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Validation Models of Turbulent Flow and Heat Transfer

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Abstract: A FEM (finite element method) based investigation is carried out for the determination of confined turbulent flow and associated heat transfer in a long straight channel. The effects of such flow in a zone close to a solid boundary have been investigated, by developing a finite element modeling technique based on near wall zone in which it replaces the traditional use of the empirical laws. The validity of the wall technique was investigated and well compared with other standard techniques.

Key words: FEM, heat transfer, internal turbulent flow, near wall zone modeling.

1. Introduction

An accurate analysis of turbulent fluid flow and heat transfer phenomena has numerous important applications in the fields of applied computer science as well as engineering. Among these are the flow in the blade passages of turbo machinery, heat exchangers and cooling system. Due to the complexity of these equations which governed the fluid motion by the N-S (Navier-Stokes) equations and the heat transfer by the energy equations, an analytical solution is intractable. During the last three decades, with the development and availability of more powerful digital computers much attention has been paid to different numerical methods for solving the resulting set of non linear partial equations which dominate the flow behaviour. One of these methods is the FEM [1-4] which established itself as a powerful tool, feasible, complementary and competitive alternative to other existing numerical methods. The aim ideal of the present work is to report a finite element based solution technique for general steady state, two dimensional, incompressible, confined, turbulent flow with heat transfer.

In dealing with confined turbulent, an effective technique is required to model the variation of the primitive variables in a zone close to a solid boundary, furthermore, the N.W.Z. (near wall zone), incorrect modeling can affect the values of the primitive variables throughout the flow domain.

In order to accommodate the rapid transfer of shear and variations in velocities, turbulent kinetic energy and temperature, within this zone, attention has been paid to model this zone accurately to obtain correct overall predictions particularly the transfer of shear from the solid wall, with associated large variations in velocity, turbulence kinetic energy, and temperature. If a conventional finite element is used to model the N.W.Z. then significant grid refinement would be required which would be costly in both computer storage and C.P.U. time.

Several solution procedures have been suggested in order to avoid such excessive spatial refinement [5-7]. A more common approach, widely adopted, is to terminate the computational domain (main domain) at some small distance away from the wall, where the gradients of independent variables are relatively small, and an appropriate technique is then used to model the flow behavior of the fluid and the heat in the N.W.Z.. In this zone, several techniques are investigated in the present work for predicting the distribution of the

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primitive variables in the N.W.Z. traditionally, in addition, the concepts of universal laws [8-9] are used, to depict the variables behaviour in near wall zone. In the present work, these concepts have been replaced by adopting a finite elements technique based N.W.Z.. The validity of the wall element technique was investigated and well compared with other standard techniques in along straight duct.

2. Mathematical Model

Equations which are commonly used to describe the momentum which implied by the Navier-Stokes equations (N-S) and the mass conservation by the continuity equation governed the state two dimensional flow of an incompressible viscous Newtonian fluid with no body forces acting are, respectively

$$\rho u_j \frac{\partial u_i}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j}\left[\mu_e\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right)\right] \quad (1)$$

and

$$\frac{\partial u_i}{\partial x_i} = 0 \quad (2)$$

where u_i is the velocities with respect to an orthogonal Cartesian coordinate systems x_i (where $i, j = 1, 2$), p is the local pressure, ρ is the fluid density, μ_e is the effective viscosity which is given by $\mu_e = \mu + \mu_t$, μ_t is the turbulent viscosity and μ is the dynamic viscosity of the fluid.

Eqs. (1) and (2) can not be solved unless a turbulence closure model can be provided to evaluate the turbulent contribution to μ_e. The simplest model is via an algebraic formula [10] which has limited application and therefore this model is not adopted in the present work, but an alternative (Prandtl [11]-kolmogorov [12]) model is used. For the present work, a one-equation model has been adopted so that,

$$\mu_t = C_\mu \rho k^{1/2} 1_\mu \quad (3)$$

where k is the turbulence kinetic energy, 1_μ is the length scale of turbulence which has been specified algebraically for the present purposes and C_μ is a constant, which is taken as 0.22. The distribution of k is depicted by the transport equation,

$$\rho u_j \frac{\partial k}{\partial x_j} = \frac{\partial}{\partial x_j}\left[\left(\mu + \frac{\mu_t}{\sigma_k}\right)\frac{\partial k}{\partial x_j}\right] + \mu_t \frac{\partial u_i}{\partial x_j}\left[\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right] - B \quad (4)$$

where B= $C_D \rho k^{3/2} / 1_\mu$, μ_t / σ_k is the turbulent diffusion coefficient, σ_k is the turbulent prandtl and C_D is a constant, which is taken as 0.418.

