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Production Scheduling and Distribution in Downstream Sector Using Block-Structured Linear Programming Solution Technique : A Comparative Analysis

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Abstract: The aim of this paper is to compare block-structured linear programming (LP) models against other practical optimization methods for solving downstream product refinery problems using a solution method different from the existing ones (like mixed integer linear programming (MILP) method). The work X-rays the Nigerian petroleum refining industries and their channel of distribution in the local setting and identifies the critical features of scheduling and allocation of refined crude products; either for distribution within the country or for exportation to the international market. Applying our model to the distribution model, the computational results reveal a better route with lowest transportation cost for the scheduling problem and the best optimal blend with higher revenue for the production problem.

Key words: Production scheduling, channel of distribution, act of vandalism, transportation technique.

1. Introduction

The block-structured linear programming (LP) is similar to the step-structured LP which is a technique suitable for solving petroleum refinery problems. In this method, the different problems of the subsectors of the refinery are formulated independently constituting the independent constraints and then later linked together by an additional constraint called the common constraint. These common constraints are usually absent in the existing formulations. One of these existing models used for solving downstream problems of product allocation, distribution and of course starting with production problem has been the mixed integer programming (which could be linear (MILP) or nonlinear (MINLP)) model [1]. Others are simulation method, statistical method and other nonlinear techniques. Talks pertaining to issues on refining have been on-going as early as in the 60's. A compilation of major advances and opportunities in Petroleum refining operations was submitted in Shah et al. [2]. Before then, some optimization techniques have already been introduced. In Fisher [3], a guide to the Lagrange decomposition method applicable to petroleum sector was presented by him involving production planning and scheduling integration. The method has been applied to Jia and Ierapetritou [4]. The method is also applicable to multi-period refinery planning as in Erdirik-Dogan and Grossman [5] and earlier in Ierapetritou and Floudas [6]. Later, Grossman [7] addressed some major challenges involved in modeling refinery problems and solution to the problems.

Although, modern petroleum refining industries have successfully accepted software solutions to enhance their profit margins and establish a steady and orderly routine in the lifting and reallocation of products. These intensions have not been well fulfilled since the end users cannot tell the same success story due to some peculiarities of the different problems associated with the software. Packages such as PIMS (Aspen Technology), RPMS (Honeywell Hi-Spec Solution), GRTMPS (Haverly Systems) and

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GAMS which make use of reduced gradient method have been applied as in Held et al. [8] while other solvers are addressed through successive LP application. These softwares allowed the development of general production plans for the whole refinery which may be unsuitable for LP Models of specific structures.

As a result of increase in demand for processed petroleum products which are also continuous, several aspects of assignment model (AM)/scheduling have been on the search light and applied by different authors. Assignment problem has wide range of applications. Hence, an efficient technique is hereby presented and compared to computational results of MILP.

Techniques like the null-space method with finite element approximations has been considered in Refs. [9-11] while the splitting scheme introduced by Dyn and Ferguson [12] and later generalized using real positive matrices in Golub and Wathen [13] have also been applied in solving downstream sector optimization problems. This is usually done by providing solution to the problem through MILP technique. However, in the mid 19th century, the work of Blanding [14], a mathematical model that was based on kinetic rate expression in catalytic cracking of petroleum was noticed. An improvement of the work was later presented in Elomari [15] for components of fuel distillate by alkylation with a view to minimizing capital cost for hydro-treating and reduction in operating cost due to low hydrogen. In order to maximizing the overall profit, there is need to determine an optimal product blend for the finished products.

Moreover, LP models have been used in analysis of scheduling and planning problems due to their ease of modeling and solution [16-18]. Bengtsson and Nonas [19] focused on modeling and analyzing different types of refinery problems using general application-package for mixed-integer solution (GAMS). This current work will look at MILP against the model proposed as presented in Ojarikre and Ekoko [20] and stepping stone method starting with Vogel's approximation method (VAM) against the same model for distribution problem. Li et al. [21] made similar comparative analysis of linear and mixed linear optimization of both theoretical and computational study of robust optimization.

