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Students' Difficulties With Multiple Representations in Introductory Mechanics*

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Research in physics education indicates that the use of multiple representations in teaching and learning helps students become better problem-solvers. We report on a study to investigate students' difficulties in solving mechanics problems presented in multiple representations. We conducted teaching/learning interviews with 20 students in a first-semester calculus-based physics course which covered introductory classical mechanics. Each student was interviewed four times during the semester, each time after they had completed an exam in the course. During these interviews, students were asked to solve a problem they had seen in the exam, followed by problems that differed in the type of representation from the exam problems. Students were provided verbal hints to solve the new problems. We discuss the common difficulties students encountered when attempting to solve problems in different representations and some common themes in students' performance.

Keywords: multiple representations, physics education research, introductory mechanics

Introduction and Background

The use of multiple representations in solving introductory physics problems has been of great interest to physics education researchers. There have been studies addressing the benefits of using different representations in solving physics problems (De Leone & Gire, 2005; Van Heuvelen & Maloney, 1999), the strategies to facilitate students' problem solving across representations (Van Heuvelen & Zou, 2001), as well as other pedagogical aspects of helping students construct representations (Dufresne, Gerace, & Leonard, 1997; Heller & Reif, 1984; Kohl, Rosengrant, & Finkelstein, 2006; Larkin & Simon, 1987; Rosengrant, Van Heuvelen, & Etkina, 2006; Van Someren, Reimann, Boshuizen, & de Jong, 1998). Meltzer (2005) found that students' performance on similar problems posed in different representations might yield significantly different results.

However, there have not been many studies addressing the specific types of difficulties students might have with different kinds of representations, as well as the difficulties they might encounter when transferring their problem-solving skills across representations. Rosengrant et al. (2009) investigated the thought processes of students when using particular representations, but did not figure out the difficulties students had with each representation. Tuminaro (2004) addressed the difficulties students had with mathematics in physics, but did not address the difficulties students might have with mathematical representations of physics problems.

In this study, we investigate the difficulties that students encounter when solving introductory mechanics

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problems in numerical, graphical and functional representations. Specifically, we address the following research questions:

(1) Research question 1: What are the common difficulties that students encounter when solving mechanics problems in different representations?

(2) Research question 2: How does the sequence of problems given to students affect their performance?

(3) Research question 3: How do the difficulties change as students progress through the semester?

Methodology

We conducted individual teaching/learning interviews, an adaptation of the teaching experiment (Steffe, 1983; Steffe & Thompson, 2000) with 20 students randomly selected from a pool of 102 volunteers enrolling in a first-semester calculus-based physics course, which covered topics in classical mechanics. A teaching/learning interview differs from a clinical interview in an important way. While the goal of a clinical interview is typically to investigate students' ideas without influencing these ideas, the goal of a teaching interview is to specifically facilitate students' development of ideas and to provide the necessary scaffolding. In a teaching/learning interview, the researcher can probe how a learner reacts to different kinds of scaffolding. In this study, teaching/learning interviews were designed to investigate as well as to scaffold students' problem-solving processes.

All of the participants were engineering majors. Each student was interviewed four times during the semester, each time after an exam in their physics class. In each interview, students were asked to solve three problems, one in each representation:

(1) Original problem: A problem from the most recent exam in students' physics class. In this problem, all of the information was given in numerical representation;

(2) Graphical problem: A modified version of the original problem in which part of the information was provided as a graph;

(3) Functional problem: A modified version of the original problem in which part of the information was provided as a function.

With three problems in each interview, students encountered two changes in representations, from numerical to graphical to functional representation or from numerical to functional to graphical representation.

The topics of each interview were one-dimensional kinematics in interview 1, work and energy without friction in interview 2, work and energy with friction in interview 3, and rotational energy with friction in interview 4. Problems in each interview are presented in the appendix.

Students were asked to think aloud as they solved the problems on paper and were given verbal hints whenever they were unable to proceed. All interviews were video and audio recorded, and students' worksheets were collected.