The temperature (T) distribution can be then obtained from an energy equation written in the form

$$\rho u_j \frac{\partial T_i}{\partial x_j} = \frac{\partial}{\partial x_j}\left[\left(\frac{\mu}{\sigma} + \frac{\mu_t}{\sigma_t}\right)\frac{\partial T}{\partial xj}\right] \quad (5)$$

In which μ_t / σ_t, σ, σ_t are respectively, the thermal molecular diffusivity of the fluid, the laminar Prandtl number which is usually taken as 0.7 and the turbulent Prandtl number which is given by:

$\sigma_t = 0.7$	$Y^+ \leq 5$
$\sigma_t = 1.4 - (0.7\,(13-Y^+))/8$	$5 < Y^+ \leq 13$
$\sigma_t = 1.4$	$13 < Y^+ \leq 17$
$\sigma_t = 0.95 + (0.45\,(25-Y^+))/8$	$17 < Y^+ \leq 25$
$\sigma_t = 0.95$	$Y^+ > 25$

where $Y^+ = (y \sqrt{(\tau_w \rho)}) / \mu$, τ_w is the wall shear stress, y is the normal distance measured away from the wall into the fluid. The above governing Eqs. (1), (2), (4) and (5) are then solved within the computational domain using a standard finite element method, in which the Galerkin weighted residual approach is adopted to solve the discretiing equations using the quadratic 8-noded elements to predict the primitive variables and linear 4-noded elements for pressure. Green theorem is used to reduce the order of the equations to unify resulting in a "weak formulation" which resulted in non-linear equation matrix which is solved then by using maching process. Within the N.W.Z. different techniques have been adopted as reported in the next section.

3. The Near Wall Zone Treatment

Once the computational domain is terminated at some distance away from the wall surface as shown in

Fig. 1, the boundary values at the interface between the computational domain (main domain) and the N.W.Z. can be estimated using conventional finite elements (i.e. 2-D elements up to the wall) to discretise the N.W.Z. and the variable values, by the following analysis, it needs an excessive mesh refinement was needed which is expensive in computer time and memory. In order to avoid such excessive refinement, semi-empirical equations, known as "wall functions" or the so-called "universal laws", are used to bridge from a solid boundary to the main domain, depicting the near wall behavior of the velocity parallel to the wall, the kinetic energy and the temperature. Ideally, the equations being used in an attempt to predict the flow field and heat transfer should be used within the N.W.Z. in a similar manner to that employed in the computational domain. Since fully developed flow was considered, a simplifying assumption can be made that the variations in the variables with respect to directions parallel to a solid wall can be considered very small when compared to those with respect to directions normal to the wall and therefore, the parallel variations can be ignored and the momentum equations can reduced. A pressure procedure, developed by Schneider [13], which implements the conservation of mass through the use of the pressure Poisson equation, in a direct manner has been utilized in this work. The inaccuracies associated with the use of one dimensional element in one direction normal to the wall have been investigated.

4. Boundary Conditions

One of the significant aspects of modeling the viscous flow is the location and the imposition of valid boundary conditions, especially at the downstream boundary. It is generally recognized that natural boundary conditions (i.e. zero gradients) should be used then either the location of the boundary at which the flow is fully developed is known with reasonable accuracy or infinite elements could be used. However, both can not be used in zones of appreciable advection. In recent years, traction boundary conditions have gained to use instead. In the present work, attention has been paid to the correct application of these conditions. Fullly developed profile for fluid and heat are assumed on all variables as forced boundary conditions at the upstream and tractions updated downstream, these tractions may written as;

