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Relations between J-, M- and N-integrals and Those with Interactions Energy in a Brittle Material

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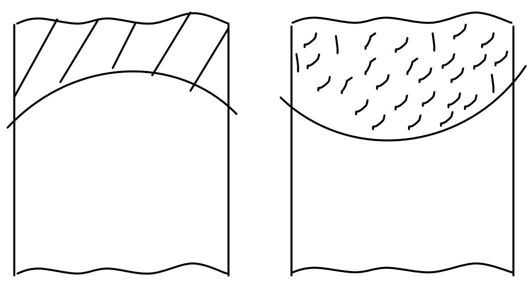
Abstract: In this study, kinematics of the Damage Zone (DZ) or the so-called Fracture Process Zone (FPZ) which often precedes the crack during its propagation and characterized by few degrees of freedom (elementary movements) such as translation, rotation, isotropic expansion and distortion are considered. On the basis of a stress field distribution obtained by the use of a Semi-Empirical Approach (SEA), which relies on the Green's functions, these driving forces corresponding to the mentioned degrees of freedom are formulated within the framework of the plane problem of elastostatics. Thus, expressions for translation (J), isotropic expansion (M), distorsion (N) and interactions effects representing the active parts of crack driving forces known as energy release rates are formulated in a purely theoretical context.

Key words: Displacement, stress, green's functions, stress intensity factors, energy release rates.

1. Introduction

Evaluation of Energy Release Rates (ERR) for a single edge notch specimen containing a crack surrounded by a Damage Zone (DZ) is considered. Sufficient experimental data has been collected in the last decade, evidencing that in most cases, a propagating crack is surrounded by a severely damaged zone [1, 2]. This zone can reveal itself as microcracks, voids, slip lines, etc.. [3]. It has been proven that different types of material exhibit varying damage zone. This difference occurs only in the morphology of the DZ. In spite of that, there are similar features in all of them such as similar global geometry of the DZ and similar kinetics of development as well [4]. A number of theoretical models have been proposed for the description of a stress field and kinetics of a DZ [5]. The traditional one identifies the DZ as a plastic zone and uses the well developed technique of the plasticity theory for the determination of its size, shape, energy release rates etc.. According to recent experimental results, some damage patterns do not yield any model of plasticity [6] and the shape of the DZ can be difficult to model. As an example, a cross section of an actual fractured specimen [7] shows that the shape of the damage is quite different (Figs. 1a and 1b). Because of this difference, a plasticity critera is not adequate



(a) (b)

Fig. 1 Plastic zone size of specimen (a) shape of plastic zone (elastoplastic solution); (b) distribution of damage through the thickness (semi-cristaline Polymer [1]).

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for damage characterization. However, elastoplastic solution is currently employed due to the lack of other approaches. In this study, a Semi Empirical Method (SEM) is proposed for evaluating the stress field and the different energy release rates. This approach is based on the representation of displacement discontinuities by means of the Green's function theory [7]. It has been used by some researchers in a purely theoretical context [8, 9]. Herein, we suggest a more realistic model (arbitrary orientations of discontinuities rather than rectilinear ones) for which the result can be obtained using the experimental data and thus avoiding the difficulties of analytical solutions.

2. Description of the Procedure

The displacement field $\tilde{U}(\tilde{x})$ at a point $\tilde{x}$ generated by a discontinuity at point $\tilde{x}^{t}$ (Fig. 2) can be represented for a plane problem as

$$\tilde{U}^{d} = \int_{\Omega} \tilde{b}(x).\Phi(\tilde{x}, \tilde{x}')\, d\tilde{x}' \qquad (1)$$

where $\tilde{b}(\tilde{x}')$ is the discontinuity displacement (or potential density), $\Phi(\tilde{x}, \tilde{x}')$ is the second Green's tensor which is defined as the displacement response at the point of observation $\tilde{x}$ due to a force dipole applied at the point of discontinuity $\tilde{x}'$ and the integration is performed along the discontinuity line Ω. This second Green's tensor for a plane stress problem is given as follows [6]:

$$\Phi(\tilde{x},\tilde{x}') = \frac{(1+\nu)}{4\pi R^2}\left\{(1-\nu)\left[\tilde{n}_{\tilde{x}'}.\tilde{R} - \tilde{R}.\tilde{n}_{\tilde{x}'} - \tilde{n}_{\tilde{x}'}.\tilde{R}.\tilde{E}\right] - 2\frac{\tilde{n}_{\tilde{x}'}.\tilde{R}}{R^2}.\tilde{R}.\tilde{R}\right\} \qquad (2)$$

where $\tilde{n}_{\tilde{x}'}$ is the unit normal vector in the dipole's direction $\tilde{x}'$, ν is the Poisson's ratio, $\tilde{E}$ is the unite tensor, and $\tilde{R}$ is the position vector $(\tilde{R} = \tilde{x}' - \tilde{x})$.