The videos of the interviews were transcribed and transcriptions were coded. A code was assigned to each difficulty encountered by students. The inter-rater reliability for coding was 80% before discussion and about 99% after discussion between coders. The codes were then sorted into categories of difficulties.

Results and Discussion

Categories of Difficulties

We found that students encountered a variety of difficulties, including those with physics context, with mathematical manipulation and with associating math and physics knowledge. More specifically, we have identified the following categories of students' difficulties. Table 1 presents all categories of difficulties students had, the codes included in each category, and the difficulties associated with each code as well as an illustrative example from our data.

Table 1

Categorized Difficulties Students Encountered in Our Interviews

Category	Code	Description	Examples
Difficulty with use of principle	D-PRIN-USE	Do not know which principle is used to solve the problem or use a principle inappropriately.	Student did not know if they could apply conservation of energy in a problem or not, or included only work done by friction in the "work-kinetic energy" theorem and not the work done by other forces.
Difficulty with formula	D-FOR-MEAN	Do not know or misinterpret the meaning of a formula.	Student did not know what ΔK and ΔU mean in the equation for conservation of energy: $\Delta K + \Delta U = 0$.
	D-FOR-WRONG	Do not know or write the wrong formula.	Student wrote $\Delta K + \Delta U = 0$ for conservation of energy in case there was friction.
Difficulty with value	D-FOR-VAL	Do not know which value to put in a formula or equation.	Student put incline length instead of vertical height into "h" when calculating gravitational potential energy "mgh".
	D-CONF-BW-VAL	Confusion between values when plugging into an equation.	Student put the total distance the object traveled instead of spring compression into "x" when calculating spring potential energy "$\frac{1}{2}kx^2$".
Difficulty with physical quantities	D-QUAN-USE	Do not know the appropriate quantities to use to describe a situation.	Student did not know what kind of energy the object had at a specific point on the trajectory.
	D-QUAN-FOR	Do not know or write incorrect formula of physical quantities.	Student did not know the formula for spring potential energy or moment of inertia of a sphere.
	D-QUAN-CALC	Do not know how to calculate a physical quantity based on given information.	Student did not know how to find work done by a force from the graph of force versus distance.
	D-QUAN-UNIT	Do not know or use wrong units of physical quantities.	Student did not know the unit for work or did not know how to convert to get the appropriate unit.
	D-CONF-BW-QUAN	Confusion between physical quantities.	Student confused about work and force, energy and force, potential energy and kinetic energy.
Difficulty with mathematics	D-MATH-PROC	Manipulate mathematical processes incorrectly.	Student confused about trigonometric functions or did not know the relation between angle and distance along a circle.
	D-MATH-MEAN	Do not know the meaning of a mathematical process.	Student did not know the physical meaning of differentiation and integration.
Difficulty with graph	D-GRA-INFO	Unable to read off information from graph.	Student was unable to read off explicit information from the graph such as vertical intercept.
	D-GRA-PROC	Unable to extract information from graph to calculate the desired quantity.	Student was unable to find work done by a force over a distance from the graph of force versus distance.
Difficulty with function	D-FUNC-WRONG	Do not know how to manage the function given to find desired quantities or use the given function inappropriately.	Given a function of force, student did not know how to find work done by that force on a certain distance.
Difficulty with calculation	D-CALC-ERROR	Student made errors in calculation.	Student forgot to take squared-root when necessary.

Besides the common difficulties experienced while solving these kinds of problems in general, we were specifically interested in those difficulties caused by the change in representation. An "R" was added in front of a code each time it appeared to indicate a difficulty caused by the change in representation of the problems. For example, the code "D-QUAN-CALC" was assigned when a student did not know how to find potential energy stored in a spring in the original problem, while the code "R-D-QUAN-CALC" was assigned if a student did not know how to do that from the given graph or function of force versus spring compression.

