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Knowledge Representation for the Geometrical Shapes

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Abstract: This paper outlines the necessity of the knowledge representation for the geometrical shapes (KRGS). We advocate that KRGS for being powerful must contain at least three major components, namely (1) fuzzy logic scheme; (2) the machine learning technique; and (3) an integrated algebraic and logical reasoning. After arguing the need for using fuzzy expressions in spatial reasoning, then inducing the spatial graph generalized and maximal common part of the expressions is discussed. Finally, the integration of approximate references into spatial reasoning using absolute measurements is outlined. The integration here means that the satisfiability of a fuzzy spatial expression is conducted by both logical and algebraic reasoning.

Key words: Knowledge representation, integrated algebraic and logical, fuzzy logic reasoning, machine learning.

1. Introduction

Referring in practical spatial description is seldom absolute. Sometimes, due to the lack of precise information, it is not possible to represent the *x*-*y* coordinate of the vanishing points. In this case, symbolic knowledge can be used as mean of expression to situate the position of an absolute point with respect to a plane [1]. For instance, in image processing by using 3D projective space [2], one can use the relation between a point *P* and an object *O* via the 3D line *PQ*, where *Q* is an ideal point. Often absolute measurements are unnecessary: if we want to know whether an object will pass through a hole, it is sufficient to know the relative size of the hole and object. Another example is the problem of soil classification, where the determination of some class is based on the above relation with respect to a particular line and the plasticity index.

The aim of this paper is to advocate in favor of three mentioned above components. Using concrete examples, we provide the evidences why these components are mandatory. The rest of the paper is organized as follows. Section 2 describes the representation of the fuzzy references. Section 3 sketches learning spatial graph. How the satisfiability process along with the integration of algebraic and logical reasoning, can be done, which is explained in Section 4. Section 5 gives conclusions and future problems.

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2. Fuzzy References

We shall call fuzzy expressions those expressions including at least one approximate references like above, below, over, and under [3].

Their intuitive meanings can be depicted by Fig. 1. To represent these predicates, we first describe how to map F_1 = {above, below} into algebraic reasoning. We then use these knowledge of F_1 with additional the predicates describing point and line relations to express the description of F_2 = {over, under}.

For mapping F_1 into algebraic reasoning, let us suppose that a fuzzy subset be characterized by a function, μ, called compatibility function, over a set of elements, called the universe of discourse, U, where $U = \{u_1, u_2, \dots, u_n\}$ and $\mu : U \to [0, 1]$. A function μ is called Π-type if there exists only one point at which monotonicity changes direction. The effect of above and below on a Π-type can be best described by.

$$\mu_{above_x(U_i)} = \begin{cases} 1-\mu_x(U_i) & \text{if } U_i \geq U_{max} \\ 0 & \text{otherwise} \end{cases}$$

$$\mu_{below_x(U_i)} = \begin{cases} 1-\mu_x(U_i) & \text{if } U_i \leq U_{min} \\ 0 & \text{otherwise} \end{cases}$$

where, $U_{max}(U_{min})$ is the value of U; where $\mu_x(U_i)$ attains its maximum (minimum) value. It is worth to mention that in many practical applications including continuous domains, the data collected in real-world experiment are discrete. Therefore, presumably, there are appropriate segments that are representative knowledge of the domain. Consequently, this observation can be best combined by Eshragh and Mamdani's [4] idea: the separation of fuzzy spreads into an appropriate number of segments with well defined characteristics.

Having the knowledge of F_1, now it is possible to represent F_2. Let us take under predicate, where by convention under (*a*, *b*) means that *a* is under *b*; where the variables *a* and *b* denote points. The representation of this predicate can be expressed by three other predicates, namely, perpendicular (or perpend for short), below and online, where online (*P*, *A*, *B*) mean that point *P* is on line segment *AB*; where perpend (*A*, *B*, *C*, *D*) represents the line segments *AB* and *CD* are perpendicular. Having the above definitions the logical representation of the predicate under can be defined.