Experimental observations of DZ surrounding the main crack are considered as being an array of microcracks confining in all sides of the crack. The problem is then formulated in terms of a system of singular equations for the unknown densities for each microcrack considered. The system of equations represents the boundary conditions both on the array of microcracks and on the main-crack as well. This system may be derived from Eq. (1) by evaluating tractions and then setting them to zero on all the cracks considered in the system (crack surfaces are traction free). Besides, the conditions on the outer border are satisfied because of the properties of the second Green's tensor. In order to avoid solving the system of integral equations, a semi-empirical procedure based on experimental results is suggested.

At first, divide the area around the crack-tip into a

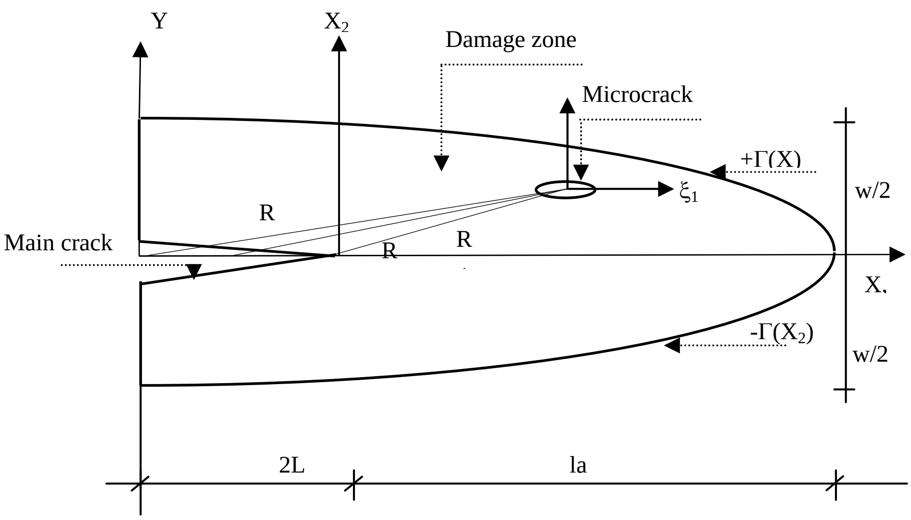

Fig. 2 Schematic representation of the damage zone.

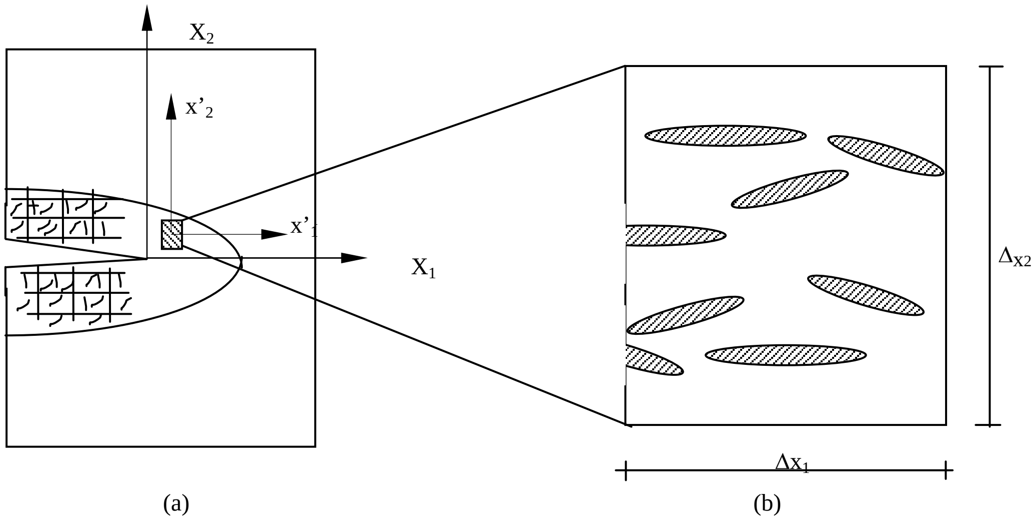

Fig. 3 Schematic representation of the damage zone (a) Subdivision of the damage zone into meshes; (b) Schematic representation of typical square for the determination of microcracks opening density.

