood leads to more generosity of second movers in a gift exchange game. For the ultimatum game, this suggests that rejections might be more likely when responders are in a negative mood, implying a propensity to punish and retaliate.

Repeated three-player ultimate games. The existence of multiple proposers allows the responder to directly assess an offer based on another offer received simultaneously and investigates the interdependence of simultaneous ultimate and corresponding behavioral dependencies. In contrast to Bohnet and Zeckhauser (2004) or Knez and Camerer (1995), responders do not evaluate whether they are treated less or more kindly compared to another player in the same position. With two simultaneous ultimate, responders can evaluate which of the two offers is in line with their social expectations and which is not. Furthermore, the information about the existence of other proposers is also provided for the proposer (Philipp, & Daniel, 2018).

Ultimatum bargaining behavior is mostly explained by social comparison and corresponding norm enforcement through costly punishment. The maximum offer in ultimatum games is usually half the cake, which in most studies coincides with the mode. An equal split of profits is generally the most prominent point of agreement in bargaining games. Deviations from this solution point occur in ultimatum games mostly in line with maximizing own profits up to a degree where average rejection rates pose a risk. For most responders the threshold is around 40% of the cake beyond which zero payment for both is preferred. One main question of interest here is if people do follow social concerns (i.e., Rabin, 1993; Fehr, & Schmidt, 1999; Bolton, & Ockenfels, 2000) or exhibit reciprocity (i.e., Dufwenberg and Kirchsteiger 2004; Falk and Fischbacher 2006). Diverse variations of the standard ultimatum game exist and experimentally support one or the other view. For distinguishing the influence of social norms from procedural influences in the form of reciprocity or rivalry, the expansion to a three player game with two different roles appears promising. For example, assume a seller is a single responder with at least two units of a good to sell, and who is simultaneously negotiating with two buyers demanding one unit each. Without competition, buyers (proposers) should simply offer the seller's (responder's) reservation value for obtaining the good. However, the literature on ultimatum bargaining shows that responders frequently reject offers simply above zero. This behavior could not only be due to relative concerns regarding income differences, but could result from the informational variety and perception of social norm violations or could reflect longer term strategic advantages (Philipp, & Daniel, 2018).

In the traditional two-player game, a proposer is provided with an original endowment x . The proposer decides on a split of the endowment among him- or herself and a second individual, the responder, so that the responder is offered a share of size s , leaving the remaining share of x minus s to the proposer. The responder decides whether to accept or reject the proposer's offer $(x-s, s)$. Upon acceptance, the responder receives s , and the proposer receives $x-s$. Upon rejection, both receive nothing. In the three-player version used here, the proposer decides how to share the endowment x among three players: a share p for the proposer, a share s for the so called active responder, and a share $t = x-p-s$ for a second (inactive) responder, called the third party. With a minimum share of one monetary unit for each player, the proposer is free to offer any distribution (p, s, t) , with $p + s + t = x$ (and all shares > 1 unit). Full information is provided about the offer (p, s, t) to the responder, and no special outside payoffs for any of the players are involved.

Furthermore, our three-option version of the ultimatum game provides three response options to the active responder. First, the responder can *accept* the proposal, whereupon each player obtains the offered share (p, s, t). Alternatively, the responder can reject the offer in one of two ways. A rejection of Type 1 (*proposer-directed* rejection) implies that neither the responder nor the proposer gets anything, but the third party keeps his or her share, yielding the payoff vector $(0, 0, t)$. A rejection of Type 2 (*third-party-directed* rejection) implies that neither the responder nor the third party gets anything, but the proposer keeps her or his share, yielding the payoff vector $(p, 0, 0)$; of course, whatever rejection the active responder chooses, his or her own payoff s is foregone (Pfister, & Bohm, 2012).

Scenario (B): Repeated Three-Player Ultimate Games

The 3-win approach. We see that, limit of a sequence’s approach: the hlp 3-Ple proposed equilibrium:

Win-win-win
Approach: the 3 sequences, converging in: $\phi \leq hlp \prec e$
The “win-win-win concept”: Sensitization Process: terms of a continuous sequence
 $u_n, \dots, \lim_{n \rightarrow \infty} u_n = h_{lp}^ = \text{community..win} - \text{The...end..of ..sensitization..process}$*

We suppose that ϕ, e, π form a NEW sequence W_n converging in hlp .