Sequencing Effect

We also investigated the effect of the sequence of problems presented to students on their performance by giving half of the students the graphical problem before the functional problem (which we called the G-F sequence) and the other half of the students the functional problem before the graphical problem (which we called the F-G sequence). In seeking the sequencing effect, we focused only on the difficulties caused by the change in representation of problems. The average numbers of difficulties caused by representational change that each student encountered in each sequence in interviews 2 and 4 are given in Tables 2 and 3, respectively.

Table 2

Average Number of Difficulties Per Student Due to Representational Change in Interview 2

	G-F sequence	F-G sequence
Graphical problem	3.50	0.17
Functional problem	1.75	1.33

Table 3

Average Number of Difficulties Per Student Due to Representational Change in Interview 4

	G-F sequence	F-G sequence
Graphical problem	0.50	0.17
Functional problem	0.25	2.67

The sequencing effect could be observed in each interview when we compared the average number of difficulties each student encountered in the G-F sequence and the F-G sequence. In interview 2, each student had an average of 5.25 difficulties per student in the G-F sequence while having only 1.5 difficulties per student in the F-G sequence. This suggested that students had fewer difficulties if they attempted the functional problem first and the graphical problem later. However, the data from interview 4 showed the opposite effect. Each student had an average of 0.75 difficulties per student in the G-F sequence while having 2.84 difficulties per student in the F-G sequence. This apparent contradiction could be explained as follows. In the graphical problem in interview 2, students could find potential energy stored in the spring by either finding the spring constant and spring compression from the graph to plug into "$\frac{1}{2}kx^2$" or finding the area under the graph. When given the graphical problem first, most students attempted the first method in which they encountered some R-D-GRA-INFO (difficulties reading off information from graph). In contrast, when given the functional problem first, students just had a few difficulties in finding work from the force function, because it was observed that students had a tendency to integrate what was changing, which was correct in this problem although they did not really understand the meaning of integration. Then, when these students moved on to the graphical problem, they were able to recognize the method of finding the area under the graph, as demonstrated in the following excerpt:

Student: "This problem also has friction, but it tells you the friction by a graph of the equation".
Interviewer: "Okay, so how would you find work in this problem?".
Student: "It would be the area".
Interviewer: "Which area?".
Student: "The area under the curve".

This decreased the number of difficulties students encountered in the graphical problem, which in turn decreased the average number of difficulties.

In interview 4, the situation was reversed when the graphical problem was more straightforward than the functional problem. The graph provided in this interview was not linear, so none of the students attempted to find "coefficient of friction" as they did with the graph in interview 2. Instead, they went on to find the area under that graph and had no difficulty in this task. After calculating the area (in unit of Newton times degree), students had some difficulties with converting units to get to the right unit of work, which is Newton times meter. The function of rolling friction force versus angle given in interview 4 was not easy to handle for those students who did not really understand the concept of function and the meaning of integration. Some students did not know what to do with such a function, some asked the interviewer whether it meant "F is a function of θ" or "F times θ". Another difficulty came from finding work done by friction, for which purposing an integral of F(θ).dθ was not enough because the correct integral for work should be that of "F(θ)ds", in which "ds = R.dθ". The following excerpt demonstrates this difficulty:

Student: "I am not sure what to do with this function… Is this 'F times θ' or 'F is a function of θ'?".
Interviewer: "F is a function of θ".
Student: "So I should take derivative or integral of F".
Interviewer: "Okay, so derivative or integral?".
Student: "Derivative… I guess".
Interviewer: "You need to integrate force to find total work done by that force. So which integral should you take?".
Student: "Integral of F".
Interviewer: "With what variable?".
Student: "Theta". (Wrote down $\int F(\theta)d\theta$).

All students had difficulties in making use of the function and could only set up the right integral after several hints from interviewer. For students following the G-F sequence in interview 4, they had no major difficulty in finding area under the graph, but had some difficulties in converting the unit afterward. These difficulties were not due to the representational change, so they were not counted in our analysis of sequencing effect. However, those difficulties helped students with the functional problem that came later in which they could take an integral of F(θ).dθ to get the area under the graph of F(θ) versus θ and then converted units to get work. For students following the F-G sequence, the difficulties with units were actually included in the function because they knew that an integral of F(θ).dθ was not the right one for work and tried to set up the right integral of F(θ).ds which then led them directly to the correct value of work. This task increased the average number of difficulties per student with the functions.