network of small squares having N_1 vertical lines and N_2 horizontal lines as in Fig. 3a. Then, the displacement field $\tilde{U}(\tilde{x})$ due to discontinuities in each mesh can be represented as follows;

$$\tilde{U}^d(\tilde{x}) = \sum_{k=1}^{N} \int_{\Omega^k} \tilde{b}^{(k)}(\tilde{x}').\Phi(\tilde{x}', \tilde{x})\, d\tilde{x}' \quad (3)$$

where N is the number of discontinuities. Taking into account that for all discontinuities (Fig. 3b), Eq. (3) becomes:

$$\tilde{U}^d(\tilde{x}) = \sum_{\alpha_1=1}^{N_1} \sum_{\alpha_2=1}^{N_2} \sum_{k=1}^{N} \int_{\Omega^{(k)} \in A_{\alpha_1 \alpha_2}} \tilde{b}^{(k)}(\tilde{x}').\Phi(\tilde{x}', \tilde{x})\, d\tilde{x}' \quad (4)$$

For small mesh size in comparison to the size of the specimen, meaning

$|\tilde{x}' - \tilde{x}| >> max\,|\Delta \tilde{x}'|$, we approximate $\Phi(\tilde{x}', \tilde{x})$ by $\Phi(\tilde{x}'_0, \tilde{x})$ where $\tilde{x}'_0$ is the position vector of the center of the mesh. With that in mind along with the mean value theorem, Eq. (4) takes the following form:

$$\tilde{U}^d(\tilde{x}) \cong \sum_{\alpha_1=1}^{N_1} \sum_{\alpha_2=1}^{N_2} \Phi(\tilde{x}'_0, \tilde{x}).\, \tilde{C}(\tilde{x}').\, \Delta x'_{\alpha_1}\, \Delta x'_{\alpha_2} \quad (5)$$

where

$$\tilde{C}(\tilde{x}') = \frac{\sum_{k=1}^{N} \int_{\Omega^{(k)} \in A_{\alpha_1 \alpha_2}} \tilde{b}^{(k)}(\tilde{x}')\, d\tilde{x}'}{\Delta x'_{\alpha_1}\, \Delta x'_{\alpha_2}}$$

is the concentration of discontinuities (or damage density). For infinitesimal squares ($N_1, N_2 \to \infty$) while $\Delta x'_{\alpha_1}$ and $\Delta x'_{\alpha_2}$ approaching zero, the sum in Eq. (5) becomes a double integral over the entire damage zone in the limit. Thus, the displacement field due to the presence of damage may be presented in the following form:

$$\tilde{U}^d(\tilde{x}') = \iint_{V_d} \Phi(\tilde{x}', \tilde{x}).\, \tilde{C}(\tilde{x}')\, d\tilde{x}' \quad (6)$$

in which V_d stands for the volume of the damage zone.

3. Energy Release Rates Evaluation

Consider a Single Edge Notch (SEN) specimen as shown in Fig. 4 in which a crack propagates surrounded by a layer of damage. Experimental measurements of the crack opening displacement and the concentration of damage in the vicinity of the crack are needed for evaluating the different energy release rates.

The work W done by an applied force F at the grips ($x_2 = H$) is given as follows:

$$W = F.\, U_{Total} \quad (7)$$

where U_{Total} is the total displacement at the grips. This latest is taken as a sum of the displacement at the grips in the initial state (with no cracks) U_I and the displacement at the grips due to the presence of the

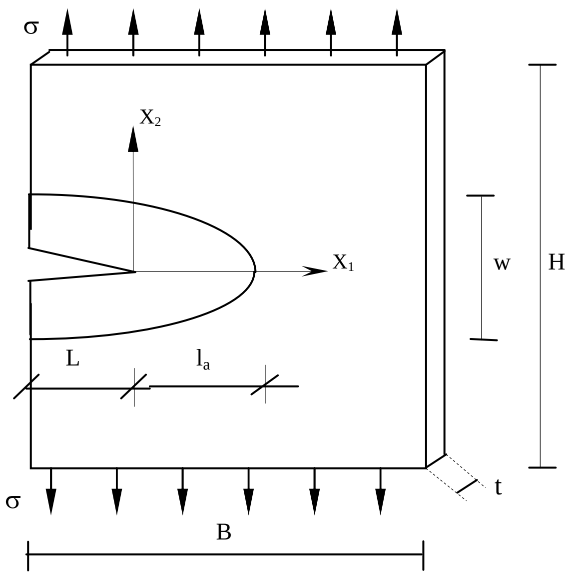

Fig. 4 Size dimensions of the proposed model.