On the issue of distribution, there are many modes of transferring processed petroleum product from the depots to various filling station in different towns and villages for discharge to consumers. It could be through pipelines, tankers (trucks) or boat vessels to riverine communities. Rejowski and Pinto [22] identified three major transfer operations of petroleum products which are by road, railways and by pipelines. The pipeline mode is supposed to be the best but it is not very safe in Nigeria due to the activities of vandals. For instance, Punch News Paper of 25th July, 2017 reported that one of the pipelines (Trans-delta) for lifting petroleum products into the international market was blown up by a group of Niger-Delta militants. However, due to the nature of crude oil and the deep water terrain of the environment where it is coming from, pipelines can only be used to supply petroleum products from oil wells to vessels for onward distribution to countries/customers. In the absence of the act of vandalism, the distribution and supply of refined petroleum products is perfect and economical through pipe lines especially crude oil from oil rigs/wells to refineries/vessels. Below is the distribution channel in Fig. 1.

Ultimately from the filling stations to the final consumers. Between the oil wells and the refineries to the depots, the mode is pipelines. However, from the depots to filling stations the mode of transfer is trucks. Section one of this work is the introduction, the distribution problem and the production problems are contained in sections two and three respectively.

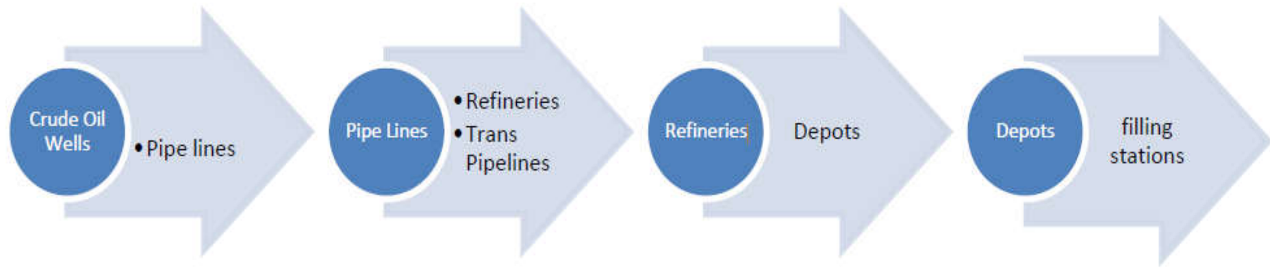

Fig. 1 Channel of product distribution.

2. Distribution Probiem

The key decision in this work is not on the best mode of delivery but on scheduling and allocation of refined petroleum products from depots to fuelling stations at reduced cost. Unlike the scheduling of machine problem where you consider processing times, due dates of finished products and penalties for completing jobs on time [23, 24] but rather the objective is to obtain the best route at minimum total transportation cost of assignments and distribution of products to the fuel stations from the depots. That is, Suppose x_{ij} is the amount of product to be moved from either the oil well to refinery or from depots to fuel stations, then the total cost of transporting the said amount is $\sum_i \sum_j c_{ij} x_{ij}$. Give that empty vessel is not transported, we restrict each $x_{ij} \geq 0$ · c_{ij} is the unit cost of each product while a_i, $i = 1(1)\ m$ is the amount demanded and b_j $j = 1(1)\ n$ is the amount available. To achieve minimum distribution cost, the mathematical formulation of the problem then becomes:

$$\text{Minimize } z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij} \tag{1}$$

Subject to

$$\sum_{j=1}^{n} x_{ij} = a_i \ , \quad i = 1(1)\,m \tag{2}$$

$$\sum_{i=1}^{m} x_{ij} = b_j \ , \quad j = 1(1)\,n \tag{3}$$

$$x_{ij} \geq 0 \tag{4}$$

We now present an illustration using comparative procedure.

