

# Origin of Particle Spins, Bosons and Fermions, Compounds of Zero Spin Particles, in Complex Spin Region

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**Abstract:** This paper shows: Bosons and Fermions maybe the compounds  $\Pi$  of two zero spin particles which are spin-conjugates each other in complex region. There are two kinds of spin-conjugates of zero spin particle: real spin-conjugate  $(\pi^{Real}, \pi)$  and imaginary spin-conjugate  $(\pi^{Imaginary}, \pi)$ . Using  $(\pi^{Real}, \pi)$ , Bosons and Fermions of Antimatter  $\Pi^R$  could be formed, and using  $(\pi^{Imaginary}, \pi)$ , Bosons and Fermions of Matter  $\Pi^I$  could be formed.

**Key words:** Zero spin particle, Casimir Operator, real region, complex region, spin-conjugate, real spin-conjugate, imaginary spin-conjugate, spin-conjugate-composite, matter, antimatter.

## 1. Introduction

It is thought that  $0\hbar$ , zero spin particle in quantum mechanics does not possess any spin rotational ability, and its angular momentum representation is  $1 \times 1$  dimension, obviously, this kind of spin representation is trivial in Math.

In STS (spin topological space) [1], however the spin angular momentum representation of zero spin particle is no longer trivial, because the first component  $\pi_1$  and the second component  $\pi_2$  of zero spin particle (Refer to Eqs. (1) and (2), [2]) can be constructed by infinite dimensional non-Hermitian operators. Further we could obtain non-trivial diagonal matrix representations of the third component  $\pi_3$  and Casimir Operator  $\pi^2$  of zero spin particle (Refer to Eqs. (3) and (4) of Ref. [2]). At the same time, the matrix elements of the third component  $\pi_3$  and Casimir Operator  $\pi^2$  of zero spin particles are all in *Real* number region. In paragraph 2, the basic properties of  $0\hbar$  zero spin particles in *Real* region are given.

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In paragraph 3, For clarity, the detailed formation of zero spin particle in *Complex* region is given through a concrete example :

In *Complex* region, two special zero spin particles ( group **a**) and group **b**) are introduced, Group **a**)  $\in \pi$  and Group **b**)  $\in \pi^{Imaginary}$ .  $\pi$  and  $\pi^{Imaginary}$  are called imaginary spin-conjugate each other, which marked by  $(\pi^{Imaginary}, \pi) = (\mathbf{b}, \mathbf{a}) = (\mathbf{a}^{Imaginary}, \mathbf{a}) = (\mathbf{b}, \mathbf{b}^{Imaginary})$ .

Expression (46)  $\pi^2_{\frac{+\sqrt{4}}{2}, \frac{+\sqrt{3}}{2}, \frac{-\sqrt{4}}{2}, \frac{+\sqrt{3}}{2}} = \{ 0 + i\sqrt{3} \} \hbar^2 I_0$  and expression (49)  $\pi^2_{\frac{+\sqrt{4}}{2}, \frac{-\sqrt{3}}{2}, \frac{-\sqrt{4}}{2}, \frac{-\sqrt{3}}{2}} = \{ 0 - i\sqrt{3} \} \hbar^2 I_0$  are Casimir Operators of zero spin particle in *Complex* region, which result from the raising operator  $\pi^+$  and the lowering operator  $\pi^-$  of group **a**) (10),(11) and group **b**) (12),(13) respectively.

Deeper researches [3] further show: Using imaginary spin-conjugate  $(\pi^{Imaginary}, \pi)$ , fermion,  $\Pi^I(1)$  (54) with one half spin  $\frac{\hbar}{2}$  of matter could be formed in paragraph 4. And in paragraph 5, similar to paragraph 3 and 4, using real spin-conjugate  $(\pi^{Real}, \pi)$ , fermion  $\Pi^R(1)$  (66) with one half spin  $i\frac{\hbar}{2}$  of antimatter could be formed.

## 2 Basic properties of of $0\hbar$ Zero Spin Particle in Real Region

In the previous paper [2], we showed the basic properties of zero spin particle in Spin Topological Space, STS as following:

Raising operator, lowering operator (diag  $\equiv$  diagonal matrix )

$$\pi_0^+ = \text{diag}\{, 5, 4, 3, 2, 1, \underline{0}, -1, -2, -3, -4, -5, , \}_{+1} \quad (1)$$

$$\pi_{-1}^- = \text{diag}\{-4, -3, -2, -1, 0, \underline{1}, 2, 3, 4, 5, 6, , \}_{-1} \quad (2)$$

$$\pi_{1;0,-1}^2 + \pi_{2;0,-1}^2 = - \text{diag}\{, 25, 16, 9, 4, 1, \underline{0}, 1, 4, 9, 16, 25, , \}_0 \quad (3)$$

$$\pi_{3;0,-1} = \text{diag}\{, 5, 4, 3, 2, 1, \underline{0}, -1, -2, -3, -4, -5, , \}_0 \quad (4)$$

$$\pi_{3;0,-1}^2 = + \text{diag}\{, 25, 16, 9, 4, 1, \underline{0}, 1, 4, 9, 16, 25, , \}_0 \quad (5)$$

Casimir Operator

$$\pi_{0,-1}^2 = \pi_{1;0,-1}^2 + \pi_{2;0,-1}^2 + \pi_{3;0,-1}^2 = 0\hbar^2 I_0 \quad (6)$$

Angular momentum commutation rules

$$\pi_0^+ \pi_{-1}^- - \pi_{-1}^- \pi_0^+ = 2\pi_{3;0,-1} \quad (7)$$

$$\pi_{3;0,-1} \pi_0^+ - \pi_0^+ \pi_{3;0,-1} = +\pi_0^+ \quad (8)$$

$$\pi_{3;0,-1} \pi_{-1}^- - \pi_{-1}^- \pi_{3;0,-1} = -\pi_{-1}^- \quad (9)$$

Note: the math elements of the third component  $\pi_{3;0,-1}$  (4) and Casimir Operator  $\pi_{0,-1}^2$  (6) are all in real number region.

In next two paragraphs, Casimir Operator  $\pi^2$  (6) will be extended to complex region ( but the third component  $\pi_3$  remain in real region), in which imaginary spin-conjugate  $(\pi^{Imaginary}, \pi)$  is concerned with, in paragraph 3, and fermion with one half spin  $\frac{\hbar}{2}$  of matter is constructed in paragraph 4.

### 3 Casimir Operator of Zero Spin Particlein with $(\pi^{Imaginary}, \pi)$ in Complex Region

In order to extend Casimir Operator (6) to complex region, the transformations of raising operator  $\pi_0^+$  (1) and lowering operator  $\pi_{-1}^-$  (2) of group **a)** and group **b)** are made as (10),(11) and (12),(13)

$$\text{a)} \quad \pi_0^+ \Rightarrow \pi_{\frac{+\sqrt{4}}{2}, \frac{+\sqrt{3}}{2}}^+ = \pi_0^+ + \frac{1}{2}(\sqrt{4} + i\sqrt{3})I_{+1} \quad (10)$$

$$\pi_{-1}^- \Rightarrow \pi_{\frac{-\sqrt{4}}{2}, \frac{+\sqrt{3}}{2}}^- = \pi_0^- + \frac{1}{2}(\sqrt{4} + i\sqrt{3})I_{-1} \quad (11)$$

$$\text{b)} \quad \pi_0^+ \Rightarrow \pi_{\frac{+\sqrt{4}}{2}, \frac{-\sqrt{3}}{2}}^+ = \pi_0^+ + \frac{1}{2}(\sqrt{4} - i\sqrt{3})I_{+1} \quad (12)$$

$$\pi_{-1}^- \Rightarrow \pi_{\frac{-\sqrt{4}}{2}, \frac{-\sqrt{3}}{2}}^- = \pi_0^- + \frac{1}{2}(\sqrt{4} - i\sqrt{3})I_{-1} \quad (13)$$

Where

$$I_{+1} = \text{diag}\{, 1, 1, 1, 1, 1, \underline{1}, 1, 1, 1, 1, 1, , \}_{+1} \quad (14)$$

$$I_{-1} = \text{diag}\{, 1, 1, 1, 1, 1, \underline{1}, 1, 1, 1, 1, 1, , \}_{-1} \quad (15)$$

**A)** We deal first with the products of (10) and (11) of group **a)**

$$\pi_{\frac{+\sqrt{4}}{2}, \frac{+\sqrt{3}}{2}}^+ \pi_{\frac{-\sqrt{4}}{2}, \frac{+\sqrt{3}}{2}}^- = \pi_{+1}^+ \pi_{-1}^- - 2I_{+1}I_{-1} + i\frac{\sqrt{3}}{2} \{ \pi_{+1}^+ I_{-1} + I_{+1} \pi_{-1}^- \} \quad (16)$$

$$\pi_{\frac{-\sqrt{4}}{2}, \frac{+\sqrt{3}}{2}}^- \pi_{\frac{+\sqrt{4}}{2}, \frac{+\sqrt{3}}{2}}^+ = \pi_{-1}^- \pi_{+1}^+ - 2I_{-1}I_{+1} + i\frac{\sqrt{3}}{2} \{ I_{-1} \pi_{+1}^+ + \pi_{-1}^- I_{+1} \} \quad (17)$$

