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Optimum Determination of Partial Transmission Ratios of Mechanical Driven Systems Using a V-belt and a Three-Step Helical Gearbox

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Abstract: This paper introduces a new study on the optimum calculation of partial transmission ratios of a mechanical drive system using a V-belt and a three-step helical gearbox in order to get the minimum size of the system. The chosen objective function was the cross section dimension of the system. In solving the optimization problem, the design equation for pitting resistance of a gear set was investigated and equations on moment equilibrium condition of a mechanic system including a V-belt and three helical gear units and their regular resistance condition were analysed. From the results of the study, effective formulas for determination of the partial ratios of the V-belt and three-step helical gearboxes were introduced. As using explicit models, the partial ratios can be determined accurately and simply.

Key words: Transmission ratio, gearbox design, optimum design, V-belt drive, helical gearbox.

1. Introduction

In optimum gearbox design, one of the most important tasks is optimum determination of partial transmission ratios of the gearbox. This is because the partial ratios are main factors affecting the size, the dimension, the weight and the cost of the gearbox [1]. Subsequently, optimum determination of the partial ratios of a gearbox has been subjected to many researches.

Up to now, many studies on the prediction of the partial ratios gearboxes have been done. The partial ratios were calculated for different gearbox types, such as for helical gearboxes [1-5], for bevel-helical gearboxes [1, 4, 6, 7] and for worm-gearboxes [4, 8]. Besides, many methods have been used for finding optimum partial ratios. These methods are the graph method [1, 2], the "practical method" [3] and modeling method [4, 6-8].

From previous studies, it is clear that there have been many studies on the calculation of the partial ratios for different types of gearboxes. However, until now, there was only a study for mechanical driven systems using a V-belt and a two-step helical gearbox [9]. There have not been studies for mechanical driven systems using a V-belt and a three-step helical gearbox. This paper introduces a result for optimal determination of partial ratios for mechanical systems using a V-belt and a three-step helical gearbox in order to get the minimum system cross-sectional dimension.

2. Calculation of Optimum Partial Transmission Ratios

For a three-step helical gearbox (Fig. 1), the cross-sectional dimension is minimum when d_{w21}, d_{w22} and d_{w23} satisfy Eq. (1) [1].

$$d_{w21} = d_{w22} = d_{w23} \tag{1}$$

From Eq. (1) and Fig. 1, for a mechanical drive system using a V-belt and a three-step helical gearbox,

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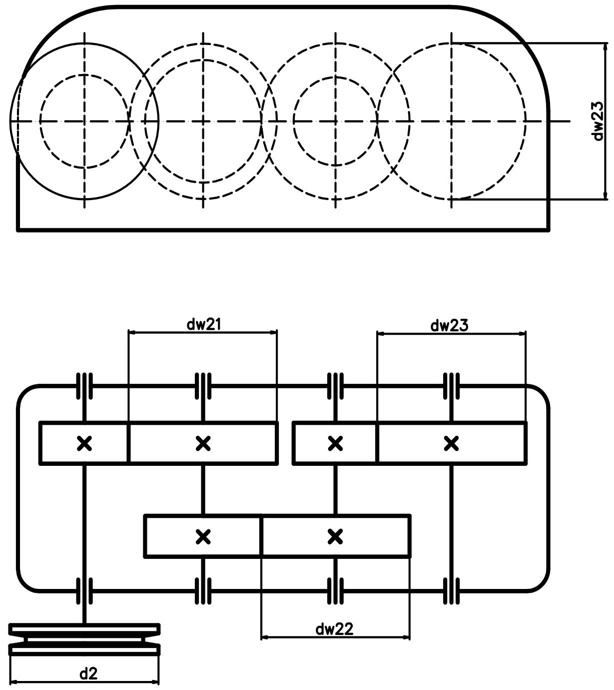

Fig. 1 Calculation chema for optimum determination of partial transmission ratios.

the cross-sectional dimension of the system is minimum when:

$$d_2 = d_{w21} = d_{w22} = d_{w23} \qquad (2)$$

In which, d_2 is driven pulley diameter (mm); d_{w21}, d_{w22} and d_{w23} are driven diameters of three gear units (mm).