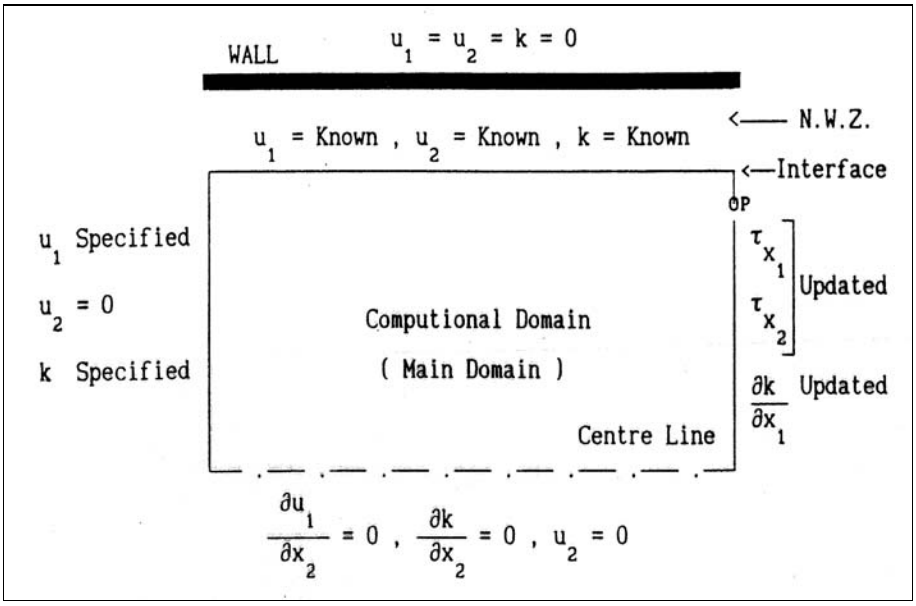

Fig. 1 Boundary conditions when the mesh is terminated at small distance away from the wall.

$$\tau_{x_1} = -p + \frac{\mu_e}{\rho}\left(\frac{\partial u_1}{\partial x_1}\right) \qquad x_1\text{—parallel to walls}$$

$$\tau_{x_2} = \frac{\mu_e}{\rho}\left(\frac{\partial u_2}{\partial x_1} + \frac{\partial u_1}{\partial x_2}\right) \qquad x_2\text{—normal to walls}$$

Zero gradients of turbulence kinetic energy as Neuman conditions are used at downstream. The updated traction technique can be equally applied to the energy equation, and is given by;

$$\left(\frac{\mu}{\sigma} + \frac{\mu_t}{\sigma_t}\right)\frac{\partial T}{\partial x_1} \qquad x_1\text{—parallel to walls}$$

On solid boundary boundaries, no slip condition was imposed and a constant heat flux is specified by imposing the temperature gradient value with respect to normal direction on the wall.

5. Results and Discussion

The validity of the wall element technique (1-D elements in one direction normal to wall (Fig. 2)) is tested by analyzing fully developed turbulent flow and associated heat transfer in a long straight duct of width *D*, which is taken as 1.0 in the present work and length *L*. Different Reynolds numbers of 50,000 and 12,000 based upon the width of *D* were investigated.

Compatible fully developed velocity and kinetic energy profiles were imposed at the upstream section when fully developed turbulent flow was considered at the first stage and the tractions were updated at downstream. These profiles were obtained by using the outlet values form each iteration as new approximations to the values at the inlet until a convergent condition is satisfied.

Convergent velocity profiles are presented in Figs. 3 and 4. Clearly, the results obtained from the adoption of the presently advocated technique (i.e. 1-D element normal to the wall) exhibit excellent agreement with the correct solution which resulted from the complete mapping (i.e. 2-D element up to the wall). These are superior to those obtained using universal laws. Fig. 5 shows an excellent agreement between the adopted technique and experimental results [14]. Once more, the "correct" values are remarkably close to those obtained from the advocated technique, as shown in Figs. 6-8 which refer to the velocity, kinetic energy and the turbulent viscosity.

It is clearly from the results obtained that the validity of the wall element technique (1-D elements in one direction) which has most advantageous owing to the number of elements used in the near wall zone is similar to those obtained from the use of 2-D

