

Egalitarian Policies, Effective Demand, and Globalization: Considering Budget Constraint^{*}

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This paper examines the effectiveness of redistribution policies under budget constraint considering government spending for the productivity improvement and effective demand. It shows that an asset-based redistribution policy is not always effective under effective demand and budget constraint. However, the increase of effective demand because of an income-based redistribution improves employment, labor productivity, and wage rates because of increased government spending for productivity improvement as the results of saving rate from profit income show. A distinctive feature of this paper is considering effective demand with political aspects. Workers' demands on unemployment compensation depend on the demand and supply condition in the goods market. The model implicitly assumes that labor is strong enough to affect political institutions and social security system is prepared enough to affect goods market. Therefore, the model built in this research is applicable to Europe in the real world.

Keywords: egalitarianism, redistribution, effective demand, globalization

Introduction

The recent expansion of inequality all over the world has been paid much attention to. Many people are eager for effective egalitarian policies. However, there is the fear that these policies may not be compatible with globalization. Redistribution under globalization may cause the decrease in international competitiveness and capital flights. Therefore, examining these effectiveness and showing the suitable policies are urgent tasks for economists. As a side note, here the author examine egalitarian policies in terms of the relationship between labor and capital.

Bowles (2012) recommended asset-based redistribution against the argument that redistribution policies are not effective under globalization. This is the origin of the “sharking model” whereby workers determine labor efficiency considering the unemployment compensation and monitoring by firms with free capital movements across borders. Bowles concluded that the strengthening of regulations for firing and expanding unemployment compensation decreases employment because of pressure for wage increases; however, asset-based redistribution increases employment because labor productivity improves because of the rise in labor incentives.

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Aspects of effective demand are excluded in Bowles (2012) because the difficulty of the redistribution of income under globalization is empirically showed in Bowles and Boyer (1995). However, Onaran and Galanis (2013) showed the feasibility of the wage-led growth depends on conditions in each country and in each era. There is also the strategy of international cooperation to achieve wage-led economies. As a side note, the increase in wages has the same meaning as the redistribution of income because the consumption propensity for wages is larger than that for capital profits.

Abe (2015) introduced effective demand factors to Bowles (2012) and reexamined the arguments. In particular, it made the goods market explicit and assumed the unemployment changes responding to market conditions on demand and supply (Bowles, 2013).

Political pressures for expanding unemployment compensation increases when there is excess supply in the goods market; then, the goods market changes accordingly.

The main conclusions of Abe's study are as follow. The improvement in labor productivity and the decrease in the labor ratio for monitoring because of the asset-based redistribution lure capital from abroad and increase wage rates. Then, in Bowles (2012), labor supply increases result in the increase of employment. In contrast, Abe (2015) proclaims the improvement in labor productivity increases employment and that the effect on employment by decreasing the monitoring ratio is vague because these cause increases in supply and demand. The results indicate asset-based redistribution under globalization is not always effective with effective demand constraints.

However, Abe (2015) does not tackle all problems in Bowles (2012). The author thinks the relationship between effective demand and redistribution policy while considering budget constraints as Bowles (2012).

The author assume the economy as follows. Goods produced by labor and capital are either for consumption or investment. Labor is homogeneous and immobile across borders. Employers extract labor efforts by monitoring and the threat of dismissal. Capital moves globally in response to the after-tax profitability. Interest and time preference rates are same across borders and each country is a small economy. Workers receive wages and unemployment compensation and spend it all. Capital consumes a fraction of the profit. When there is excess supply in the goods market, there is political pressure for increasing unemployment compensation, and vice versa. Government funds unemployment compensation and improvement in productivity from its capital gains tax revenues.

This research is organized as follows. Section 1 explains the Bowles model, and section 2 introduces the effective demand factors to the basic model. Section 3 includes a comparative statics analysis, followed by our conclusion.

Bowles Model

In this section, the author explains the Bowles (2012) model as the basic one.

The gross production Q is

$$Q = yeh(1 - m) \quad (1)$$

where h , e , y , and m are labor time, labor effort per hour, production per effort unit, and the ratio of monitoring labor, respectively. The author normalizes h to $0 < h < 1$ and assume that workers can choose effort unit 0 or 1.

