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The Interstorey Drift Sensitivity Coefficient in Calculating Seismic Building Frames

J. M. Martínez Valle[1], J. M. Martínez Jiménez[1] and P. Martínez Jiménez[2]

1. Department of Mechanics, Leonardo Da Vinci Building, University of Córdoba, Córdoba 14071, Spain

2. Department of Applied Physics, Albert Einstein Building, University of Córdoba, Córdoba 14071, Spain

Abstract: In calculating the seismic response of a building, the Spanish Instructions NCSE-02 and CTE, paragraph 3.7.7 (also EUROCODE 8 paragraph 1.2 part 1-1), establish that if for all storeys the interstorey drift sensitivity coefficient, ξ, is less than or equal to 0.1, then it will not be necessary to consider the effects of the 2nd order ($P-\Delta$ effects). In this paper the authors review this claim, because even for $\xi \le 0.1$, increases of the bending moment at the ends of the columns due to the inclusion of second order effects can account for between 15% and 34% of its value for static service loads. This is significant since most adverse effects are shown in the lower height buildings (up to 5 floors) which it is precisely the range in which most of the housing stock of Spain is located. Finally, the authors delimit the coefficient for buildings of lesser height (up to 5 floors), proposing to lower it generally to $\xi \le 0.06$.

Key words: Seismic analysis, interstorey drift sensitivity coefficient.

1. Description of the Problem and the Building Used as an Example

The NCSE-02 [1] and CTE [2] Spanish Instructions are instructions that allow us to study dynamic effects in different structural elements. Spain is a country that has different provinces in which dynamic studies are important and mandatory. In this article, it is analyzed the interstorey drift sensitivity coefficient, and consider whether or not the 2nd order effects are important in building structures to dynamic stresses.

This coefficient is defined in Eurocode 8 [3], but one of the conditions of whether or not to consider the effects of the 2nd order of coefficient, is formulated identically in Spanish Instructions NCSE 02 and the CTE.

The interstory drift sensitivity coefficient depends on the total gravity load at and above the story considered in the seismic design situation P, and the design interstory drift, evaluated as the difference of the average lateral displacements d_s at the top and bottom of the story under consideration d, which is directly proportional, and the total seismic story shear F and interstory height h, which is inversely proportional. To analyze the problem, we take a building type, as shown in Fig. 1, and study the influence of 2nd order effects using the corresponding theoretical model we develop in the body of the article.

The example in Fig. 1 is, a regular rectangular concrete building of dimension 25 × 25 m with 25 columns arranged to give rise to 5 frames in each direction with spacing of 5.50 m. For the analysis, we adopt the hypotheses that it might have 2, 4, 7, and 10 floors, the loadings considered are for medium duty use, and that the location of the building is in an average Spanish earthquake zone.

The section of the beams of the floor is 0.30 × 0.40 m (Fig. 2). The dead load is the corresponding to a 0.28 m thick slab, including flooring and ceiling. Overloading is the appropriate use of public spaces with chairs and tables. The basic seismic acceleration is 0.102 g and the coefficients of terrain features and contribution are C = 1.4 and K = 1.

With these dimensions, loads and frequently used materials (concrete HA-20 or 25 and reinforcement steels B-500) the obtained mechanical loading

Corresponding author: Martinez Valle J. M., Ph.D., assistant professor, research fields: structural dynamics, non linear finite elements for plates and shells.

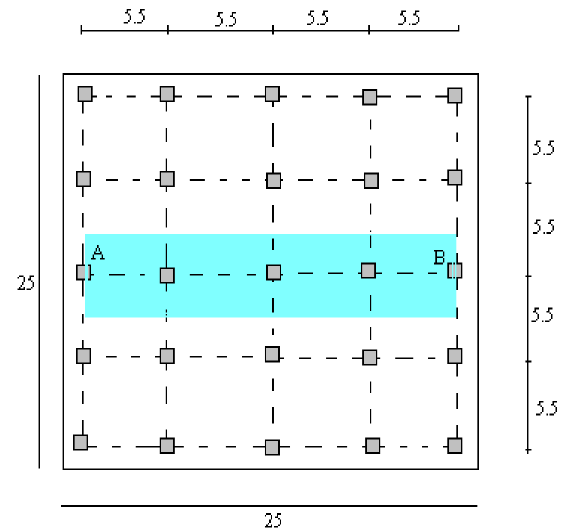

Fig. 1 Rectangular plant of building of size 25 m × 25 m with the situation of the columns.

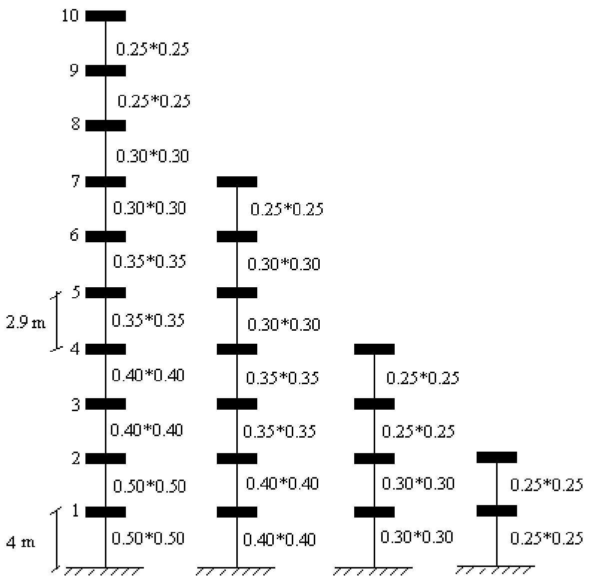

Fig. 2 Simple shear model for the frame A-B under the assumptions of 2, 4, 7, and 10 storeys.

amounts of a permanent reinforcement are quite normal (e.g., 4-6 ϕ16-20, diameter of the concrete reinforcing bar, for columns of 25 × 25 cm on two floors, or 10-12 ϕ20 for columns, 40-45 × 40-45 cm in seven storeys).