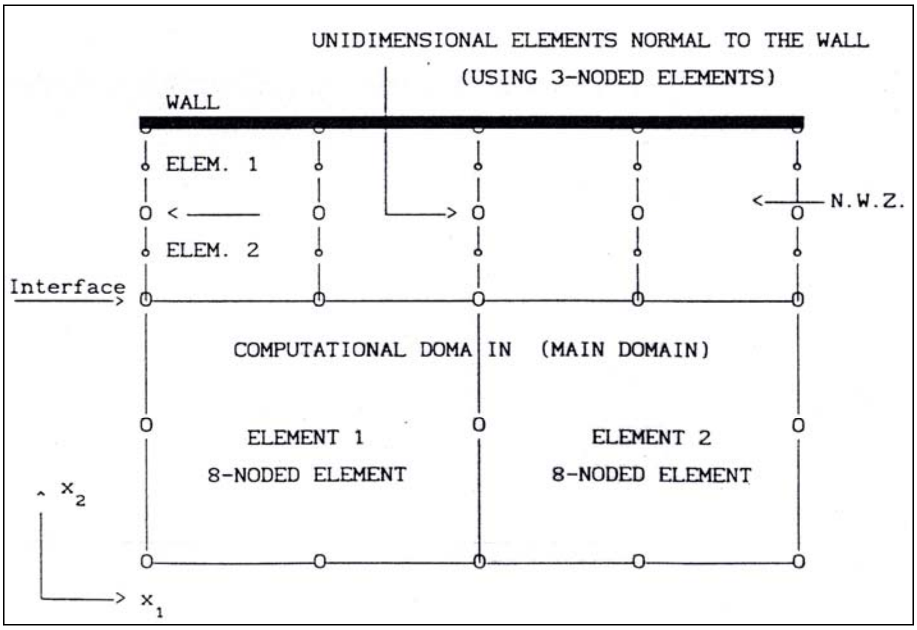

Fig. 2 One-dimensional elements in one-direction normal to the wall used in the N.W.Z. .

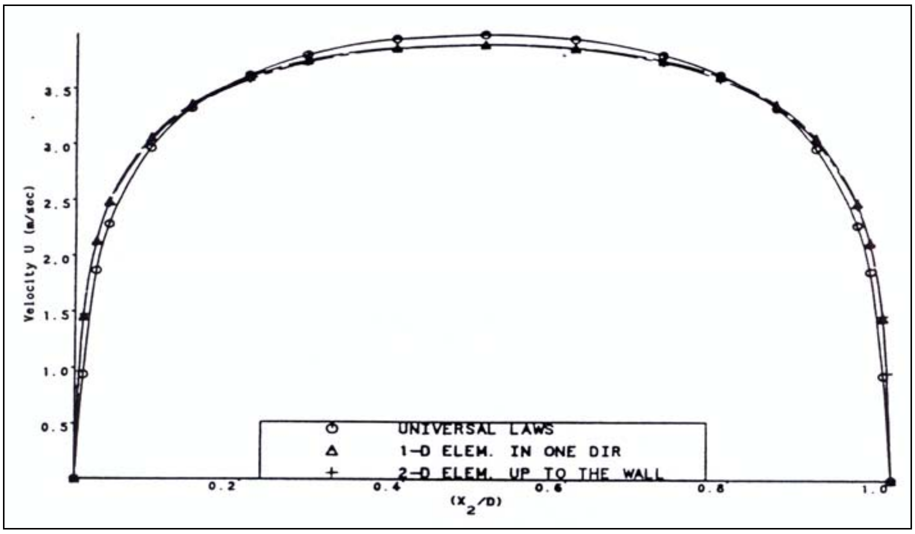

Fig. 3 Turbulent velocity profiles for fully developed flow, at 8*D* downstream, *L* = 8*D*, *Re* = 12,000.

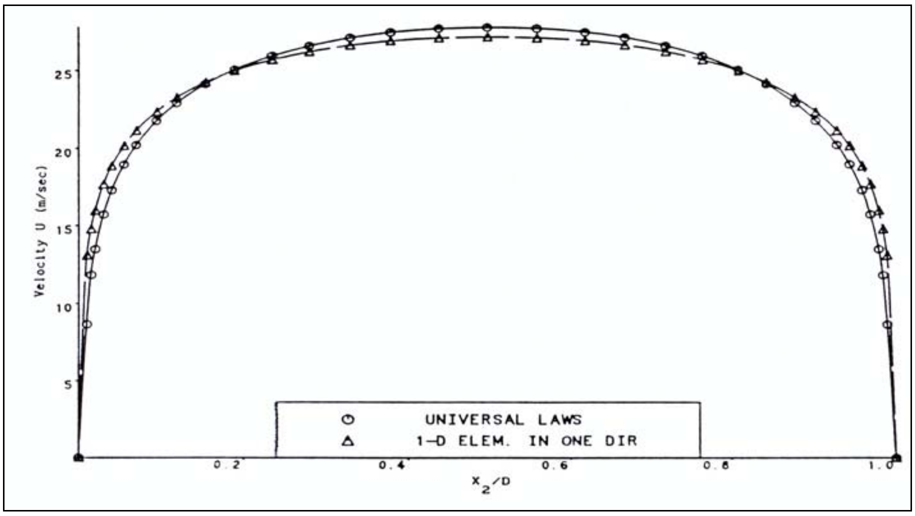

Fig. 4 Turbulent velocity profiles for fully developed flow, at 8*D* downstream, *L* = 8*D*, *Re* = 50,000.