For a three-step helical gearbox, based on the relations between gear ratios and driven diameters as well as based on Eq. (1), the optimum partial gear ratios u_1, u_2 and u_3 have been determined as Eqs. (3)-(5) [4].

$$u_1 = 0.8527 \cdot u_g^{0.5714} \qquad (3)$$

$$u_2 = 1.0324 \cdot u_g^{0.2857} \qquad (4)$$

$$u_3 = 1.136 \cdot u_g^{0.1429} \qquad (5)$$

In which, u_g is the transmission ratio of the gearbox.

From above analysis, it is clear that for finding the optimum partial ratios of the systems in order to get the minimum system cross section, it is necessary to determine the diameters d_2 and d_{w21}.

2.1 Determining the Driver Pulley Diameter d_2

For a V-belt set, from tabulated data for determining allowable power [10], the regression model for calculation of driver diameter d_1 (with the determination coefficient R^2 = 0.9156) was found:

$$d_1 = 269.7721 \cdot [P_1]^{0.7042} / v^{0.5067} \qquad (6)$$

Theoretically, the peripheral velocity of the belt can determined as Eq. (7):

$$v = \pi \cdot d_1 \cdot n_1 / 60000 \qquad (7)$$

From Eqs. (6) and (7), the diameter of the driver pulley can be determined by Eq. (8):

$$d_1 = 1093.8 \cdot [P_1]^{0.7923} / n_1^{0.6369} \qquad (8)$$

Also, the diameter of driven pulley of a V-belt drive is calculated by Eq. (9) [10]:

$$d_2 = u_b \cdot d_1 \cdot (1 - \varepsilon) \qquad (9)$$

Substituting Eq. (8) into Eq. (9) gives Eq. (10):

$$d_2 = 1093.8 \cdot u_b \cdot (1 - \varepsilon) \cdot [p_1]^{0.7923} / n_1^{0.6369} \qquad (10)$$

Where ε is slippage coefficient; $\varepsilon = 0.01 \ldots 0.02$ [10]; u_b is the transmission ratio of the V-belt set; $[P_1]$ is the allowable power of the drive (kW); $[P_1]$ is calculated by Eq. (11):

$$[P_1] = n_1 \cdot [T_1] / \left(9.55 \cdot 10^6\right) \qquad (11)$$

Choosing $\varepsilon = 0.015$ and substituting it and Eq. (11) into Eq. (10) gives Eq. (12):

$$d_2 = 0.0032 \cdot u_b \cdot n_1^{0.1554} \cdot [T_1]^{0.7923} \qquad (12)$$

2.2 Determining the Driven Diameter d_{w21}

For the first step of a helical gear unit, Eq. (13) is used for the pitting resistance [10]:

$$\sigma_{H1} = Z_{M1} \cdot Z_{H1} \cdot Z_{\varepsilon 1} \sqrt{2 \cdot T_{11} \cdot K_{H1} \cdot \sqrt{u_1 + 1} / \left(b_{w1} \cdot d_{w11}^2 \cdot u_1\right)} \le [\sigma_{H1}] \qquad (13)$$

It follows from Eq. (13) that:

$$[T_{11}] = b_{w1} \cdot d_{w11}^2 \cdot u_1 \cdot [\sigma_{H1}]^2 / \left(2 \cdot (u_1 + 1) \cdot K_{H1} \cdot (Z_{M1} \cdot Z_{H1} \cdot Z_{\varepsilon 1})^2\right) \qquad (14)$$

where, b_{w1} and d_{w11} are determined by Eqs. (15)

and (16):

$$b_{w1} = \psi_{ba1} \cdot a_{w1} = \psi_{ba1} \cdot d_{w11} \cdot (u_1 + 1)/2 \quad (15)$$

$$d_{w11} = d_{w21} / u_1 \quad (16)$$

Substituting Eqs. (15) and (16) into Eq. (14) gives Eq. (17):

$$[T_{11}] = \psi_{ba1} \cdot d_{w21}^3 \cdot [K_{01}] / \left(4 \cdot u_1^2\right) \quad (17)$$

From Eq. (17), the driven diameter d_{w21} can be calculated by Eq. (18):

$$d_{w21} = \sqrt[3]{4u_1^2 [T_{11}] / \left(\psi_{ba1} [K_{01}]\right)} \quad (18)$$

Where

$$K_{01} = [\sigma_{H1}]^2 / \left(K_{H1} (Z_{M1} Z_{H1} Z_{\varepsilon 1})^2\right) \quad (19)$$