Following results are useful to calculate (16) and (17)

Obtainment 1

$$\begin{aligned} \pi_{+1}^+ \pi_{-1}^- &= \text{diag}\{, 5+1, 4+1, 3+1, 2+1, 1+1, \underline{0+1}, -1+1, -2+1, -3+1, -4+1, -5+1, \}_0 \\ &\quad \{, -5+1, -4+1, -3+1, -2+1, -1+1, \underline{0+1}, 1+1, 2+1, 3+1, 4+1, 5+1, \}_0 \\ &= \text{diag}\{, 6, 5, 4, 3, 2, \underline{1}, 0, -1, -2, -3, -4, \}_0 \\ &\quad \{, -4, -3, -2, -1, 0, \underline{1}, 2, 3, 4, 5, 6, \}_0 \end{aligned}$$

$$\pi_{+1}^+ \pi_{-1}^- = \text{diag}\{, -24, -15, -8, -3, 0, \underline{1}, 0, -3, -8, -15, -24, \}_0 \quad (18)$$

$$\pi_{-1}^- \pi_{+1}^+ = \text{diag}\{, -35, -24, -15, -8, -3, \underline{0}, 1, 0, -3, -8, -15, \}_0 \quad (19)$$

then

$$\begin{aligned} \pi_{+1}^+ \pi_{-1}^- - \pi_{-1}^- \pi_{+1}^+ &= + \text{diag}\{, 11, 9, 7, 5, 3, \underline{1}, -1, -3, -5, -7, -9, \}_0 \quad (20) \end{aligned}$$

$$\begin{aligned} \pi_{+1}^+ \pi_{-1}^- + \pi_{-1}^- \pi_{+1}^+ &= + \text{diag}\{, -59, -39, -23, -11, -3, \underline{1}, 1, -3, -11, -23, -59, \}_0 \quad (21) \end{aligned}$$

Obtainment 2

$$I_{+1}I_{-1} = I_{-1}I_{+1} = I_0 \quad (22)$$

$$I_0 = \text{diag}\{, 1, 1, 1, 1, 1, \underline{1}, 1, 1, 1, 1, 1, , \}_0 \quad (23)$$

then

$$I_{+1}I_{-1} - I_{-1}I_{+1} = 0I_0 \quad (24)$$

$$I_{+1}I_{-1} + I_{-1}I_{+1} = 2I_0 \quad (25)$$

Obtainment 3

$$\pi_{+1}^+ I_{-1} = \text{diag}\{, 6, 5, 4, 3, 2, \underline{1}, 0, -1, -2, -3, -4, , \}_0 \quad (26)$$

$$I_{-1} \pi_{+1}^+ = \text{diag}\{, 7, 6, 5, 4, 3, \underline{2}, 1, 0, -1, -2, -3, , \}_0 \quad (27)$$

then

$$\pi_{+1}^+ I_{-1} - I_{-1} \pi_{+1}^+ = -I_0 \quad (28)$$

$$\begin{aligned} & \pi_{+1}^+ I_{-1} + I_{-1} \pi_{+1}^+ \\ & = \text{diag}\{, 13, 11, 9, 7, 5, \underline{3}, 1, -1, -3, -5, -7, , \}_0 \end{aligned} \quad (29)$$

Obtainment 4

$$I_{+1} \pi_{-1}^- = \text{diag}\{, -4, -3, -2, -1, 0, \underline{1}, 2, 3, 4, 5, 6, , \}_0 \quad (30)$$

$$\pi_{-1}^- I_{+1} = \text{diag}\{, -5, -4, -3, -2, -1, \underline{0}, 1, 2, 3, 4, 5, , \}_0 \quad (31)$$

then

$$I_{+1} \pi_{-1}^- - \pi_{-1}^- I_{+1} = +I_0 \quad (32)$$

$$\begin{aligned} & I_{+1} \pi_{-1}^- + \pi_{-1}^- I_{+1} \\ & = \text{diag}\{, -9, -7, -5, -3, -1, \underline{1}, 3, 5, 7, 9, 11, , \}_0 \end{aligned} \quad (33)$$

Or

Obtainment 5

$$(\pi_{+1}^+ I_{-1} - I_{-1} \pi_{+1}^+) - (I_{+1} \pi_{-1}^- - \pi_{-1}^- I_{+1}) = -2I_0 \quad (34)$$

$$(\pi_{+1}^+ I_{-1} - I_{-1} \pi_{+1}^+) + (I_{+1} \pi_{-1}^- - \pi_{-1}^- I_{+1}) = 0I_0 \quad (35)$$

Obtainment 6

$$\begin{aligned} & (\pi_{+1}^+ I_{-1} + I_{-1} \pi_{+1}^+) - (I_{+1} \pi_{-1}^- + \pi_{-1}^- I_{+1}) \\ & = 2 \text{diag}\{, 11, 9, 7, 5, 3, \underline{1}, -1, -3, -5, -7, -9, , \}_0 \end{aligned} \quad (36)$$

$$(\pi_{+2}^+ I_{-1} + I_{-1} \pi_{+2}^+) + (I_{+1} \pi_{-1}^- + \pi_{-1}^- I_{+1}) = 4I_0 \quad (37)$$

**B)** Having gotten the above preliminary results, it is now turn to discuss the third component and Casimir operator of zero spin particle

**B1)** (16) minus (17), obtain

$$\begin{aligned} & \pi_{\frac{+\sqrt{4}}{2}, \frac{+\sqrt{3}}{2}}^+ \pi_{\frac{-\sqrt{4}}{2}, \frac{+\sqrt{3}}{2}}^- - \pi_{\frac{-\sqrt{4}}{2}, \frac{+\sqrt{3}}{2}}^- \pi_{\frac{+\sqrt{4}}{2}, \frac{+\sqrt{3}}{2}}^+ \\ & = +2 \frac{1}{2} \text{diag}\{, 11, 9, 7, 5, 3, \underline{1}, -1, -3, -5, -7, -9, \}_0 \end{aligned} \quad (38)$$

Finally we have the third component

$$\pi_{3; \frac{+\sqrt{4}}{2}, \frac{+\sqrt{3}}{2}, \frac{-\sqrt{4}}{2}, \frac{+\sqrt{3}}{2}} = \frac{1}{2} (\pi_{\frac{+\sqrt{4}}{2}, \frac{+\sqrt{3}}{2}}^+ \pi_{\frac{-\sqrt{4}}{2}, \frac{+\sqrt{3}}{2}}^- - \pi_{\frac{-\sqrt{4}}{2}, \frac{+\sqrt{3}}{2}}^- \pi_{\frac{+\sqrt{4}}{2}, \frac{+\sqrt{3}}{2}}^+) \quad (39)$$

$$= \frac{1}{2} \text{diag}\{, 11, 9, 7, 5, 3, \underline{1}, -1, -3, -5, -7, -9, \}_0 = \pi_{3; +1, -1} \quad (40)$$

And the square of the third component

$$\begin{aligned} & \pi_{3; \frac{+\sqrt{4}}{2}, \frac{+\sqrt{3}}{2}, \frac{-\sqrt{4}}{2}, \frac{+\sqrt{3}}{2}}^2 \\ & = \frac{1}{4} \text{diag}\{, 121, 81, 49, 25, 9, \underline{1}, 1, 9, 25, 49, 81, , \}_0 = \pi_{3; +1, -1}^2 \end{aligned} \quad (41)$$

**B2)** (16) add (17), obtain

$$\begin{aligned} & \pi_{\frac{+\sqrt{4}}{2}, \frac{+\sqrt{3}}{2}}^+ \pi_{\frac{-\sqrt{4}}{2}, \frac{+\sqrt{3}}{2}}^- + \pi_{\frac{-\sqrt{4}}{2}, \frac{+\sqrt{3}}{2}}^- \pi_{\frac{+\sqrt{4}}{2}, \frac{+\sqrt{3}}{2}}^+ \\ & = 2 \left\{ - \frac{1}{2} \text{diag}\{, -59, -39, -23, -11, -3, \underline{1}, 1, -3, -11, -23, -59, \}_0 \right. \\ & \quad \left. - \frac{3}{4} I_0 + i\sqrt{3} I_0 \right\} \end{aligned} \quad (42)$$

Then square sum of the first component and the second component

$$\pi_{1; \frac{+\sqrt{4}}{2}, \frac{+\sqrt{3}}{2}, \frac{-\sqrt{4}}{2}, \frac{+\sqrt{3}}{2}}^2 + \pi_{2; \frac{+\sqrt{4}}{2}, \frac{+\sqrt{3}}{2}, \frac{-\sqrt{4}}{2}, \frac{+\sqrt{3}}{2}}^2$$

$$= \frac{1}{2} \{ \pi_{\frac{+\sqrt{4}}{2}, \frac{+\sqrt{3}}{2}}^+ \pi_{\frac{-\sqrt{4}}{2}, \frac{+\sqrt{3}}{2}}^- + \pi_{\frac{-\sqrt{4}}{2}, \frac{+\sqrt{3}}{2}}^- \pi_{\frac{+\sqrt{4}}{2}, \frac{+\sqrt{3}}{2}}^+ \}$$
(43)

$$= -\frac{1}{4} \text{diag}\{, -118, -78, -46, -22, -6, \underline{2}, 2, -6, -22, -46, -78, \}_0$$

$$- \frac{3}{4} I_0 + i\sqrt{3} I_0$$
(44)