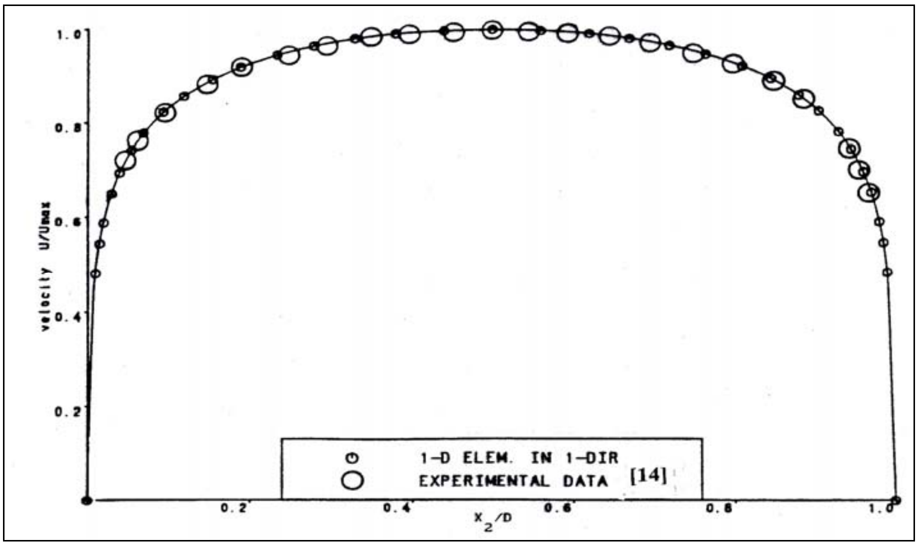

Fig. 5 Turbulent velocity profiles for fully developed flow, at 8*D* downstream, *L*=8*D*, *Re*=50,000.

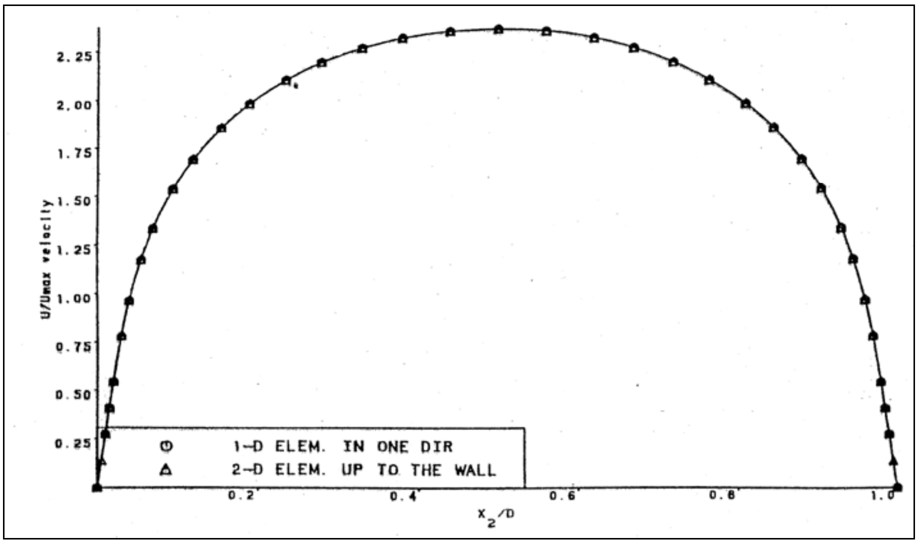

Fig. 6 Fully developed velocity profiles for turbulent flow, at 1.4 *D* downstream, *L* = 1.4*D*, *Re* = 12,000.