2. Geometric Stiffness Matrix of First Order of Beams of 2d Frames

In order to consider the 2nd order effects (effects), we must know the stiffness matrix, including the axial force in local coordinates of a beam belonging to a planar structure of rigid nodes, connected rigidly at both ends. A very appropriate methodology for this deduction is based on the consideration of the equilibrium of the slice in the deformed geometry. Alternatively, we can consider nonlinear expressions of deformations of a beam in bending (valid for small deformations and moderate or large displacements) [4]. In this paper, we choose the former for its better description of the problem. In the following paragraphs we expose the non-linear theory of beams including shear deformation (first order shear deformation theory) which is the basis for the description of the problem.

Let us consider a beam whose initial neutral axis and deformed neutral axis initial imperfections are represented as in Fig. 3a. As a consequence of geometric initial imperfections, the centerline of the beam is deflected by the amount. The transverse sections are rotated at an angle.

Consider a slice of this beam of length dx, unstressed axial, bending and shear, and its geometry deformed as in Fig. 3b, where the slope of the centerline is given by the shift v along the axis relative to one side, and on the other is given by

$$OO_1 = \vartheta\cdot dx + \hat{\vartheta}\cdot dx = \frac{dv}{dx}\cdot dx + \hat{\vartheta}\cdot dx = dv + \hat{\vartheta}\cdot dx \quad (1)$$

We can express the sum as a fraction loading of the displacement v and consequently, we obtain

$$OO_1 = \vartheta\cdot dx + \hat{\vartheta}\cdot dx = \frac{dv}{dx}\cdot dx + \hat{\vartheta}\cdot dx = (1+\alpha)\cdot dv \quad (2)$$

If we use the bending theory of thin beams, where $M_\varphi = E \bullet I\, v''$ and establish the equilibrium of the slice into the deformed geometry, we obtain

$$\Sigma Fy = 0 \Rightarrow P\cdot dx - Q + Q + \Delta Q = 0 \quad (3)$$

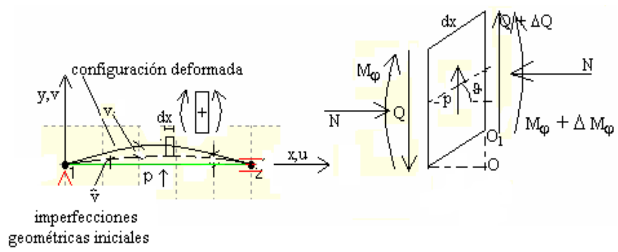

Fig. 3 Deformed configuration of a beam (left), equilibrium of a slice of the beam in its deformed configuration (right).

$$\Sigma M = 0 \Rightarrow M_\varphi + \Delta M_\varphi -$$

$$M_\phi + Q \cdot dx + N \cdot (1+\alpha) \cdot dv = 0 \tag{4}$$

The coefficient gives us an idea, as a percentage, of the level of imperfections on the de-formed configuration. Solving for Q in Eq. (2), and deriving and substituting in Eq. (1), we reach the following differential equation.

$$P(x) = E \cdot I \cdot \frac{d^4 v}{dx^4} + (1+\alpha) \cdot N \cdot \frac{d^2 v}{dx^2} \tag{5}$$

In studying free transverse oscillations P = 0, such that the equation reduces to

$$E \cdot I \cdot \frac{d^4 v}{dx^4} + (1+\alpha) \cdot N \cdot \frac{d^2 v}{dx^2} = 0 \tag{6}$$

This is usually written as $\omega^2 = (1+\alpha)$N/(E·I), as $\frac{d^4 v}{dx^4} + \omega^2 \cdot \frac{d^2 v}{dx^2}$, whose general solution is given by the following expression:

$$v = -\frac{A}{\omega^2}\cos(\omega x) - \frac{B}{\omega^2}\operatorname{sen}(\omega x) + Cx + D \tag{7}$$

Integration constants A, B, C, and D are provided by our boundary conditions:

$$v_{x=0} = v_1,\ v_{x=L} = v_2,\ v'_{x=0} = \vartheta_1\ v'_{x=L} = \vartheta_2 \tag{8}$$

where v_i and ϑ_i , are respectively the deflection and rotation of the end i. If we particularize Eq. (7) on the values x = 0 and x = L, we obtain:

$$\begin{aligned} v_1 &= -\frac{A}{\omega^2} + D \\ v_2 &= -\frac{A}{\omega^2}\cos(\omega L) - \frac{B}{\omega^2}\operatorname{sen}(\omega L) + C\,L + D \\ \vartheta_1 &= -\frac{B}{\omega} + C \\ \vartheta_2 &= \frac{A}{\omega}\operatorname{sen}(\omega L) - \frac{B}{\omega}\cos(\omega L) + C \end{aligned} \tag{9}$$

which we can write in matrix form, as follows:

$$\begin{pmatrix} v_1 \\ \vartheta_1 \\ v_2 \\ \vartheta_2 \end{pmatrix} = \begin{pmatrix} -\frac{1}{\omega^2} & 0 & 0 & 1 \\ 0 & -\frac{1}{\omega} & 1 & 0 \\ -\frac{\cos(\omega L)}{\omega^2} & -\frac{\operatorname{sen}(\omega L)}{\omega^2} & L & 1 \\ \frac{\operatorname{sen}(\omega L)}{\omega} & -\frac{\cos(\omega L)}{\omega} & 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} A \\ B \\ C \\ D \end{pmatrix} = \Im \cdot \begin{pmatrix} A \\ B \\ C \\ D \end{pmatrix} \tag{10}$$

If we suppose that the inverse matrix is partitioned in the form

$$\Im^{-1} = \begin{pmatrix} \Im^{-1}{}_{11} & \Im^{-1}{}_{12} \\ \Im^{-1}{}_{21} & \Im^{-1}{}_{22} \end{pmatrix} \tag{11}$$

then, if we incorporate the horizontal displacements u_1 and u_2 from the beam ends, and we use the usual notation in the theory of structures in which displacements and rotations of the ends of the beam are expressed by

$$\begin{pmatrix} d^{e1x} \\ d^{e1y} \\ \vartheta^{e1} \\ d^{e2x} \\ d^{e2y} \\ \vartheta^{e2} \end{pmatrix} = \begin{pmatrix} u_1 \\ v_1 \\ \vartheta_1 \\ u_2 \\ v_2 \\ \vartheta_2 \end{pmatrix} \tag{12}$$

the equation of the deformed beam is:

$$v = -\frac{A}{\omega^2}\cos(\omega x) - \frac{B}{\omega^2}\operatorname{sen}(\omega \cdot x) + C \cdot x + D =$$

$$\begin{pmatrix} \frac{-\cos(\omega \cdot x)}{\omega^2} & \frac{-\operatorname{sen}(\omega \cdot x)}{\omega^2} & x & 1 \end{pmatrix} \cdot \begin{pmatrix} A \\ B \\ C \\ D \end{pmatrix} =$$

$$\begin{pmatrix} \frac{-\cos(\omega \cdot x)}{\omega^2} & \frac{-\operatorname{sen}(\omega \cdot x)}{\omega^2} & x & 1 \end{pmatrix} \begin{pmatrix} \Omega & \Im^{-1}{}_{11} & \Omega & \Im^{-1}{}_{12} \\ \Omega & \Im^{-1}{}_{21} & \Omega & \Im^{-1}{}_{22} \end{pmatrix} \begin{pmatrix} d^{e1x} \\ d^{e1y} \\ \vartheta^{e1} \\ d^{e2x} \\ d^{e2y} \\ \vartheta^{e2} \end{pmatrix} \tag{13}$$

According to Eq. (10), the matrix $\Im^{-1}$, can be written as:

$$\Im^{-1} = \varsigma$$

$$\left\langle \begin{matrix} -\omega^2 \cdot (1-\cos(\omega \cdot L)) & \omega \cdot (-\sin(\omega \cdot L) + \omega \cdot L \cdot \cos(\omega \cdot L)) \\ \omega^2 \cdot \sin(\omega \cdot L) & \omega \cdot (-1 + \cos(\omega \cdot L) + \omega \cdot L \cdot \sin(\omega \cdot L)) \\ \omega \cdot \sin(\omega \cdot L) & 1-\cos(\omega \cdot L) \\ 1-\cos(\omega \cdot L) - \omega \cdot L \cdot \sin(\omega \cdot L) & -\sin(\omega \cdot L) + \omega \cdot L \cdot \cos(\omega \cdot L) \end{matrix} \right. \tag{14}$$

$$\left. \begin{matrix} \omega^2 \cdot (1-\cos(\omega \cdot L) & \omega \cdot (\sin(\omega \cdot L) - \omega \cdot L) \\ -\omega^3 \cdot \sin(\omega \cdot L) & \omega^2 \cdot (1-\cos(\omega \cdot L) \\ -\omega \cdot \sin(\omega \cdot L) & (1-\cos(\omega \cdot L) \\ 1-\cos(\omega \cdot L) & \sin(\omega \cdot L) - \omega \cdot L \end{matrix} \right\rangle$$

where ς is $\varsigma = \frac{1}{2 - 2 \cdot \cos(\omega L) - \omega L \cdot \operatorname{sen}(\omega L)}$

If the loads at the ends of the beam along the x-axis are designated by p^x , the matrix k_{viga} can be deduced from the bending moment expressions (M_φ), shear force (Q) and axial force (N) by

$$p^{e1x} = -p^{e2x} = \frac{E\cdot A}{L}?\, d^{e1x} - d^{e2x})$$

$$(Q)_{x=0} = -\, p^{e1y} = [-N\cdot\frac{dv}{dx} - E\cdot I\cdot\frac{d^3v}{dx^3}]_{x=0} =$$

$$-N\cdot\vartheta_1 - E\cdot I\cdot(\frac{d^3v}{dx^3})_{x=0} = E\cdot I\cdot[-\omega^2\cdot\vartheta_1 - (\frac{d^3v}{dx^3})_{x=0}] \quad (15)$$

$$(\,M_\varphi\,)_{x=0} = -M_1 = EI(\,v''\,)_{x=0}$$

$$(Q)_{x=L} = p^{e2y} = [-N\cdot\frac{dv}{dx} - E\cdot I\cdot\frac{d^3v}{dx^3}]_{x=L} =$$

$$-N\cdot\vartheta_2 - E\cdot I\cdot(\frac{d^3v}{dx^3})_{x=L} = E\cdot I\cdot[-\omega^2\cdot\vartheta_2 - (\frac{d^3v}{dx^3})_{x=L}]$$

$$(\,M_\varphi\,)_{x=0} = M_2 = E\cdot I(\,v''\,)_{x=L}$$

Substituting in the above equations the derivatives of the deflection v, calculated from Eq. (13), and particularizing at x equals zero or L, as appropriate, we obtain:

$$\begin{pmatrix} p^{e1x} \\ p^{e1y} \\ m^{e1} \\ p^{e2x} \\ p^{e2y} \\ m^{e2} \end{pmatrix} = \left\{ E\cdot I\cdot \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \omega & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \omega\cdot sen(\omega\cdot L) & -\omega\cdot cos(\omega\cdot L) & 0 & 0 \\ cos(\omega\cdot L) & sen(\omega\cdot L) & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} \Omega & \Im^{-1}{}_{11} & \Omega & \Im^{-1}{}_{12} \\ \Omega & \Im^{-1}{}_{21} & \Omega & \Im^{-1}{}_{22} \end{pmatrix} + \right.$$

$$\left. \begin{pmatrix} \frac{E\cdot A}{L} & 0 & 0 & -\frac{E\cdot A}{L} & 0 & 0 \\ 0 & 0 & \omega^2\cdot E\cdot I & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{E\cdot A}{L} & 0 & 0 & \frac{E\cdot A}{L} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\omega^2\cdot E\cdot I \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \right\} \cdot \begin{pmatrix} d^{e1x} \\ d^{e1y} \\ \vartheta^{e1} \\ d^{e2x} \\ d^{e2y} \\ \vartheta^{e2} \end{pmatrix} \quad (16)$$