**B3)** Finally Casimir operator, or the total square of spin angular momentum of zero spin particle with imaginary number is given as below:

$$\pi_{\frac{+\sqrt{4}}{2}, \frac{+\sqrt{3}}{2}, \frac{-\sqrt{4}}{2}, \frac{+\sqrt{3}}{2}}^2$$

$$= \pi_{1; \frac{+\sqrt{4}}{2}, \frac{+\sqrt{3}}{2}, \frac{-\sqrt{4}}{2}, \frac{+\sqrt{3}}{2}}^2 + \pi_{2; \frac{+\sqrt{4}}{2}, \frac{+\sqrt{3}}{2}, \frac{-\sqrt{4}}{2}, \frac{+\sqrt{3}}{2}}^2 + \pi_{3; \frac{+\sqrt{4}}{2}, \frac{+\sqrt{3}}{2}, \frac{-\sqrt{4}}{2}, \frac{+\sqrt{3}}{2}}^2$$
(45)

$$= -\frac{1}{4} \text{diag}\{, -118, -78, -46, -22, -6, \underline{2}, 2, -6, -22, -46, -78, \}_0$$

$$+ \frac{1}{4} \text{diag}\{, 121, 81, 49, 25, 9, \underline{1}, 1, 9, 25, 49, 81, \}_0$$

$$- \frac{3}{4} I_0 + i\sqrt{3} I_0$$

$$= +\frac{3}{4} I_0 - \frac{3}{4} I_0 + i\sqrt{3} I_0 = \{ \underline{0 + i\sqrt{3}} \} \hbar^2 I_0$$
(46)

**C)** For (12) and (13) of group **b)**, it is very similar to previous group **a)**, we get

$$\pi_{3; \frac{+\sqrt{4}}{2}, \frac{-\sqrt{3}}{2}, \frac{-\sqrt{4}}{2}, \frac{-\sqrt{3}}{2}} = \pi_{3; +1, -1}$$
(47)

$$\pi_{\frac{+\sqrt{4}}{2}, \frac{-\sqrt{3}}{2}, \frac{-\sqrt{4}}{2}, \frac{-\sqrt{3}}{2}}^2$$

$$= \pi_{1; \frac{+\sqrt{4}}{2}, \frac{-\sqrt{3}}{2}, \frac{-\sqrt{4}}{2}, \frac{-\sqrt{3}}{2}}^2 + \pi_{2; \frac{+\sqrt{4}}{2}, \frac{-\sqrt{3}}{2}, \frac{-\sqrt{4}}{2}, \frac{-\sqrt{3}}{2}}^2 + \pi_{3; \frac{+\sqrt{4}}{2}, \frac{-\sqrt{3}}{2}, \frac{-\sqrt{4}}{2}, \frac{-\sqrt{3}}{2}}^2$$
(48)

$$= +\frac{3}{4} I_0 - \frac{3}{4} I_0 - i\sqrt{3} I_0 = \{ \underline{0 - i\sqrt{3}} \} \hbar^2 I_0$$
(49)

**D)** It can prove:

$$\mathbf{a1)} \quad \pi_{\frac{+\sqrt{4}}{2}, \frac{+\sqrt{3}}{2}}^+, \pi_{\frac{-\sqrt{4}}{2}, \frac{+\sqrt{3}}{2}}^-, \pi_{3; \frac{+\sqrt{4}}{2}, \frac{+\sqrt{3}}{2}, \frac{-\sqrt{4}}{2}, \frac{+\sqrt{3}}{2}}$$
(50)

$$\mathbf{b1)} \quad \pi_{\frac{+\sqrt{4}}{2}, \frac{-\sqrt{3}}{2}}^+, \pi_{\frac{-\sqrt{4}}{2}, \frac{-\sqrt{3}}{2}}^-, \pi_{3; \frac{+\sqrt{4}}{2}, \frac{-\sqrt{3}}{2}, \frac{-\sqrt{4}}{2}, \frac{-\sqrt{3}}{2}}$$
(51)

Or

$$\mathbf{a2)} \quad \vec{\pi}_{\frac{+\sqrt{4}}{2}, \frac{+\sqrt{3}}{2}, \frac{-\sqrt{4}}{2}, \frac{+\sqrt{3}}{2}}$$

$$= ( \pi_{1; \frac{+\sqrt{4}}{2}, \frac{+\sqrt{3}}{2}, \frac{-\sqrt{4}}{2}, \frac{+\sqrt{3}}{2}}, \pi_{2; \frac{+\sqrt{4}}{2}, \frac{+\sqrt{3}}{2}, \frac{-\sqrt{4}}{2}, \frac{+\sqrt{3}}{2}}, \pi_{3; \frac{+\sqrt{4}}{2}, \frac{+\sqrt{3}}{2}, \frac{-\sqrt{4}}{2}, \frac{+\sqrt{3}}{2}} )$$
(52)

$$\mathbf{b2)} \quad \vec{\pi}_{\frac{+\sqrt{4}}{2}, \frac{-\sqrt{3}}{2}, \frac{-\sqrt{4}}{2}, \frac{-\sqrt{3}}{2}}$$

$$= ( \pi_{1; \frac{+\sqrt{4}}{2}, \frac{-\sqrt{3}}{2}, \frac{-\sqrt{4}}{2}, \frac{-\sqrt{3}}{2}}, \pi_{2; \frac{+\sqrt{4}}{2}, \frac{-\sqrt{3}}{2}, \frac{-\sqrt{4}}{2}, \frac{-\sqrt{3}}{2}}, \pi_{3; \frac{+\sqrt{4}}{2}, \frac{-\sqrt{3}}{2}, \frac{-\sqrt{4}}{2}, \frac{-\sqrt{3}}{2}} )$$
(53)

**a1), a2)** and **b1), b2)** satisfy angular momentum commutation rules

Pay attention to (6) and (46),(49), the former is in real spin region and the two latters are in complex spin region.

Group **a)**  $\in \pi$  and Group **b)**  $\in \pi^{Imaginary}$ .  $\pi$  and  $\pi^{Imaginary}$  are called imaginary spin-conjugate each other ( $\pi^{Imaginary}, \pi$ )

**4 Using  $(\pi^{Imaginary}, \pi)$ , to construct fermion with one half spin  $\frac{\hbar}{2}$  of matter** marked by  $\Pi^I(1)$  ( for clear and convenience, use  $\Pi^I \equiv \Pi^I(1)$  ), which named **Imaginary Spin-Conjugate-Composite** that consist of  $\pi^{Imaginary}$  and  $\pi$  as below

$$\vec{\Pi}^I = \frac{1}{2} \{ \vec{\pi}^{Imaginary} + \vec{\pi} \} \quad (54)$$

$\Pi^I$  satisfies angular momentum commutation rule

$$\vec{\Pi}^I \times \vec{\Pi}^I = i\vec{\Pi}^I \quad (55)$$

Here

$$\vec{\pi} = \pi_{\frac{+\sqrt{4}}{2}, \frac{+\sqrt{3}}{2}; \frac{-\sqrt{4}}{2}, \frac{+\sqrt{3}}{2}} \quad (52)$$

$$\vec{\pi}^{Imaginary} = \pi_{\frac{+\sqrt{4}}{2}, \frac{-\sqrt{3}}{2}; \frac{-\sqrt{4}}{2}, \frac{-\sqrt{3}}{2}} \quad (53)$$

Then combine (52) with (53), we have raising operators  $\Pi^{I+}$ , lowering operator  $\Pi^{I-}$  and the third component  $\Pi_{3,I}$  and Casimir Operator  $\Pi^{2,I}$  of  $\vec{\Pi}^I$  (54)

$$\Pi^{I+} = \frac{1}{2} \{ 2\pi_0^+ + \frac{2}{2} \sqrt{4} I_{+1} \} = \pi_0^+ + I_{+1} = \pi_{+1}^+ \quad (56)$$

$$\Pi^{I-} = \frac{1}{2} \{ 2\pi_0^- + \frac{2}{2} \sqrt{4} I_{-1} \} = \pi_0^- + I_{-1} = \pi_{-1}^- \quad (57)$$

$$\Pi_{3,I} = \pi_{3; +1, -1} \in \mathcal{R} \quad (58)$$

$$\Pi^{2,I} = \frac{\hbar}{2} \left( \frac{\hbar}{2} + 1\hbar \right) = +\frac{3}{4} \hbar^2 \in \mathcal{R} \quad (59)$$

Above four expresses show: Formular (55) is just the representation of fermion with one half spin  $\frac{\hbar}{2}$  of matter. **Diagram 1** demonstrates the whole formation of (55) due to Imaginary spin-conjugate  $(\pi^{Imaginary}, \pi)$ .