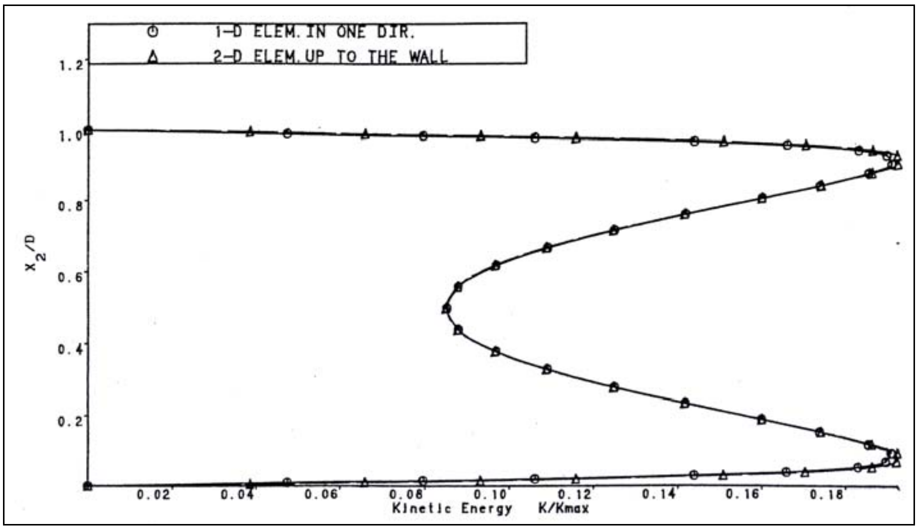

Fig. 7 Fully developed kinetic energy profiles for turbulent flow, at 1.4*D* downstream, *L* = 1.4*D*, *Re* =12,000.

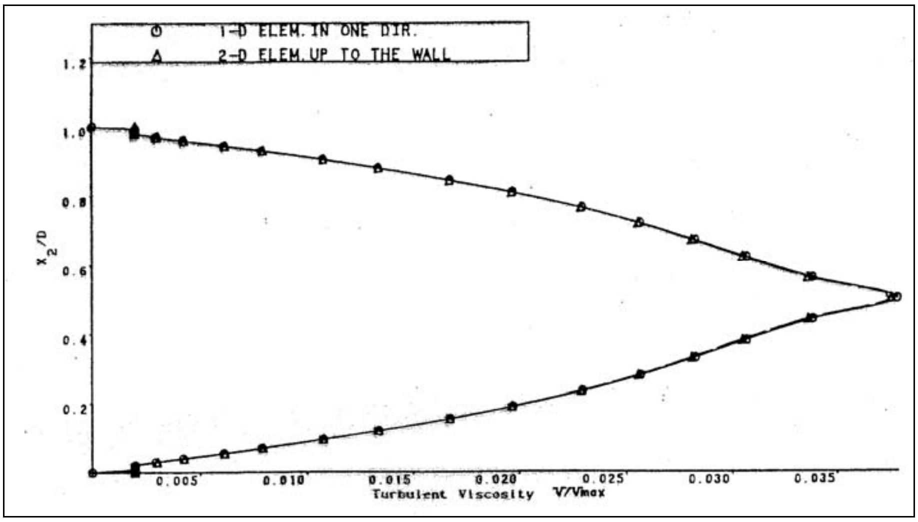

Fig. 8 Fully developed viscosity distribution profiles for turbulent flow, at 1.4*D* downstream, *L* = 1.4*D*, *R e*= 12,000.

element up to the wall which is not economic technique and needs excessive refinement when fully-developed turbulent flow was considered. The validity of the wall element technique (one-dimensional normal) has been tested and approved [15]. In the present work, this validity will be proved again when heat transfer is considered.

Temperature and velocity distributions were imposed at the duct inlet. Temperature values were those obtained after several solutions were undertaken [16-17], with constant heat flux boundary conditions at the solid walls, and downstream values of temperature re-imposed at the upstream for the following iteration. This follows the technique when updated tractions were used. This gave a smooth distribution of temperature compatible with the upstream flow conditions. Fig. 9 shows that the adopted technique and complete mapping are almost identical whilst approximately 5% discrepancy, maximum, exists when comparing these to the temperature profile obtained using universal laws and also shows a good agreement with the experimental results [18]. For lower *L*/*D* values, it says 4.5D in Fig. 10, the disparity is less but obviously increases with increasing distance downstream. Therefore, for this simple test case it is apparent that the introduction of a better mapping method is more important than for the flow analysis. Fig. 11 shows a large difference in values when considering temperature profiles in the longitudinal direction.