In which,

$[\sigma_{H1}]$—Allowable contact stress of the first-step gear set (MPa); For a helical gear set, $[\sigma_{H1}]$ can be calculated by Eq. (20) [10]:

$$[\sigma_{H1}] = \left([\sigma_H]_1 + [\sigma_H]_2\right)/2 \quad (20)$$

Where, $[\sigma_H]_1$ and $[\sigma_H]_2$ are the allowable contact stress of pinion and gear of the first-step gear set (MPa). $[\sigma_H]_1$ is determined by Eq. (21) [10]:

$$[\sigma_H]_1 = \sigma_{H\lim 1}^0 \cdot K_{HL} / S_H \quad (21)$$

In Eq. (21), $\sigma_{H\lim 1}^0$ is allowable contact stress for based stress cycle life of the pinion: $\sigma_{H\lim 1}^0 = 2 \cdot HB_1 + 70$; HB_1 is Brinell hardness of the pinion; K_{HL} is the stress cycle life factor; S_H is the safety factor. Since the material of the pinion is medium carbon steel 1045, it was found $HB_1 = 250$, $K_{HL} = 1$ and $S_H = 1.1$ [10].

Substituting $\sigma_{H\lim 1}^0$, K_{HL} and S_H into Eq. (21) gives $[\sigma_H]_1 = 518.18$ (MPa).

Calculating the same way, for the gear it gives $[\sigma_H]_2 = 490.91$ (MPa).

Substituting values of $[\sigma_H]_1$ and $[\sigma_H]_2$ into Eq. (20) gives $[\sigma_{H1}] = 504.55$ (MPa).

ψ_{ba1}—Coefficients of helical gear face width of step 1; since $\psi_{ba1} = 0.3 \div 0.35$ [10], it can be the average of these values, that is, $\psi_{ba1} = 0.325$;

K_{H1}—Contact load factor for pitting resistance; Since $K_{H1} = 1.1 - 1.3$ [9], authors can choose $K_{H1} = 1.2$;

Z_{M1}—Material factor; Since the pinion and gear are made from steel, Z_{M1} is 274 (MPa$^{1/3}$) [10];

Z_{H1}—Surface condition factor; As the pinion and gear are standard and the helix angles are $8^0 \div 20^0$, Z_{H1} = 1.74 ÷ 1.67 [10]. Therefore, authors can choose $Z_{H1} = 1.71$;

$Z_{\varepsilon 1}$—Loading sharing factor: $Z_{\varepsilon 1} = (1/\varepsilon_\alpha)^{1/2}$ [10] with ε_α is contact ratio. ε_α can be calculated by Eq. (22) [10]:

$$\varepsilon_\alpha = \left[1.88 - 3.2\left(1/z_1 + 1/z_2\right)\right] \cdot \cos\beta \quad (22)$$

With the helix angles are $8^0 \div 20^0$ and the number of teeth of pinion and gear are 15 to 90, the values of the transverse contact ratio is $Z_{\varepsilon 1} = 0.7628 \div 0.8344$. Therefore, the value of $Z_{\varepsilon 1}$ can be chosen as the average of these values, that is, $Z_{\varepsilon 1} = 0.7986$.

Substituting values of $[\sigma_{H1}]$, K_{H1}, Z_{M1}, Z_{H1} and $Z_{\varepsilon 1}$ into Eq. (19) gives Eq. (23):

$$K_{01} = 504.55^2 / \left(1.2 \cdot (274 \cdot 1.71 \cdot 0.7986)^2\right) = 1.5152 \quad (23)$$

Substituting $\psi_{ba1} = 0.325$ and $K_{01} = 1.5152$ into Eq. (16) gives Eq. (24):

$$d_{w21} = 2.0102 \cdot [T_{11}]^{1/3} \cdot u_1^{2/3} \quad (24)$$

2.3 Determining the Partial Ratios

Eqs. (2), (12) and (20) gives Eq. (25):

$$0.0032 \cdot u_d \cdot n_1^{0.1554} \cdot [T_1]^{0.7923} = 2.0102 \cdot [T_{11}]^{1/3} \cdot u_1^{2/3} \quad (25)$$

Based on the moment equilibrium condition of a mechanic system including V-belt and two helical gear units, the allowable torque on the drive shaft $[T_{11}]$ is determined by Eq. (26):

$$[T_{11}] = [T_1] \cdot u_d \cdot \eta_d \quad (26)$$

In which, $[T_1]$ is the permissible torque on the drive shaft; $[T_1]$ can be calculated from permissible torque on the output shaft $[T_r]$ by:

$$\left[T_1\right] = \left[T_r\right] / \left(u_\Sigma \cdot \eta_\Sigma\right) \quad (27)$$

