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Analyzing the R-Implications of Weber and Fodor in the Counting of Mycorrhizal Fungi Spores

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Abstract*:* The Mathematical Morphology presents a systematic model to extract geometric features of images using morphological operators that transform the original image into another, using a third image called structuring element. The fuzzy mathematic morphology extends the morphological operators for images in shades of gray and colored, using the theory of fuzzy sets, especially the fuzzy logic, where the definition of the operators are defined using the notions of implications and T-norms. In this work, it was proposed an automatic method for spores counting of mycorrhizal fungi, the spores counting is done manually, by using a ribbed plate and with the aid of a stereomicroscope. The proposed method uses Gödel fuzzy morphological operators and in addition, the article makes a comparison to other existing methods, showing their efficiency.

Key words: Fuzzy morphology, fuzzy implications, counting spores.

1. Introduction

Fuzzy Mathematical Morphology is a very powerful tool in image processing that began in 1990 by Goetcherian [1]. There are several ways to extend the binary images for images grayscale and colors. In this case we used the fuzzy logic. Several authors have worked with this approach, for example, Dougherty and Sinhá [2, 3], Bloch e Maître [4, 5], De Beats, Nachtegael e Kerre [6-8], Deng e Heijmans [9] and Andrade et al. [10]. This approach uses logical connective as implications and conjunctions. A very different class of implication, the R-implications, because it has a T-norm associated. These implications play an important role in fuzzy morphology. In the work of Andrade et al. [10] were presented morphological operators from the R-implications of Lukasiewicz, Godel and Goguen, however the R-implications of Weber and Fodor developed functions named Epsilon. These functions were used in this work as an object of study to analyze their respective performances in image processing, a very particular case, the counting of mycorrhizal fungi spores. However, mathematically R-implications of Weber and Fodor do not form an adjunct each, which gives rise to the pair of morphological operators (erosion, dilation). In this case, only we got the epsilon function of each R-implication. The counting of mycorrhizal fungi spores with fuzzy morphology studies is already being targeted as can be seen in works by Andrade et al. [11-13].

Mycorrhizal is an association between a group of soil fungi and most "terrestrial vascular plants, epiphytes, water and also bryophytes and rhizoids stalks and other plant basal" [14]. However, for the purpose of this study we found the focus only on MAs (arbuscular mycorrhizae) and the FMAs (mycorrhizal fungi) which act as an extension of the roots, enhancing better water absorption and nutrients [15], especially phosphorus, improving the state nutritional plan and promoting the reduction of losses by stresses,

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whether biotic or abiotic [16]. There are several methods for the extraction of spores in the literature [17-24], the end of the implementation of these we will have a certain amount of to quantify. Thus, after the extraction procedure, the spores are counted by hand, but only by making use of a microscope stereoscopic (magnifying glass) to assist in viewing these structures ranging from 22 to 1,050 μm. At the time of actual count, the spores may be placed in a common petri dish, but it should be further put a checkered sheet underneath to provide guidance. However, in laboratories where there is already a routine count spores FMAs, use is made of a petri dish with circles concentric (corrugated board), made of acrylic. These circles facilitate the separation of a particular portion of spores to be counted, and the amounts recorded along the count are done by so-called manuals and or digital counters, common in clinical laboratories for blood cell count.

This work aims to analyze the R-implications of Weber and Fodor, more specifically the Epsilon Weber and Epsilon Fodor functions in fungal spore counts mycorrhizal grounded by a statistical study of the Bland-Altman method. The paper is organized as follows. In Section 2 the preliminary definitions are presented. In Section 3 we provided a description of the experiments followed the statistical study and the end Section 4 is expressed in the final considerations.

2. Preliminaries

This section will contain the necessary settings for the best understanding of fuzzy morphology. For this two important concepts were used: implications and T-norm that make an adjunct. For details, see Andrade et al. [10].

In order to introduce the notion of fuzzy morphology one has to speak about two fuzzy connectives of much importance: the conjunctions and the implications.

Definition 1: (Residual Functions and Adjunction)[25,26] Given two partial orders A and B, a function $f: A \to B$is said **residual**, if $f$is isotonic and there is also an isotonic function, f^R,$f^R: B \to A$ called the **residue** of f so that $f^R \circ f > id_A$ and $f \circ f^R > id_B$.The pair $(f, f^R)$is called **Adjunction.** (f is called **Left Adjunction** and f^R **Right Adjunction**).