Replacing the four submatrices $\Im_{ij}^{-1}$ of order 2 × 2, in the above Eq. (13) and operating, we obtain the equation of beam loads movements, $p_{viga} = \bar{k}_{viga} d_{viga}$ in local coordinates:

$$\begin{pmatrix} p^{e1x} \\ p^{e1y} \\ m^{e1} \\ p^{e2x} \\ p^{e2y} \\ m^{e2} \end{pmatrix} = \bar{k}_{viga} \cdot \begin{pmatrix} d^{e1x} \\ d^{e1y} \\ \vartheta^{e1} \\ d^{e2x} \\ d^{e2y} \\ \vartheta^{e2} \end{pmatrix} \quad (17)$$

with $\bar{k}_{viga}$ equal to

$$\bar{k}_V = \begin{pmatrix} \frac{E\cdot A}{L} & 0 & 0 & -\frac{E\cdot A}{L} & 0 & 0 \\ 0 & \frac{\omega^3\, sen(\omega\cdot L)}{f(\omega)} & \frac{\omega^2\cdot(1-cos(\omega\cdot L))}{f(\omega)} & 0 & -\frac{\omega^3\, sen(\omega\cdot L)}{f(\omega)} & \frac{\omega^2\cdot(1-cos(\omega\cdot L))}{f(\omega)} \\ 0 & \frac{\omega^2\cdot(1-cos(\omega\cdot L))}{f(\omega)} & \frac{\omega\cdot(sen(\omega\cdot L)-\omega\cdot L\cdot cos(\omega\cdot L))}{f(\omega)} & 0 & -\frac{\omega^2\cdot(1-cos(\omega\cdot L))}{f(\omega)} & \frac{\omega\cdot(\omega\cdot L - sen(\omega\cdot L))}{f(\omega)} \\ -\frac{E\cdot A}{L} & 0 & 0 & \frac{E\cdot A}{L} & 0 & 0 \\ 0 & -\frac{\omega^3\, sen(\omega\cdot L)}{f(\omega)} & -\frac{\omega^2\cdot(1-cos(\omega\cdot L))}{f(\omega)} & 0 & \frac{\omega^3\, sen(\omega\cdot L)}{f(\omega)} & -\frac{\omega^2\cdot(1-cos(\omega\cdot L))}{f(\omega)} \\ 0 & \frac{\omega^2\cdot(1-cos(\omega\cdot L))}{f(\omega)} & \frac{\omega\cdot(\omega\cdot L - sen(\omega\cdot L))}{f(\omega)} & 0 & -\frac{\omega^2\cdot(1-cos(\omega\cdot L))}{f(\omega)} & \frac{\omega\cdot(sen(\omega\cdot L)-\omega\cdot L\cdot cos(\omega\cdot L))}{f(\omega)} \end{pmatrix} \quad (18)$$

or in a more compact form

$$p_{viga} = \bar{k}_{viga} \cdot d_{viga} \quad (19)$$

Where $f(\omega) = \frac{2 - 2\cos(\omega L) - \omega L\cdot sen(\omega L)}{E\cdot I}$ and $\bar{k}_{viga}$ is the beam stiffness matrix in local coordinates, including the influence of the axial force N. Let us note that we have identified, for the differentiation of linear stiffness matrix k_{viga}, by the stroke located on k, and which depends, Via ω, on the value of the axial force on the beams (N).

Apparently, the expression of the stiffness matrix of the beam expressed in Eq. (18) $\bar{k}_{viga}$, is quite different from the known k_{viga}, without the inclusion of the axial force. However, we can find a similar expression when we consider that

$$tg\frac{\omega\cdot L}{2} = \frac{1-cos(\omega\cdot L)}{sin(\omega\cdot L)} \quad (20)$$

and if we call to

$$\Psi_1 = \frac{s\cdot(1+c)}{6} - \frac{N\cdot L^2}{12\cdot E\cdot I};\ \Psi_2 = \frac{s\cdot(1+c)}{6};\ \Psi_3 = \frac{s}{4};\ \Psi_4 = \frac{s\cdot c}{2} \qquad (21)$$

where c and s are adimensional functions, called *stability functions*

$$c = \frac{\omega\cdot L - \sin(\omega\cdot L)}{\sin(\omega\cdot L) - \omega\cdot L\cdot\cos(\omega\cdot L)};$$

$$s = \frac{[1-\omega L\cdot \cot g(\omega L)]\cdot \omega L/2}{tg\frac{\omega L}{2} - \frac{\omega L}{2}} \qquad (22)$$

which were originally derived by Lundquist and Kroll [5], and later developed by Merchant [6] under different methodology and in another context, Fig. 4. We prefer the new original approach presented here [7], because we want to show the identity of procedures and goals achieved over time, even though at times the results appear to be different and have been obtained by using very different methodologies.

Operating conveniently we found that the matrix Eq. (16), for a beam rigidly connected at both ends and belonging to a 2D frame, is written as

$$\begin{aligned} p^{e1} &= \bar{k}^{11}\cdot d^{e1} + \bar{k}^{12}\cdot d^{e} \\ p^{e2} &= \bar{k}^{21}\cdot d^{e1} + \bar{k}^{22}\cdot d^{e2} \end{aligned} \qquad (23)$$

or in matrix form

$$\begin{pmatrix} p^{e1} \\ p^{e2} \end{pmatrix} = \begin{pmatrix} \bar{k}^{11} & \bar{k}^{12} \\ \bar{k}^{21} & \bar{k}^{22} \end{pmatrix} \cdot \begin{pmatrix} d^{e1} \\ d^{e2} \end{pmatrix} \qquad (24)$$

being

$$\bar{k}^{11} = \begin{pmatrix} \frac{E\cdot A}{L} & 0 & 0 \\ 0 & \frac{12\cdot E\cdot I\cdot \Psi_1}{L^3} & \frac{6\cdot E\cdot I\cdot \Psi_2}{L^2} \\ 0 & \frac{6\cdot E\cdot I\cdot \Psi_2}{L^2} & \frac{4\cdot E\cdot I\cdot \Psi_3}{L} \end{pmatrix};$$

$$\bar{k}^{12} = (\bar{k}^{21})^T = \begin{pmatrix} \frac{-E\cdot A}{L} & 0 & 0 \\ 0 & \frac{-12\cdot E\cdot I\cdot \Psi_1}{L^3} & \frac{6\cdot E\cdot I\cdot \Psi_2}{L^2} \\ 0 & \frac{-6\cdot E\cdot I\cdot \Psi_2}{L^2} & \frac{2\cdot E\cdot I\cdot \Psi_4}{L} \end{pmatrix}$$

$$\bar{k}^{22} = \begin{pmatrix} \frac{E\cdot A}{L} & 0 & 0 \\ 0 & \frac{12\cdot E\cdot I\cdot \Psi_1}{L^3} & \frac{-6\cdot E\cdot I\cdot \Psi_2}{L^2} \\ 0 & \frac{-6\cdot E\cdot I\cdot \Psi_2}{L^2} & \frac{4\cdot E\cdot I\cdot \Psi_3}{L} \end{pmatrix} \qquad (25)$$

Comparing these expressions with the known k_{ij}, it is clear that the functions are simply multiplier factors of the coefficients of the stiffness matrix of a beam without axial forces and can conveniently be expressed as functions of the relationship between axial force N and the Euler critical load $Ncr = \frac{\pi^2\cdot E\cdot I}{L^2}$, as shown in Fig. 5. Note that all are 1 for N = 0, so that $\bar{k}_{viga} = k_{viga}$ for N = 0.