Firms monitor workers and determine the wage rate to equate payoff for those working and those sharking. Thus, the author gets

$$w - a = (1 - \tau)w + \tau h w + \tau(1 - h)b \quad (2)$$

where w , a , τ , and b are wage rate, disutility of labor, the probability of firing, and the unemployment compensation, respectively. The left hand shows payoff for those working, and the right hand shows payoff for those sharking. The first term in the right hand is the case of continued employment, the second term is the case where the employee is dismissed and finds a new job, and the third term is the case where the employee is dismissed and is unemployed.

From (2), the author gets

$$w = \frac{a}{r(1 - h)} + b \quad (3)$$

This wage is the minimum level to prevent workers from sharking, and profits and utility of workers are optimal under the wage. In (3), wage rate w is the increasing function on disutility of labor a , employment rate h , and unemployment compensation b is the equilibrium condition for labor supply.

The profit rate is

$$r = \frac{y - k - \frac{w}{1 - m}}{k} \quad (4)$$

where k is capital per labor hour, k as the intermediate goods is removed in numerator of (4) because the production goods have characteristics of both investment and consumption. It should be noted that workers for monitoring receive wages.

The after-tax profit rate π is

$$\pi = r(1 - t) = \frac{(1 - t)(y - k - \frac{w}{1 - m})}{k} \quad (5)$$

where t is the tax rate for profit.

The expectation of after-tax profit rate $E(\pi)$ is

$$E(\pi) = \pi(1 - d) \quad (6)$$

where the probability of confiscation is d , which depends on the macroeconomic policies and political factors in each country.

The author denotes interest rate of safe asset ρ where it is equal across borders. Thus, the arbitrage equation of capital is

$$E(\pi) = \rho \quad (7)$$

The author assumes $1/(1 - d) = \mu$. Thus, from (5)-(7), the author gets

$$w = (1 - m)(y - k - \frac{k\rho\mu}{1 - t}) \quad (8)$$

(8) is the equilibrium equation for labor demand.

The author denotes government spending for labor productivity p , which includes nutrition, medication, education, and infrastructure. When the author assumes the effectiveness is λ , the author gets

$$y = y(\lambda p) \quad (9)$$

Next, the author takes up budget constraint. Tax revenue from only profit is $th\{(1-m)[y(\lambda p) - k]\}$. Government spending is used for unemployment compensation $b(1-h)$ and spending for productivity p . Thus, the author gets

$$b(1-h) + p = th\{(1-m)[y(\lambda p) - k] - w\} \quad (10)$$

Substituting (8) for (10), the author gets

$$p = th(1-m) \frac{k\rho\mu}{1-t} - b(1-h) \quad (11)$$

The author can sum up the model using equations (3), (8), and (11) and three endogenous variables w , h , and p .

Figure 1 shows the determination of wage w and employment h .

The curve in (3) is an increasing function because wages increase with the increase in employment. The curve in (8) is also an increasing function because productivity increases because of the increase in government spending for productivity with the increase in employment. Only equilibrium value E in Figure 1 is stable.

The results of the comparative statics analysis are listed in Table 1.