Definition 2: (Fuzzy Conjunction) [26] A function $C: [0,1]^2 \to [0,1]$ is called **fuzzy conjunction** if the following conditions are satisfied:

(1) $C$is an isotonic application;

(2)$C(0,0) = C(0,1) = C(1,0) = 0$, $C(1,1) = 1$

An special case of conjunction is called T-norms.

Definition 3: [27] The **T-norm** is a function $T: [0,1]^2 \to [0,1]$ that satisfies the properties:

(1) T is a conjunction;

(2) T is commutative;

(3) T(1, x) = x.

The Table1 gives some examples of fuzzy T-norms that are used in this work.

Definition 4: (Fuzzy Implication)[27] A function $I: [0,1]^2 \to [0,1]$is called **fuzzy implication**, if the following conditions are satisfied for all$x_1, x_2, y, y_1, y_2 \in [0,1]$:

(i) $x_1 \le x_2 \Rightarrow I(x_1, y) \ge I(x_2, y)$;

(ii) $y_1 \le y_2 \Rightarrow I(x, y_1) \le I(x, y_2)$;

(iii) I(0, 0) = 1;

(iv) I (1,1) = 1;

(v) I(1,0) = 0;

(vi) I(0,1) = 1.

Table 2 contains some fuzzy implications.

According to Fodor and Rubens [28] some fuzzy implications have some interesting additional properties; to know;

Table 1 Some fuzzy T-norms.

Name	T-norm
Lukasiewicz	$T_{LK}(x, y) = \max(0, x + y - 1)$
Gödel	$T_{GD}(x,y) = min(x,y)$
Weber	$T_{WB}(x, y) = \begin{cases} 1 & \text{se } x < 1, y < 1 \\ \min(x, y) & \text{c.c.} \end{cases}$
Fodor	$T_{FD}(x, y) = \begin{cases} 1 & \text{se } x + y < 1 \\ \min(x, y) & \text{c.c.} \end{cases}$

Table 2 Some fuzzy implications.

Name	Implication
Lukasiewicz	$I_{LK}(x,y) = \mathrm{MIN}(1, 1-x+y)$
Gödel	$I_{GD}(x,y) = \begin{cases} 1 & \text{se } x \le y \\ y & \text{se } x > y \end{cases}$
Weber	$I_{WB}(x,y) = \begin{cases} 1 & \text{se } x < 1 \\ y & \text{se } x = 1 \end{cases}$
Fodor	$I_{FD}(x,y) = \begin{cases} 1 & \text{se } x \le y \\ \max(1-x, y) & \text{se } x > y \end{cases}$

(1) $I(1,x) = x, x \in [0,1]$;
(2) $I(x, I(y,z)) = I(y, I(x,z)), x,y,z \in [0,1]$;
(3) $I(x,x) = 1, x \in [0,1]$;
(4) $I(x,y) = 1 \Leftrightarrow x \le y, x, y \in [0,1]$;
(5) $I(x,y) \ge y; x, y \in [0,1]$;
(6) $I(x,y)$ is a continuous function.

Definition 5: [27] A function $I: [0,1]^2 \rightarrow [0,1]$ is called **R-implication** if the there is a T-norm T so that

$$I(x,y) = \sup\{t \in [0,1]; T(x, t \le y)\} \quad (1)$$

The implications I_{LK}, I_{GD} ,I_{WB}eI_{FD} are R-implications. Within such implications there is a very important class, the **residual implications**, which are R-implications where the supreme in Eq. (1) coincides with the maximum of the set, i.e.,

$$I(x,y) = \max\{t \in [0,1]; T(x, t \le y)\}\}$$

According to Ronse [29], the basic morphological operators can be defined in any complete lattice opening the possibility to be modeled like this: binary images, grayscale or color. It is noticed that the basic operators of erosion and dilation are defined using logical operators implication, I , and T-norm, T , respectively as can be seen in definition 23. However it is not any pair (I,T) which gives result in adjuncts, as will be seen in the following, the implication must be residual. An analysis was done about R-implications in the work by Andrade et al. [10] in which Lukasiewicz and Gödel were included as residual whereas the Weber and Fodor were not included.