3. Geometric Stiffness of a Column Belonging to Simple Shear Model

If the beam is one of the groups of carriers of a frame modeled as a simple shear model, Fig. 6, and we take the global axes of the figure.

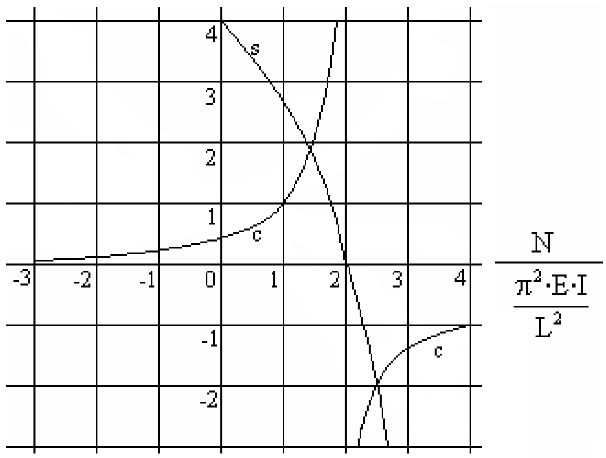

Fig. 4 Stability Functions.

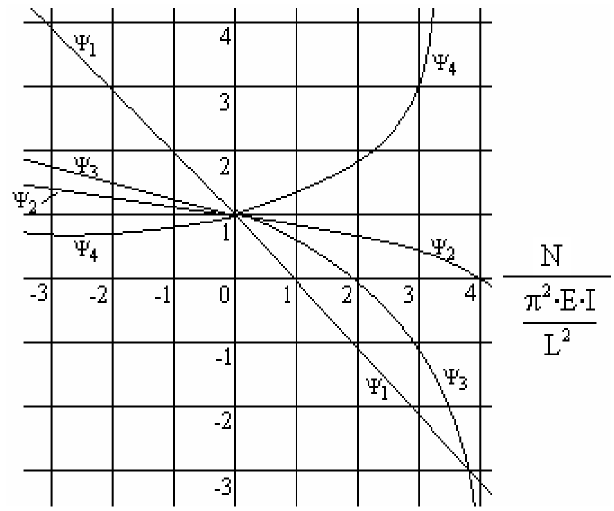

Fig. 5 Graphical representation of the functions Ψ.

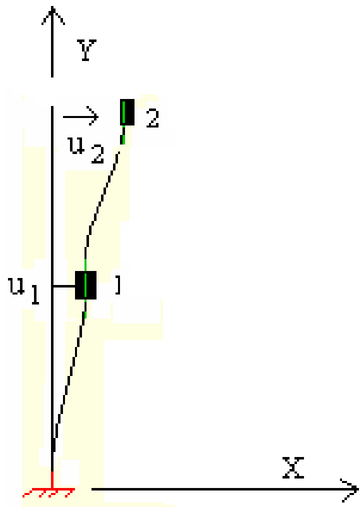

Fig. 6 Simple shear model for a frame of 2 storeys.

then Eq. (18) can be written as

$$\begin{pmatrix} p^{e1x} \\ p^{e1y} \\ m^{e1} \\ p^{e2x} \\ p^{e2y} \\ m^{e2} \end{pmatrix} = \begin{pmatrix} \frac{E\cdot A}{L} & 0 & 0 & -\frac{E\cdot A}{L} & 0 & 0 \\ 0 & \frac{\omega^3\,\mathrm{sen}(\omega\cdot L)}{f(\omega)} & \frac{\omega^2\cdot(1-\cos(\omega\cdot L))}{f(\omega)} & 0 & -\frac{\omega^3\,\mathrm{sen}(\omega\cdot L)}{f(\omega)} & \frac{\omega^2\cdot(1-\cos(\omega\cdot L))}{f(\omega)} \\ 0 & \frac{\omega^2\cdot(1-\cos(\omega\cdot L))}{f(\omega)} & \frac{\omega\cdot(\mathrm{sen}(\omega\cdot L)-\omega\cdot L\cdot\cos(\omega\cdot L))}{f(\omega)} & 0 & -\frac{\omega^2\cdot(1-\cos(\omega\cdot L))}{f(\omega)} & \frac{\omega\cdot(\omega\cdot L-\mathrm{sen}(\omega\cdot L))}{f(\omega)} \\ -\frac{E\cdot A}{L} & 0 & 0 & \frac{E\cdot A}{L} & 0 & 0 \\ 0 & -\frac{\omega^3\,\mathrm{sen}(\omega\cdot L)}{f(\omega)} & -\frac{\omega^2\cdot(1-\cos(\omega\cdot L))}{f(\omega)} & 0 & \frac{\omega^3\,\mathrm{sen}(\omega\cdot L)}{f(\omega)} & -\frac{\omega^2\cdot(1-\cos(\omega\cdot L))}{f(\omega)} \\ 0 & \frac{\omega^2\cdot(1-\cos(\omega\cdot L))}{f(\omega)} & \frac{\omega\cdot(\omega\cdot L-\mathrm{sen}(\omega\cdot L))}{f(\omega)} & 0 & -\frac{\omega^2\cdot(1-\cos(\omega\cdot L))}{f(\omega)} & \frac{\omega\cdot(\mathrm{sen}(\omega\cdot L)-\omega\cdot L\cdot\cos(\omega\cdot L))}{f(\omega)} \end{pmatrix} \cdot \begin{Bmatrix} d^{e1x}=0 \\ d^{e1y}=U^1 \\ \vartheta^1=0 \\ d^{e2x}=0 \\ d^{e2y}=U^2 \\ \vartheta^2=0 \end{Bmatrix} \quad (26)$$