### Summary

$$\vec{\Pi}^I(1) = \frac{1}{2} (\vec{\pi}^{Imaginary} + \vec{\pi}) \quad (54)$$

$$\frac{\hbar}{2} \Leftrightarrow \frac{1}{2} \{ 0\hbar + 0\hbar \} \quad (60)$$

$$\left\{ \frac{\hbar}{2}, \Pi^{2,I}(1) \in \mathcal{R} \right\} = \frac{1}{2} \left( \{ 0\hbar, (\pi^{Imaginary})^2 \in \mathcal{C} \} + \{ 0\hbar, \pi^2 \in \mathcal{C} \} \right) \quad (61)$$

### 5 Casimir Operator of Zero Spin Particle with $(\pi^{Real}, \pi)$ in Complex Region and Fermion with one half spin $i\frac{\hbar}{2}$ of Antimatter

First, raising operator  $\pi_0^+$  (1) and lowering operator  $\pi_{-1}^-$  (2) of group **c**) and group **d**) in Complex Region are given as (62),(63) and (64),(65)

$$\text{c) } \pi_0^+ \Rightarrow \pi_{\frac{+\sqrt{3}}{2}, \frac{\sqrt{2}}{2}}^+ = \pi_0^+ + \frac{1}{2} (+\sqrt{3} + i\sqrt{2}) I_{+1} \quad (62)$$

$$\pi_{-1}^- \Rightarrow \pi_{\frac{-\sqrt{3}}{2}, \frac{\sqrt{2}}{2}}^- = \pi_0^- + \frac{1}{2} (+\sqrt{3} + i\sqrt{2}) I_{-1} \quad (63)$$

$$\text{d) } \pi_0^+ \Rightarrow \pi_{\frac{-\sqrt{3}}{2}, \frac{\sqrt{2}}{2}}^+ = \pi_0^+ + \frac{1}{2} (-\sqrt{3} + i\sqrt{2}) I_{+1} \quad (64)$$

$$\pi_{-1}^- \Rightarrow \pi_{\frac{+\sqrt{3}}{2}, \frac{\sqrt{2}}{2}}^- = \pi_0^- + \frac{1}{2} (-\sqrt{3} + i\sqrt{2}) I_{-1} \quad (65)$$

Group **c**)  $\in \pi$  and Group **d**)  $\in \pi^{Real}$ .  $\pi$  and  $\pi^{Real}$  are called real spin-conjugate each other ( $\pi^{Real}, \pi$ ). Then in the way analogous to Group **a**)  $\in \pi$  and Group **b**), the third component and Casimir Operators of zero spin particle with real spin-conjugate ( $\pi^{Real}, \pi$ ) in *Complex* region are given as follows

$$\pi_{3; \frac{+\sqrt{3}}{2}, \frac{\sqrt{2}}{2}; \frac{-\sqrt{3}}{2}, \frac{\sqrt{2}}{2}} = \pi_{3; \frac{+\sqrt{3}}{2}, \frac{-\sqrt{3}}{2}} = \pi_{3; 0, 0} \quad (66)$$

$$\pi_{\frac{+\sqrt{3}}{2}, \frac{\sqrt{2}}{2}; \frac{-\sqrt{3}}{2}, \frac{\sqrt{2}}{2}}^2 = \{ \underline{0 + i\sqrt{3/2}} \} \hbar^2 I_0 \quad (67)$$

$$\pi_{3; \frac{-\sqrt{3}}{2}, \frac{\sqrt{2}}{2}; \frac{+\sqrt{3}}{2}, \frac{\sqrt{2}}{2}} = \pi_{3; \frac{-\sqrt{3}}{2}, \frac{+\sqrt{3}}{2}} = \pi_{3; 0, 0} \quad (68)$$

$$\pi_{\frac{-\sqrt{3}}{2}, \frac{\sqrt{2}}{2}; \frac{+\sqrt{3}}{2}, \frac{\sqrt{2}}{2}}^2 = \{ \underline{0 - i\sqrt{3/2}} \} \hbar^2 I_0 \quad (69)$$

Then, using ( $\pi^{Real}, \pi$ ), to construct fermion with one half spin  $i\frac{\hbar}{2}$  of antimatter marked by  $\Pi^R$ , which named **Real Spin-Conjugate-Composite** that consist of  $\pi^{Real}$  and  $\pi$  as below

$$\vec{\Pi}^R = \frac{1}{2} \{ \vec{\pi}^{Real} + \vec{\pi} \} \quad (70)$$

$\Pi^R$  ( $\vec{\Pi}^R \equiv \vec{\Pi}^R(1)$ ) satisfies angular momentum commutation rule

$$\vec{\Pi}^R \times \vec{\Pi}^R = i\vec{\Pi}^R \quad (71)$$

Here

$$\vec{\pi} = \pi_{\frac{+\sqrt{3}}{2}, \frac{\sqrt{2}}{2}; \frac{-\sqrt{3}}{2}, \frac{\sqrt{2}}{2}} \quad (72)$$

$$\vec{\pi}^{Real} = \pi_{\frac{-\sqrt{3}}{2}, \frac{\sqrt{2}}{2}; \frac{+\sqrt{3}}{2}, \frac{\sqrt{2}}{2}} \quad (73)$$

Then combine (72) with (73), we have raising operators  $\Pi^{R,+}$ , lowering operator  $\Pi^{R,-}$  and the third component  $\Pi_{3,R}$  and Casimir Operator  $\Pi^{2,R}$  of  $\vec{\Pi}^R$  (70)

$$\Pi^{R,+} = \frac{1}{2} \{ 2\pi_0^+ + i\frac{2}{2}\sqrt{2}I_{+1} \} = \pi_0^+ + i\frac{\sqrt{2}}{2}I_{+1} = \pi_{0, \frac{\sqrt{2}}{2}}^+ \quad (74)$$

$$\Pi^{R,-} = \frac{1}{2} \{ 2\pi_0^- + i\frac{2}{2}\sqrt{2}I_{-1} \} = \pi_0^- + i\frac{\sqrt{2}}{2}I_{-1} = \pi_{0, \frac{\sqrt{2}}{2}}^- \quad (75)$$

$$\Pi_{3,R}(1) = \pi_{3; 0, 0} \in \mathcal{R} \quad (76)$$

$$\Pi^{2,R}(1) = i\frac{\hbar}{2}(i\frac{\hbar}{2} + i\hbar) = -\frac{3}{4}\hbar^2 \in \mathcal{R} \quad (77)$$

Above four expresses show: Formular (70) is just the representation of fermion with one half spin  $i\frac{\hbar}{2}$  of antimatter. **Diagram 2** demonstrates the whole formation of (66) due to Real spin-conjugate ( $\pi^{Real}, \pi$ ).

### Summary

$$\vec{\Pi}^R(1) = \frac{1}{2} (\vec{\pi}^{Real} + \vec{\pi}) \quad (70)$$

$$\frac{\hbar}{2} \Leftrightarrow \frac{1}{2} \{ 0\hbar + 0\hbar \} \quad (78)$$

$$\{ i\frac{\hbar}{2}, \Pi^{2,R}(1) \in \mathcal{R} \} = \frac{1}{2} ( \{ 0\hbar, (\pi^{Real})^2 \in \mathcal{C} \} + \{ 0\hbar, \pi^2 \in \mathcal{C} \} ) \quad (79)$$

## Two Zero Spin Particles in Complex Region Compose a Matter Fermion with One Half Spin

$$\boxed{\vec{\Pi}^I(1)} = \frac{1}{2} \left( \vec{\pi}_{\frac{\sqrt{4}}{2}, \frac{+\sqrt{3}}{2}; \frac{-\sqrt{4}}{2}, \frac{+\sqrt{3}}{2}}^{Imaginary}(1) + \vec{\pi}_{\frac{\sqrt{4}}{2}, \frac{+\sqrt{3}}{2}; \frac{-\sqrt{4}}{2}, \frac{+\sqrt{3}}{2}}(1) \right)$$

Real Spin Number :

$$\frac{1}{2} \hbar \Leftrightarrow \frac{1}{2} ( \quad 0\hbar \quad + \quad 0\hbar \quad )$$

Casimir Operator :  $\Pi^2 \cdot I(1) = \Pi_{+1, 0; -1, 0}^2 = \frac{1}{2} \left( \frac{1}{2} + 1 \right) \hbar^2$   
 $= +\frac{3}{4} \hbar^2 > 0$

Matter One Half Spin Particle  
in Real Region

$$\boxed{\vec{\Pi}^I(1)}, +\frac{3}{4} \hbar^2, \frac{1}{2} \hbar \text{ ————— }$$



$$\boxed{\vec{\pi}_{+1, \frac{+\sqrt{3}}{2}; -1, \frac{+\sqrt{3}}{2}} 0\hbar}$$

—————

$$0\hbar \boxed{\vec{\pi}_{+1, \frac{-\sqrt{3}}{2}; -1, \frac{-\sqrt{3}}{2}}}$$

Zero Spin Particle

,

Zero Spin Particle

in Complex Region

,

in Complex Region

$$\pi_{\frac{\sqrt{4}}{2}, \frac{+\sqrt{3}}{2}; \frac{-\sqrt{4}}{2}, \frac{+\sqrt{3}}{2}}^2 = (0 + i\sqrt{3}) I_0 \hbar^2 \quad , \quad \pi_{\frac{\sqrt{4}}{2}, \frac{-\sqrt{3}}{2}; \frac{-\sqrt{4}}{2}, \frac{-\sqrt{3}}{2}}^2 = (0 - i\sqrt{3}) I_0 \hbar^2$$