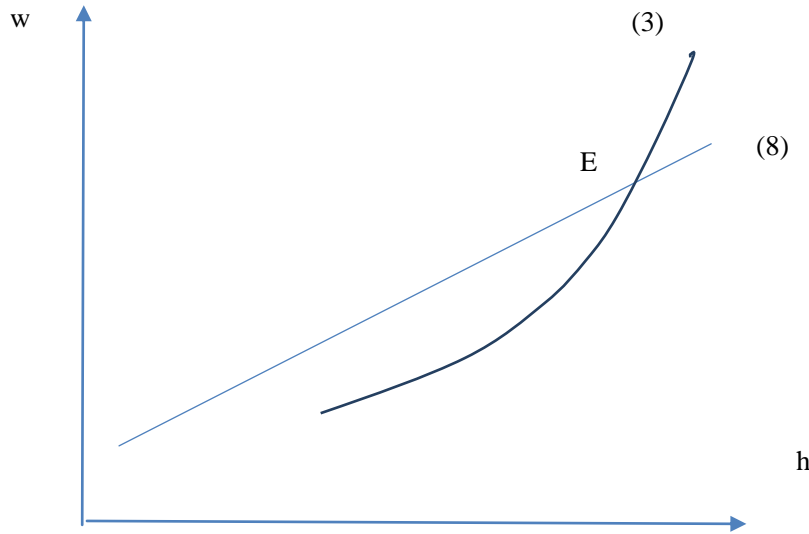


Figure 1. Determination of wage and employment.

Table 1

Results of Comparative Statics Analysis

	h	w	y	p
m	—	—	—	—
t	\mp	\mp	\mp	\mp
b	—	—	—	—
τ	+	+	+	+
λ	+	+	+	+
a	—	—	—	—
k	\mp	\mp	\mp	\mp
ρ	\mp	\mp	\mp	\mp
μ	\mp	\mp	\mp	\mp

The notable results are as follow. On the one hand, anti-worker's policies, the decrease in the unemployment compensation ($b \downarrow$), and the strengthening of dismissal regulations ($\tau \downarrow$) all increase wages and employment. On the other hand, a decrease in the ratio of monitoring labor ($m \downarrow$) causes wages and employment to increase. Table 2-4 show these results.

Decreases in the ratio of monitoring labor mean asset-based redistribution decreases the need for the monitoring.

As mentioned above, these results ignore the effect of effective demand. Therefore, the author considers it in the next section.

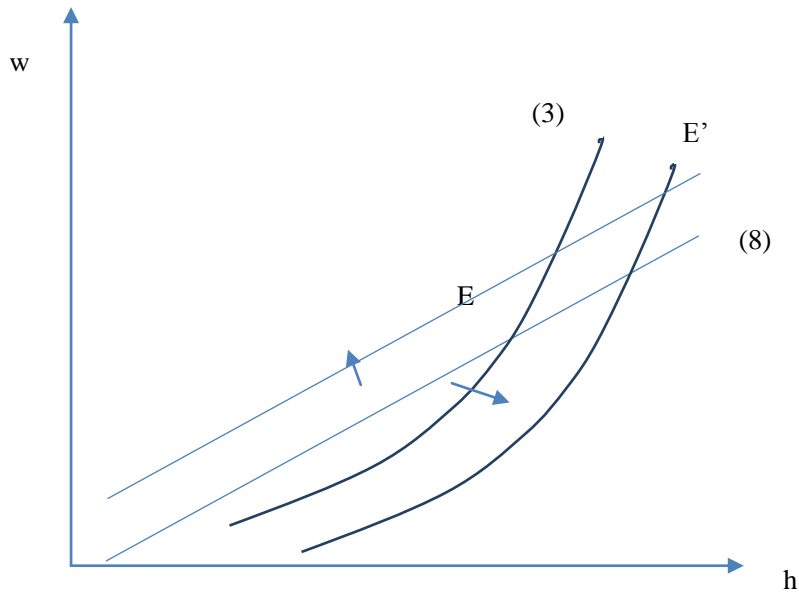


Figure 2. Decrease in b .