Illustrations 1.

A transportation problem has the following Table 1 with unit transportation costs in Naira.

Table 1 Transportation table of supply and demand requirements.

		Filling stations				Supply of petrol. products (000 litres)
		1	2	3	4	
	1	40	40	45	35	18
Depots	2	20	25	30	25	26
	3	50	40	30	35	20
Demand for petrol products (000 litres)		19	15	12	18	

(1) Solution by existing methods (Vogel's approximation and stepping stone),

Starting with the initial basic feasible solution of Vogel approximation method (VAM) and finally using the stepping stone method we obtain the optimal solution as x_{12} = 8,000, x_{14} = 10,000, x_{21} = 19,000, x_{22} = 7,000, x_{33} = 12,000, x_{34} = 8,000, and z = $1,865,000.00;

(b) Solution by block-structured LP approach [24].

The block-structured LP model of the example above is formulated thus:

$$\begin{aligned} \textit{Minimize} \quad z = {} & 40x_{11} + 40x_{12} + 45x_{13} + 35x_{14} + 20x_{21} + 25x_{22} + 30x_{23} + 25x_{24} + \\ & 50x_{31} + 40x_{32} + 30x_{33} + 35x_{34} \end{aligned}$$

Subject to

$$\left.\begin{array}{llll} \boxed{x_{11} + x_{12} + x_{13} + x_{14}} & & & = 18 \\ & \boxed{x_{21} + x_{22} + x_{23} + x_{24}} & & = 26 \\ & & \boxed{x_{31} + x_{32} + x_{33} + x_{34}} & = 20 \end{array}\right\} \text{Row constraints in indep. blocks} \quad (4)$$

$$\left.\begin{array}{lllr} x_{11} & + x_{21} & + x_{31} & = 19 \\ x_{12} & + x_{22} & + x_{32} & = 15 \\ x_{13} & + x_{23} & + x_{33} & = 12 \end{array}\right\} \text{Column constraints}$$

$$x_{ij} \geq 0, \quad i = 1(1)3, \quad j = 1(1)4$$

Re-denoting the variables, the above system can be rewritten as:

$$\begin{aligned} \textit{Minimize} \quad z = {} & 40x_1 + 40x_2 + 45x_3 + 35x_4 + 20x_5 + 25x_6 + 30x_7 + 25x_8 + \\ & 50x_9 + 40x_{10} + 30x_{11} + 35x_{12} \end{aligned}$$

Subject to

$$\begin{array}{llll} \boxed{x_1 + x_2 + x_3 + x_4} & & & = 18 \\ & \boxed{x_5 + x_6 + x_7 + x_8} & & = 26 \\ & & \boxed{x_9 + x_{10} + x_{11} + x_{12}} & = 20 \end{array}$$

$$\begin{array}{lllr} x_1 & + x_5 & + x_9 & = 19 \\ x_2 & + x_6 & + x_{10} & = 15 \\ x_3 & + x_7 & + x_{11} & = 12 \end{array}$$

$$x_j \geq 0, \quad j = 1(1)12$$

Solving this, yields the optimal solution as x_4 = 18,000 = x_{14}, x_5 = 19,000 = x_{21}, x_6 = 7,000 = x_{22}, x_{10} = 8,000 = x_{32}, x_{11} = 12,000 = x_{33}, x_{12} = 0 = x_{34} and z = \$1,865,000.00. From the optimal solution, we observed that out of the 12 different routes only the routes (1, 4), (2, 1), (2, 2), (3, 2), (3, 3), (3, 4) are to be used if the minimum total transportation cost of \$1,865,000.00 is to be achieved. It is worth noting that this optimal solution is the same as that obtained earlier using stepping stone method.