Diagram 1



## Two Zero Spin Particles in Complex Region Compose an Antimatter Fermion with One Half Spin

$$\boxed{\vec{\Pi}^{.R}(1)} = \frac{1}{2} \left( \vec{\pi}_{\frac{+\sqrt{3}}{2}, \frac{\sqrt{2}}{2}; \frac{-\sqrt{3}}{2}, \frac{\sqrt{2}}{2}}(1) + \vec{\pi}_{\frac{+\sqrt{3}}{2}, \frac{\sqrt{2}}{2}; \frac{-\sqrt{3}}{2}, \frac{\sqrt{2}}{2}}^{Real}(1) \right)$$

Imaginary Spin Number :

$$i \frac{1}{2} \hbar \Leftrightarrow \frac{1}{2} ( 0\hbar + 0\hbar )$$

$$\begin{aligned} \text{Casimir Operator : } \Pi^{2.R}(1) &= \Pi_{0, \frac{\sqrt{2}}{2}; 0, \frac{\sqrt{2}}{2}}^2 = i \frac{1}{2} (i \frac{1}{2} + i) \hbar^2 \\ &= -\frac{3}{4} \hbar^2 < 0 \end{aligned}$$

Zero Spin Particle

,

Zero Spin Particle

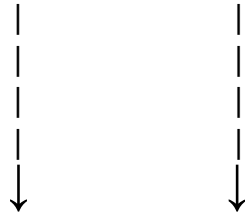
in Complex Region

,

in Complex Region

$$\pi_{\frac{+\sqrt{3}}{2}, \frac{\sqrt{2}}{2}; \frac{-\sqrt{3}}{2}, \frac{\sqrt{2}}{2}}^2 = (0 + i\sqrt{3/2}) I_0 \hbar^2, \quad \pi_{\frac{-\sqrt{3}}{2}, \frac{\sqrt{2}}{2}; \frac{+\sqrt{3}}{2}, \frac{\sqrt{2}}{2}}^2 = (0 - i\sqrt{3/2}) I_0 \hbar^2$$

$$\boxed{\vec{\pi}_{\frac{+\sqrt{3}}{2}, \frac{\sqrt{2}}{2}; \frac{-\sqrt{3}}{2}, \frac{\sqrt{2}}{2}} 0\hbar} \quad \text{-----} \quad \text{-----} \quad \boxed{0\hbar \vec{\pi}_{\frac{-\sqrt{3}}{2}, \frac{\sqrt{2}}{2}; \frac{+\sqrt{3}}{2}, \frac{\sqrt{2}}{2}}}$$



$$\text{-----} i \frac{1}{2} \hbar, -\frac{3}{4} \hbar^2, \boxed{\vec{\Pi}^{.R}(1)}$$

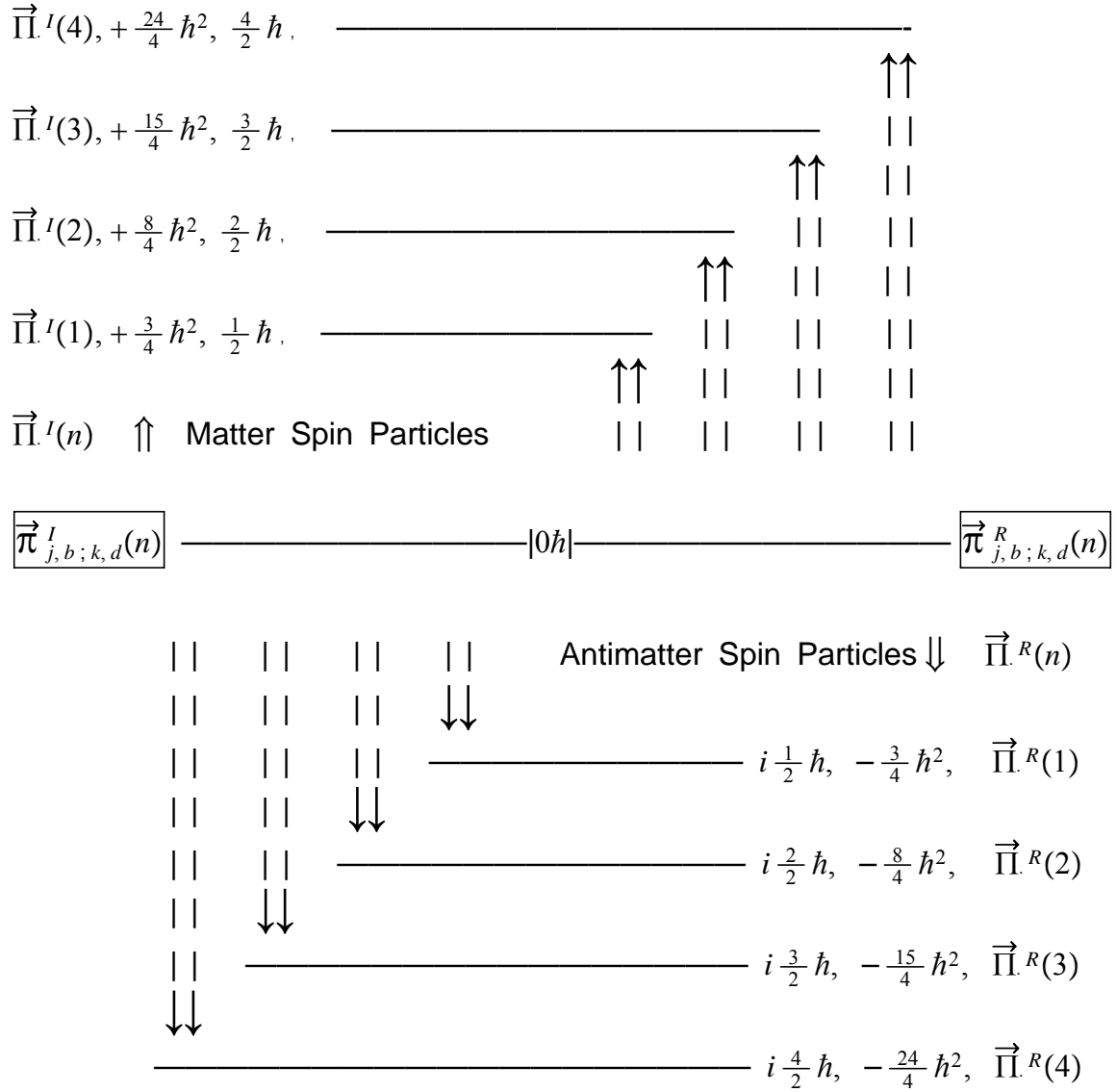
Antimatter One Half Spin Particle

in Real Region

$$\Pi^{2.R}(1) = \Pi_{0, \frac{\sqrt{2}}{2}; 0, \frac{\sqrt{2}}{2}}^2 = -\frac{3}{4} \hbar^2$$

Diagram 2

## Excited Levels of Spin-Interactions between Two Zero Spin Particles in Complex Region



$$\boxed{\vec{\pi}_{j,b;k,d}^R(n)}, \quad \boxed{\vec{\pi}_{j,b;k,d}^I(n)} :$$

Zero Spin Particles in Complex Region

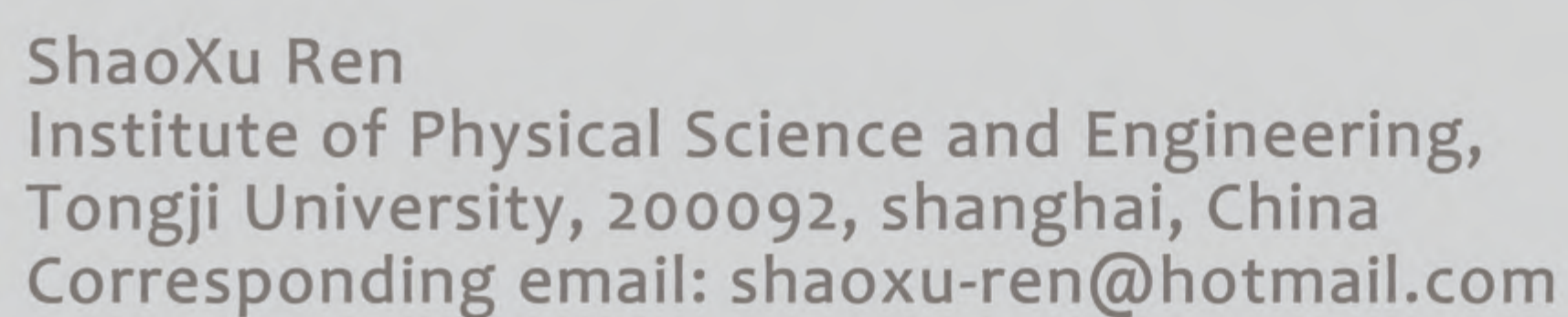
Casimir Operators  $\pi_{j,b;k,d}^2 = \pi_{j,b;k,d}^I = (0 \pm i(j-k)(b+d)/2) I_0 \hbar^2$

for  $\pi_{j,b;k,d}^R$  :  $j-k = \sqrt{(n+1)^2-1} = \pm(\sqrt{3}, \sqrt{8}, \sqrt{15}, \sqrt{24}, \dots)$   
 $n = 1, 2, 3, 4, \dots$   $b+d = \pm\sqrt{(n+1)^2-2} = \pm(\sqrt{2}, \sqrt{7}, \sqrt{14}, \sqrt{23}, \dots)$

for  $\pi_{j,b;k,d}^I$  :  $j-k = \pm\sqrt{(n+1)^2-0} = \pm(\sqrt{1}, \sqrt{4}, \sqrt{9}, \sqrt{16}, \dots)$   
 $n = 0, 1, 2, 3, \dots$   $b+d = \pm\sqrt{(n+1)^2-1} = \pm(\sqrt{0}, \sqrt{3}, \sqrt{8}, \sqrt{15}, \dots)$