Table 1 shows the definition of under predicate expressed in terms of those predicates taking points as their arguments. Those relations whose algebraic representation include inequalities are called order relations. As is clear from the definitions in Table 1, these predicates are non-order relations since they are defined in terms of non-order relations on, eqseg and noteq. Several redundant noteqs are included in Table 1 to clarify non-degenerated case specifications.

Note that the valid algebraic representations of the order relations cannot be obtained in the Gröbner basis method. This is also true in the geometric domain, where for instance, between and eqang whose meanings will be given later, are order relations. As pointed in Ref. [5], all geometric theorems proved so far by the Gröbner basis method do not include any order relations. This is also the case in Wu's method [6].

3. Learning Spatial Grap

Any geometrical shape can be expressed by a logical expression (*Exp*). In order to speed up the reasoning process, it is desirable to find a way for determining the common part of two or more geometrical shapes. In other words, learning the generalized common maximal (GCM) for the current expressions is required.

An *n*-ary predicate will be represented by $t_1(t_2, ..., t_n)$. Each t_i is a term, which may be either a constant, represented by lower case Roman letters, or a variable shown by upper case Roman letters. A literal is a list of terms, optionally prefixed by the logical negation (¬) operator. For instance, on(o1, o2) and red(X) are both literals. If we consider two following expressions:

Exp_1 = on(o1, o2) ∧ sphere(o1) ∧ red(o1) ∧ cube(o2) ∧ red(o2)

Exp_2 = on(o3, o4) ∧ pyram(o3) ∧ blue(o3) ∧ cube(o4) ∧ red(o4)

where, the predicate on (*X*, *Y*) means that *Y* is on *X*, the meaning other ones are self meaning. Then we obtain the following output expression: *GCM*(*Exp*1, *Exp*2) = *on*(*X*, *Y*) ∧ *red*(*Y*) ∧ *cube*(*Y*) which is obtained by the linearization of the spatial graph shown in Fig. 2.

An expression graph is a 6-tuple [7] $(L, C, \sigma, \theta, K, a)$ where *L* (resp. *C*) is a finite set of literal (resp. constant) nodes; σ is the literal dimension function $L \rightarrow Z_+$ (i.e. the set of positive integers; θ is the literal sign function $L \rightarrow \{+, -\}$; *K* is the literal partial content function $I \times Z_+ \rightarrow C$, such that, if $(\ell, i, c) \in K$, then $i \in \{1, 2, \ldots, \sigma(\ell)\}$; and finally, *a* is the literal adjacency relation, a finite subset of $Z_+ \times Z_+ \times L \times L$ along with the following properties:

(1) Symmetry: $(i_1, i_2, \ell_1, \ell_2) \in a$ if $(i_2, i_1, \ell_2, \ell_1) \in a$;

(2) Transitivity: if $(i_1, i_2, \ell_1, \ell_2) \in a$ and $(i_1, i_3, \ell_1, \ell_3) \in a$, then $(i_1, i_3, \ell_1, \ell_3) \in a$;

(3) Consistency: $\forall \ell_1, \ell_2 \in L$ and $i_1, i_2 \in Z_+$, if $(\ell_1, i_1, c_1) \in K$ and $(\ell_2, i_2, c_2) \in K$, then $(i_1, i_2, \ell_1, \ell_2) \in a$ if $c_1 = c_2$.

In the method given in Ref. [7], the generlaization replaces just two expressions. We have developped a method, not reported here, to accepts more than two expresions. A common LISP software has been writen which implements and confirms the method.

It is interesting to point out that the expression graph can also be used for one expression, as in Fig. 4 by a combination of ways, including above mentionned properties, done for the evaluation of the predicate para(a, b, c, d) of Fig. 3 under the hypotheses depicted at the head part of Fig. 4, except $\wedge$ ¬para(a, b, c, d).