ϕ, e, π relationship’s manipulation

We can take the ϕ, e, π interactive relations in more than one way:

$$\binom{n}{k} = \frac{n!}{k!(n-k)!} = \frac{(n(n-1)..)(n-k-1)}{k(k-1)*...*1} = \frac{3*2*1}{2*1(3-1)*(3-2)} = 3 \text{ ways for } \phi, e, \pi \text{ combination.}$$

Taking into account the three mathematical constants (π, ϕ, e), as the limits of $(u_n, \dots, v_n, \dots, z_n)$ i.e $\lim_{n \rightarrow \infty} (u_n, \dots, v_n, \dots, z_n)$, then it is easy to manipulate their combination One of these math combinations seems to be

$$\phi: \frac{e}{\pi}$$

$$\frac{e}{\pi} = \frac{2.718}{3.14} = \frac{2.71}{3.14} = 0.8569 \dots = \omega$$

$$\frac{\phi}{\omega} = \frac{1.618}{0.8569} = 1.888 \dots$$

Even,

$$\phi: \frac{e}{\pi} = \left(\frac{\phi}{\frac{e}{\pi}} \right) = \left(\frac{\phi}{\frac{1}{\pi}} \right) = \frac{\phi \pi}{e} = \left(\frac{1.618 * 3.14159}{2.7182818} \right) = \frac{5.083092}{2.7182818} \approx 1.888 \dots$$

$$\lim_{n \rightarrow \infty} F_n = \phi \leq hlp = 1.888 \dots \prec \lim \left(1 + \frac{1}{n} \right)^n = e, \dots$$

The intermediate Sequence: The hlp equi-harmony point -1.888.

The intermediate sequence $g(x)$ between two sequences:
 Suppose that $f(x) \leq g(x) \leq h(x)$ in the area of x_0 .
 If $\lim_{x \rightarrow x_0} g(x) = \lim_{x \rightarrow x_0} h(x) = L \in R$, then $\lim_{x \rightarrow x_0} g(x) = L$

$\ln 2 < \phi < \lim_{n \rightarrow \infty} u_n \leq \lim_{n \rightarrow \infty} v_n \leq \lim_{n \rightarrow \infty} z_n < e$

$\ln 2 \approx \frac{1}{n} + \frac{1}{n+1} + \dots + \frac{1}{2n-1} < \phi \leq \lim_{n \rightarrow \infty} \left(\frac{\phi \pi}{e} \right) \approx 1.888.. \leq \frac{1}{n} + \frac{1}{n+1} + \dots < \left(1 + \frac{1}{n} \right)^n \approx e$

In their “win-win-win” ideal situation (perfect sensitization, or the angels’ moment),
 $\phi < hlp = \lim_{n \rightarrow \infty} u_n = \lim_{n \rightarrow \infty} v_n = \lim_{n \rightarrow \infty} z_n < e$

The “3-win Lemma¹:

*if ..for.the..sequences.. $u_n, v_n, z_n, \dots, n = 1, 2, \dots, n$
 we.have.,.that.. $u_n < v_n < z_n, \dots$ for.all. n .and.that.they.are.all.converging, .i.e
 we.have.that.. $a = \lim_{n \rightarrow \infty} u_n, \dots, b = \lim_{n \rightarrow \infty} v_n, \dots$ and.. $c = \lim_{n \rightarrow \infty} z_n$.,.then.(conclusion)
 we..will..have.. $\lim_{n \rightarrow \infty} u_n \leq \lim_{n \rightarrow \infty} v_n \leq \lim_{n \rightarrow \infty} z_n, \dots$.i.e.,.. $a \leq b \leq c$
 wewill.use.the.. $\ln 2$.,.to.start..with*

If ..c.is.the.. $\lim_{n \rightarrow \infty}$..of.. $z_n = \left(1 + \frac{1}{n} + \frac{1}{n+1} + \frac{1}{n+2} + \dots + \frac{1}{2n} \right) = \left(1 + \frac{1}{n} \right)^n \leq e \approx 2,7182816 \dots$

If ...a...is.the.. $\lim_{n \rightarrow 0}$..of.. $u_n = \left(\frac{1}{n} + \frac{1}{n+1} + \dots + \frac{1}{2n-1} \right) \geq \ln 2$

then,

..... $\ln 2 \leq u_n, v_n, z_n \leq e$

We will now show that the sequence²:

$$u_n = \frac{1}{n+1} + \frac{1}{n+2} + \dots + \frac{1}{2n}, \dots, n = 1, 2, \dots$$

converges and we have:

$$\lim_{n \rightarrow \infty} \left(\frac{1}{n+1} + \frac{1}{n+2} + \dots + \frac{1}{2n} \right) = \log_e 2 = \ln 2$$

