

# The Comparative Study of High School Mathematics Textbooks in Mainland China and Taiwan—Take “Plane Vector” for Example

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By using the methods of qualitative description and quantitative description, this paper compared the compilation characteristics of “plane vector” content in three versions of high school mathematics textbooks in mainland China and Taiwan from the aspects of content structure, style column, content breadth, and depth. The People’s Education A version has a strong inquisitiveness; the Beijing Normal University version highlights the subject compatibility and the student subjectivity; and the Hanlin version emphasizes the mathematics knowledge intuitiveness and correlation. Thus, it is suggested that the construction and compilation of high school mathematics textbooks in China need to integrate and link the internal relationship between mathematical knowledge and the design and compilation of mathematics textbooks based on core literacy.

*Keywords:* mathematics textbook, plane vector, comparative study

## Introduction

There are many similarities between the mainland China and Taiwan in mathematics education. However, there are also differences in the compilation of mathematics textbooks. Johnsen B. (2001), a Norwegian scholar, proposed that the study of textbook mainly includes the static study of textbook entity and the dynamic study of its formation and use process. Zhou Pei-yi (2005), a scholar in Taiwan, suggested to divide textbook research into four categories: content analysis, evaluation and selection, editing and development, and application. This study based on the viewpoints of these two scholars, to compares the similarities and differences of “plane vector” in the three versions of high school mathematics textbooks in mainland China and Taiwan from the perspectives of content structure, style column, content breadth and depth, exercise setting, and other dimensions, and then summarizes the characteristics of the two sides’ mathematics textbooks, aiming to inspire the construction and compilation of high school mathematics textbooks in mainland China.

By choosing the chapter of “plane vector” as the comparison object, because all three textbook versions include it in the compulsory content, and they are independent chapters. Vector is one of the important and basic mathematical concepts in modern mathematics. It is the bridge between algebra and geometry (Ministry

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of Education of the People's Republic of China, 2003). Vector was brought into high school mathematics curriculum since 1996, and has profound connotation of mathematics: Vector theory is straight line, curve, surface, surface, and high dimensional space basic tool for mathematical problems, it is the foundation of further study on other mathematics problems, it is also depicting reality and important mathematical model to describe the real world, playing an important role in solving practical problems.

### Methods

Select the experimental textbook of Curriculum Standards for Ordinary High Schools 4 (version A) published by People's Education Press (2007 version), the experimental textbook of Curriculum Standards for Ordinary High Schools 4 published by Beijing Normal University Press (2014 version), and Ordinary High School Mathematics 3 published by Hanlin Publishing Company Limited (2015 version). The three versions of mathematics textbooks was widely used and influential in mainland China and Taiwan.

Qualitative description and quantitative description were adopted. Qualitative description refers to the comparison and analysis of the content structure and style columns of the three versions of "plane vector", while quantitative description refers to the comparison and analysis of the content breadth and depth.

### Research Results and Analysis

#### Content Structure

Content structure is the epitome of the whole chapter and has a function of introduction. The content structure of the chapter "plane vector" of the three versions is presented as a whole, as shown in Table 1.

From Table 1, it can be seen that the content structure of the three versions of "plane vector" has different focuses. Both People's Education A version and Beijing Normal University version are carried out in accordance with the process of "practical background, representation and operation, and application examples". Beijing Normal University version pays more attention on the combination with other disciplines. Relevant knowledge is introduced through physical situations, such as displacement, velocity, force to vector concept, displacement synthesis to vector addition, velocity multiple to vector multiplication, force work to vector dot product, and so on. Horizontal comparison, Hanlin version connects "Cauchy inequality", "the parameters of the straight line type", "area formula", "determinants", and "Kramer formula" with plane vector, the content such as fusion reflects Taiwan Senior High School Compulsory Subject "mathematics syllabus in compiling textbooks" advocated the integration of mathematical knowledge between and link (Taiwan Education Authorities, 2009).

#### Style Column

The style column reflects the compilation characteristics of textbooks, and its function aims to present the structure and content of textbooks. The style columns of the three versions are shown in Table 2.

As can be seen from Table 2, the style columns of "plane vector" in the three versions are rich and diverse, with differences in the same.

All three versions of textbooks present the contents successively in the order of "continuity", that is, chapter head diagram, chapter introduction, chapter table of contents, chapter core knowledge (related concepts, theorems, algorithms, example exercises, etc.), and chapter review problem at the end. In addition, "reading and thinking", "reading material", and "biography of mathematician" are set in the three versions, respectively. All of them infiltrate the mathematics culture into the "plane vector" content through the form of reading material, which further reflects the organic combination of mathematics culture and high school mathematics curriculum

content. Furthermore, chapter summary is set in three versions of textbooks, which are respectively: summary, summary suggestions, and content summary.