4. Satisfiability of Fuzzy Spatial Expression

Definition: Let *Expr* be the set of spatial references of the following form: $Expr = \text{Pred}_1 \wedge \text{Pred}_2 \ldots \wedge \text{Pred}_n$, where Pred_i for $i \leq 1 \leq n$ is a spatial predicate. If at least one above predicate is a fuzzy one, then the expression is called fuzzy one. An example of such expression is the following one.

$$Expr = \underbrace{\texttt{online}(Z, O, E)}_{Pred_1} \wedge \underbrace{\texttt{above}(Z, O)}_{Pred_2} \wedge \underbrace{\texttt{under}(D, O)}_{Pred_3}$$

where, online(Z, O, E) means that the point Z is on segment line OE. This example can be used in the interpretation of laser-material experiments where before perforating Z, we would like to be sure of the following information:

- Z is above O and also on Zapata's line;
- D is under O.

where, Zapata's line is a nickname visualized in Fig. 1 by $L = [O, E]$. Let us suppose *Expr* can be divided into two sub-expressions, such that $Expr \equiv Expr_h \wedge (\neg Expr_c)$. By convention $Expr_h$ and $Expr_c$ will be called problem hypotheses and conclusion, respectively.

Satisfiability: Let axioms denote the set of application's axioms. Then the prof of domain-dependent property $Expr_c$ under a given set of hypotheses Expr_h is formalized as follows:

$$\text{Axioms} \cup \text{Expr}_c \vdash \text{Expr}_c \quad (1)$$

$$\text{Axioms} \models \text{Expr}_h \rightarrow \text{Expr}_c \quad (2)$$

$$\text{Expr}_h \rightarrow \text{Expr}_c \quad (3)$$

$$\neg(\text{Expr}_h \rightarrow \text{Expr}_c) \equiv \text{Expr}_h \wedge (\neg\text{Expr}_c) \equiv \text{Expr} \quad (4)$$

The formula (1) is equivalent to the formula (2), which implies that all logical models of Axioms satisfy formula (3). In refutational reasoning, formula (4) is proved by showing that the negation of the *Expr* is not satisfied by any logical models of axioms. However, since it is known that axioms is categorical and all its logical models are isomorphics, it is sufficient to show that logical formula of *Expr* is not satisfied by a specific logical model of axioms. For complete details of the integrated algebraic and logical reasoning and termination/correctness proofs, as well as the limitations of that method, see Ref. [5].

Evaluation: in addition to our four fuzzy relations, in our work, nine predicates taking points as their arguments are used: eqseg, eqang, collinear, online, midpoint, para, rangle, perpen and line. The predicate line take a plain list of points and declare the existence of a straight line as well as the fact that the points in the list are aligned on that line. Furthermore, line is an order relation like between, where between (P, A, B) means that point P is located between a pair points A and B.

Therefore, by this definition, line can appear only in *Expr* to maintain the soundness of the integrated reasoning. Moreover, since line subsumes non-order relations collinear and online, we do not use the latter predicate to describe the hypotheses; collinear and online are used within a spatial expression to describe the conclusion. Among the four following predicates on, between, eqseg and eqang that can be used to describe Expr, the predicate “between” issubstituted by the predictae “line”.

The predicate on is excluded, because no information useful for reasoning is specified by describing either on(A, L) or on(A, L) ∧ on(B, L), and because the collinearity among more than three points, on(A, L) ∧ on(B, L) ∧ on (C, L) ∧ … can be described using the predicate collinear. The predicate on(A, L) means that the point A is on line L and it is used internally by the evaluator.

To facilitate the evaluation process, it is often useful to define higher-level predicates. As pointed out in Refs. [5, 8], however, their meanings must be specified very strictly; careless loose definitions. Table 1 shows the strict definitions of six higher-level predicates, which are used in Ref. [5] following the method described in Ref. [8]. Here the predicate noteq (x, y) implies that two points x and y are different. This predicate is non-order relation and is often used to specify subsidiary conditions to exclude degenerated case. Table 2 shows the algebraic representation of under (x, y) predicate depending on the seven predicates of Table 1.

