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Role of Metacognition in Mathematical Problem Solving Process of Permutations and Combinations—Basic Study for CAI Software Development

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Abstract: When solving a mathematical problem, we sometimes encounter a situation where we can not reach a correct answer in spite of acquiring knowledge and formula necessary for the solution. The reason can be attributed to the lack in metacognitive abilities. Metacognitive abilities consist of comparing the difficulty of problem with own ability, proper plan of solution process, and conscious monitoring and control of solution process. The role and importance of metacognitive ability in mathematical problem solving of permutations and combinations was explored. Participants were required to solve five practical problems related to permutations and combinations. For each problem, the solution process was divided into: (1) understanding (recognition) of mathematical problem; (2) plan of solution; (3) execution of solution. Participants were also required to rate the anticipation whether they could solve it or not, and to rate the confidence of their own answer. According to the total score of five problems, the participants were categorized into the group of the high test score and the group of the low test score. As a result, at the plan and the execution processes, statistically significant differences were detected between the high and the low score groups. As for the rating on the anticipation of result and the confidence of own answer, no significant differences were found between both groups. Moreover, the relationship between the score of plan process and the score of execution process was statistically correlated. In other words, the more proper the plan process was conducted, the more proper solution the participants reached. In such a way, the importance of metacognitive ability in the solving process, especially the plan ability, was suggested.

Keywords: Permutation and combination problem, metacognition, understanding, plan, execution, CAI (computer assisted instruction).

1. Introduction

When solving a science problem such as a mathematical problem or a basic electric circuit problem, we sometimes encounter a situation where we can not reach correct answers in spite of mastering knowledge and formula necessary for the solution. The reason can be contributed to the lack in metacognitive abilities. "Metacognition" refers to one's knowledge on one's own cognitive processes. The role of metacognition in a variety of mathematical problems is explored [1-7]. It has been conceptually shown that successful problem solving depends on not only adequate knowledge related to the problem but also sufficient monitoring (awareness) and control of the cognitive process during problem solving, that is, metacognition.

Lawanto [8] developed a lecture system using EGN (enhanced guided notes) in an electric circuit course for beginners. The difference between the traditional lecture system and the proposed one was that the proposed one made an attempt to prompt students to evaluate their metacognitive knowledge. Although he showed that the proposed lecture system led to higher performance of students, the problem solving process was not analyzed at all.

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It is sure that there exist learners who can not solve practical problems at all in spite of mastering necessary and basic knowledge and how to solve basic problems. One of the reasons why learned knowledge can not be applied sufficiently for solving practical problems must be the lack in metacognitive ability which tells learners when and how the learned knowledge or strategy is used. Flavell [1, 2] pointed out the importance of metacognitive ability in problem solving. Therefore, we assumed that higher ability of problem solving is attributed to high metacognitive ability.

It is generally regarded to be difficult to master the ability to solve practical problems in mathematics learned at high school, although the basic problems described in the text book can be easily solved. As mentioned above, in order to overcome such a situation, enhancing the metacognitive ability, which corresponds to the control and the monitoring of own solving process (mainly, understanding of the problem, plan of the solution, and execution of the solution), must be effective.

The deep and detailed understanding of problem solving process would be helpful for the development of an interactive CAI (computer assisted instruction). The aim of this study was to verify the role of metacognition in the mathematical problem solving of permutations and combinations. The paper is organized as follows: Section 2 summarizes the understanding process of mathematical problems and its relevance to metacognition. Section 3 shows the experimental method. In particular, the work sheet developed for observing metacognitive process was explained in detail. Section 4 shows the experimental results, and compares the score obtained from the analysis of the work sheet among high and low score groups. Section 5 discusses the result and considers how the metacognition plays a role in the mathematical problem solving. Section 6 summarizes and concludes the results.

2. Understanding Process of Mathematical Problems and Metacognition

Metacognition represents knowledge related to cognitive activities, and is also a process that monitors and controls cognitive activities. Flavell [1, 2] and Brown [3] suggest that two aspects, that is, the metacognitive knowledge and the control of cognition, are included in metacognition. While the metacognitive knowledge refers to the knowledge on what kind of factors or strategies affect the cognitive process or when and how the knowledge should be applied to the problem solving, the control of cognition monitors and checks whether cognitive process for the problem solving is proceeding smoothly and properly, and controls the process if the process is not properly conducted.