Table 1

*Comparison of “Plane Vector” Content Structures in Three Versions*

People's Education A version	Beijing Normal University version	Hanlin version
Plane vector 2.1 Actual background and basic concepts of plane vectors 2.1.1 Vector physics background and concepts 2.1.2 Geometric representation of vectors 2.1.3 Equal vectors and collinear vectors 2.2 Linear operation of plane vectors 2.2.1 Vector addition operation and its geometric significance 2.2.2 Vector subtraction and its geometric significance 2.2.3 Vector multiplication and its geometric significance 2.3 Fundamental theorem of plane vector and coordinate representation 2.3.1 Fundamental theorem of plane vectors 2.3.2 Orthogonal decomposition and coordinate representation of plane vectors 2.3.3 Coordinate operation of plane vectors 2.3.4 Coordinate representation of plane vectors collinear 2.4 Dot product of plane vectors 2.4.1 Physical background and meaning of plane vector dot product 2.4.2 Coordinate representation, modulus and included Angle of plane vector dot product 2.5 Examples of plane vector applications 2.5.1 Vector methods in plane geometry 2.5.2 Examples of vector application in physics	Plane vector §1 From displacement, velocity, and force to vector 1.1 Displacement, velocity, and force 1.2 Vector concept §2 From composition of displacements to addition of vectors 2.1 Vector addition 2.2 Vector subtraction §3 From multiples of velocity to multiples of vectors 3.1 Multiplication by vector 3.2 Fundamental theorem of plane vectors §4 Coordinates of plane vectors 4.1 Coordinate representation of plane vectors 4.2 Coordinate representation of linear operations of plane vectors 4.3 Coordinate representation of parallel vectors §5 From the work done by the force to the dot product of the vector §6 Coordinate representation of the dot product of plane vectors §7 Vector application example 7.1 Distance formula between point and straight line 7.2 Examples of application of vector	Plane vector 3-1 Representation of plane vector 1. Geometric representation and coordinate representation of vectors 2. Addition, subtraction, and multiplication of vectors 3. Linear combinations of vectors 4. Equinoctial formula 3-2 The inner product of plane vectors 1. The inner product of an Angle with a vector 2. Properties of inner products 3. Cauchy inequality 4. Orthography 5. Application of inner product to geometry 3-3 Straight lines in the plane 1. Parametric expression of a line 2. The intersection of two lines 3. The distance from the point to the line 3-4 Area and second order determinant 1. Area formula and second order determinant 2. Property of determinant 3. Algebraic determination of geometric relations between two straight lines and kramer formula

Table 2

*Comparison of Three Versions of “Plane Vector”*

Version	People's Education A version	Beijing Normal University version	Hanlin version
Style column	Chapter head map, chapter introduction, and chapter table of contents; Problem situation; Inquiry and thinking; Examples, exercises, and quizzes; Reading and thinking; Summary; Review reference questions.	Chapter head map, chapter introduction, and chapter table of contents; The example analysis questions; Thinking about communication; Think, explain, and expand; Examples, exercises, and quizzes; Reading material; This chapter summarizes the recommendations; Review questions.	Chapter head map, chapter introduction, and chapter table of contents; The relevant content is given by intuitive description; Examples, class exercises, and quizzes; Comprehensive practice; Content in this paper; Biography of mathematician.

However, each section of People's Education version A and Beijing Normal University version introduces new knowledge with questions as the context. For example, People's Education Press (PEP) textbook sets up

columns, such as “problem context”, “inquiry”, “thinking”, etc., while Beijing Normal University version sets up columns, such as “case analysis”, “question raising”, “thinking”, and “communication”. But the academician version presents the correlation concept, the theorem, and so on through the intuitive description. In addition, the infiltration of mathematics culture in the three versions has different emphases: The chapter of the People’s Education version A presents “reading and thinking” for two times, which respectively introduces the origin of vector and vector symbols, vector operations (laws of operation) and graphical properties, and establishes corresponding relations between vector operations (laws of operation) and geometric figures through the combination of number and shape. The first chapter of Beijing Normal University presents “reading material”, introducing the mathematical connotation of vector. Hanlin version of a chapter presents a “biography of mathematicians”, introduce of Cauchy, who plays a significant role in the mathematics history, reflecting the role of mathematics in the progress of human society. The general chapter summary also has different forms of expression as shown in Table 3.

Table 3

*Chapter Summary Comparison of Three Versions of “Plane Vector”*

People’s Education A version	Beijing Normal University version	Hanlin version
Summary 1.The knowledge structure framework of this chapter; 2. Review and reflection (7).	This chapter summarizes the recommendations; Learning requirements (7); Review the knowledge of this chapter, sort out notes, suggest the following questions to think, summarize, summarize, and write a review report (9).	Chapter 3 summary; Presenting relevant core knowledge in terms (19).