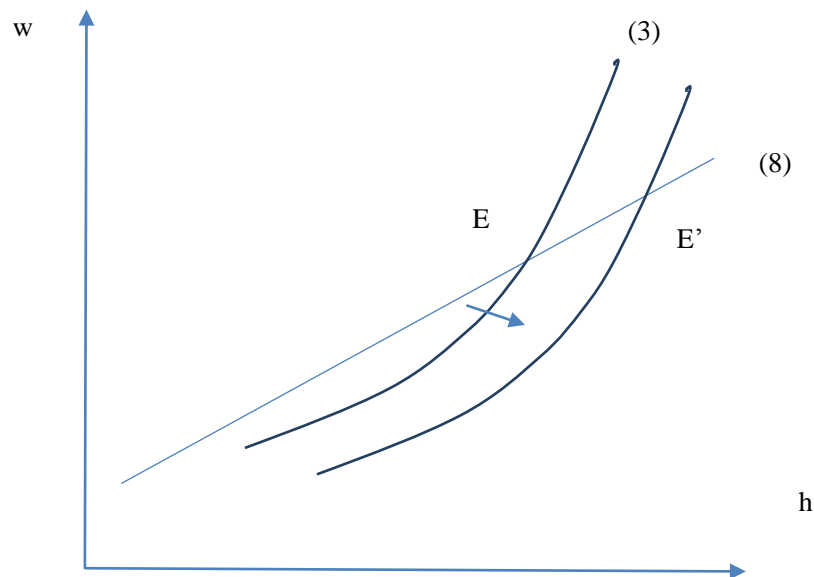
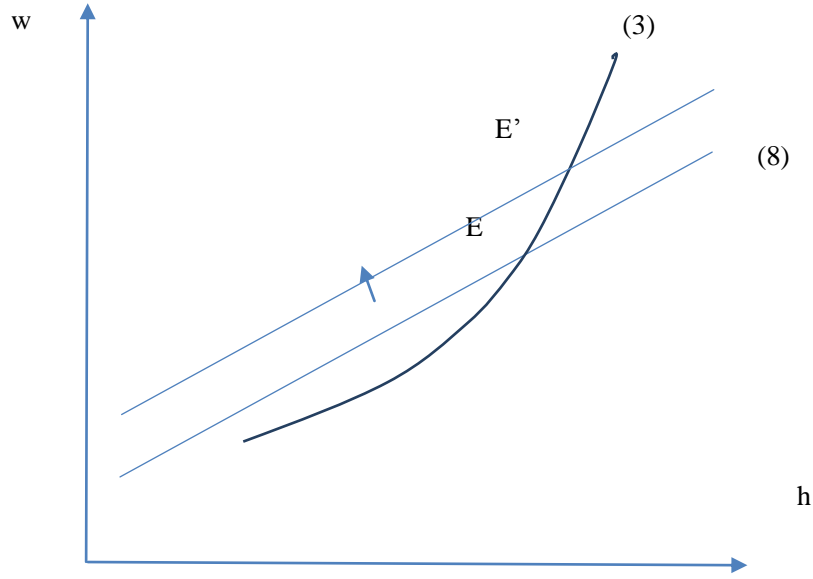


Figure 3. Decrease in τ .

Figure 4. Decrease in m .

Considering Effective Demand

In this section, the author build a model considering effective demand.

First of all, the author takes up the goods market. The equilibrium equation in the goods market is

$$(y - k)(1 - m)h = i + c + g + x \quad (12)$$

where i , c , g , and x are investment, consumption, governmental spending, and net export, respectively.

Next, the author assumes that investment depends on the after-tax profit as Bowles and Boyer (1988). The investment function is

$$i = i_0 + i_r rk(1 - m)(1 - t), \quad i_0 > 0, \quad i_r > 0 \quad (13)$$

where i_0 , i_r , and $k(1 - m)h$ are animal spirits, responsiveness of investment on profit, and the amount of capital, respectively.

The author assumes all wages income and part of profit income are spent. Thus, the consumption function is

$$c = [w + (1 - s_r)r(1 - t)k(1 - m)]h \quad (14)$$

Government spending g is used for unemployment compensation and productivity improvement.

$$g = b(1 - h) + p \quad (15)$$

The author assumes that export f is constant and the constant ratio β of demand $i + c + g + x$ is import. Thus, net exports x is

$$x = f - \beta(i + c + g + x) \quad (16)$$

The author assumes that unemployment compensation decreases in excess demand of goods market, and vice versa. Thus, the dynamic equation for unemployment compensation is

$$\dot{b} = a[(y - k)(i - m)h - (i + c + g + x)] \quad (17)$$

This shows that workers' demands on unemployment compensation depend on the demand and supply condition in the goods market like Bowles (2013).¹

Next, the author will sum up the model.