Training Effect

In each sequence, we observed a significant difference in the average number of difficulties in the problem that came first (i.e., the graphical problem in the G-F sequence and the functional problem in the F-G sequence) and the one that came later in the interview. For example, in interview 2, students who followed the G-F sequence had an average of 3.50 difficulties per student with the graphical problems while having 1.75 difficulties per student with the functional problem. This trend was also observed in the F-G sequence in interview 2 and both sequences in interview 4. This appeared to indicate a training effect, meaning that students were trained to deal with change in representation of problems when they encountered the first change in representation, which then helped them deal with the second change more easily. In other words, in our interviews, the second transfer across representations occurred more easily than the first one. This result is particularly interesting in that it appears to indicate that when students learn to transfer their problem-solving skills from one representation to another, the first time they encounter a new representation prepares them to do so the second time as well, even though the actual representation (graphical or functional) may be different.

Students' Progress Through Interviews

In terms of difficulties caused by representational change, we could see progress of students throughout interviews (Tables 2 and 3). In the G-F sequence, students had an average of 5.25 difficulties per student in interview 2 while having only 0.75 difficulties per student with the same sequence in interview 4. This effect was not shown in the F-G sequence, however, mainly because the function in interview 4 was more difficult to handle than the function in interview 2.

When asked why they performed better in interview 4 compared to interview 2, students said that was because they had learned to work with graphical and functional representations through our interviews, as demonstrated in the following excerpt:

> Interviewer: "In the previous interview you didn't recognize that you have to calculate the integral and the area, but in this interview you recognized those methods. So is that what you learned from our previous interviews or from your calculus class?".
> Student: "From our interviews".

There were some increases in the number of difficulties in some categories as seen in Figure 1. More detailed analysis of the source of the difficulties indicated that those increases were due to the increase in complexity of the problems and not because of the change in representation. For example, the most obvious increase can be seen in the quantity category, which is due to the fact that in interview 4, students dealt with rotational motion including rotational energy and rolling friction, which were not included in interview 2.

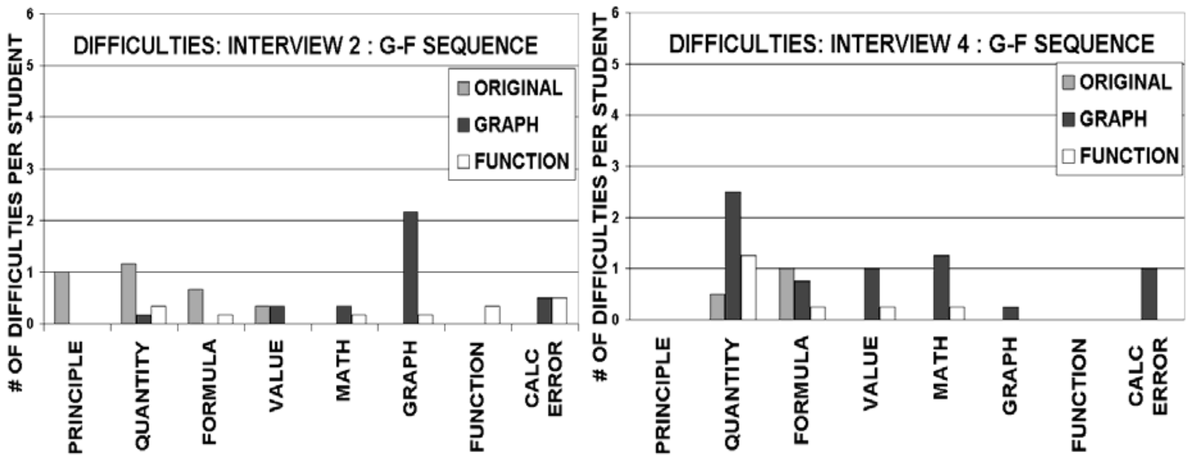

Figure 1. Average number of difficulties per student in interview 2 and interview 4 in the G-F sequence.