3. The Production Model

Let the incomes per barrel of the refined products be denoted by C_A, C_B, C_C, C_D and C_N for the different refined products A, B, C, D can be obtained by subtracting costs per barrel from the selling price per barrel. Now, $b_j^{(i)}$ is the total available volumetric property of the jth stream blend of the refined products and $x_j^{(k)}$ is the volume of stream j needed to blend each of the k (LPG(A), Premium Motor Spirit (B), Kerosene (C) and Diesel (D)) different products. $x_j^{(k)} \geq 0$ since it is the volume of stream j produced per day, it must not be negative. a_{ij} is the ith component property that is mixed with the jth stream. We consider only two of the common properties of the four refined products, octane number and Reid volumetric property (RVP). Now, to obtain the required properties of the different product mix, we have as follows:

$$\sum_{j=1}^{4} a_{ij}^{(i)} x_j^{(k)} \leq b_k^{(i)} \sum_{j=1}^{4} x_j^{(k)} \quad (5)$$

$$i = 1, 2 \quad and \quad k = A, B, C, D$$

Which is broken down as:

$$\sum_{j=1}^{4} d_{kj}^{(i)} x_j^{(k)} \leq 0 \quad \text{for RVP} \quad (6)$$

and

$$\sum_{j=1}^{4} d_{kj}^{(i)} x_j^{(k)} \geq 0 \quad \text{for property Octane number.} \quad (7)$$

Two major properties observed.

$$d_{kj}^{(i)} = \left(a_{ij}^{(i)} - b_k^{(i)}\right), \quad \forall \; i = 1, 2 \;\; and \;\; j = 1, 2, 3, 4 \;\; k = A, B, C, D \quad (8)$$

Objective function is modeled as:

$$Maximize\ z = C_A V_A + C_B V_B + C_C V_C + C_D V_D + C_N\left(\left(b_1 + b_2 + b_3 + b_4\right) - \left(V_A + V_B + V_C + V_D\right)\right) \quad (9)$$

The volumetric propertires V_A, V_B, V_C, V_D can be expressed in terms of $x_j^{(k)}$, Eq. (9) to obtain

$$Maximize\ z = C_A\left(x_1^{(A)} + x_2^{(A)} + x_3^{(A)} + x_4^{(A)}\right) + C_B\left(x_1^{(B)} + x_2^{(B)} + x_3^{(B)} + x_4^{(B)}\right) + C_C\left(x_1^{(C)} + x_2^{(C)} + x_3^{(C)} + x_4^{(C)}\right) + C_D\left(x_1^{(D)} + x_2^{(D)} + x_3^{(D)} + x_4^{(D)}\right) + C_N\left\{\left(b_1 + b_2 + b_3 + b_4\right) - \left(x_1^{(A)} + \cdots + x_4^{(A)} + x_1^{(B)} + \cdots + x_4^{(B)} + x_1^{(C)} + \cdots + x_4^{(C)} + x_1^{(D)} + \cdots + x_4^{(D)}\right)\right\}$$

or

$$Maximize\ z = C'_N + \sum_{j=1}^{4} C'_A x_j^{(A)} + \sum_{j=1}^{4} C'_B x_j^{(B)} + \sum_{j=1}^{4} C'_C x_j^{(C)} + \sum_{j=1}^{4} C'_D x_j^{(D)} \quad (10)$$

Which is subject to Eqs. (6) and (7)

where,

$$C'_N = C_N\left(b_1 + b_2 + b_3 + b_4\right), \; C'_A = \left(C_A - C_N\right), \; C'_B = \left(C_B - C_N\right), \; C'_C = \left(C_C - C_N\right), \; C'_D = \left(C_D - C_N\right)$$

In Illustration 2, the above steps are applied.

Illustration 2.

The data in Tables 2 and 3 are obtained from Warri Refinery and Petrochemical Company (WRPC), NNPC Warri Office and 2008 NNPC annual statistical bulletin.

Table 2 Streams property and availability.