Therefore, we obtain

$$p^{e1x}=p^{e2x}=0\,;\ p^{e1y}=-p^{e2y}=-\frac{\omega^3\cdot\mathrm{sen}(\omega L)}{f(\omega)}\left(U^2-U^1\right)\,;\ m^{e1y}=m^{e2y}=-\frac{\omega^2\cdot(1-\cos(\omega L))}{f(\omega)}\left(U^2-U^1\right) \quad (27)$$

As for N = 0, the functions $\frac{\omega^3\cdot\mathrm{sen}(\omega L)}{f(\omega)}$ y $\frac{\omega^2\cdot(1-\cos(\omega L))}{f(\omega)}$ are preferred indeterminate, and we prefer to develop them using Taylor series and limiting the developments to the first two terms, we obtain

$$\frac{1}{f(\omega)}=\frac{E\cdot I}{2-2\cdot\cos(\omega L)-\omega L\cdot\mathrm{sen}(\omega L)}=\frac{12\cdot E\cdot I}{\omega^4\cdot L^4}\cdot\left(1+\frac{\omega^2\cdot L^2}{15}+\frac{67\cdot\omega^4\cdot L^4}{25200}+\ldots\right)$$

$$\frac{\omega^3\cdot\mathrm{sen}(\omega\cdot L)}{f(\omega)}=\omega^3\cdot\left(\omega\cdot L-\frac{\omega^3\cdot L^3}{3!}+\frac{\omega^5\cdot L^5}{5!}+\ldots\right)\cdot\frac{12\cdot E\cdot I}{\omega^4\cdot L^4}\cdot\left(1+\frac{\omega^2\cdot L^2}{15}+\ldots\right)=\frac{12\cdot E\cdot I}{L^3}-\frac{36\cdot(1+\alpha)\cdot N}{30\cdot L}$$

$$\frac{\omega^2\cdot(1-\cos(\omega L))}{f(\omega)}=\omega^2\cdot\left(1-1+\frac{\omega^2\cdot L^2}{2!}-\frac{\omega^4\cdot L^4}{4!}+\ldots\right)\cdot\frac{12\cdot E\cdot I}{\omega^4\cdot L^4}\cdot\left(1+\frac{\omega^2\cdot L^2}{15}+\ldots\right)=\frac{6\cdot E\cdot I}{L^2}-\frac{(1+\alpha)\cdot N}{10} \quad (28)$$

If we proceed similarly for all matrix elements, the second addends constitute the geometric stiffness matrix of the first order. If we increase the terms of the development, the third summands constitute the geometric stiffness matrix of second order and so on.

For the shear efforts at beam ends we have

$$p^{e1y} = -p^{e2y} = -\left(\frac{12\cdot E\cdot I}{L^3} - \frac{36\cdot(1+\alpha)\cdot N}{30\cdot L}\right)\cdot\left(U^2 - U^1\right) \quad (29)$$

Thus, the stiffness of the column of the media set of a frame modeled as a simple shear model is

$$\bar{k} = \left(\sum \frac{12\cdot E\cdot I}{L^3}\right) - \frac{36\cdot(1+\alpha)\cdot N}{30\cdot L} \quad (30)$$

where is the stiffness of each of the columns forming the group and N is the area weight condition of all floors above the considered column. As already discussed, α marks the level of initial imperfections, in percentage terms, relative to the deformed column.

To circumvent the approach that involves limiting the Taylor series expansion of ψ_1 to the first two terms, and also based on the representation of Fig. 5, we can choose to use a very approximate analytical expression for ψ_1 . If we call $\beta = \frac{N}{\pi^2 \cdot E \cdot I / L^2}$ the function ψ_1 is given by $\psi_1 = -\beta + 1$, in which case the stiffness of the media set of columns of a frame modeled as a simple shear model is

$$\bar{k} = \sum_{i=1a\ n^o pilares} \frac{12\cdot E\cdot I_i}{L_i^3}\cdot\left(1 - \beta_i\right) \quad (31)$$

Although the differences obtained by taking Eq. (30) or (31) are not significant, we have opted for the latter as being more accurate, even slightly, and is implemented in the program for use with equal ease.

4. Results of Calculating the Building Taken as an Example

The following cases have been solved with the help of a program written in Matlab (following the usual methodology for dynamic calculation of structures) whose key steps are:

- Calculation of the stiffnesses of the columns of the media set of a frame by Eq. (30).
- Calculation of the interstorey drift sensitivity coefficients for each floor according to the equation:

$$\xi = \frac{P\cdot d}{F\cdot h} = \frac{P\cdot d}{\bar{k}\cdot d\cdot h} = \frac{P}{\bar{k}_p\cdot L_p} \quad (32)$$

where all the variables have been defined previously; the subscript "p" refers to the stiffness and length of the column or group of columns respectively.

- Calculation of the natural frequencies of the structural model.
- Calculation of the maximum values of the variables of the equations resulting from de-coupling the initial system of differential equations, using the response spectrum proposed by the Spanish instruction NCSE 02.
- Calculation of the maximum values of the displacements by the method of least squares. These maximum values of the displacements will allow us to calculate the shear stresses and the resulting bending moments.

Case A Two Storey Type Building

For the ratio of stiffness supposed in paragraph 1, the bending moments at the top of the column in the ground floor and at the end of the lintel for static loads are

$$M_{\varphi}^{pilar} \simeq \frac{q\cdot L^2}{40} = \frac{41\cdot 5\cdot 5^2}{40} = 31.2 \text{ kN}\cdot\text{m};$$

$$M_{\varphi}^{dintel} \simeq \frac{q\cdot L^2}{25} = \frac{41\cdot 5\cdot 5^2}{25} = 50 \text{ kN}\cdot\text{m} \quad (33)$$

The Interstorey Drift Sensitivity Coefficients, ξ , and maximum deflections are in Table 1.