Definition 6: [26] Let I be a residual implication (obtained from a left-continuous T-norm T) and the family$\{(\delta_B^T, \varepsilon_B^I); B \in [0,1]\}$ of associated disjunctions. **The fuzzy erosion** of the image A by the structuring element B on the point x is defined as

$$\varepsilon_B^I(A(x)) = inf_{x\in A}\{I(B(x), A(x))\} \quad (2)$$

where as **the fuzzy dilation** of the image A by the structuring element B on the point x is defined as

$$\delta_B^T(A(x)) = sup_{x\in A}\{T(B(x), A(x))\} \quad (3)$$

Based on the above definition settings 7 can be drawn up lying in more detail in Ref. [10] and that were used in the experiments of the next section.

Definition 7: Sets up the Gödel's erosion and Lukasiewicz's erosion by the following equations.

(1) **Lukasiewicz's erosion** of an image A by structuring element B at the point x, is denoted by ε_B^{LK} A is given by:

$$\varepsilon_B^{LK} A(x) = \wedge_{y\in A}\, [1, 1 - B_x(y) + A(y)] \quad (4)$$

(2) **Godel's erosion** of an image A by structuring element B at point x, is denoted by ε_B^{GD} A is given by:

$$\varepsilon_B^{GD} A(x) = \wedge_{y\in A} \begin{cases} 1 & if\ B_x(y) \le A(y) \\ A(y) & if\ B_x(y) > A(y) \end{cases} \quad (5)$$

There are R-implications that are not residual but which can provide an interesting property for the image processing. In this case, the implications of Weber and Fodor provide functions that behave like erosions. They are called Epsilon functions.

Definition 8: Sets up the Epsilon Weber and Epsilon Fodor functions by the following equations.

(1) **The Epsilon Weber Function** is given by$\varepsilon_B^I = inf_{x\in A} I_{WB}[B(x), A(x)]$

$$= inf_{x\in A} \begin{cases} 1 & if\ B(x) \le 1 \\ A(x) & if\ B(x) = 1 \end{cases} \quad (6)$$

(2) **The Epsilon Fodor Function** is given by$\varepsilon_B^I = inf_{x\in A} I_{FD}[B(x), A(x)] =$

$$inf_{x\in A} \begin{cases} 1 & if\ B(x) \le A(x) \\ \max(1 - B(x), A(x))\ if & B(x) > A(x) \end{cases} \quad (7)$$

3. Journal of Experiments

In the paper by Andrade et al. [10] one presented and analyzed three groups of morphological operators from their respective R-implications: Lukasiewicz, Gödel and Goguen and concomitantly the Epsilon Weber and Epsilon Fodor functions. This work aims to analyze the Lukasiewicz's operator, Gödel's operator and Epsilon Weber and Epsilon Fodor functions in 37 images obtained in Ref. [30], with extension jpg, 2,054 KB and 430 × 311 pixels to analyze the applicability of these operators in the counting of mycorrhizal fungi spores. Fig. 1 presents two examples of spores images.

Experiments were made with four different structural elements illustrated in Fig. 2 in the counting of mycorrhizal fungi spores to analyze the impact of the erosion of Gödel and Lukasiewicz and Epsilon functions of Weber and Fodor. With these experiments were a statistical study that one described concomitantly. In this study was proceeded the Bland-Altman [31]. The proposed methodology evaluates the correlation between two variables $(X-Y)$ and the variables between $\frac{(X+Y)}{2}$. In this graphic you can see the bias (how far to withdraw zero differences), the error (the dispersion of the points of differences around the mean), outliers and trends. For this, the bias is calculated (d) and its SD (standard deviation) is possible to obtain the limits of agreement: d + 1.96sd. These limits represent the region in which 95% of the differences in the cases studied are found [32]. To acquire the graphics we used the Free Software R (7) to the data manipulated in Excel with csv extension of 37 images taken from [30] with the standard data.

3.1 Experiments

3.1.1 Experiment

With the element 1, the processing is made and the same result obtained for the four implications. It was found that the methods have a strong correlation because the correlation coefficient is 0.87 which can be seen in Fig. 3(a). They also had a good agreement, as the graphic of Bland-Altman methodology according to Fig. 3(b) it can be noticed that the bias is 37.84. The limits of agreement indicate that the differences between the two methods are less than 7.68.