The metacognition can be rephrased as consisting of the following processes: (1) Own solving ability is compared with the difficulty level of the problem; (2) The type of the problem is analyzed (understanding of problem); (3) The measure or strategy for solving is planned properly (plan); (4) The planned solution is executed. The monitoring and the control of these processes are included in the function of metacognition. Here, it must be noted that the metacognition is one of the models to explain the process of problem solving. Our problem solving process is not necessarily performed and explained perfectly according to the model or concept of metacognition.

In this study, the three basic processes, that is, the understanding, the plan, and the execution, were assumed. Glasser [9] pointed out that the prediction of results of the problem solving, and the evaluation of confidence of own answer affected the problem solving performance. As shown in Fig. 1, an attempt was made to observe the problem solving process and model the process on the basis of five processes: (1) prediction of results; (2) understanding of problem; (3) plan of solution; (4) execution of solution; (5) evaluation of confidence of own answer.

The understanding of the problem requires us to make the aim of the problem solving clear, identify the necessary information, and clarify the relevance among various information. The monitoring and the control of

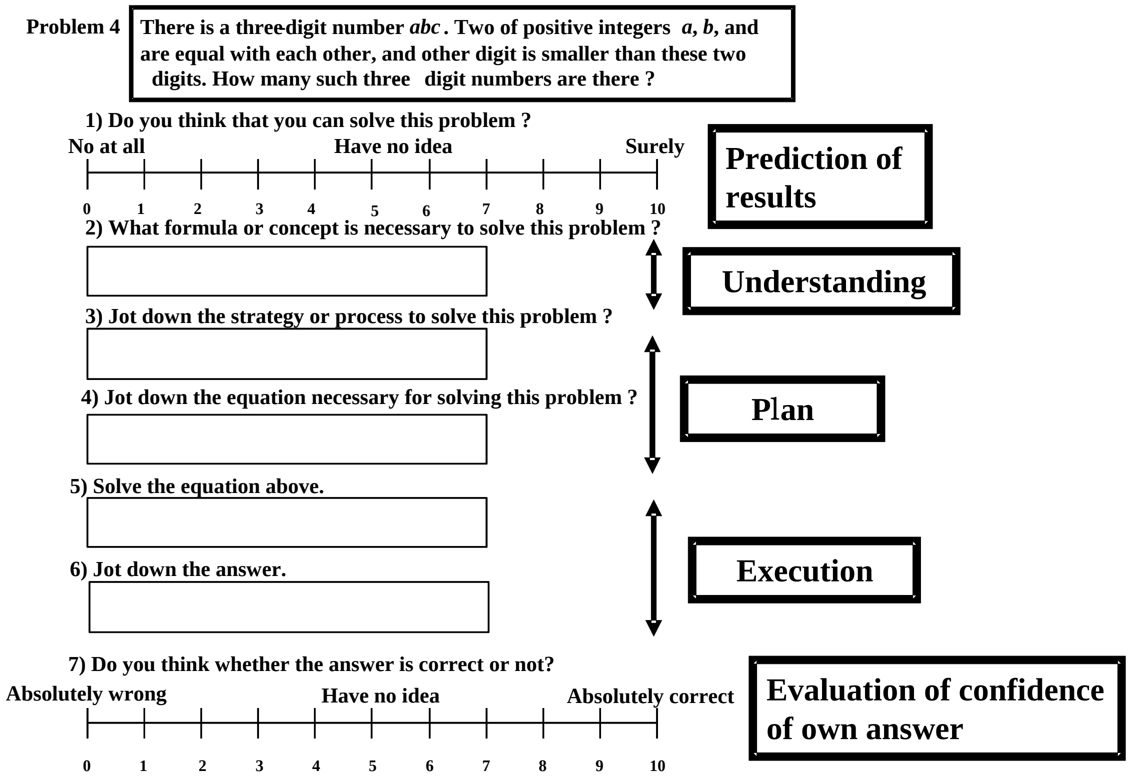

Fig. 1 Work sheet for problem 4.

this process are necessary for the effective problem solving. Such function of monitoring and control of the solving process is also a prerequisite condition in the plan and the execution processes.