damage U_{DZ}; in other terms:

$$U_{Total} = U_I + U_{DZ} \tag{8}$$

The elastic potential energy P is given as:

$$P = -\frac{1}{2}.W = -\frac{1}{2}.F.U_{Total} \tag{9}$$

Then, the total energy release rate A_I takes the following form:

$$A_I = -\frac{1}{2 \cdot t}\frac{\partial P}{\partial L} = \frac{1}{2t}F.\frac{\partial U_{Total}}{\partial L} \tag{10}$$

where t is considered as the specimen thickness. Since U_I (no cracks) is independent of the crack length, Eq. (10) is reduced to:

$$A_I = \frac{1}{2 \cdot t} \cdot F.\frac{\partial U_{DZ}}{\partial L} \tag{11}$$

The displacement U_{DZ} is an average of the displacement along the grips for a specimen of width B,

$$U_{DZ} = \frac{2}{B}\int U_2(\tilde{x})\Big|_{x_2 = H} dx_1 \tag{12}$$

where $U_2(\tilde{x})$ is derived by the use of the aforementioned Eq. (6) given in Section 2.

$$U_2(\tilde{x}) = \int_0^L \Phi_{22}(x_1' - x_1, x_2) b_2(x_1')dx_1' + \iint_{V_d} \Phi_{2j}(x_1' - x_1, x_2' - x_2)\tilde{C}_j(\tilde{x}')d\tilde{x}' \tag{13}$$

Variation of $\tilde{U}_{DZ}$ due to DZ growth can lead us to the expression of the work increment δP and correspondingly for the elastic energy increment. Carrying out this procedure, expressions for active parts of crack driving forces J, M and N can be obtained. Then, the variation of the displacement U_{DZ} given in Eq. (12) takes the form;

$$\frac{\partial U_{DZ}}{\partial L} = \frac{2}{B}\int_0^B \frac{\partial U_2^C(\tilde{x})}{\partial L}\Big|_{x_2=H} dx_1 + \frac{2}{B}\int_0^B \frac{\partial U_2^d(\tilde{x})}{\partial L}\Big|_{x_2=H} dx_1 \tag{14}$$

$$= \frac{2}{B}\hat{\Phi}_{22}.\frac{\partial \hat{b}_2}{\partial L} + \frac{2}{B}\hat{\Phi}_{2j}\hat{C}_j\frac{\partial V_d}{\partial L} + \frac{2}{B}\hat{\Phi}_{2j}V_d\frac{\partial \hat{C}_j}{\partial L}$$

where $\hat{\Phi}_{22}$, $\hat{b}_2$, $\hat{\Phi}_{2j}$ and $\hat{C}_j$ represent the average expression of Φ_{22}, b_2, Φ_{2j}, C_j after integration, respectively. The total ERR A_I in Eq. (11) becomes by the use of Eq. (14);

$$A_I = \frac{F}{B \cdot t}\hat{\Phi}_{22}\frac{\partial \hat{b}_2}{\partial L} + \frac{F}{B \cdot t}\hat{\Phi}_{2j}.\hat{C}_j.\frac{\partial V_d}{\partial L} + \frac{F}{B \cdot t}\hat{\Phi}_{2j}.V_L.\frac{\partial \hat{C}_j}{\partial L} \tag{15}$$

The above Eq. (15) can be rewritten in a simplified way as;

$$A_I = \left[J_I + M\partial_L \ell + \tilde{N}\partial_L \tilde{d}\right] + \Gamma \partial_L t \tag{16}$$

where J_I corresponds to the translational energy of the active zone, M and N are the isotropic expansion and distorsion energy of the active zone, respectively. Here, Γ correspond to the change in concentration (flaws and new crazes) and interaction effects (time dependency).

4. Conclusions

Theoretical expressions for translation (J), isotropic expansion (M), distorsion (N) representing the active parts of crack driving forces are formulated. It is also shown in a number of cases that J has a significant statistical distribution. It is the expenditure of energy into various modes of crack propagation meaning the translational motion of the crack with the process zone unchanging on one hand and the expansion as well as the distorsion of the DZ on the other hand. These latest along with the change in concentration and interaction effects constitute an important percentage of the total energy release rate. Besides, the distribution of energy into modes varies size from one experiment to the other as being a loading history dependant quantity.

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