3.1.2 Experiment

With the element 2, it was found that the method by the naked eye and Epsilon Fodor function have a strong correlation because the correlation coefficient 0.85. They also had a good agreement, as the graphic of Bland-Altman methodology in Fig. 4(a) it can be noticed that the bias 44.54. The limits of agreement pointed out that the differences between the two methods are less than 89.09. With the naked eye method

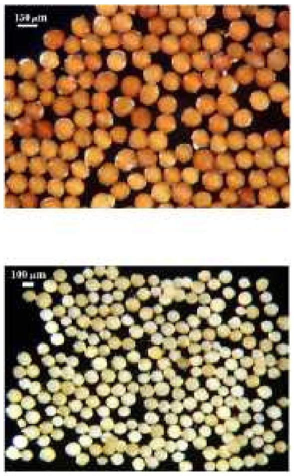
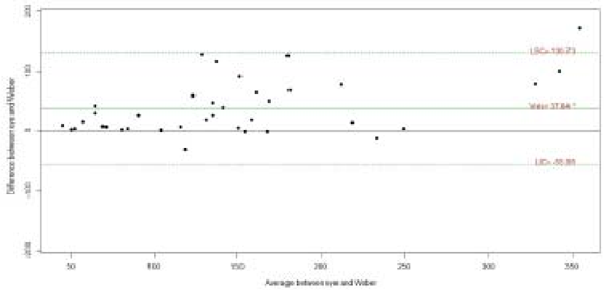


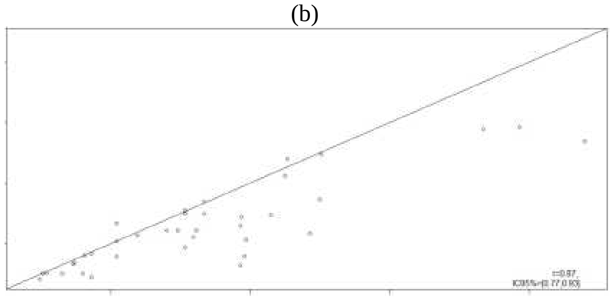
Fig. 1 Exemples of mycorrhizal fungi spores.

(1) (2) (3) (4)

Fig. 2 Structuring elements.



(b)



(a)

Fig. 3 (a) Graphic of naked eye correlation and Epsilon Weber Function; (b) Graphic of Bland-Altman method of Epsilon Weber Function.

and Gödel morphology have a strong correlation because the correlation coefficient is 0.85. They also presented a good agreement, since the graphic of Bland-Altman methodology in Fig. 4(b) one can perceive that the bias is 45.35. The limits of agreement show that the differences between the two methods are less than 90.7. Then it was observed that the naked eye method and Lukasiewicz morphology have a strong correlation because the correlation coefficient is 0.84. They also had a good agreement, as the graphic of Bland-Altman methodology according to Fig. 4(c) where the bias is 45.84. The limits of agreement indicate that the differences between the two methods are less than 91.68. And finally, the eye and Weber Epsilon function method have a strong correlation because the correlation coefficient is 0.81. They also had a good agreement, as the graphic of Bland-Altman methodology according to Fig. 4(d) with bias 46.86. The limits of agreement indicate that the differences between the two methods are less than 93.73.

3.1.3 Experiment

With the element 3, it was found that the method by the naked eye and Epsilon Fodor function have a strong correlation because the correlation coefficient is 0, 89. They also had a good agreement, as the graphic of Bland-Altman methodology in Fig. 5(a) one can perceive that the bias is 37.68. The limits of agreement indicate that the differences between the two methods are less that 75.35.With the naked eye method and Gdel morphology have a strong correlation because the correlation coefficient is 0.88. They also had a good agreement, as the graphic of Bland-Altman methodology according to Fig. 5(b) where the bias is 37.84. The limits of agreement indicate that the differences between the two methods are less than 75.68. Then, there was the naked eye method and Lukasiewicz morphology have a strong correlation because the correlation coefficient is 0.84. They also had a good agreement, as the graphic of Bland-Altman methodology that can be seen in Fig. 5(c) one can perceive that the bias is 44.59. The limits of agreement indicate that the differences between the two methods are less than 89.19. And finally, the naked eye method and Epsilon Weber function have a strong correlation because the correlation coefficient is 0.82. They also had a good agreement, as