Glasser [9] showed that to the more extent learners understand the problem and the higher their test score are, the more accurate the evaluation whether the answer is correct or not get. Glasser [9] also showed that the more skilled learners are in the problem solving, the more accurate the prediction whether learners can solve the problem or not gets. Thus, the processes (1) and (5) above were added to the work sheet in Fig. 1. We assumed that the prediction of results and the evaluation of confidence of own answer need to be accurate for the purpose of properly monitoring and controlling the solving process.

3. Method

3.1 Participants

Fourteen graduates or undergraduates aged from 21 to 24 years old who majored in system engineering participated in the experiment. All participants signed the document on informed consent after receiving a brief explanation on the objective, the contents, and the procedure of the experiment.

3.2 Confirmation Test

Ten problems in the confirmation test are listed below. These are basic problems of permutations and combinations learned during a high-school mathematics class. In the scoring (grading), we paid attention to only whether the answer was correct or not (one point was given to the correct answer. Thus, the score of 10 points corresponded to the full marks).

(1) How many ways are there to select one chairman, one vice-chairman, and one secretary out of 12 committee members? It must be noted that the concurrent act of two or more positions is not allowed.

(2) How many ways are there to make a four-digit number from five digits 0, 1, 2, 3, and 4?

(3) Five vowels a, i, u, e, and o and three consonants k, s, and t are lined up. In how many ways

can eight characters be lined up so that all of five vowels are lined up in a row?

(4) In how many ways can five boys and two girls be lined up so that boys are located at both ends of the line and a girl is located between two boys?

(5) Two adults and four children sit on a round table. How many ways are there so that two adults do not adjoin?

(6) Five cards are selected from a pair of heart cards (13 cards). How many ways are there when just two of three (jack, queen, and king) cards are included in the selected five cards?

(7) A letter string is made using all of six letters of “banana”. How many letter strings are there in this case?

(8) Three members are selected from a group of six boys and four girls. How many ways are there so that at least one girl is included in the three members?

(9) Using eight letters of “Internet”, permutations are created. How many permutations are there so that every t is located to the left of every e?

(10) How many ways are there to divide 15 members into three groups of six members, six members, and three members?

3.3 Practical Problems of Permutations and Combinations and Worksheet for Analyzing Metacognitive Process

The following five practical problems on permutations and combinations learned during a high-school mathematics class were prepared.

Problem 1: Calculate the permutation when four letters are arbitrarily picked up from “mathematics”.

Problem 2: One climbs up 15 stairs (steps) every two or three stairs (steps). How many ways are there in this case?

Problem 3: There are five white cards, two red cards, and one black card. It is assumed that the card with the same color can not be discriminated. Eight cards are lined up. In how many ways can two cards with different colors be located on both ends of eight cards that were lined up?

Problem 4: There is a three-digit number *abc*. Two of positive integers *a*, *b*, and *c* are equal with each other, and another digit is smaller than these two digits. How many such three-digit numbers are there?

Problem 5: There are two round tables (big and small ones). The big round table and the small round table have four and three seats, respectively. Six persons A, B, C, D, E, and F sit on one of these round tables. In how many ways can A and B sit on the same round table?

In order to observe the problem solving process, the work sheet such as shown in Fig. 1 (problem 4) was prepared. The three processes are directly related to the problem solving “(2) understanding of problem”, “(3) plan”, and “(4) execution” were assigned. The item 2), the items 3) and 4), and the items 5) and 6) correspond to “understanding of problem”, “plan” and “execution”, respectively.

3.4 Procedure

The experiment was conducted all at once for 15 participants in a classroom. First, the confirmation test consisted of 10 problems (Section 3.2) was carried out for 40 mins. After the confirmation test, the participants were allowed to take a 5 mins break.

Next, the participants were required to take a course on basics and application of permutation and combination. The aim was to review the contents of permutations and combinations learned at high school, and remind the participants of the knowledge necessary for solving the problems 1-5 mentioned in Section 3.3. How to describe on the work sheet in Fig. 1 was also demonstrated to the participants.