Among the nine mentioned predicates, eqseg, eqang and para are equivalent relations. It is often possible to express a fuzzy spatial expression using one of the mentioned predicate, like this one: para(A, B, C, D) ∧ below (F, E), where the predicate para(A, B, C, D) means that the line segments AB and CD are parallel. This predicate can be logically expressed by the following representation: ¬(∃P) (online(P, A, B) ∧ online(P, C, D)). As pointed out in Ref. [5], this predicate is defined as a non-order relation. If para is used in the problem hypotheses, it must be possible to evaluate the directions of the pair of line segments AB and CD coincide with each other or not. To solve this, we report the definition of ordered parallel (opara for short).

$$\text{Opera}(A, B, C, D) \Leftrightarrow \text{para}(A, B, C, D) \wedge$$
$$(\exists P)\ (\text{between } (P, A, D) \wedge \text{between } (P, B, C))$$
$$\text{Para}(A, B, C, D) \Leftrightarrow \neg(\exists \text{P})\ (\text{online}(P, A, B)$$
$$\wedge\ \text{online}(P, C, D))$$

When the predicate “between” is included in the spatial expression, first it is transformed into the non-order relations.

$$\text{Between } (P, A, B) \equiv (\exists L) \wedge \text{on}(P, L) \wedge \text{on}(A, L)$$
$$\wedge\ \text{on}(B, L) \wedge (\text{A} \neq \text{B}) \wedge (\text{A} \neq \text{P}) \wedge (\text{B} \neq \text{P})$$

As appear from the above relation, the reasoning capacity of the predicate between is very limited. With the integration of the algebraic representation of para(A, B, C, D) and between (P, A, B) the above question can be done by the evaluation of the predicate opera(A, B, C, D) of the following algebraic representation.

$$\text{Para } (A, B, C, D) \Leftrightarrow (x_A \neq x_B \vee y_A \neq y_B\ ) \wedge$$
$$(x_C \neq x_D \vee y_C \neq y_D) \wedge (y_B - y_A)x_C +$$
$$(x_A - x_B\ )y_C + (x_B y_A - x_A y_B\ ) \neq \wedge\ 0$$
$$(x_B - x_A)(y_D - y_C) - (x_D - x_C)(y_B - y_A) = 0$$

Since the evaluation of a fuzzy spatial expression Expr by essence is equivalent to the problem of unsatisfiability in refutational logic then the expansions of completeness and the validation of soundness concerning the inclusions of four mentioned fuzzy relations into the reasoning method of Matsuyama and Nitta can be done provided that the domain-dependent axioms be carefully defined.

It is intersting to point out that the expression graph can also be used just for one expression as in Fig. 4 by way of the mentionned properties, done just for the evaluation of the predicate para(a, b, c, d) of Fig. 3 under the hypotheses depicted at the head part of Fig. 4, except ¬para(a, b, c, d).

5. Conclusions

The aim of this paper was to introduce the needs for the integration of three components of the knowledge represntation for the geometrical shapes. The experiments done on a limited numbers of simples shapes are satisfactory. Exploration of its capabilities is in progress.

This paper partially is based on Eshragh and Mamdani’s idea [4]: the sep-aration of fuzzy spreads into an appropriate number of segments with well defined characteristics. Fortunately, in many practical applications including continuous domains, the data collected in real-world experiment are discrete. Therefore, presumably, there are appropriate segments that are representa-tive knowledge of the domain. If the data are not discrete, by symbolic constraints, the knowledge represntation can be conduted [9].

The followings are among future problems to be studied.

Representation of approximative references like toward, from, etc. (see Fig. 1). To capture the intuitive meanings of these relations we have to analyze the fundamental conceptual structure of the shapes in which using these references make senses.

Representation of complex shapes by means of logical combination of the simple ones.

Elaboration of set of spatial axioms (axioms) for the practical spatial applications like earthquake engineering, soil classification, etc.