The stiffness k_p of the ground floor columns group is k_p = 7.0182 KN/m and the increases of shear force and bending moment, as a result of considering the coupling of axial bending, are

$$\Delta Q = \frac{12\cdot E\cdot I}{L^3}\cdot\left(U_1 - \bar{U}_1\right) =$$

$$\frac{7018.2\cdot 10^3}{5}\cdot(0.0353 - 0.0335) = 2.53 \text{ kN} \quad (34)$$

$$\Delta M_{\varphi} = \frac{\Delta Q\cdot L}{2} = 5.05 \text{ kN}\cdot\text{m} \quad (35)$$

The latter increase in the bending moment on the column represents an increase of 16.2% from the value of the moment due to gravity loads, and an increase of

Table 1 Interstory drift sensitivity coefficient for 2 story frame.

Interstory Drift Sensitivity Coefficient	
$\xi 1 = 0.0735$ Without consideration to axial force	$\xi 2 = 0.0193$ Coupling of bending moment and axial forces
U_1	$\overline{U_1}$
0.0335	0.0353
0.0405	0.0421

5% compared with the sum of the value of the moments due to static loads more dynamic loading. However, increases incurred as a result of considering a typical level of initial imperfections are not significant.

Case B Four Storey Type Building

For the ratio of stiffness supposed in paragraph 2, the bending moments at the top of the column in the ground floor and at the end of the lintel for gravity loads are

$$M_{\varphi}^{pilar} \simeq \frac{q \cdot L^2}{40} = \frac{41 \cdot 5 \cdot 5^2}{40} = 31.2 \text{ kN·m};$$

$$M_{\varphi}^{dintel} \simeq \frac{q \cdot L^2}{20} = \frac{41 \cdot 5 \cdot 5^2}{20} = 62 \text{ kN·m} \quad (36)$$

The Interstory Drift Sensitivity Coefficients, ξ, and maximum deflections are in Table 2.

The stiffness k_p of the ground floor columns group is k_p = 14,555 kN/m and the increases of shear force and bending moment, as a result of considering the coupling of axial bending, are

$$\Delta Q = \frac{12 \cdot E \cdot I}{L^3} \cdot \left(U_1 - \bar{U}_1\right) =$$

$$\frac{14555 \cdot 1}{5} \cdot (0.0282 - 0.0264) = 5.24 \text{ kN} \quad (37)$$

$$\Delta M_{\varphi} = \frac{\Delta Q \cdot L}{2} = 10.48 \text{ kN·m} \quad (38)$$

This latest increase of the bending moment on the column represents an increase of 33.59% from the value of the moment due to gravity loads, and an increase of 5.7% from the sum of the value of the moments due to gravity loads more dynamic loading. However, increases incurred as a result of considering a typical level of initial imperfections are not significant.

Table 2 Interstory drift sensitivity coefficient for 4 story frame.

Interstory Drift Sensitivity Coefficient	
$\xi 1 = 0.0709$ $\xi 2 = 0.0279$ Without consideration to axial force	$\xi 3 = 0.0386$ $\xi 4 = 0.0193$ Coupling of bending moment and axial forces
U_1	$\overline{U_1}$
0.0264	0.0282
0.0346	0.0364
0.0474	0.0493
0.0544	0.0562

Case C Seven Story Type Building

For the ratio of stiffness supposed in paragraph 2, the bending moments at the top of the column in the ground floor and at the end of the lintel for gravity loads are

$$M_{\varphi}^{pilar} \simeq 1.15 \cdot \frac{q \cdot L^2}{35} = 1.15 \cdot \frac{41 \cdot 5 \cdot 5^2}{35} = 40.7 \text{ kN·m}$$

$$M_{\varphi}^{dintel} \simeq \frac{q \cdot L^2}{17.5} = \frac{41 \cdot 5 \cdot 5^2}{17.5} = 70.8 \text{ kN·m} \quad (39)$$

The Interstory Drift Sensitivity Coefficients, ξ, and maximum deflections are in Table 3.

The stiffness k_p of the ground floor columns group is k_p = 45,999 kN/m and the increases in bending moment and shear forces, as a result of considering the coupling of axial deflection, are

$$\Delta Q = \frac{12 \cdot E \cdot I}{L^3} \cdot \left(U_1 - \bar{U}_1\right) =$$

$$\frac{45999}{5} \cdot (0.0130 - 0.0126) = 3.68 \text{ kN} \quad (40)$$

$$\Delta M_{\varphi} = \frac{\Delta Q \cdot L}{2} = 7.36 \text{ kN·m} \quad (41)$$

This latest increase of the bending moment on the column represents an increase of 18.08% over the value of the moment due to gravity loads, and an increase of 2.7% from the sum of the value of the moments due to static loads more dynamic loading. However, increases incurred as a result of considering a typical level of initial imperfections are not significant.

Case D Ten Story Type building.

For the ratio of stiffness supposed in paragraph 2, the

Table 3 Interstory drift sensitivity coefficient for 7 story frame.

Interstory Drift Sensitivity Coefficient	
$\xi 1 = 0.0392$ $\xi 2 = 0.0302$ $\xi 5 = 0.0279$	$\xi 3 = 0.0251$ $\xi 4 = 0.0193$ $\xi 6 = 0.0386$ $\xi 7 = 0.0193$
Without consideration to axial force	Coupling of bending moment and axial forces
U_1	$\overline{U}_1$
0.0126	0.0130
0.0201	0.0206
0.0266	0.0272
0.0370	0.0378
0.0454	0.0463
0.0588	0.0601
0.0668	0.0681

bending moments at the top of the column in ground floor and at the end of the lintel for gravity loads are

$$M_{\varphi}^{pilar} \simeq \frac{q \cdot L^2}{30} = \frac{41 \cdot 5 \cdot 5^2}{30} = 41.3 \text{ kN}\cdot\text{m};$$

$$M_{\varphi}^{dintel} \simeq \frac{q \cdot L^2}{15} = \frac{41 \cdot 5 \cdot 5^2}{15} = 82.6 \text{ kN}\cdot\text{m} \quad (42)$$

The Interstory Drift Sensitivity Coefficients, ξ, and maximum deflections are in Table 4.