After all procedures were completed, the main experimental procedure to solve problems 1-5 in Section 3.3 was started. The experiment was self-paced, and no time limitation was imposed on the participants. Almost all of the participants completed the procedure in about 1 h. The longest duration of the experiment was about 2 h.

3.5 Scoring Procedure of Work Sheet

The answers for prediction of result (item 1)), understanding of problems (item 2)), plan (items 3) and 4)), execution (items 5) and 6)), and evaluation of confidence of own answer (item 7)) were scored using a 5-point scale.

The sample answers for (2) understanding, (3) plan, and (4) execution processes were prepared, and this was checked by an expert of probability and statistics. The answer was found to be valid. When nothing was written on the answer sheet, or the answer was apparently wrong, we scored this as 0. When the participant's answer corresponded well with the sample answer, the score of 5 was given. The scores between 0 and 5 (1, 2, 3, and 4) was marked according to the situation of the answer. The results of the scoring was checked and validated by the same expert of probability and statistics.

The prediction of results was scored according to the following procedure. On the basis of whether the prediction of the result corresponded with the actual result (correct or wrong answer), and the 10-point rating on prediction of result, the following scoring was carried out. When the prediction of the result did not correspond with the actual result (correct or wrong answer), the score was 0. When prediction of the result corresponded with the actual result (correct or wrong answer), the score of 2 was marked. The scoring rule above can be summarized as follows. When the participant predicted that he could not reach a correct answer (rating of prediction of result: 0-4), and actually could not solve the problem, 2 points were provided. When the participant predicted that he could reach a correct answer (rating of prediction of result: 6-10), and actually could solve the problem, 2 points were provided.

When prediction of the result corresponded with the actual result (correct or wrong answer) and the rating of prediction of results was high, extra points (2 points or 3 points) were added to the score. When the rating of prediction of results was 8 or 9, 2 points were added to the score. For the maximum rating (10) of prediction of results, 3 points were added to the score.

The evaluation of confidence of own result was scored as follows. On the basis of whether the evaluation of confidence of own answer corresponded with the actual result (correct or wrong answer), and the 10-point evaluation of confidence of own answer, the scoring was carried out according to the following procedure. When the evaluation of confidence of own answer did not correspond with the actual result (correct or wrong answer), the score was 0. When the evaluation of confidence of own answer corresponded with the actual result (correct or wrong answer), 2 points were given. The scoring rule of the evaluation of confidence of own answer above can be summarized as follows. When the participant evaluated that he had no confidence of a correct answer (rating of prediction of result: 0-4), and actually could not solve the problem, 2 points were provided. When the participant convinced that own answer was correct (rating of prediction of result: 6-10), and actually could solve the problem, 2 points were provided.

When the evaluation of confidence of own answer corresponded with the actual result (correct or wrong answer) and the corresponding score was high, extra points (2 points or 3 points) were added to the score. When the rating of evaluation of confidence of own answer was 8 or 9, 2 points were added to the score. For the maximum rating (10), 3 points were added to the score.

4. Results

4.1 Comparison of Score of Basic Ability Confirmation Test between High and Low Score Groups

The summed score (full mark: 5 points × 3 = 15 points) of three items in the work sheet (understanding, plan, and execution) was calculated for each problem. This was added for problems1-5 (full mark: 15 points × 5 = 75 points). The mean of the total score was 46.1 (S.D. (standard deviation) = 14.62). Using 50 points as a threshold value, the participants were divided into the high score and the low score groups. The mean total scores of the high score group and the low score group were 62.80 (n = 5, S.D. = 8.16) and 36.89 (n = 9, S.D. = 7.48), respectively.

The mean score of the confirmation test in Section 3.2 was 5.14. The scores of confirmation test were 5.6 (S.D. = 3.21) and 4.8 (S.D. = 2.20) for the high and the low score groups, respectively (Fig. 2). As a result of a statistical t-test conducted on the mean score of the confirmation test, no significant difference of the mean score was detected between the high and the low score groups.