As can be seen from Table 3, the chapter summary of “plane vector” in different versions has different presentation forms. A version of People’s Education guides students to review the knowledge structure of the whole chapter by presenting the structure diagram and setting up seven questions. Beijing Normal University version defines the learning requirements of each key point of knowledge through “understanding”, “mastering”, “experiencing”, “using”, and other action verbs, and then suggests students to construct the knowledge structure of this chapter in the form of summary report in line with personal cognitive characteristics, and pay more attention to cultivate students’ subjective initiative. The Hanlin version presents 19 “knowledge” content summaries in the form of entries.

**Breadth and Depth of Content**

Content breadth is quantified by the number of knowledge points included in the chapter “plane vector”. Now, the knowledge points of the whole chapter of the three versions “plane vector” are summarized as shown in Table 4.

The depth of content is mainly measured according to the presentation modes of concepts contained in “knowledge point”, which is divided into three levels of intuitive description, analogical induction, and deduction, respectively assigning values 1, 2, and 3 (Cao & Wu, 2015). Intuitive description means describe the relevant concepts through the presentation of only one case; analogical induction means generalizing through at least two or more cases to obtain concepts; deduction means from the universal mathematical theory knowledge, to understand the individual, special mathematical concepts of a logical reasoning method.

The content depth of the  $i$  knowledge point is  $S_i = \frac{1 \times A + 2 \times B + 3 \times C}{A + B + C}$ ,  $A$  is the sum of the concepts of “intuitive description” with the  $i$  knowledge point,  $B$  is the sum of the concepts of “analogical induction”

with the  $i$  knowledge point, and  $C$  is the sum of the concepts of “deduction” with the  $i$  knowledge point.

Use the content depth formula,  $S = \frac{\sum_{i=1}^n S_i}{n}$ ,  $n$  is the number of knowledge points,  $S_i$  is the content depth of the  $i$  knowledge point (Wang & Cao, 2016).

Table 4

*Comparison of Knowledge Points of “Plane Vector” in Three Versions*

	People's Education A version	Beijing Normal University version	Hanlin version
Knowledge points	1. Vector-A 2. Directed line segment starting point/direction/length)-A 3. Norm-A 4. Zero vector-A 5. Unit vector-A 6. Parallel vectors-A 7. Zero vector is parallel to any vector-A 8. Equal vector-A 9. Collinear vectors-C 10. Vector addition-B 11. Triangle rule-A 12. Parallelogram rule-A 13. Zero vector plus any vector is equal to any vector-A 14. Opposite vectors-A 15. Vector subtraction-A 16. The opposite vector of the zero vector is the zero vector-A 17. Scalar multiplication-B 18. Scalar multiplication of a vector-A 19. Vector collinear theorem-B 20. Fundamental theorem of plane vectors (base/included angle/perpendicular)-B 21. Orthogonal decomposition-B 22. Coordinate representation of vectors-A 23. Coordinate operations on vectors-A 24. Scalar product/inner product-A 25. Projection-A 26. The geometric meaning of the dot product-B 27. Coordinate representation of the dot product-A	1. Vector-B 2. Directed line segment-B 3. Norm-A 4. Zero vector-A 5. Unit vector-A 6. Free vector-A 7. Equal vector-A 8. Parallel vectors-A 9. Zero vector is parallel to any vector-A 10. Vector addition-B 11. Triangle rule-B 12. Parallelogram rule-B 13. The law of vector addition-A 14. Vector subtraction-B 15. Scalar multiplication of vectors-B 16. The law of vector scalar multiplication-A 17. Linear operation/linear combination-A 18. Fundamental theorem of collinear vectors-A 19. Fundamental theorem of plane vectors (base)-B 20. Coordinate representation of vectors-A 21. Coordinate representation of vector linear operations-A 22. Coordinate representation of vectors parallel to each other-A 23. Scalar product/included angle/perpendicular-B 24. Zero vector is perpendicular to any vector-A 25. The geometric meaning of the dot product-A 26. The law of the dot product-A 27. Coordinate representation of the dot product-A 28. Vector direction-A 29. Formula for the distance between point and line-A	1. Vector-B 2. Directed line segment-A 3. Equal vector-A 4. Zero vector-A 5. Coordinate representation-A 6. Length of vector-A 7. Coordinate representation of vectors-B 8. Vector addition-A 9. Coordinate representation of vector addition-B 10. Basic properties of vector addition-A 11. Opposite vectors-A 12. Coordinate representation of vector subtraction-A 13. Scalar multiplication of vectors-A 14. Parallel vectors-A 15. Linear combination-A 16. Equinoctial formula-C 17. The angle between the vectors-A 18. The inner product of vectors-B 19. Coordinate representation of the inner product-A 20. Decision rules of perpendicular vectors-A 21. Fundamental properties of inner products-B 22. Cauchy inequality-B 23. Orthograph-B 24. Parallelogram rule-B 25. Triangle inequality-B 26. Parametric formula for line L-A 27. Vector direction-A 28. Normal vector-A 29. The crossing angle of two lines-A 30. Formula for the distance between point and line-B 31. Area formula-A 32. Second order determinant-A 33. Area formula of triangle and parallelogram-B 34. Rules for parallel vectors-A 35. Property of determinant-B 36. Kramer formula-B 37. Algebraic determination of the geometric relationship between two lines-B
Number	27	29	37