Conclusions

We summarize our findings which address each of the research questions as follows:

(1) Research question 1: What are the common difficulties that students encounter when solving mechanics problems in different representations?

Students encountered a variety of difficulties when solving mechanics problems in different representations as described in Table 1. Specifically, the difficulties with graphical and functional representations were due to students' inability to apply mathematical knowledge into physics contexts. With appropriate hints from the interviewer, all students were able to solve the problems correctly, and learned the methods of solving problems in representations other than numerical representation, which they did not have many opportunities to do in their physics class;

(2) Research question 2: How does the sequence of problems given to students affect their performance?

It appears from our results that different sequences of problems in different representations presented to students might lead to dissimilar performances of students. Our data also appear to indicate that students appear to get better trained at transferring from one representation to another. It seems to indicate that a problem in one representation might facilitate the ability to solve similar problems in other representations. Invariably, students had fewer difficulties with the second change of representation compared to the first one, regardless of the sequence of problems they got;

(3) Research question 3: How do the difficulties change as students progress through the semester?

We observed a decrease in the average number of difficulties each student encountered in each interview. There were some increases in the numbers of difficulties in some categories, but those were due to the increase in complexity of the problems and were not due to the representational change. As some students stated when asked, they did not learn to work with problems in different representations in their calculus or physics class, but from our interviews. This might suggest that the inclusion of problems in multiple representations in teaching would help students build their representational competence.

Potential Significance

This study provides a closer look at students' difficulties when solving physics problems in multiple representations. It constitutes a research-based database on which an online system can be built to better address students' needs for assistance when solving physics problems in numerical, graphic and functional representations.

This study also contributes to the body of knowledge in the area of use of multiple representations in solving problems in introductory physics. It informs us of the barriers that students encounter as they progress to representational competence, which is an important skill that future scientists and engineers should have.

We have found that students have significant difficulties in transferring their problem solving skills across representations. Our comparison of sequences of problems in different representations appears to indicate that no one particular sequence is better than the other; rather, it depends upon the context of the problem. However, we have also found evidence that students improve their ability to transfer across representations as they solve more problems in different representations, as well as over a longer period of time. This study underscores the importance of learning experiences that would facilitate students' transfer of problem-solving skills across representations. It also calls for further research in investigating these issues across other problem contexts and other domains.

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Appendix

Interview 1

(1) Problem 1

The position of an object moving along an x axis is given by $x = 3t^3 - 2t + 4$, where x is in meters and t in seconds.

(a) Find at least one time when the velocity is zero.

(b) What is the average acceleration between 0 and 3 seconds?

(c)What is the acceleration at $t = 3$ seconds?

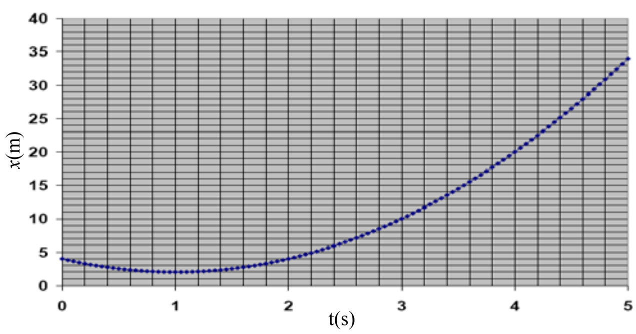
(2) Problem 2

The position of an object moving along an x axis versus time is given by the graph below, where x is in meters and t in seconds.

(a)Find at least one time when the velocity is zero.

(b)What is the average acceleration between 0 and 5 seconds?