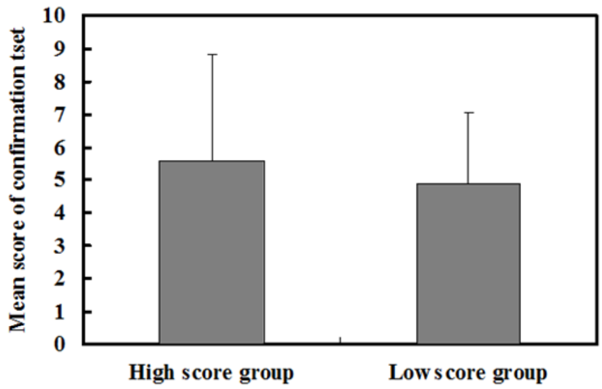

Fig. 2 Comparison of confirmation test score between high and low score groups.

4.2 Comparison of Score at Each Process between High and Low Score Groups

The results for problems1-5 are shown below.

4.2.1 Problem 1

The mean scores of the low score and the high score groups were 2.89 (S.D. = 0.314) and 6.0 (S.D. = 5.367), respectively. In Fig. 3, the score for each solving process is compared between the high and the low score groups. As a result, a two-way (process of problem solving by score group) ANOVA (analysis of variance) conducted on the score revealed only a main effect of process of problem solving (F(4, 48) = 11.168, p < 0.01). Fisher's multiple comparison revealed significant (p < 0.01) differences between the following pairs: (prediction, understanding), (prediction, plan), (understanding, plan), (understanding, execution), (understanding, evaluation), and (plan, execution).

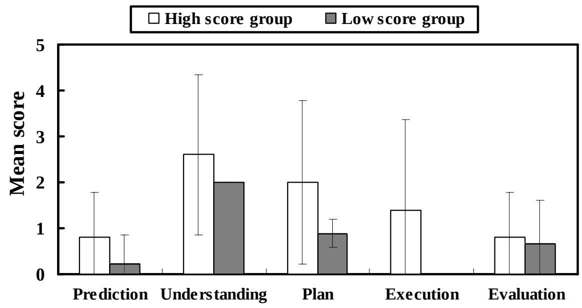

Fig. 3 Comparison of score for each solving process between high and low score groups (problem 1).

4.2.2 Problem 2

The mean scores of the low score and the high score groups were 10.9 (S.D. = 5.195) and 15.0 (S.D. = 0), respectively. In Fig. 4, the score for each solving process is compared between the high and the low score groups. As a result, a similar two-way (process of problem solving by score group) ANOVA conducted on the score revealed main effects of process of problem solving (F(4, 48) = 24.959, p < 0.01) and score group (F(1, 12) = 5.024, p < 0.05). Fisher's multiple comparison revealed significant (p < 0.01) differences between the following pairs: (prediction, understanding), (prediction, plan), (prediction, execution), (prediction, evaluation), (understanding, execution), (understanding, evaluation), (plan, evaluation), and (execution, evaluation).

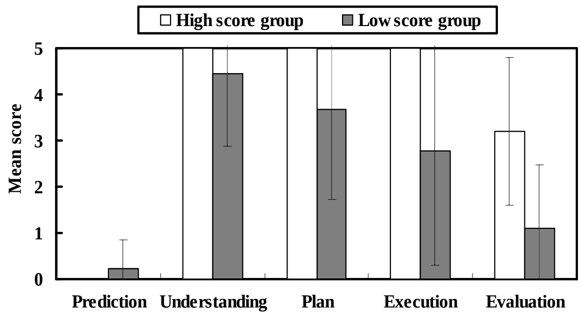

Fig. 4 Comparison of score for each solving process between high and low score groups (problem 2).

4.2.3 Problem 3

The mean scores of the low score and the high score groups were 11.4 (S.D. = 4.374) and 12.4 (S.D. =

4.716), respectively. In Fig. 5, the score for each solving process is compared between the high and the low score groups. As a result, a similar two-way (process of problem solving by score group) ANOVA conducted on the score revealed only a main effect of process of problem solving ($F(4, 48) = 21.846$, $p < 0.01$). Fisher's multiple comparison revealed significant ($p < 0.01$) differences between the following pairs: (prediction, understanding), (prediction, plan), (prediction, execution), (prediction, evaluation), (understanding, evaluation), (plan, evaluation), and (execution, evaluation).