As can be seen from Table 4, the number of knowledge points in the three versions of “plane vector” is 27,

29, and 37, respectively, which is the highest in the Hanlin version and the lowest in the People's Education A version. According to the content depth calculation formula, the content depth values of the chapter "plane vector" in People's Education A version, Beijing Normal University version, and Hanlin version can be calculated respectively as 1.296, 1.310, and 1.432. In terms of content depth, the Hanlin version is the highest; the People's Education A version is the lowest.

### Research Conclusions and Implications

#### Three Version Textbook Content Design Each has the Characteristic

From a macro-point of view, the textbook content of "plane vector" in the three versions of high school mathematics textbooks on both sides of the straits is basically the same. However, through deep static comparison, it is found that the three versions have their own characteristics and there are still many differences.

**People's Education version A is very inquiry-based.** Each section provides inquiry-based case activities before the introduction of new knowledge and integrates A lot of "inquiry and thinking" into the core content of the whole chapter of "plane vector", enabling students to explore the operation and algorithm of "plane vector" under the guidance of teachers. Version, on the one hand, manifests the taught A pay attention to training students' independent thinking, cooperation, and communication, the consciousness of hands-on practice, on the other hand, physical reflects the basic idea of "the ordinary high school curriculum standard", advocating the students initiative, enterprising approach to learning, this way of learning helps to exert students' learning initiative, make students' learning process under teachers' guide of "re-creation" process.

**Beijing Normal University version highlights the compatibility of subjects as well as the subjectivity of students.** It can be clearly seen from chapter directory that the "plane vector" chapter structure system, the introduction of the content of each section and the background of the intimate contact with the physical discipline. From the displacement, velocity, and force to the vector concept, from the synthesis of displacement to the vector addition, from the speed ratio to the number of take vector, from forces work to vector product, etc. the number of the textbooks has very strong compatibility disciplines. In addition, it can be seen from the "chapter summary" that students are advised to construct the knowledge structure of this chapter in the form of summary report in line with personal cognitive characteristics and pay attention to students' subjectivity of mathematical knowledge.

**Hanlin version's biggest characteristic is the mathematics knowledge intuitiveness and the correlation.** Each section of the "plane vector" chapter presents relevant contents in turn by means of intuitive description, highlighting the key points of knowledge with blue boxes, and presenting the content summary in the form of items. Although concise and intuitive, students lack the understanding of the occurrence and development process of mathematical knowledge. In addition, the chapter of "plane vector" integrates "Cauchy inequality", "parametric formula of straight line", "area formula", "determinant", "Kramer formula", and other contents, showing the connection between related contents, and promoting students to comprehensively understand mathematics.

#### Enlightenment to the Compilation of High School Mathematics Textbooks in Mainland China

Academic rigor and systematicness of mathematics decide its profound intrinsic relations among knowledge, Hanlin version fully reflected the integration of mathematical knowledge, connect "Cauchy

inequality”, “area formula”, “Kramer formula”, and “plane vector”. It is good for student to understand the mathematics internal relations. Under the background of intellectual education, innovation, and development of dynamic interaction of digital textbooks, with the traditional textbooks, will the fusion of information technology and mathematics curriculum content, make the link between the mathematical knowledge more closely, in the process of teachers’ teaching and students’ learning, and interest in May, according to the characteristics of students, through digital textbook or electronic schoolbag dynamic interaction mathematical correlation knowledge, has certain elasticity among makes the related content.

Under the background of global educational trend, the further design and compilation of mathematics textbooks will propose, integrate, determine, and reorganize mathematics contents according to students’ mathematical core accomplishment. On the one hand, the value of mathematical contents should be considered; on the other hand, the value of students’ mathematical core accomplishment should be considered (Shi & Zhang, 2016). Mathematics textbooks with core qualities should no longer be a tool of knowledge mechanical transmission, but should pay more attention to guiding students to actively explore, cooperate, and exchange, subject construction, and acquire knowledge independently. In addition, mathematics textbooks with core qualities need to be combined with other disciplines. For example, the content of “plane vector” in Beijing Normal University is introduced through the physics background to further open mathematics textbooks, so as to cultivate the core qualities of Chinese students in a real sense.

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