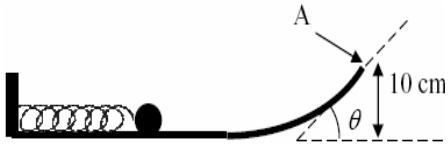
(c)What is the acceleration at $t = 3$ seconds?



Interview 2

(1) Problem 1

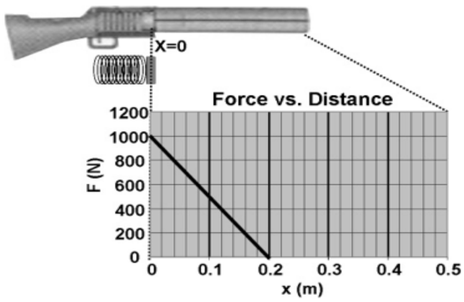
A spring of spring constant 3.0 kN/m is compressed a distance of 1.5 cm and a small ball is placed in front of it. The spring is then released and the small ball, mass 0.1 kg, is fired along the slope and launched into the air at point A which is 10 cm above the spring. The angle θ of velocity at launch is 30°. Friction is negligible.



What is the speed of the ball at the launch point (point A)?

(2) Problem 2

A 0.1 kg bullet is loaded into a gun (muzzle length 0.5 m) compressing a spring as shown. The gun is then tilted at an angle of 30° and fired.

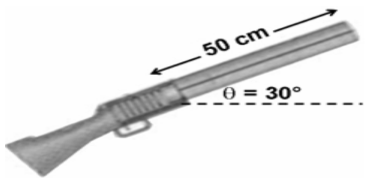


The only information you are given about the gun is that the barrel of the gun is frictionless and when the gun is held horizontal, the net force F (N) exerted on a bullet by the spring as it leaves the fully compressed position varies as a function of its position x (m) in the barrel as shown in the graph below.

What is the muzzle velocity of the bullet as it leaves the gun, when the gun is fired at the 30° angle as shown above?

(3) Problem 3

A 0.1 kg bullet is loaded into a gun (muzzle length 0.5 m) compressing a spring to a maximum of 0.2 m as shown. The gun is then tilted at an angle of 30° and fired.



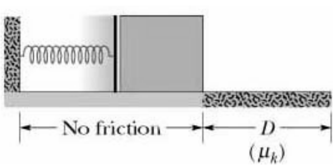
The only information you are given about the gun is that the barrel of the gun is frictionless and that the gun contains a non-linear spring such that when the held horizontal, the net force, F (N) exerted on a bullet by the spring as it leaves the fully compressed position varies as a function of the spring compression, x (m) as given by: $F = 1000\ x + 3000\ x^2$.

What is the muzzle velocity of the bullet as it leaves the gun, when the gun is fired at the 30° angle as shown above?

Interview 3

(1) Problem 1

A 3.5 kg block is accelerated from rest by a spring, spring constant 632 N/m that was compressed by an amount x. After the block leaves the spring it travels over a horizontal floor with a coefficient of kinetic friction $\mu_k = 0.25$. The frictional force stops the block in distance $D = 7.8$ m.

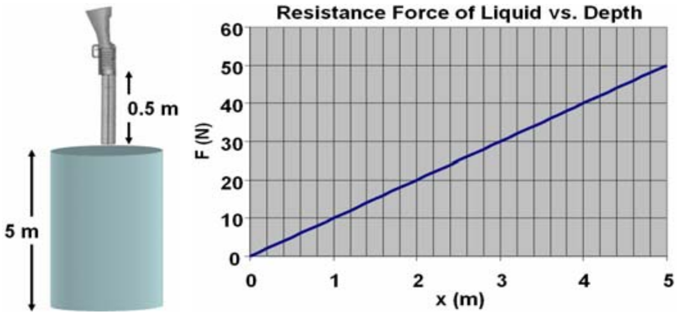


What was the spring compression x?

(2) Problem 2

A 0.1 kg bullet is loaded into a gun compressing a spring of spring constant k = 6,000 N/m. The gun is tilted vertically downward and the bullet is fired into a drum 5.0 m deep, filled with a liquid.