Heat transfer from a fluid is usually depicted in terms of a dimensionless Nusselt number (*Nu*). For present purposes, the local Nusselt number is defined as:

$$Nu = (q/ K_n) * (D/ \Delta T)$$

in which q is the local flux, K_n is the thermal conductivity of the fluid, D is the duct width and ΔT is the difference between the wall temperature at the wall/fluid interface and the local bulk fluid temperature. Fig. 12 presented the Nusselt numbers which indicate the advocated near wall element technique is far superior to the usually adopted universal profiles. The effect of small variations in temperature is now exaggerated when comparing Nusselt numbers. The differences are now approximately 18%; when considering universal laws and 3%; when one-dimensional elements are used.

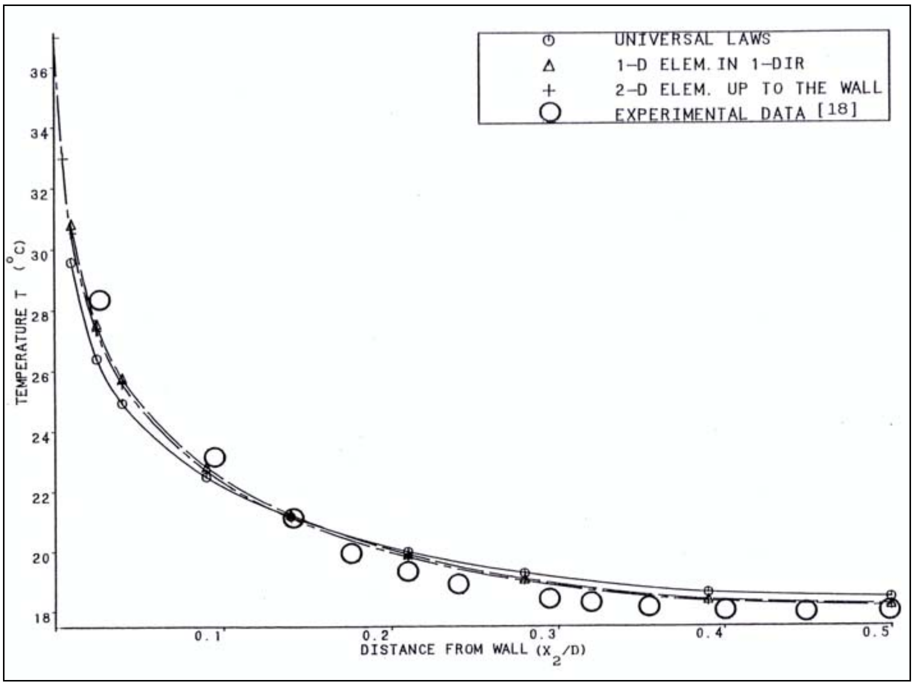

Fig. 9 Temperature profiles for fully developed flow, *L* = 10*D*, at interface 0.49*D*, *R e*= 12,000.

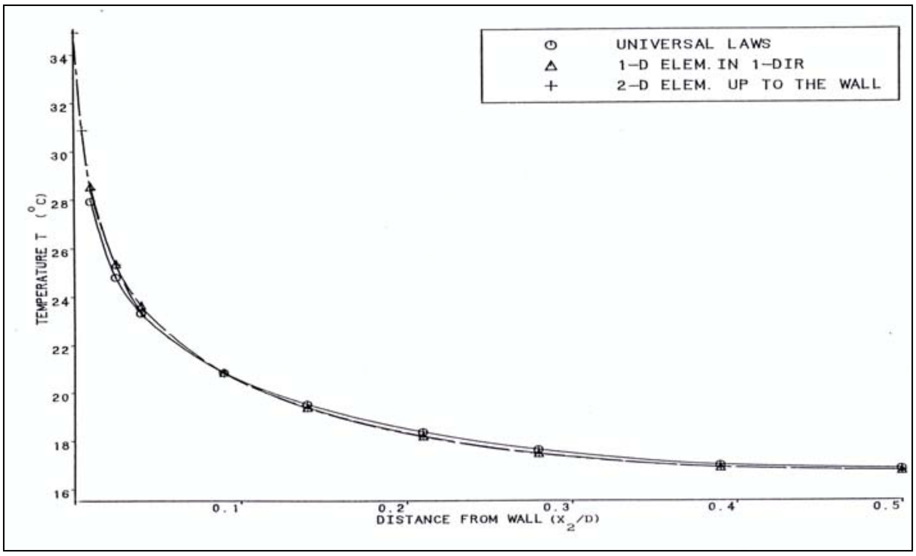

Fig. 10 Downstream temperature profiles for fully developed flow, $L = 4.2D$, at interface 0.49D, Re = 12,000.