**Diagram 3**





# Origin of Particle Spins, Bosons and Fermions, Compounds of Zero Spin Particles, in Complex Spin Region

$$\vec{\pi}_{+1, \frac{+\sqrt{3}}{2}, -1, \frac{+\sqrt{3}}{2}} (1) \ 0\hbar$$
$$\begin{aligned}\pi_{+1, \frac{+\sqrt{3}}{2}}^+ &= \pi_0^+ + \frac{1}{2}(\sqrt{4} + i\sqrt{3})I_{+1} \\ \pi_{-1, \frac{+\sqrt{3}}{2}}^- &= \pi_0^- + \frac{1}{2}(\sqrt{4} + i\sqrt{3})I_{-1}\end{aligned}$$
$$\boxed{\vec{\Pi}^{I(1)}} = \frac{1}{2} \left( \vec{\pi}_{+1, \frac{+\sqrt{3}}{2}}; -1, \frac{+\sqrt{3}}{2} (1) + \vec{\pi}_{+1, \frac{+\sqrt{3}}{2}}^{Imaginary}; -1, \frac{+\sqrt{3}}{2} (1) \right)$$

Casimir Operator :  $\Pi^2.I(1) = \Pi_{+1,0;-1,0}^2 = +\frac{3}{4}\hbar^2 > 0$

$$\boxed{\overline{\Pi}_\bullet}'(1) + \frac{3}{4} \hbar^2, \frac{1}{2} \hbar \longrightarrow$$

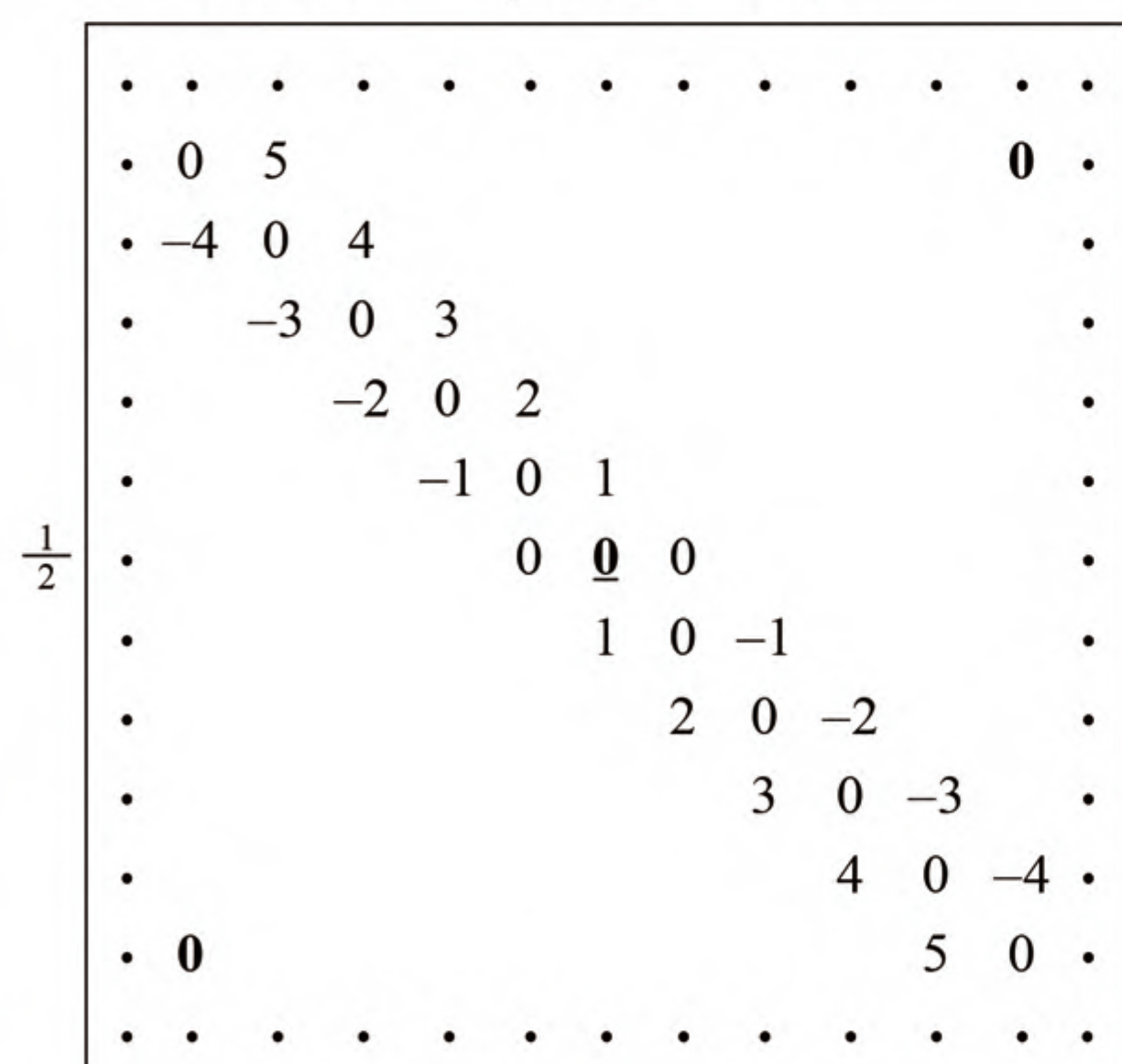
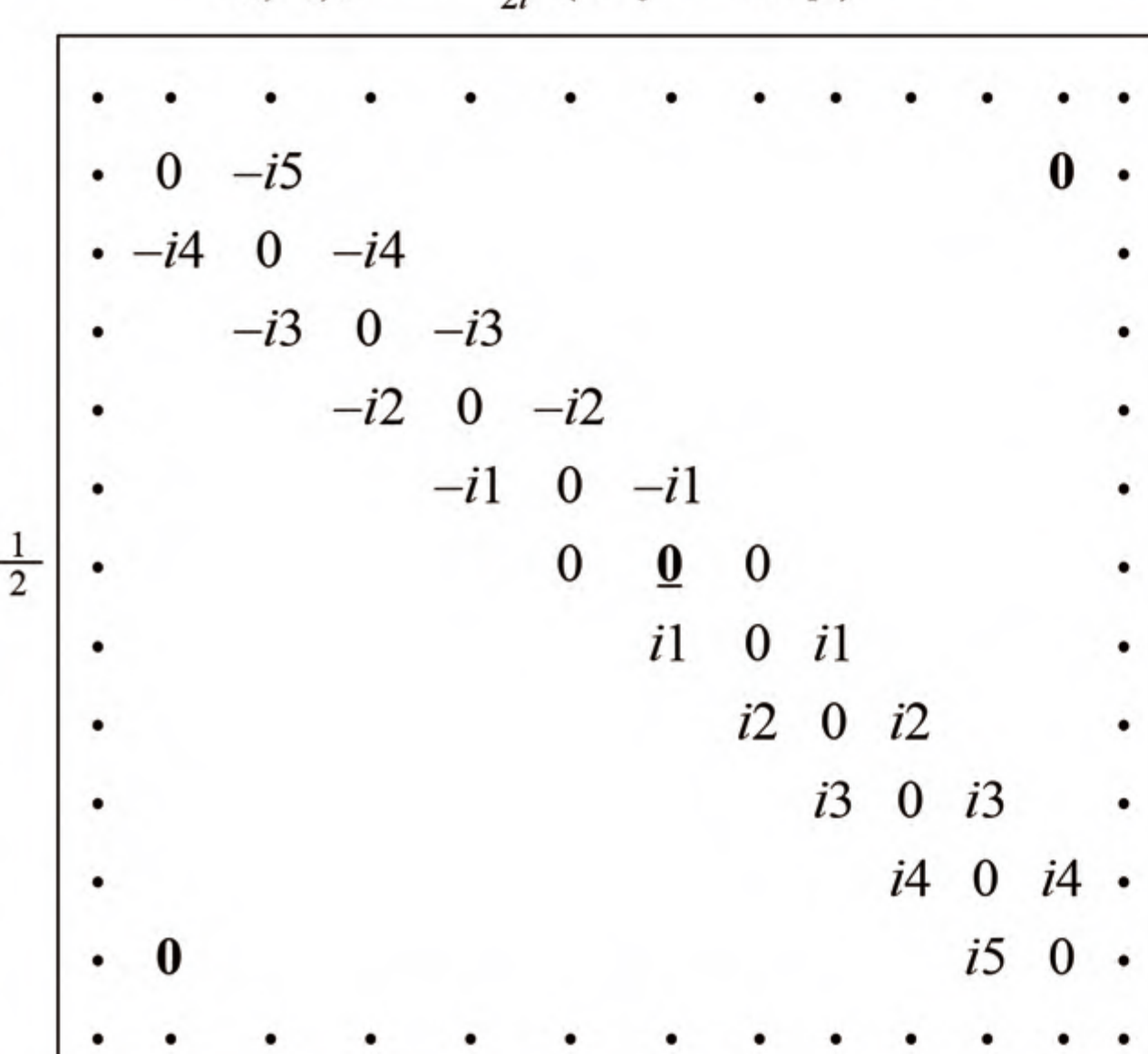
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⋮  
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$$\begin{array}{ccc}
 \boxed{\vec{\pi}_{+1, \frac{+\sqrt{3}}{2}; -1, \frac{+\sqrt{3}}{2}} 0h} & \text{---} & \boxed{0h \vec{\pi}_{+1, \frac{-\sqrt{3}}{2}; -1, \frac{-\sqrt{3}}{2}}} \\
 \text{Zero Spin Particle} & , & \text{Zero Spin Particle} \\
 \text{in Complex Region} & , & \text{in Complex Region} \\
 \pi_{+1, \frac{+\sqrt{3}}{2}; -1, \frac{+\sqrt{3}}{2}}^2 = (0 + i\sqrt{3})I_0h^2 & , & \pi_{+1, \frac{-\sqrt{3}}{2}; -1, \frac{-\sqrt{3}}{2}}^2 = (0 - i\sqrt{3})I_0h^2
 \end{array}$$