Property *i* \ Stream *j*		1 Alkylate.	2 Catalytic Ref. gasoline	3 Straight run gasoline	4 Fluid cat. gasoline
1	RVP	6	9	8	10.4
2	ON	86	93	72	98

bls/day	3,370	7,750	5,460	8,010

Table 3 Properties specification of the petroleum products.

k \ i		1 RVP	2 ON	$/bls
A	LPG	≤ 7	≥ 94	3,275
B	PMS	≤ 7	≥ 97	9,450
C	Kero	≤ 7	≥ 78	7,717.5
D	Diesel	≤ 7	≥ 85	16,852.5
N	Naphta	-	-	1,300

The LP problem is now formulated as:

A.

$$Maximize\, z = 1975\sum_{j=1}^{4} x_j^{(A)} + 8150\sum_{j=1}^{4} x_j^{(B)} + 6417.5\sum_{j=1}^{4} x_j^{(C)} + 15552.5\sum_{j=1}^{4} x_j^{(D)}$$

Subject to

$$\left.\begin{array}{ll}
-x_1^{(A)} + 2x_2^{(A)} + x_3^{(A)} + 3.4x_4^{(A)} & \leq 0 \\
-8x_1^{(A)} - x_2^{(A)} - 22x_3^{(A)} + 4x_4^{(A)} & \geq 0 \\
-x_1^{(B)} + 2x_2^{(B)} + x_3^{(B)} + 3.4x_4^{(B)} & \leq 0 \\
-11x_1^{(B)} - 4x_2^{(B)} - 25x_3^{(B)} + x_4^{(B)} & \geq 0 \\
-x_1^{(C)} + 2x_2^{(C)} + x_3^{(C)} + 3.4x_4^{(C)} & \leq 0 \\
10x_1^{(C)} + 17x_2^{(C)} - 4x_3^{(C)} + 12x_4^{(C)} & \geq 0 \\
-x_1^{(D)} + 2x_2^{(D)} + x_3^{(D)} + 3.4x_4^{(D)} & \leq 0 \\
x_1^{(D)} + 8x_2^{(D)} - 13x_3^{(D)} + 13x_4^{(D)} & \geq 0
\end{array}\right\}$$

$$\begin{array}{l}
x_1^{(A)} + x_1^{(B)} + x_1^{(C)} + x_1^{(D)} \leq 3370 \\
x_2^{(A)} + x_2^{(B)} + x_2^{(C)} + x_2^{(D)} \leq 7750 \\
x_3^{(A)} + x_3^{(B)} + x_3^{(C)} + x_3^{(D)} \leq 5466 \\
x_4^{(A)} + x_4^{(B)} + x_4^{(C)} + x_4^{(D)} \leq 8010 \\
x_1^{(A)}, x_2^{(A)}, x_3^{(A)}, x_4^{(A)}, x_1^{(B)}, x_2^{(B)}, x_3^{(B)}, x_4^{(B)}, x_1^{(C)}, x_2^{(C)}, x_3^{(C)}, x_4^{(C)}, x_1^{(D)}, x_2^{(D)}, x_3^{(D)}, x_4^{(D)} \geq 0
\end{array}$$

This was solved using two-phase simplex method to obtain the following optimal results;

$$x_2 = x_6 = x_{10} = x_3 = x_7 = x_{11} = 0,$$

$$x_{14} = 1189.41 = x_2^{(D)}, x_{15} = 991.18 = x_3^{(D)}, x_{13} = 3370 = x_1^{(D)},$$

$$x_{26} = 6560.59,\ x_{27} = 4468.82, x_{28} = 8010,\ and\ z = \#86{,}325{,}523.53$$

To maximize total income, the optimal solution suggests production of more of product (D),