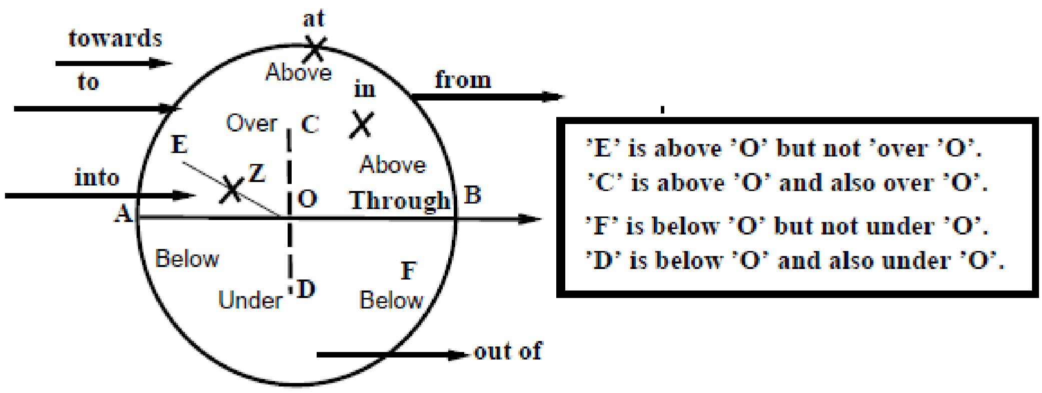

Fig. 1 Points and approximate references.

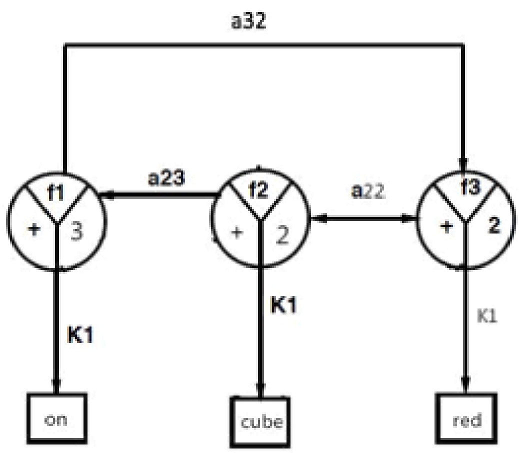

Fig. 2 Generalized spatial graph of Exp_1 and Exp_2.

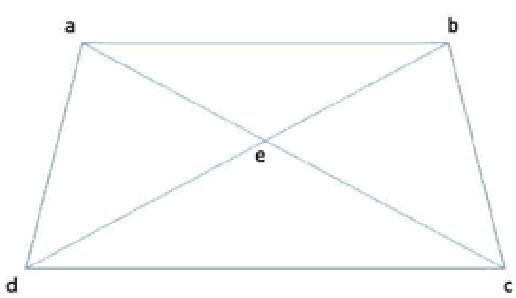

Fig. 3 Hypotheses: eqang(*e*, *a*, *d*, *e*, *b*, *c*) and eqseg(*e*, *a*, *e*, *b*). Task: para(*a*, *b*, *d*).