Where, η_Σ is the total efficiency of the system:

$$\eta_\Sigma = \eta_d \cdot \eta_{br}^3 \eta_o^4 \quad (28)$$

In which, η_d is V-belt efficiency (η_d is from 0.956 to 0.96 [2]); η_{br} is helical gear transmission efficiency (η_{br} is from 0.96 to 0.98 [2]); η_o is transmission efficiency of a pair of rolling bearing (η_o is from 0.99 to 0.995 [2]). Choosing $\eta_d = 0.955$, $\eta_{br} = 0.97$ and $\eta_o = 0.992$ [10] and substituting Eqs. (3) and (26)-(28) into Eq. (25) with the note that $u_g = u_t / u_b$ gives Eq. (29):

$$u_b = 387.5054 \cdot \left[T_{out}\right]^{-0.4755} \cdot n_1^{-0.1691} \cdot u_t^{0.8466} \quad (29)$$

Eq. (29) is used to calculate the speed ratio of the V-belt driver. After having u_d , the ratio of the gearbox is calculated by $u_g = u_t / u_b$ and the partial speed ratios of the gearbox u_1, u_2 and u_3 can be found by Eqs. (3), (4) and (5), respectively.

3. Conclusions

The minimum system cross-sectional dimension of a mechanical drive system using a V-belt and a three-step helical gearbox can be obtained by optimum determination of partial transmission ratios of units of the system.

Models for calculating the partial ratios of the V-belt and the three-step helical gearbox for getting the minimum cross-sectional dimension of the system were proposed.

By using explicit models, the partial ratios of the V-belt driver and three-step helical gear units can be determined accurately and simply.

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References

[1] Kudreavtev, V. N., Gierzaves, I. A., and Glukharev, E. G. 1971. "Design and Calculus of Gearboxes (in Russian)." Mashinostroenie Publishing, Sankt Petersburg.

[2] Petrovski, A. N., Sapiro, B. A., and Saphonova, N. K. 1987. "About Optimal Problem for Multi-Step Gearboxes (in Russian)." *Vestnik Mashinostroenie* 10: 13-4.

[3] Milou, G., Dobre, G., Visa, F., and Vintila, H. 1996. "Optimal Design of Two Step Gear Units, regarding the Main Parameters." *VDI BERICHTE* 1230: 227-44.

[4] Pi, V. N., and Tuan, N. K. 2016. "Optimum Determination of Partial Transmission Ratios of Three-Step Helical Gearboxes for Getting Minimum Cross Section Dimension." *Journal of Environmental Science and Engineering A* 5: 570-3.

[5] Pi, V. N. 2008. "A New Study on Optimal Calculation of Partial Transmission Ratio of Three-Step Helical Reducers." In *Accepted Paper for the 3rd IASME/WSEAS International Conference on Continuum Mechanics*, 23-5, Cambridge, UK.

[6] Pi, V. N. 2000. "A New and Effective Method for Optimal Calculation of Total Transmission Ratio of Two Step Bevel-Helical Gearboxes." In *International Colloquium on Mechannics of Solids, Fluids, Structures & Interaction Nha Trang, Vietnam*, 716-9.

[7] Pi, V. N., Binh, N. D., Dac, V. Q., and The, P. Q. 2005, "A New and Effective Method for Optimal Splitting of Total Transmission Ratio of Three Step Bevel-Helical Gearboxes." In *The Sixth Vietnam Conference on Automation*, 175-80, Hanoi.

[8] Pi, V. N., and Dac, V. Q. 2005. "Optimum Calculation of Transmission Ratios for Helical-Worm Reducers (in Vietnamese)." *Journal of Science and Technology, Thai Nguyen University* 4 (36): 70-3.

[9] Pi, V. N., Thao, T. T. P., and Thao, L. T. P. 2015. "A New Study on Optimum Determination of Partial Transmission Ratios of Mechanical Driven Systems Using a V-belt and Two-Step Helical Gearbox." *Vietnam Mechanical Engineering Journal* 10: 123-5.

[10] Chat, T., and Le Van Uyen, D. 1998. "Calculus of Mechanical Transmissions (in Vietnamese)." *Educational Republishing House, Hanoi.*