The stiffness k_p of the ground floor columns group is k_p =112,150 KN/m and the increases in bending moment and shear forces, as a result of considering the coupling of axial deflection, are

$$\Delta Q = \frac{12 \cdot E \cdot I}{L^3} \cdot \left(U_1 - \overline{U}_1\right) = \frac{112150}{5} \cdot (0.0067 - 0.0066) = 2.24 \text{ kN} \quad (43)$$

$$\Delta M_{\varphi} = \frac{\Delta Q \cdot L}{2} = 4.49 \text{ kN}\cdot\text{m} \quad (44)$$

This latest increase of the bending moment on the column represents an increase of 10.87% from the value of the time due to gravity loads, and an increase in 1.35% from the sum of the value of the moments due to gravity loads more dynamic loading. However, increases incurred as a result of considering a typical level of initial imperfections are not significant.

5. Dimensioning Interstory Drift Sensitivity Coefficient

In the seismic calculation of a building, Spanish Instructions NCSE-02 and therefore, the CTE,

Table 4 Interstory Drift Sensitivity Coefficient for 10 Story Frame.

Interstory Drift Sensitivity Coefficient	
$\xi 1 = 0.0230$ $\xi 2 = 0.0109$ $\xi 3 = 0.0236$ $\xi 4 = 0.0206$ $\xi 5 = 0.0302$	$\xi 6 = 0.0251$ $\xi 7 = 0.0372$ $\xi 8 = 0.0279$ $\xi 9 = 0.0386$ $\xi 10 = 0.0193$
Without consideration to axial force	Coupling of bending moment and axial forces
U_1	$\overline{U}_1$
0.0066	0.0067
0.009	0.0091
0.0145	0.0146
0.0195	0.0196
0.027	0.0273
0.0335	0.0339
0.0438	0.0444
0.0522	0.053
0.0661	0.0673
0.0746	0.0759

establish in paragraph 3.7.7 that: "As long as the collapse of the head of the building does not exceed two per thousand of the height, it is not necessary to consider the 2nd order effects".

On the other hand, it will not be necessary to consider the 2nd order effects, in line with that set by Eurocode Nº 8 (Paragraph1-2 Part 1-1), if for all storeys the Interstory Drift Sensitivity Coefficient is less than 0.1.

As we can see, what is actually limiting the destabilizing moment is that the column is less than 10% of the stabilizing moment. Because the criterion of the 10% limit seems to be arbitrary, we propose to replace it with another limit that is more in line with the structural reality.

If we assume a square section column of side "b", for frequently used materials and for the dimensions set out in paragraph 1, the relationship between the critical load and the axial calculation is $\frac{N_{crit}}{N_{cálc}} \simeq 150 \cdot b^2$, so that we obtain $N_{calc} = 0.075\ N_{crit}$ for b = 0.30 m, and for b = 0.35 m (corresponding to buildings of lesser heights), we have $N_{calc} = 0.055\ N_{crit}$. Accordingly, we adopt

$$N_{calc} < 0.075\, N_{crit} \tag{45}$$

Under this assumption, coefficient, defined in section 3, $\beta = \dfrac{N}{\pi^2 \cdot E \cdot I / L^2} = \dfrac{N_{calc}}{N_{crit}}$, gives

$$\beta \cdot N_{crit} = N_{calc} < 0.075 \cdot N_{crit} \tag{46}$$

Keeping the definition for the interstory drift sensitivity coefficient and by taking into account Eq. (27), we can write

$$\xi = \frac{N \cdot d}{F \cdot h} = \frac{N \cdot d}{\frac{12 \cdot E \cdot I}{L^3} \cdot \psi_1 \cdot d \cdot L} = \frac{\pi^2 \cdot \beta}{12 \cdot \psi_1} \tag{47}$$

From which we deduce $\beta = \dfrac{12 \cdot \psi_1 \cdot \xi}{\pi^2}$ and as $\psi_1 = -\beta + 1$, we obtain $1 - \psi_1 = \dfrac{12 \cdot \psi_1 \cdot \xi}{\pi^2}$. So,

$$\psi_1 = \frac{1}{1.216 \cdot \xi + 1} \tag{48}$$

By virtue of Eq. (46), it must be satisfied that $\beta < 0.075$, i.e., $1 - \dfrac{1}{1.216\xi +} < 0.075$. From which we deduce

$$\xi < 0.0667 \tag{49}$$

which is consistent with the results obtained in the practical **cases A** and **B** of section 4.

6. Conclusions

In the seismic calculation of a building, Spanish Instructions NCSE-02 and thus, the CTE, establish in paragraph 3.7.7 that: "As long as the collapse of the head of the building does not exceed two per thousand of the height, it is not necessary to consider the effects of 2nd order".

On the other hand, it will not be necessary to consider the 2nd order effects (s effects), in line with that set by Eurocode 8 (Paragraph1-2 Part 1-1), if for all storeys the interstory drift sensitivity coefficient fulfills:

$$\xi = \frac{P \cdot d}{F \cdot h} \leq 0.1 \tag{50}$$

We show in this discussion, based on the current state of knowledge, it is not justified to ignore the 2nd order effects ($P - \Delta$ effects):

(1) Because it is not justified that the spectral modal analysis be simplified, because a high-level program such as MATLAB can very easily address a multimodal spectral analysis study for frames, the simple model of shear or associating the nodes of the structure of the inertial properties of the frame beams and performing a calculation of stiffness.

(2) Because the inclusion of 2nd order effects (effects), does not add any conceptual effort nor complicate the calculation. Additionally, if we follow the line of thought of Bazant [8] and truly think that the equilibrium, static or dynamic loading, occurs in the deformed geometry, it is logical that the calculation includes this.

(3) Because we show that although the Interstory Drift Sensitivity Coefficient does not reach the value of 0.1, increases of the bending moment at the ends of the columns resulting from the refinement of the calculation including the second order effects, can account for between 15% and 34% of its value for static service loads.

(4) Because most adverse effects are shown in lower height buildings (up to 5 floors), which is precisely the range in which most of the housing stock of Spain is located, to be in the highest limit allowed by the various planning regulations (PGOU).

(5) As in section 5, we limited the Interstory Drift Sensitivity Coefficient to 0.0667 for buildings of lower height (up to 5 floors), so we propose that it is generally lowered to $\xi \leq 0.06$ instead of $\xi \leq 0.1$ as proposed in Eurocode 8 and the Spanish Instruction NCSE-02.

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