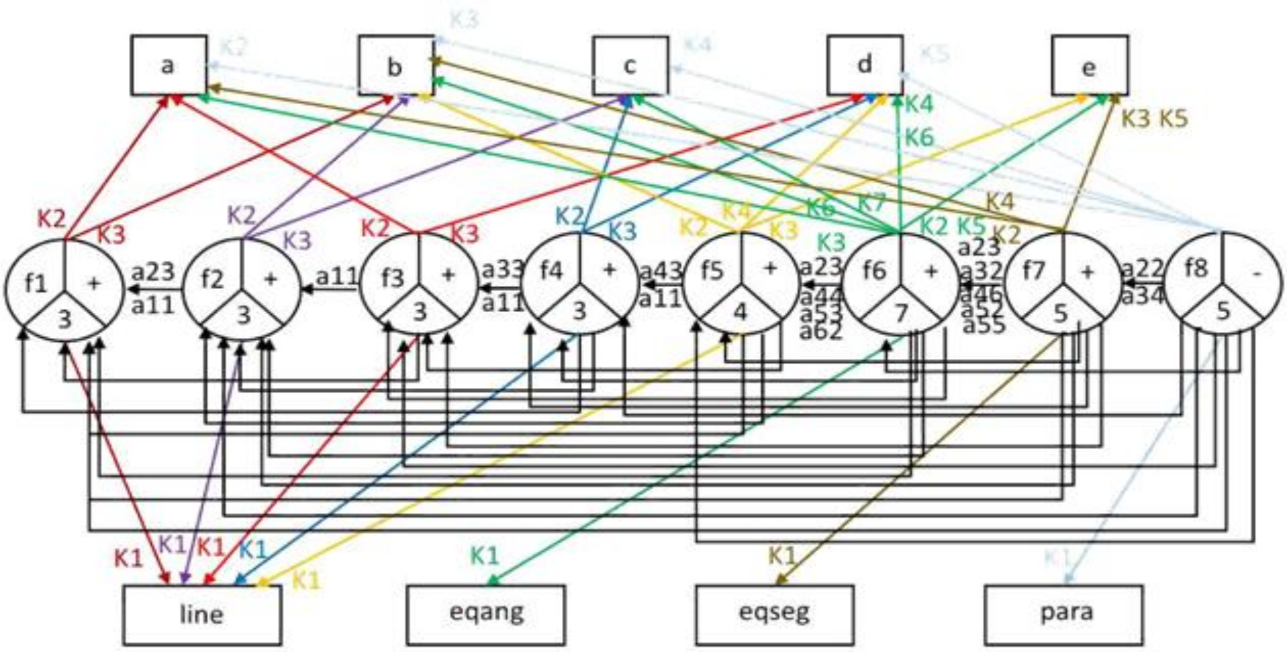

Fig. 4 Expression graph for the task para(*a*, *b*, *c*, *d*).

Table 1 Logical representation of the predicate under.

under(D,C)	$(\exists A, B)$ below(D,C) $\wedge$ perpen(A,B,C,D)
perpen(A,B,C,D)	$noteq(A,B) \wedge noteq(C,D) \wedge ((\neg noteq(A,C) \wedge rangle(B,A,D)) \vee (noteq(A,C) \wedge online(A,C,D) \wedge rangle(B,A,C)) \vee (noteq(A,C) \wedge \neg online(A,C,D) \wedge online(C,A,B) \wedge rangle(A,C,D)) \vee (noteq(A,C) \wedge \neg online(A,C,D) \wedge \neg rangle(C,A,B) \wedge (\exists O)(online(O,A,B) \wedge online(O,C,D) \wedge online(A,O,C))))$
rangle(A,B,C)	$noteq(A,B) \wedge noteq(B,C)(\exists O)(midpoint(B,A,O) \wedge eqseq(A,C,C,O))$
midpoint(O,A,B)	$noteq(A,B) \wedge collinar(O,A,B)$
eqseg(A,B,C,D)	length(AB) = length(CD)
collinear(O,A,B)	$(\exists L)\ on(A,L) \wedge on(B,L) \wedge on(C,L) \wedge noteq(O,A) \wedge noteq(O,B) \wedge noteq(A,B)$
online(O,A,B)	$noteq(A,B) \wedge collinear(O,A,B)$

Table 2 Algebraic representations of the predicate under.

under(D,C)	$(\exists A, B)$ below(D,C) $\wedge$ perpen(A,B,C,D)
below(A,B)	$(\exists \prod -type)(AB \subset \prod -type)\ (U_i \leq U_{min}) \wedge (1 - \mu_x(U_i))$
perpen(A,B,C,D)	$(x_A \neq x_B \vee y_A \neq y_B) \wedge (x_C \neq x_D \vee y_C \neq y_D) \wedge (x_B - x_A)(x_D - x_C) + (y_B - y_A)(y_D - y_C) = 0$
noteq(A,B)	$x_A \neq x_B \wedge y_A \neq y_B$
eqseg(A,B,C,D)	$(x_B - x_A)^2 - (y_B - y_A)^2 = (x_D - x_C)^2 - (y_D - y_C)^2$
collinear(A,B,C)	$(y_B - x_A)x_C + (x_A - x_B)y_C + (x_B y_A - x_A y_B) = 0$
online(P,A,B)	$(x_A \neq x_B \vee y_A \neq y_B) \wedge (y_A - y_P y_B + x_A y_P - x_P y_A) = 0$
midpoint(P,A,B)	$(x_A \neq x_B \vee y_A \neq y_B) \wedge (2x_P - x_A - x_B = 0) \wedge (2y_P - y_A - y_B = 0)$

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