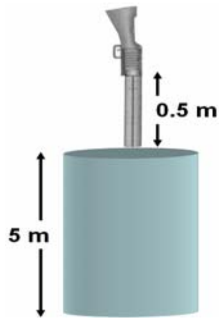
The barrel of the gun is frictionless. The resistance force provided by the liquid changes with depth as shown in the graph below. The bullet comes to rest at the bottom of the drum.



What is the spring compression x?

(3) Problem 3

A 0.1 kg bullet is loaded into a gun compressing a spring of spring constant k = 6,000 N/m. The gun is tilted vertically downward and the bullet is fired into a drum 5.0 m deep, filled with a liquid.



The barrel of the gun is frictionless. The frictional force F(N) provided by the liquid changes with depth x(m) as per the following function.

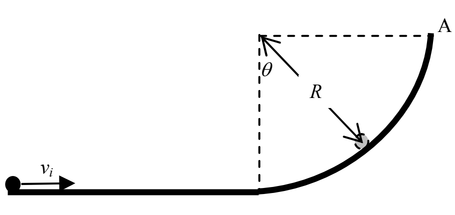
$$F = 10x + 0.6x^2$$

The bullet comes to rest at the bottom of the drum. What is the spring compression x?

Interview 4

(1) Problem 1

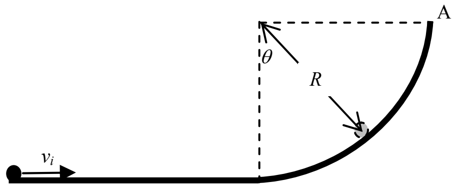
A hoop radius r = 1 cm and mass m = 2 kg is rolling at an initial speed v_i of 10 m/s along a track as shown. It hits a curved section (radius R = 2.0 m) and is launched vertically at point A.



What is the launch speed of the hoop as it leaves the slope at point A?

(2) Problem 2

A sphere radius r = 1 cm and mass m = 2 kg is rolling at an initial speed v_i of 5 m/s along a track as shown. It hits a curved section (radius R = 1.0 m) and is launched vertically at point A. The rolling friction on the straight section is negligible.



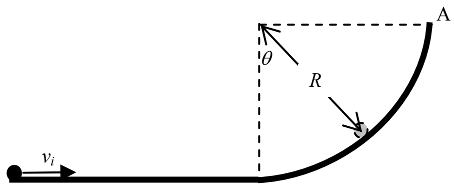
The magnitude of the rolling friction force F_{roll} (N) acting on the sphere varies as angle θ (radians) as per the following function:

$$F_{roll}(\theta) = -0.7\theta^2 - 1.2\theta + 4.5$$

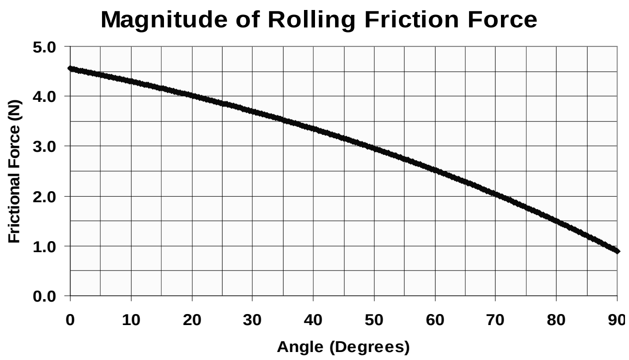
What is the launch speed of the hoop as it leaves the curve at point A?

(3) Problem 3

A sphere radius r = 1 cm and mass m = 2 kg is rolling at an initial speed v_i of 5 m/s along a track as shown. It hits a curved section (radius R = 1.0 m) and is launched vertically at point A. The rolling friction on the straight section is negligible.



The magnitude of the rolling friction force acting on the sphere varies as angle θ as per the graph shown below:



What is the launch speed of the hoop as it leaves the curve at point A?