When the author assumes $\dot{b} = 0$ in (17), the author gets

$$1 - m[\beta(y - k) + p\mu k(s_r) - i_r]h = i_0 + f \quad (18)$$

From (3) and (8), the author gets

$$\frac{q}{\tau(1 - h)} + b = (1 - m)(y - k - \frac{kp\mu}{1 - t}) \quad (19)$$

Thus, the author summarizes the model to three equations (11), (18), and (19) and three endogenous variables, i.e., h , p , and b .

The results of the comparative statics analysis are listed in Table 2.²

Table 2

Results of comparative statics analysis

	h	w	y	p	b
m	\mp	\mp	\mp	\mp	\mp
t	0	$+$	$+$	$+$	$+$
τ	$-$	$-$	$-$	$-$	$+$
λ	\mp	\mp	\mp	\mp	\mp
a	$-$	$+$	$+$	$+$	$-$
k	\mp	\mp	\mp	\mp	\mp
ρ	\mp	\mp	\mp	\mp	\mp
μ	\mp	\mp	\mp	\mp	\mp
s_r	$-$	$-$	$-$	$-$	$+$
i_r	$+$	$+$	$+$	$+$	$-$
i_{s_r}	$+$	$+$	$+$	$+$	$-$

The notable results are as follow.

When m increases, employment h increases because of excess demand in the goods market.

Tax revenue increases because of the increase in employment, but the decrease in the profit because of the increase in the monitoring labor decreases tax revenue. Therefore, the effect to p is vague, and the effects to y and w are unclear.

¹ Refer to Appendix 1 for a stability condition.

² Refer to Appendix 2 for major calculations.

When t increases, employment h decreases because of the decrease in investment. The decrease in employment decreases tax revenue; however, the increase of t increases tax revenue. Thus, the effect to p is ambiguous as are the effects to y and w .

The increase in τ is pressure for the decrease in wage rate. Thus, unemployment compensation can increase under constant employment to compensate for the decrease.

The increase in unemployment compensation makes p decrease because of the budget constraint, which results in excess demand in the goods market. Finally, employment h increases.

The increase in λ decrease employment h because of excess supply in the goods market.

This results in the decrease of p , but the effect to the productivity is ambiguous as are the effects to w and p .

The increase in a causes the pressure for the increase of wage rate. The unemployment compensation has to decrease to prevent capital flight under constant employment. Then p increases because of budget constraint, which results in excess supply in the goods market.

Therefore, h decreases, whereas y and w increase.

The increase in saving rate on profit income s_r decreases employment h because of excess supply in the goods market, which results in an increase in the unemployment compensation b because of political pressure and a decrease of p because of budget constraint. Thus, y and w decrease.

The decreases of r_r and r_0 are the same as the increase of s_r .

Conclusion

The author examined the effectiveness of redistribution policies under budget constraint considering government spending for the productivity improvement as Bowles (2012) and effective demand based on Abe (2015).

The author showed that an asset-based redistribution policy is not always effective under effective demand and budget constraint. The decrease in the ratio of monitoring labor decreases employment because of excess supply in the goods market. The result is typical in a demand-constrained economy.

Egalitarian policies, such as strengthening dismissal regulations, are also not effective like Bowles (2012). It increases the wages because of the increase in government spending for labor productivity. On the other hand, employment decreases resulted from in excess supply in the goods market.

However, the increase of effective demand because of income-based redistribution improves employment, labor productivity, and wage rates because of increased government spending for productivity improvement as the results of saving rate from profit income show. As a side note, the decrease in saving rate from profit income can be regarded as a kind of income-based redistribution.

A distinctive feature of this paper is considering effective demand with political aspects. Workers' demands on unemployment compensation depend on the demand and supply condition in the goods market. The story implicitly assumes that labor is strong enough to affect political institutions and social security system is prepared enough to affect goods market. Therefore, the model built in this research is applicable to Europe in the real world. Countries except Europe should be researched in another model.

Another distinctive feature is the assumption that capital moves across borders immediately. The extreme assumption is adopted because features of globalization can be showed apparently. However, there are many barriers against capital in the real world.

In addition, risk premium is endogenized in Bowles (2012). These tasks remain.