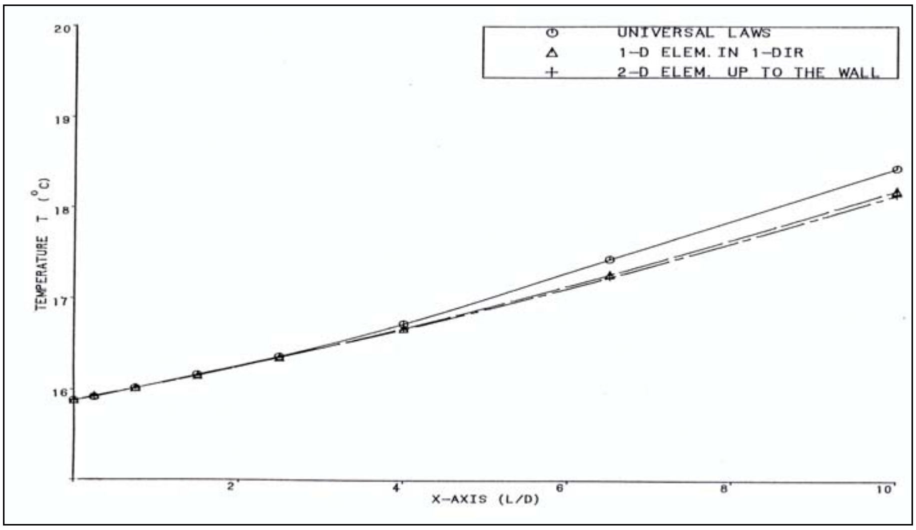

Fig. 11 Temperature distribution along the centre line for fully-developed flow, $L = 10D$, Re = 12,000.

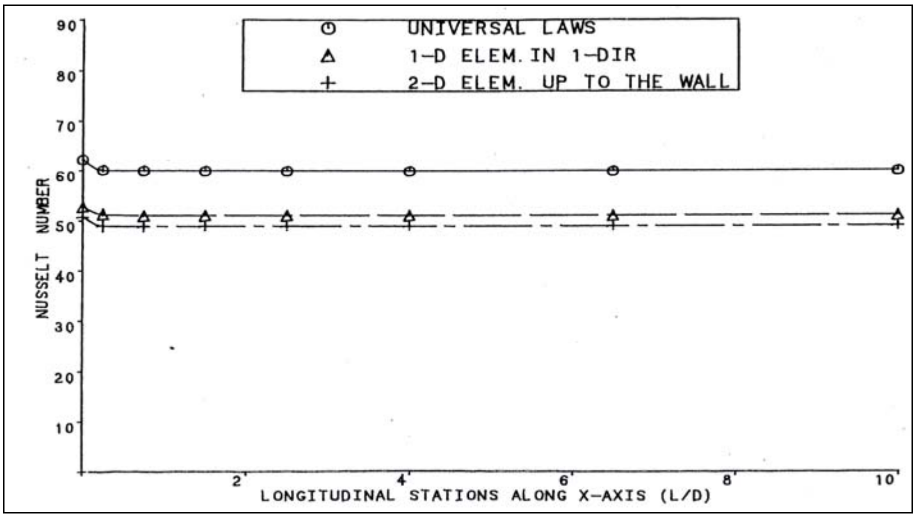

Fig. 12 Nusselt number variation along the channel for fully developed flow, interface at 0.49D, $L = 10D$, Re = 12,000.

Again, the advocated wall element technique has been demonstrated to be superior to that usually used and compares very favorably with an accurate, alternative calculation. This is primarily due to the inaccuracies in gradients of temperature when universal laws are used which reflects the same trend as when velocities are evaluated.

6. Conclusions

A near wall element technique, based on the use of F.E.M., has been used and demonstrated to be superior to the utilization of empirical universal laws. By introducing such a technique, the use of conventional elements (2-D elements up to the wall) in the near wall zone and associated excessive refinement can also be avoided. The near wall element technique can be used with confidence for fully developed turbulent flow and heat transfer.

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