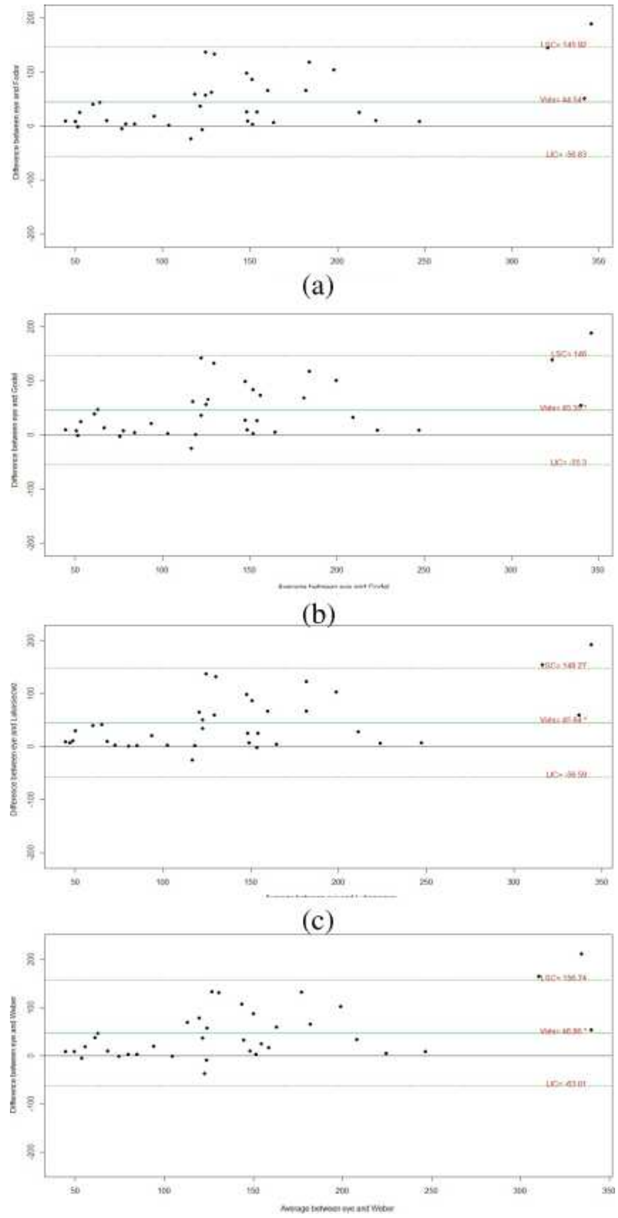


Fig. 4(a) Graphic of Bland-Altman methodology of naked eye and the Epsilon Fodor function; (b) Graphic of Bland-Altman methodology of naked eye and the Gödel's Erosion; (c) Graphic of Bland-Altman methodology of naked eye and the Lukasiewicz's Erosion; (d) Graphic of Bland-Altman methodology of naked eye and the Epsilon Weber function.