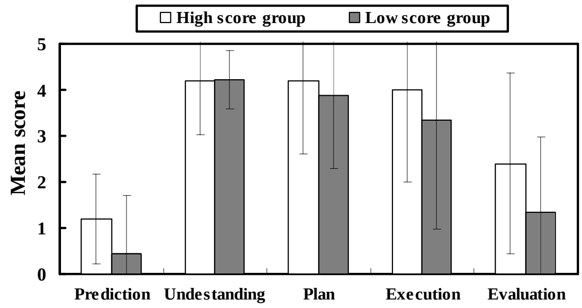

Fig. 5 Comparison of score for each solving process between high and low score groups (problem 3).

4.2.4 Problem 4

The mean scores of the low score and the high score groups were 7.2 (S.D. = 3.705) and 14.6 (S.D. = 0.490), respectively. In Fig. 6, the score for each solving process is compared between the high and the low score groups. As a result, a similar two-way (process of problem solving by score group) ANOVA conducted on the score revealed main effects of process of problem solving ($F(4, 48) = 33.947$, $p < 0.01$) and score group ($F(1, 12) = 4.772$, $p < 0.05$). Fisher's multiple comparison revealed significant ($p < 0.01$) differences between the following pairs: (prediction, understanding), (prediction, plan), (prediction, execution), (prediction, evaluation), (understanding, evaluation), (plan, evaluation), and (execution, evaluation). For each problem solving process, the t-test was carried out on the score. The significant difference of mean score between the high and the low score groups was detected for the plan process ($|t| = 3.133$, $p < 0.05$) and the execution process ($|t| = 5.403$, $p < 0.05$).

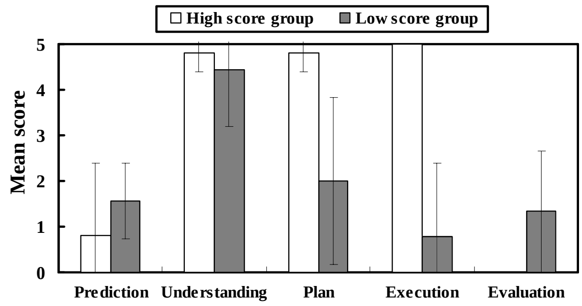

Fig. 6 Comparison of score for each solving process between high and low score groups (problem 4).

4.2.5 Problem 5

The mean scores of the low score and the high score groups were 4.4 (S.D. = 1.832) and 14.8 (S.D. = 0.4), respectively. In Fig. 7, the score for each solving process is compared between the high and the low score groups. As a result, a similar two-way (process of problem solving by score group) ANOVA conducted on the score revealed main effects of process of problem solving ($F(4, 48) = 31.527$, $p < 0.01$) and score group ($F(1, 12) = 64.398$, $p < 0.01$). Fisher's multiple comparison revealed significant ($p < 0.01$) differences between all combinations except (prediction, evaluation). For each problem solving process, as a result of a similar t-test, significant difference of mean score between the high and the low score group was detected for the plan process ($|t| = 7.590$, $p < 0.05$) and the execution process ($|t| = 2.287$, $p < 0.05$).

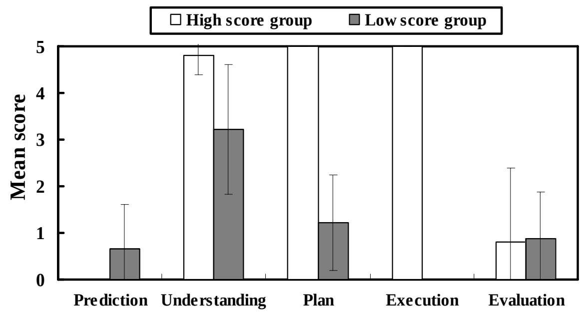

Fig. 7 Comparison of score for each solving process between high and low score groups (problem 5).

4.2.6 Mean Score of Problems 1-5

The mean scores of five problems were calculated for each solving process. The mean scores of the low score and the high score groups were 7.4 (S.D. = 4.918) and 12.6 (S.D. = 4.344), respectively. In Fig. 8, the score for each solving process is compared between the high and the low score groups. As a result, a similar two-way (process of problem solving by score group) ANOVA conducted on the score revealed main effects of process of problem solving ($F(4, 48) = 83.133$, $p < 0.01$) and score group ($F(1, 12) = 21.352$, $p < 0.01$). Fisher's multiple comparison revealed significant ($p < 0.01$) differences between all combinations except (prediction, evaluation). For each problem solving process, as a result of a similar *t*-test, significant difference of mean score between the high and the low score group was detected for the plan process ($|t| = 6.849$, $p < 0.05$) and the execution process ($|t| = 5.382$, $p < 0.05$).