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Appendix 1

From (8), (10), and (13)-(16):

$$\dot{b} = \frac{\alpha(1-m)}{\alpha+\beta} [\beta(1-m)(y-k)h + \rho\mu k(1-m)h(s_r - i_r) - i_0 - f] \quad (20)$$

Thus,

$$\frac{\dot{b}}{b} = \frac{\alpha(1-m)}{1+\beta} \left[\beta(y-k) + \rho\mu k(s_r - i_r) + \beta h \lambda y' \frac{dp}{dh} \right] \frac{dh}{db} \quad (21)$$

From (3), (8), and (11):

$$\frac{dp}{dh} = \frac{\tau \frac{t\rho(1-m)\mu k}{1-t} + (1-m)\tau \left(y - k - \frac{k\rho\mu}{1-t} \right)}{(1-m)\tau(1-h)\lambda y' + \tau} > 0 \quad (22)$$

From (3), (8), and (11):

$$\frac{\alpha}{\tau(1-h)^2} dh + db = (1-m)\lambda y' dp \quad (23)$$

Therefore, from (22) and (23):

$$\frac{dh}{db} = \frac{-1 - (1-m)\lambda y'(1-h)}{\frac{\alpha}{\tau(1-h)^2 - (1-m)\lambda y' \left[\frac{tk\rho\mu(1-m)}{1-t} + b \right]}} \quad (24)$$

When the author assumes $S_r > \dot{i}_r$ in (19), $\frac{\alpha}{\tau(1-h)^2} - (1-m)\lambda y' \left[\frac{tk\rho\mu(1-m)}{1-t} + b \right]$ is a stable condition.

Appendix 2

 Calculation on m

From (11):

$$(1-h)db - b dh + dp = \frac{t(1-m)\rho\mu k}{1-t} dh - th \frac{\rho\mu k}{1-t} dm \quad (25)$$

From (19):

$$\frac{\alpha}{\tau(1-h)^2} dh + db = (1-m)y' \lambda dp - (y-k - \frac{kp\mu}{1-t}) dm \quad (26)$$

Substituting (26) for (25), the author gets

$$[(1-h)(1-m)y' \lambda + 1] dp = \left[\frac{t(1-m)\rho\mu k}{1-t} + b + \frac{\alpha}{\tau(1-h)} \right] dh + \left[(1-h) \left(y-k - \frac{kp\mu}{1-t} \right) - th \frac{kp\mu}{1-t} \right] dm \quad (27)$$

From (18), the author gets

$$(1-m)\{\beta(y-k) + k\rho\mu(s_r - i_r)\} dh = -(1-m)\beta h y' \lambda dp + \{\beta(y-k) + k\rho\mu(s_r - i_r)\} h dm \quad (28)$$

Substituting (27) for (28), the author gets

$$\frac{dh}{dm} = \frac{\beta(y-k)h + k\rho\mu h(s_r - i_r) \left[(1-h)(1-m)y' \lambda + 1 \right] - (1-m)\beta h y' \lambda \left[(1-h) \left(y-k - \frac{kp\mu}{1-t} \right) - th \frac{kp\mu}{1-t} \right]}{(1-m) \left[(1-h)(1-m)y' \lambda + 1 \right] \left[\beta(y-k) + k\rho\mu(s_r - i_r) \right] + \beta h y' \lambda \left[\frac{t(1-m)\rho\mu k}{1-t} + b + \frac{\alpha}{\tau(1-h)} \right]} \quad (29)$$

 Calculation on t

From (18), the author gets $\frac{dh}{dt}$

Thus, from (11):

$$(1-h)db + dp = \frac{(1-m)\rho\mu k h}{(1-t)^2} dt \quad (30)$$

From (19):

$$db = (1-m)y' \lambda dp \quad (31)$$

Substituting (31) for (30), the author gets

$$\frac{dp}{dt} = \frac{(1-m)kp\mu h}{(1-t)^2 [(1-h)(1-m)y' \lambda + 1]} > 0 \quad (32)$$

Thus, $\frac{db}{dt} > 0$ holds from (31).