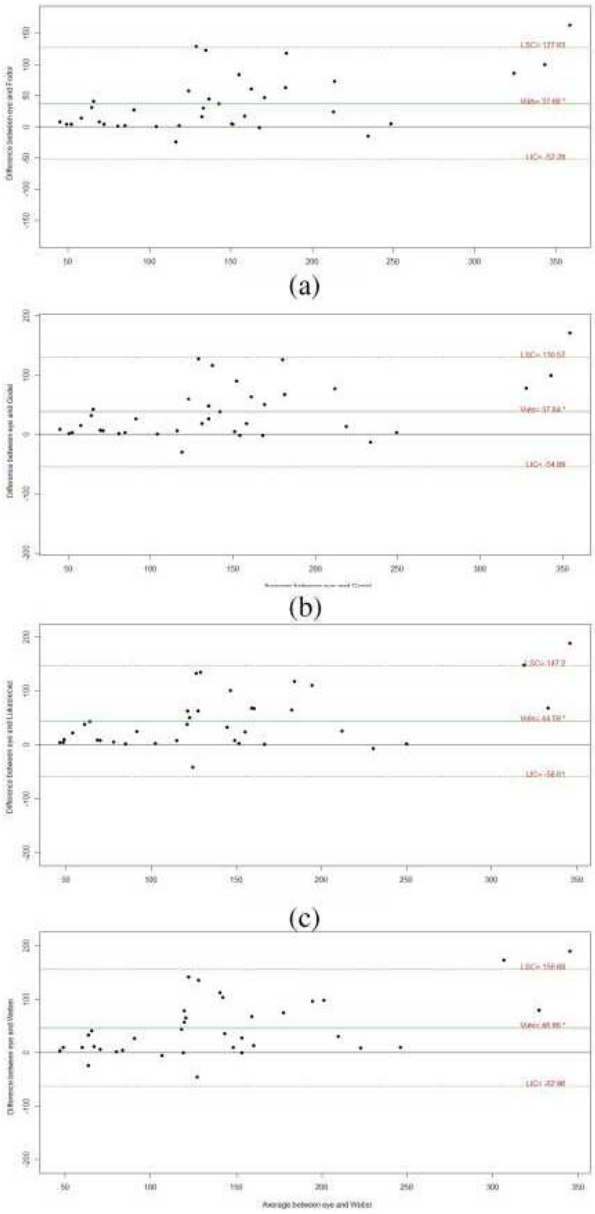


Fig. 5(a) Graphic of Bland-Altman methodology of naked eye and the Epsilon Fodor function; (b) Graphic of Bland-Altman methodology of naked eye and the Gödel's Erosion; (c) Graphic of Bland-Altman methodology of naked eye and Lukasiewicz's Erosion; (d) Graphic of Bland-Altman methodology of naked eye and the Epsilon Weber function.

the graphic of Bland-Altman methodology according to Fig. 5(d) the bias is 46.86. The limits of agreement indicate that the differences between the two methods are less than 93.73.

3.1.4. Experiment

Using the element 4, it was found that the naked eye method and Gödel morphology have a strong correlation because the correlation coefficient is 0.88 which can be seen in Fig. 6(a). They also had a good agreement, as the graphic of Bland-Altman methodology seen in Fig. 6(b) with the bias 30.68. The limits of agreement indicate that the differences between the two methods are less than 61.35. With the methods naked eye and Epsilon Weber function do not have a strong correlation because the correlation coefficient is 0.03 according to Fig. 6(c). They did not show agreement, as can be seen in the graphic of the Bland-Altman method in Fig. 6(d), according to the bias 102.89 which was discarded. The limits of agreement indicate that the differences between the two methods are less than 389.68, which is considered too large for this method. Then, it was observed that the naked eye method and Epsilon Fodor function does not have a strong correlation because the correlation coefficient is 0.29.They did not show agreement, as can be seen in the graphic of the Bland-Altman method in Fig. 7(a), one can perceive that the bias is 104.89 which was discarded. The limits of agreement indicate that the differences between the two methods are less than 356.18, which is considered too large for the method. And finally, the naked eye method and Lukasiewicz morphology do not have a strong correlation because the correlation coefficient is 0.04. They did not show agreement, as can be seen in the graphic of the Bland-Altman method in Fig. 7(b), with the bias 109.19 discarded by the method. The limits of agreement indicate that the differences between the two methods are less than 398.68, which is considered too large for the method.

3.2 Analysis of Results

The first, second and third experiments, with the Bland-Altman method led to concluding that any operator: Lukasiewicz's erosion, Gdel's erosion, Epsilon Weber Function and Epsilon Fodor Function can be used in spore counts. In the case of the fourth experiment, it is concluded that only the operator:

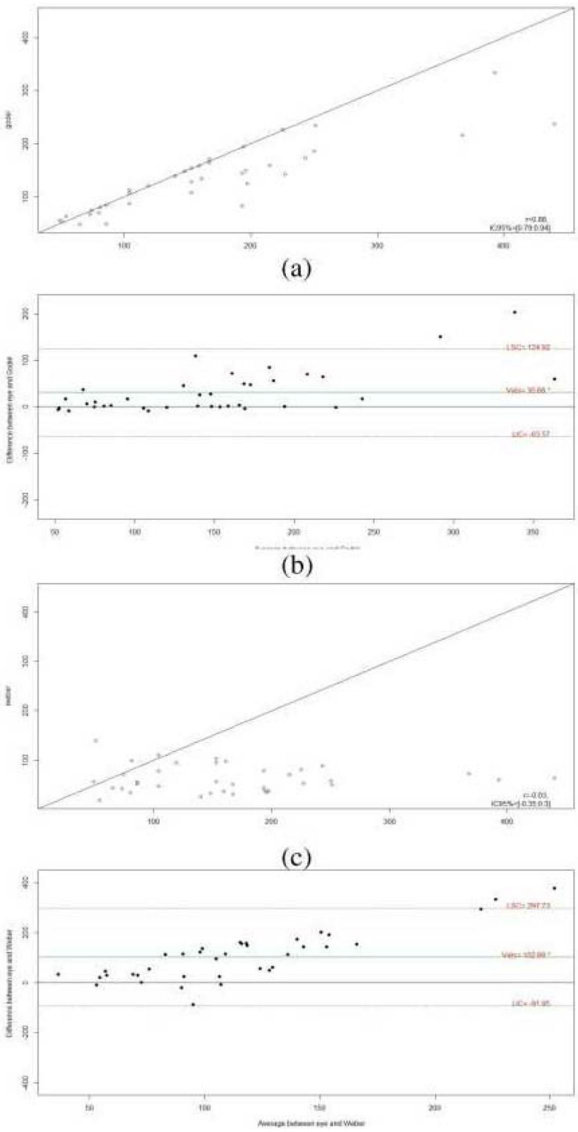


Fig. 6(a) Graphic of naked eye correlation and Gdel's Erosion; (b) Graphic of Bland-Altman methodology of naked eye and Gdel's Erosion; (c) Graphic of naked eye correlation and Epsilon Weber Function; (d) Graphic of Bland- Altman methodology of naked eye and the Epsilon Weber function.

Godel's erosion can be used in the spore counts. This means the requirement that applications used in image processing are residual to obtain expansion and fuzzy erosion is very strong.

4. Final Considerations

In this article, an analysis was made of the repercussions of Epsilon Fodor and Epsilon Weber functions in image processing, more specifically

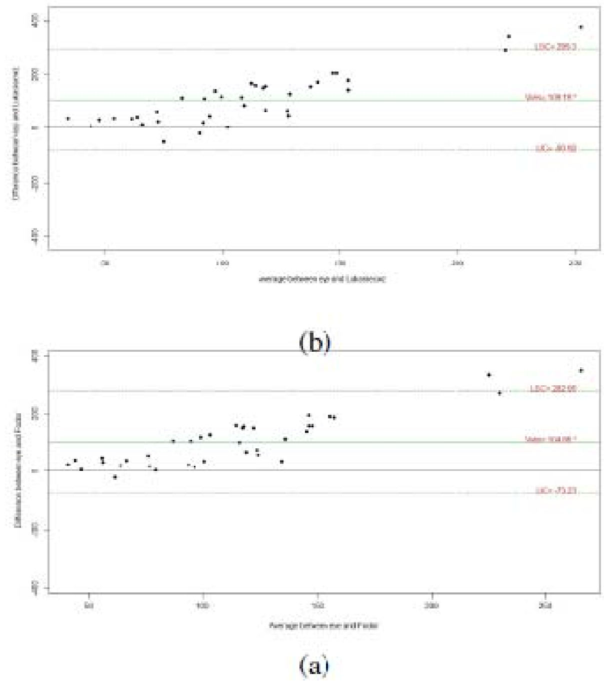


Fig. 7(a) Graphic of Bland-Altman methodology of naked eye and the Epsilon Fodor function; (b) Graphic of Bland-Altman methodology of naked eye and Lukasiewicz's Erosion.

in mycorrhizal fungal spores count. It was verified that these functions can be used depending on the structural element. With the structural elements 1, 2, 3 the functions Epsilon Fodor and Epsilon Weber were considered for processing. Only with the structural element 4, the only option for processing Gödel's erosion. Although Epsilon Fodor and Epsilon Weber functions are not erosions, they worked as tools in image processing, specifically, the mycorrhizal fungi spore count.

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