STATEMENT
 The sensitization process-which is the core of this Work -may be formulated by the three (3) world constants, as limits of u_n, v_n, z_n : ϕ, e, π
 W_n .,.so..that.. $\lim_{n \rightarrow \infty} W_n = h_p^* = 1.888..$ such that³:
 $\ln 2 < \phi = 1.618.. \leq hlp = 1.888.. < e = 2.7182818..$

MEASUREMENT: DEVIATION % from the hlp suggested Equilibrium

¹ Starting from: Lambros Iossif-Leonidas A. Papakonstantinidis (1990). Observation on a Limit. Mathematical Review, issue 37.
² The same for the other two, v_n and z_n
³ We use $\ln 2$ and e to “describe” the W_n as a sequence bounded up and down, by $\ln 2$ and the e sequences.

The win-win-win papakonstantinidis model is, thus, the limit-up of a continuous sensitization procedure, at any (A-B) bargainers and community “C” symbolized by the three sequences’, i.e

$$\begin{aligned}
 &u_n \dots \text{for...the..BARGAINER...}A \\
 &v_n \dots \text{for...the..BARGAINER...}B \\
 &z_n \dots \text{for...the...COMMUNITY, AS...THE..THIRD..BARGAINER}
 \end{aligned}$$

Indeed, you can imagine the successive steps towards sensitization as a series of sequence terms that converges to a limit.

The boundaries of these sequences, since they represent “sensitization levels”, also form a new sequence W_n and let a a real number:

$$\begin{aligned}
 &W_n, \dots \text{with } \lim_{n \rightarrow \infty} W_n = hlp \\
 &|W_n - a| < \varepsilon \\
 &\text{and} \\
 &\varepsilon > 0
 \end{aligned}$$

W_n “sensitization sequence” converges at some point to the ideal limit -up (the Angels’ society) then all three limits of the u_n, v_n, z_n sequences a, b and the community, C, respectively forming a new-unique sequence, which converges in a new hlp

Welfare can be measured, as the deviation from hlp suggested Equilibrium.

Tending toward an ideal (angels) situation, i.e the limit of the sensitization process, the win-win-win papakonstantinidis model is the limit-up of a continuous sensitization procedure, at any (A-B) bargainers symbolized by the three sequences’, i.e

$$\lim_{n \rightarrow \infty} u_n \leq \lim_{n \rightarrow \infty} v_n \leq \lim_{n \leftarrow \infty} z_n$$

In their limit, we have:

$$\lim_{n \rightarrow \infty} u_n = \lim_{n \rightarrow \infty} v_n = \lim_{n \rightarrow \infty} z_n = \lim_{n \rightarrow \infty} W_n = 1.888..$$

Indeed, you can imagine the successive steps towards sensitization as a series of sequence terms that converges to a limit.

The boundaries of these sequences, since they represent “sensitization levels”, also form a new “Win” sequence.

We prove that this new sequence consists of their limits as “sensitization sequences” converges to the ideal limit -up (the perfect society, or the society of angels) then all three limits of the u_n, v_n, z_n sequences a, b and the community, C, respectively forming a new-unique sequence, which converges in a new L limit.

Conclusions

The combination of *the mathematical constants* φ, π, e provides us with the possible percentages that the proposer could propose and the receptors could accept, so that the ultimatum bargain could result in agreement.

It is appreciated that the % percentage of acceptance differs from the one use to other (i.e salaries, loan, cake).

In experimental base, the Hypothesis that people are-about-willing to accept a salary of just over 40% or a loan of around 15% but a share of cake at 50%, has been accepted.

It seems, therefore, that the bargaining power at this particular moment of the ultimatum is crucial. However, they are willing to accept a salary just above that 40% of those who ask.

It is generally argued here that 60-40 is the most likely and not the 61.8-38.2 suggested by Suleiman.

The existence of multiple proposers allows the responder to directly assess an offer based on another offer received simultaneously and investigates the interdependence of simultaneous ultimate and corresponding behavioral dependencies.

In the case of repeated three-player ultimate games, or low bargaining power (for example a loan), the responders could accept a smaller percentage striking agreement with observations, in the base of 0.888-0.112.

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