$$\vec{\pi}_{0,0,-1,0}(0) \quad 0\hbar$$

$$\begin{aligned}\pi_{0,0}^+ &= \pi_0^+ + \frac{1}{2}(\sqrt{0} + i\sqrt{0})I_{+1} \\ \pi_{-1}^- &= \pi_0^- + \frac{1}{2}(\sqrt{0} + i\sqrt{0})I_{-1}\end{aligned}$$

$$\pi_{1;-1,0} = \frac{1}{2} (\pi_0^+ + \pi_{-1}^-) =$$

$$\pi_{2;-1,0} = \frac{1}{2i} (\pi_0^+ - \pi_{-1}^-) =$$


Particle Spins, Where do they come from? Bosons and Fermions maybe the compounds  $\Pi(n)$  ( $n=1, 2, 3, 4, \dots$ ) of two zero spin particles which are Spin-conjugates each other in complex spin region of Spin Topological Space, STS [\*].

Using  $(\pi^{Real}, \pi) \blacktriangledown$ , Antimatter Bosons and Fermions  $\Pi^R(n)$  could be formed, Using  $(\pi^{Imaginary}, \pi) \blacktriangle$ , Matter Bosons and Fermions  $\Pi^I(n)$  could be formed

**between Two Zero Spin Rarticles in Complex Region.**

The diagram illustrates the construction of zero spin particles in the complex region. It consists of several parts:

- Matter Spin Particles (Left Side):** Four levels are shown, each with a horizontal line representing an energy level. Above each line is a label:  $\vec{\pi}^I(4), +\frac{24}{4}\hbar^2, \frac{4}{2}\hbar$ ;  $\vec{\pi}^I(3), +\frac{15}{4}\hbar^2, \frac{3}{2}\hbar$ ;  $\vec{\pi}^I(2), +\frac{8}{4}\hbar^2, \frac{2}{2}\hbar$ ; and  $\vec{\pi}^I(1), +\frac{3}{4}\hbar^2, \frac{1}{2}\hbar$ . Arrows indicate transitions between these levels.
- Antimatter Spin Particles (Right Side):** Five levels are shown, each with a horizontal line. To the right of each line is a label:  $\vec{\pi}^R(n)$ ,  $\vec{\pi}^R(1)$ ,  $\vec{\pi}^R(2)$ ,  $\vec{\pi}^R(3)$ , and  $\vec{\pi}^R(4)$ . Arrows indicate transitions between these levels.
- Complex Region (Bottom):** A box contains the expression  $\vec{\pi}_{j,b;k,d}(n)^I$  followed by a horizontal line and another box containing  $\vec{\pi}_{j,b;k,d}(n)^R$ .
- Zero Spin Particles in Complex Region:** This is the final result of the process, indicated by a large arrow pointing from the complex region towards it.

All sorts of spin particles are attributed to one spin space, STS. The first component  $\pi_1$  and the second component  $\pi_2$  of all spin particles are Non-Hermitian operators, and the matrix representations of  $\pi_1$  and  $\pi_2$  are infinite dimensional. The third component  $\pi_3$  and Casimir Operator  $\pi^2$  of all spin particles are infinite diagonal matrices.  $\vec{\pi}(\pi_1, \pi_2, \pi_3)$ , or  $(\pi^+, \pi^-, \pi_3)$  satisfy the spin angular momentum commutation rules in Math.

Zero spin particle,  $0\hbar$ , has Non-Trivial spin matrix representations that manifests the existence of its possible spin rotation construction, because the third component of zero spin particle possesses non-zero eigenvalues.

**Casimir Operators** of zero spin particle in Complex Spin Region could account for the origin of Bosons and Fermions.

[\*] ShaoXu Ren (2015) *Interaction of the Origins of Spin Angular Momentum*

ISBN 978-988-14902-0-9 (2016 2nd edition).

ShaoXu Ren (2016) *Journal of Modern Physics*, **7**, 2257-2265

<http://dx.doi.org/10.4236/jmp.2016.716194>

$$\vec{\pi}_{\frac{+\sqrt{3}}{2}, \frac{\sqrt{2}}{2}, \frac{-\sqrt{3}}{2}, \frac{\sqrt{2}}{2}} (1) 0\hbar$$
$$\begin{aligned}\pi_{+\frac{\sqrt{3}}{2}, \frac{\sqrt{2}}{2}}^+ &= \pi_0^+ + \frac{1}{2} (+\sqrt{3} + i\sqrt{2})I_{+1} \\ \pi_{-\frac{\sqrt{3}}{2}, \frac{\sqrt{2}}{2}}^- &= \pi_0^- + \frac{1}{2} (+\sqrt{3} + i\sqrt{2})I_{-1}\end{aligned}$$
$$\boxed{\vec{\pi}^{R(1)}} = \frac{1}{2} (\vec{\pi}_{+\sqrt{3}}^{\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}}(1) + \vec{\pi}_{+\sqrt{3}}^{Real, \frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}}(1))$$