In Table 1, the correlation coefficients between scores of arbitrary two solving process are separately shown for the high score and the low score groups.

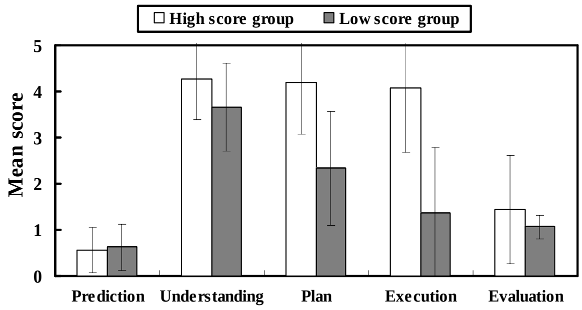

Fig. 8 Comparison of score for each solving process between high and low score groups (mean score of problems 1-5).

5. Discussion

5.1 Confirmation Test

As shown in Fig. 2, no significant difference of the confirmation test score was detected between the high score and the low score groups. As far as the participants in this study are concerned, the scores of practical problems 1-5 were different between the high and the low score groups irrespective of no significant difference of confirmation (basic) test score (Figs. 3-7). When it comes to solving practical problems, the difference of solving ability seems to be definitely reflected in the score. Such a difference was not reflected in the basic problems.

This must suggest that how the basic knowledge such as formulas or strategies should be applied must be based on the metacognitive ability to monitor and control the problem solving processes. Therefore, an attempt was made to model the difference of score of (1) prediction of results, (2) understanding of problem, (3) plan of solution, (4) execution of solution, and (5) evaluation of confidence of own answer between the high and the low score groups using the model or concept of metacognition to monitor and control the problem solving process.

5.2 Problem Solving Process of Permutation and Combination

As for problems 1 and 3, it tended that the scores of the understanding, the plan, and the execution processes for the high score group were significantly higher than those for the low score group. A similar result was observed for problem 2. Such a tendency was further reinforced for problems 4 and 5. Although the tendency was not completely consistent or coherent for five problems, it can be inferred, on the basis of the data that the high score group marked higher score for each solving process than the low score group, that the participants classified as the high score group excels in the metacognitive ability to monitor and control the solving process.

Throughout problems 1-5, no significant differences of scores for the prediction of result and the evaluation of confidence of own answer were detected between the high score and the low score groups. The reason is discussed later. On the basis of Fig. 8, the finding above is further discussed.

5.2.1 Understanding, Plan, and Execution

Even from Fig. 8, it is also possible to infer that the high score group excels metacognitive ability to monitor and control the problem solving process. Although no significant difference between the high and the low score groups was observed for the understanding

Table 1 Correlation matrix of score at each solving stage: (a) high score group; (b) low score group.

(a) High score group

	Prediction	Understanding	Plan	Execution	Evaluation
Prediction					
Understanding	0.6				
Plan	0.172	0.941**			
Execution	0.138	0.918**	0.984**		
Evaluation	0.161	0.231	0.233	0.199	

(b) Low score group

	Prediction	Understanding	Plan	Execution	Evaluation
Prediction					
Understanding	-0.01				
Plan	-0.083	0.556**			
Execution	-0.045	0.449**	0.868**		
Evaluation	0.169	0.3	0.354	0.477**	

**: $p < 0.01$.

process, significant differences of the score between the high and the low score groups were detected for the plan and the execution processes.

It is also seen from Fig. 8 that the score of the high score group did not differ among three solving processes, that is, understanding, plan, and execution. On the other hand, the score of the low score group tended to decrease as the solving process proceeded from understanding to plan, and from plan to execution. While the higher score group must have monitored and controlled a series of solving processes properly, the low score group must be lacking in such a metacognitive ability.