BV	b_i	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}	x_{13}	x_{14}	x_{15}	x_{16}	x_{17}	x_{18}	x_{19}	x_{20}	x_{21}	x_{22}	x_{23}	x_{24}	x_{25}	x_{26}	x_{27}	x_{28}	x_{29}
x_{29}	(-0.41)	(-0.35)	0	0	0	(-0.35)	0	0	0	(-0.35)	0	0	0	0	0	0	-1.68	0	0	0	(-0.02)	0	0	0	(-0.38)	(-0.35)	0	0	0	1
*		23043.79				15694.14				11240.68							11161.21				457.43				9605.96	25615.88				
Absolute		[65,839				44,840				32,116							6,643				22,871				25,276	73,188] = 6,643 from				

$\mathbf{X}_{16}$ column

Fig. 2 Extract of the non-integral value of x29 required to be made integer.

B. Existing Formulation Method.

(1) Simplex method

The problem is formulated as a maximization with slacks introduced to standardized the LPP. This same example introduced above for the production problem is solved using the original simplex method without the common constraints and after the 4th iteration, yielded a trivial solution. The result from block-structured model is certainly better than the trivial one of the existing models since Z = \$0 < Z (of proposed model) = \$86,325,523.53. The objective function is maximization of the total income to be received.

(2) Mixed integer linear programming (MILP)

Using the optimal solution obtained from illustration 2A and incorporating MILP into the non-integral optimal tableau, we now solve the same problem using the MILP approach. Suppose x_{14} = 1,189.41 is required to be an integer. Initiating Gomory's computational steps for MILP as stated in Ekoko [25]. We have the initial entries of the additional constraint with x_{29} in its basic variable column (BV) in Fig. 2.

The objective function entries in Fig. 2 that correspond to the negative entries in the new row are all indicated in the starred row while the absolute value of their ratios $\left(i.e. \left| \frac{z_j - c_j}{a_{rj}} \right| \right)$ are as shown in the last row of Fig. 2. The minimum absolute value is 6,643 from column × 16, an indication that × 16 column is the pivot column and -1.68 is the pivot element. The new row is incorporated into a new tableau and we obtain a super-optimal tableau.

Applying the dual simplex method to the new tableau and iterating, a new objective function value of the MILP tableau is obtained as:

$$Z' = 53\ .86325523 - \frac{(-0.41)(11161.21)}{-1.68}$$

= \$86,322,799.66

But Z' = \$86,322,799.66 $<$ z = \$86,325,523.53

Hence, our optimal value (z) is a better result compared to z'.

4. Conclusions

The model presented in this work is suitable for solving scheduling (for production or distribution), distribution and production blending problems. Moreover, the optimal solution obtained using our model formulation is better than that obtained from MILP technique. From the Illustrations 1 also, our model is good for solving distribution problems and the optimal solutions can be compared to those of stepping stone method or the modified distribution method. It was also observed that some refinery problems cannot be solved using the original simplex method.

The number one objective of any refining outfit is to process the crude constituents into its refined components. Thus, depending on the volatility of the crude oil stream, a higher percentage of some of its mixtures are obtained through many processes. Consequently, several problems are identified resulting from each of the processes. This current work handles just a fraction of these problems. Hence, there is need to proffer solution using optimization technique even in the area of bridging. This occurs whenever there is a short fall in the volume/quantity of refined product produced for internal use. The difference in cost price is made up by the federal government which they call subsidy. Under production/unavailability of petroleum product is the genesis of petroleum crisis in the country which is currently being experienced in almost every state of the federation.

All the channels of distributions of petroleum products have its own merits and shortcomings. Each of them could be means of oil theft and are all subject to the act of vandalism. According to several findings, the pipeline system is adjudged to be the best. On the high sea, the oil vessels are used for international supply. For local supply and distribution, oil tanker/trucks are used because more risk is attached to pipelines but these trucks could also break down or be involved in an accident on their way between the depots and the filling stations. These accidents are usually very serious and often times involved loss of human lives as a result of fire outbreak.

In conclusion, the formulation of the problems of downstream sector of the petroleum industry by integrating the independent constraints and the common constraints always yields a better solution.

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