 Calculation on τ

From (11):

$$(1-h)db - b dh + dp = \frac{t(1-m)\rho\mu k}{1-t} dh \quad (33)$$

From (19):

$$\frac{a}{\tau(1-h)^2} dh - \frac{a}{(1-h)\tau^2} + db = (1-m)y'\lambda dp \quad (34)$$

Substituting (34) for (33), the author gets

$$\left[(1-h)(1-m)y'\lambda + 1 \right] dp = \left[\frac{t(1-m)\rho\mu k}{1-t} + b + \frac{\alpha}{\tau(1-h)} \right] dh - \frac{\alpha}{\tau^2} d\tau \quad (35)$$

From (18), the author gets

$$(1-m)\{\beta(y-k) + k\rho\mu(s_r - i_r)\}dh = -(1-m)\beta hy'\lambda dp \quad (36)$$

Substituting (35) for (36), the author gets

$$\frac{dh}{dt} = \frac{\beta hy' \lambda \frac{\alpha}{t^2}}{(1-m)[(1-h)(1-m)y'\lambda + 1][\beta(y-k) + k\rho\mu(s_r - i_r)] + \beta hy' \lambda \left[\frac{t(1-m)\rho\mu k}{1-t} + b + \frac{\alpha}{\tau(1-h)} \right]} > 0 \quad (37)$$

From (35) and (36), the author gets

$$\frac{dp}{d\tau} = \frac{\beta(y-k) + k\rho\mu(s_r - i_r)}{\beta hy'} \frac{dh}{d\tau} < 0 \quad (38)$$

According to the results of (37) and (38), $\frac{db}{dt} > 0$ from (33).

Calculation on λ

From (11):

$$(1-h)db - b dh + dp = \frac{t(1-m)\rho\mu k}{1-t} dh \quad (39)$$

From (19):

$$\frac{\alpha}{\tau(1-h)^2} dh - \frac{\alpha}{(1-h)\tau^2} + db = (1-m)y'\lambda dp + (1-m)y'pd\lambda \quad (40)$$

Substituting (40) for (39), the author gets

$$\left[(1-h)(1-m)y'\lambda + 1 \right] dp = \left[\frac{t(1-m)\rho\mu k}{1-t} + b + \frac{\alpha}{\tau(1-h)} \right] dh - (1-h)(1-m)y'pd\lambda \quad (41)$$

From (18), the author gets

$$(1-m)\{\beta(y-k) + k\rho\mu(s_r - i_r)\}dh = -(1-m)\beta hy'\lambda dp - (1-m)\beta hy'pd\lambda \quad (42)$$

Substituting (41) for (42), the author gets

$$\frac{dh}{d\lambda} = \frac{\beta hy' \lambda \{ (1-h)(1-m)y'p - (1-m)\beta hy'p[(1-h)(1-m)y'\lambda + 1] \}}{(1-m)[(1-h)(1-m)y'\lambda + 1][\beta(y-k) + kp\mu(s_r - i_r) + \beta hy' \lambda [\frac{t(1-m)p\mu k}{1-t} + b + \frac{a}{\tau(1-h)}]} \quad (43)$$

Calculation on τ

From (11):

$$(1-h)db - dbh + dp = \frac{t(1-m)p\mu k}{1-t} dh \quad (44)$$

From (19):

$$\frac{a}{\tau(1-h)^2} dh + \frac{1}{(1-h)\tau^2} da + db = (1-m)y'\lambda dp \quad (45)$$

Substituting (45) for (44), the author gets

$$[(1-h)(1-m)y'\lambda + 1]dp = [\frac{t(1-m)p\mu k}{1-t} + b + \frac{q}{\tau(1-h)}]dh + \frac{1}{\tau} da \quad (46)$$

From (18), the author gets

$$(1-m)\{\beta(y-k) + kp\mu(s_r - i_r)\}dh = -(1-m)\beta hy'\lambda dp \quad (47)$$

Substituting (46) for (47), the author gets

$$\frac{dh}{da} = - \frac{\beta hy' \lambda}{(1-m)[(1-h)(1-m)y'\lambda + 1][\beta(y-k) + kp\mu(s_r - i_r)] + \beta hy' \lambda [\frac{t(1-m)p\mu k}{1-t} + b + \frac{q}{\tau(1-h)}]} < 0 \quad (48)$$

From (47) and (48), the author gets

$$\frac{dp}{da} = - \frac{\beta(y-k) + kp\mu(s_r - i_r)}{\beta hy' \lambda} \frac{dh}{da} > 0 \quad (49)$$

The results of (48) and (49) indicate that $\frac{dp}{da} < 0$ holds in (44).