From Table 1, significant correlations between the understanding score and the plan score, and between the understanding score and the execution score were detected for both high and low score groups. The correlation coefficient between the plan score and the execution score was by far higher (more than 0.86) for both high score and low score groups, which indicates the possibility that the proper plan leads to the correct execution (answer). This also suggests that the plan process is especially important for the proper problem solving. This finding could be incorporated into the design or the development of CAI with high learnability.

The correlation coefficient of the high score group was higher than that of the low score group (Table 1). The higher correlation coefficient of the high score group might be interpreted as follows. The higher correlation coefficients between the understanding and the plan scores, between the understanding and the execution scores, and between the plan and the execution scores mean that the linkages between these solving processes are very strong. The strong linkages between the understanding and the plan scores, between the understanding and the execution scores, and between the plan and the execution scores suggest that the solving process is carried out smoothly and properly due to the metacognitive monitoring and control functions of the process as a whole.

While the correlation coefficients between the understanding score and the plan score, between the understanding score and the execution score, and between the plan score and the execution score were higher for the high score group than for the low score group, such a tendency was reversed between the execution score and the evaluation score. In other words, the correlation between the execution score and the evaluation score was higher for the low score group than for the high score group. Although it is difficult to identify the reason accurately, it might be inferred as follows. Some psychological factor such as excessive under-confidence of high score group might have contributed to the result. As discussed later in Section 5.2.2, if the solving ability of the high score group is much higher than that of the low score group, the excessive under-confidence of the high score group might disappear. The confidence of own answer will grow, and consequently the evaluation score will get higher rating, which leads to the higher correlation coefficient between the execution score and the evaluation score for the high score group.

5.2.2 Prediction of Result and Evaluation of Confidence of Own Answer

The prediction of result and the evaluation of confidence of own answer are not directly related to the solving of problems. Glasser [9] pointed out that the prediction of results of the problem solving, and the confidence of own answer affected the problem solving performance. Glasser [9] also showed that the more skilled learners are in the problem solving, the more accurate the prediction whether they can solve the problem or not gets. Thus, the processes (1) and (5) were added to the work sheet in Fig. 1. For the purpose of properly monitoring and controlling the solving process, the prediction of results and the evaluation of the answer need to be accurate. Thus, it was hypothesized that the score of these two items was higher for the high score group (assumed to have higher metacognitive ability and can get higher score) than for the low score group.

However, different from the understanding, plan, and execution scores, no significant differences of prediction and evaluation scores were detected (Fig. 8). In the range of this experiment, as shown in Figs. 3-8, the difference of understanding, plan, and execution scores between two groups are not remarkably large. This might mean that the difference of metacognitive ability is not so remarkably large. As stated in Glasser [9], it might be possible that the higher the metacognitive ability is, the more accurate the prediction of result is and the more confident the evaluation of own answer is. If the difference of metacognitive ability for problem solving gets larger between the two groups, the difference might appear in these items (prediction of result and evaluation of confidence of own answer).

The high score group showed higher scores of understanding, plan, and execution than the low score group. We judged that this must be explained by high metacognitive ability stated in Flavell [1, 2]. If the participants with much higher ability in permutation and combination are compared with the low score group, it might be possible that the difference appears not only in understanding, plan, and execution processes, but also in prediction and evaluation processes. Future work should verify the validity of this inference.

6. Conclusions

An attempt was made to explain, using the metacognitive model, the problem solving process of permutations and combinations. The results can be summarized as follows.

(1) Irrespective of no significant difference of confirmation (basic) test score among the participants, the abilities to deal with practical problems differed among participants, and the participants were divided into the high score and the low score groups.

(2) Analysis of problem solving process using a work sheet in Fig. 1 revealed that the understanding, plan, and execution scores differed between the high score and the low score groups. This was explained by the metacognitive model which attributed the higher score of problems to the proper monitoring and control of the solving process.

Although the metacognitive modeling was conducted through problem solving of permutations and combinations, the role of metacognition must be explored for other types of mathematical problem solving [10] such as sequences and limits, or differential and integral calculus. As suggested in this study, the plan plays a crucial role in the solving process of permutation and combination problems. A proper learning method on how to properly monitor and control the process from the understanding of the problem to the plan of strategy or solution must be proposed. This must surely lead to the development of more useful CAI systems.

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