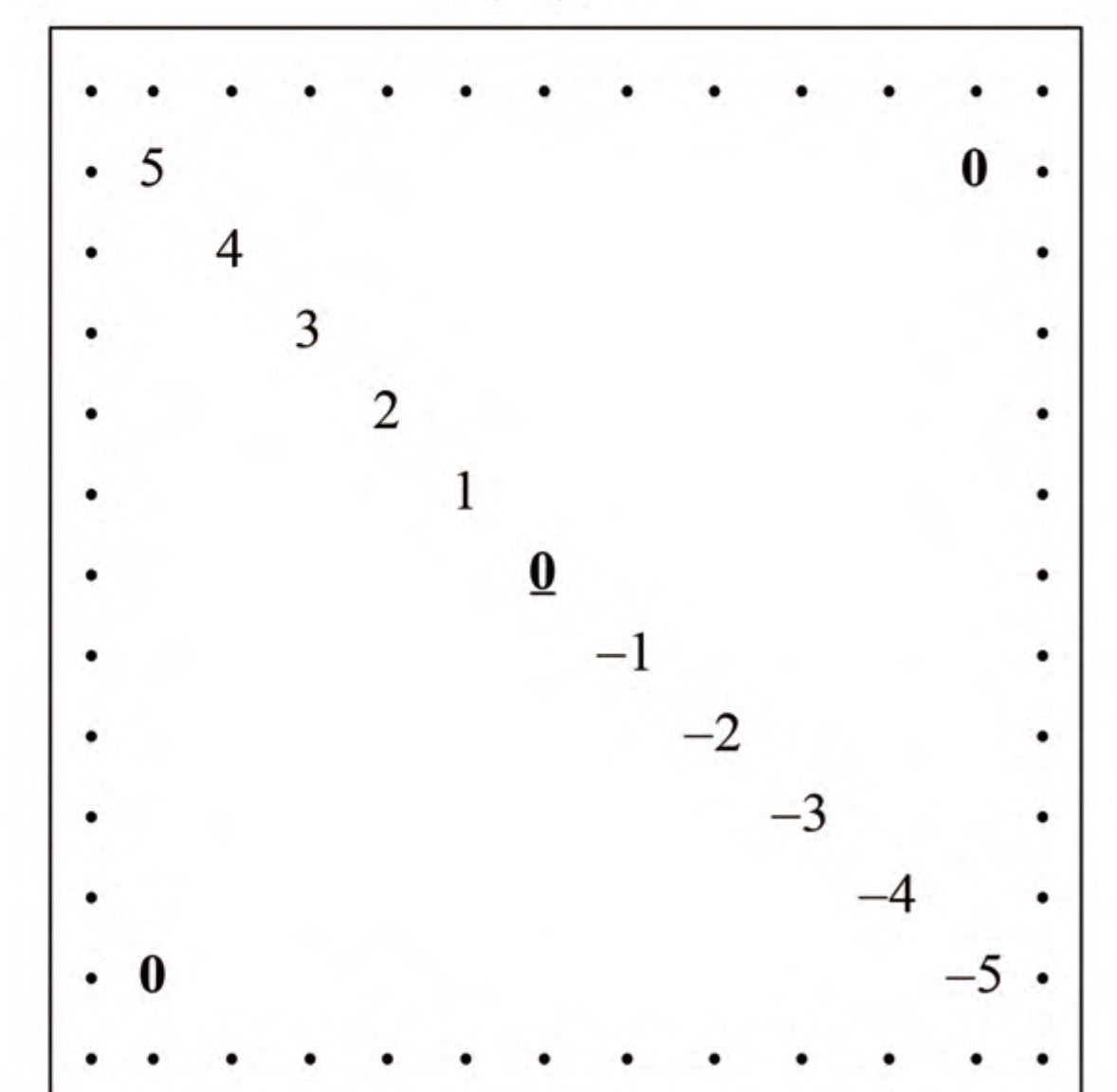
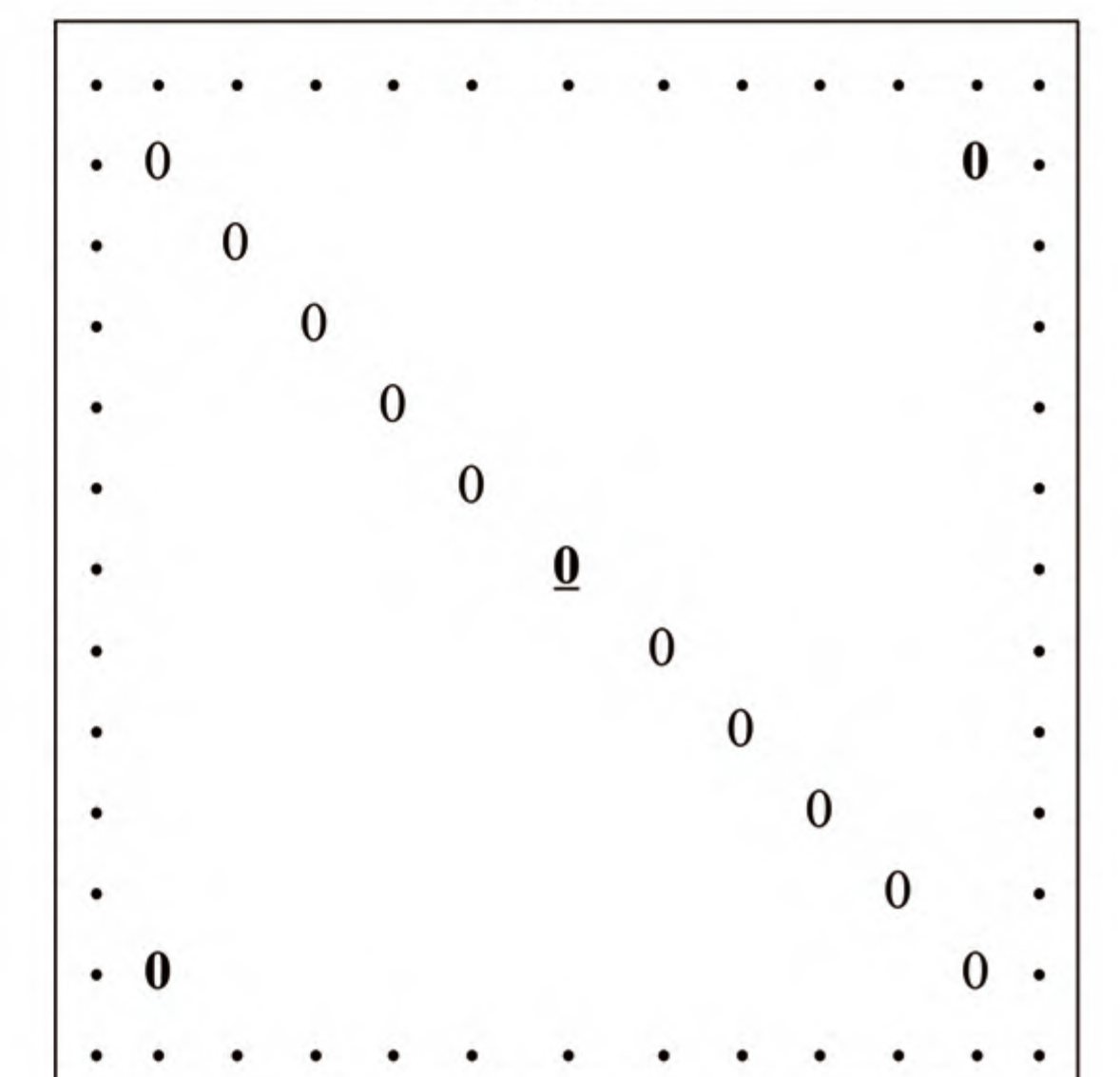
Casimir Operator :  $\Pi^{2,R}(1) = \Pi_{\sqrt{2}, 0, \sqrt{2}}^2 = -\frac{3}{4}\hbar^2 < 0$

$$\pi_{+\sqrt{3}}^2 \pi_{-\sqrt{2}} \pi_{-\sqrt{3}} \pi_{-\sqrt{2}} = (0 + i\sqrt{3/2}) I_0 \hbar^2 \quad , \quad \pi_{-\sqrt{3}}^2 \pi_{-\sqrt{2}} \pi_{+\sqrt{3}} \pi_{-\sqrt{2}} = (0 - i\sqrt{3/2}) I_0 \hbar^2$$

$$\boxed{\vec{\pi}_{+\sqrt{3}, \frac{\sqrt{2}}{2}}; -\frac{\sqrt{3}}{2}, \frac{\sqrt{2}}{2}} 0h \quad \text{---} \quad \text{---} \quad 0h \quad \boxed{0h \vec{\pi}_{-\frac{\sqrt{3}}{2}, \frac{\sqrt{2}}{2}}; +\frac{\sqrt{3}}{2}, \frac{\sqrt{2}}{2}}$$

$i\frac{1}{2}\hbar, -\frac{3}{4}\hbar^2, \boxed{\bar{\pi}^R(1)}$

Antimatter One Half Spin Particle  
in Real Region

$$\Pi^{2,R}(1) = \Pi_{0, \frac{-\sqrt{2}}{2}, 0, \frac{-\sqrt{2}}{2}}^2 = -\frac{3}{4}\hbar^2$$
$$\vec{\pi}_{0,0,-1,0}(0) = 0\hbar$$
$$\pi_{3;-1,0} =$$

$$\pi_{-1,0}^2 =$$




**Diagram 3** consists of **Diagram 1** and **Diagram 2**. Which composes fermions with one half spin of matter and antimatter. *The same programe* could explain " Origin of Particles, Bosons and Fermions, Compounds of Zero Spin Particles, in Complex Spin Region " [3]

#### 4 Conclusions

Basing on the current point of view of quantum machenics, there is no any spin-interactions between two zero spin particles.

But " 0 " doesn't mean "no reality", if the concept of Spin Topological Space, STS, is introduced in particle spin angular momentum theory, zero spin particle will no longer be "The invisible man", it would give rise to many spin-interaction phenomenon never expected before.

This paper shows that in complex region of Casimir Operator of zero spin particle: two zero spin particles which stay in the case of imaginary spin-conjugates each other, would combine into a matter fermion  $\vec{\Pi}^I(1)$  with one half spin; and in the case of real spin-conjugates each other, an antimatter fermion  $\vec{\Pi}^R(1)$  with one half spin would be formed.

The details of cases of  $\vec{\Pi}^I(n)$  and  $\vec{\Pi}^R(n)$ ,  $n \geq 2$  in **Diagram 3**, will be given in the next paper.

#### References

- [1] ShaoXu Ren (2015) *Interaction of the Origins of Spin Angular Momentum* ISBN 978-988-14902-0-9 (2016 2nd edition)
- [2] ShaoXu Ren (2016) *Journal of Modern Physics*, **7**, 2257-2265  
<http://dx.doi.org/10.4236/jmp.2016.716194>
- [3] ShaoXu Ren 642. WE-Heraeus-Seminar, *Non-Hermitian Hamiltonians in Physics: Theory and Experiment* May 15-19, 2017, physikzentrum Bad Honnef Germany