Calculation on k

From (11):

$$(1-h)db - b dh + dp = \frac{t(1-m)p\mu k}{1-t} dh + \frac{t(1-m)p\mu h}{1-t} dk \quad (50)$$

From (19):

$$\frac{a}{\tau(1-h)^2} dh + \frac{1}{(1-h)\tau^2} da + db = (1-m)y'\lambda dp - (1-m)(1 + \frac{p\mu}{1-t})dk \quad (51)$$

Substituting (51) for (50), the author gets

$$[(1-h)(1-m)y'\lambda + 1]dp = [\frac{t(1-m)p\mu k}{1-t} + b + \frac{a}{\tau(1-h)}]dh + [(1-h)(1-m)(1 + \frac{p\mu}{1-t}) + \frac{t(1-m)p\mu h}{1-t}]dk \quad (52)$$

From (18), the author gets

$$(1-m)\{\beta(y-k) + kp\mu(s_r - i_r)\}dh = -(1-m)\beta hy'\lambda dp \quad (53)$$

Substituting (52) for (53), the author gets

$$\frac{dh}{dk} = - \frac{[(1-h)(1-m)y'\lambda + 1][\beta - p\mu(s_r - i_r)] - \beta hy'\lambda[(1-h)(1-m)(1 + \frac{p\mu}{1-t}) + \frac{t(1-m)p\mu h}{1-t}]}{(1-m)[(1-h)(1-m)y'\lambda + 1][\beta(y-k) + kp\mu(s_r - i_r)] + \beta hy'\lambda[\frac{t(1-m)p\mu k}{1-t} + b + \frac{a}{\tau(1-h)}]} \quad (54)$$

Calculations on ρ and μ are basically same as k .

Calculations on s_r

From (11):

$$(1-h)db - b dh + dp = \frac{t(1-m)p\mu k}{1-t} dh \quad (55)$$

From (19):

$$\frac{a}{\tau(1-h)^2} dh + db = (1-m)y'\lambda dp \quad (56)$$

Substituting (56) for (55), the author gets

$$[(1-h)(1-m)y'\lambda + 1]dp = [\frac{t(1-m)p\mu k}{1-t} + b + \frac{a}{\tau(1-h)}]dh \quad (57)$$

From (18), the author gets

$$(1-m)\{(y-k) + kp\mu(s_r - i_r)\}dh = -(1-m)\beta hy'\lambda dp - (1-m)kp\mu h ds_r \quad (58)$$

Substituting (59) for (60), the author gets

$$\frac{dh}{ds_r} = - \frac{-(1-\beta)kp\mu h}{(1-m)[(1-h)(1-m)y'\lambda + 1][\beta(y-k) + kp\mu(s_r - i_r)] + \beta hy'\lambda[\frac{t(1-m)p\mu k}{1-t} + b + \frac{a}{\tau(1-h)}]} + b + \frac{a}{\tau(1-h)} < 0 \quad (59)$$

The result of (59) indicates $\frac{dh}{ds_r} < 0$ in (57).

From (55) and (56), the author gets

$$\frac{db}{ds_r} = \frac{(1-m)y'\lambda[\frac{t(1-m)p\mu k}{1-t} + b] - \frac{a}{\tau(1-h)^2}}{(1-h)(1-m)y'\lambda + 1} \frac{dh}{ds_r} > 0 \quad (60)$$

The numerator in the right hand in (60) is negative because of the stability condition.

The calculations on \dot{i}_r and \dot{i}_0 are basically the same as s_r .