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Reconstruction of the Decimal Number Line Concept Through Teacher's Scaffolding Practices

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This research investigated the reconstruction of the decimal line concept by Alice, the research subject, after she had received the four stages of scaffolding assistance: "multiple representations scenario", "explanation and demonstration", "concept reconstruction", and "concept extension". The research discovered that there were two reasons for Alice's incorrect understanding of the decimal line concept. First, Alice did not understand the concept of the counting system for decimals; and Second, she could not distinguish between the different decimal units on the various line segments after the decimal line was segmented. The research further discovered that under a scenario with multiple representations, Alice was able to effectively reconstruct the decimal counting system. With the help of explanation and demonstration of the equivalent decimal concept, she was also able to effectively use the counting system and equivalent decimal concept to reconstruct the decimal number line system and develop conceptual thinking to solve non-routine problems.

Keywords: decimal classification, decimal number line, equivalent decimal, scaffolding instruction, visual representation

Introduction

Representations are often employed in elementary mathematics teaching to help students reconstruct the link between different relevant concepts (Goldin, 1998). Compared to other representations (images, dienes blocks, and fraction board), however, a number line looks even more abstract, as it consists of both numeric symbols and a visual representation simultaneously. Nonetheless, number line representations are often used to link the relationship between integers, fractions, and decimals (Wiegel, 1998). Among these, the numeral system on the decimal line is similar to the integer system, whereas its visual representation is similar to that of the fractional system. The decimal line concept is greatly affected by the integer and fraction concepts, which is also shown by the decimal line concept representations of students through monitoring of their learning. However, research has shown that many students perform unsatisfactorily in their learning of the decimal line concept (Diezmann, Lowrie, & Sugars, 2010; Liang, 2003; Liu & Yang, 2009). This phenomenon has also occurred in the classes taught by the present paper's first author: The probability of students making mistakes becomes higher when there are varying numbers of decimal places on the decimal line.

Research has shown that teachers who use scaffolding instructions as part of their teaching strategies can help to unleash the potential of students, allowing them to attain maximum learning performance (Chien & Tsai,

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2012; Huang, 2006). This study hopes to build the understanding of the decimal line concept in a single student, Alice, through scaffolding instruction, which can help to effectively enhance the learning concepts of students.

Literature Review

Scaffolding Theory

The term "scaffolding" was developed from the concept of "the ZPD (zone of proximal development)" proposed by Vygotsky (Chen & Leou, 2005). Vygotsky (1978) posited that the development of a student has two levels: the level of actual development and the level of potential development. The gap separating these two levels is the ZPD, and students can bridge this gap with the help of adults or teachers.

Wood, Bruner, and Ross (1976) came up with the concept of scaffolding functions, which states that scaffolding teaching consists of six critical elements: recruitment, reduction in degrees of freedom, direction maintenance, marking critical features, frustration control, and demonstration. Wood, Bruner, and Ross (1976) further pointed out that these six elements can help new learners to effectively unleash their potential and build up capabilities with the active and planned guidance of experts, such as parents and teachers. From the scaffolding theory of Wood, Bruner, and Ross (1976), Anghileri (2006) developed "scaffolding practices" supportive of mathematics learning and possessing the three levels of: "environmental provisions", "explaining, reviewing, and restructuring", and "developing conceptual thinking". Teachers, who play an important role in the scaffolding process, can provide the best and most flexible scaffolding teaching method to students. It can be seen from the scaffolding teaching principles of Wood, Bruner, and Ross (1976) and Anghileri (2006) that those of the former are more inclined towards the technological aspects of the implementation, whereas those of the latter are more focused on scenario creation.

The decimal line scaffolding construction in this research shall be based upon the principles of Wood, Bruner, and Ross (1976) and Anghileri (2006). This research divided the scaffolding structure used for decimal line teaching into four stages: Stage 1 was the multiple representations scenario; Stage 2 included explanation and demonstration; Stage 3 entailed concept reconstruction; and Stage 4 consisted of concept extension.

The Structure of Decimal Notation

The notation rule for decimal symbols is an extension of the decimal classification and uses the 10 digits of 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9 in conjunction with decimal classification and the place value concept. Each place value can only record one digit, with each neighboring place value indirectly fulfilling a 10-time proportional relationship. Due to the decimal classification principle, the subsequent digit after nine has to be shifted to the left place value. As the numbering rule for decimal symbols fulfills the requirements of the decimal classification principle, the place value for each digit in the decimal system is 10 times that of the digit on its right, and 1/10 that of the digit on its left (Hiebert, 1992; Steinle & Stacey, 1998). Using 11.11 as an example, the value of 1 in the tenths place is 10 times that of the value of 1 in the ones place, whereas the value of 1 in the ones place is 1/10 that of the value of 1 in the tens place; the value of 1 in the ones place is 10 times that of the value of 1 in the tenths place, whereas the value of 1 in the tenths place is 1/10 that of the value of 1 in the ones place; the value of 1 in the tenths place is 10 times that of the value of 1 in the hundredths place, whereas the value of 1 in the hundredths place is 1/10 that of the value of 1 in the tenths place (see Figure 1).

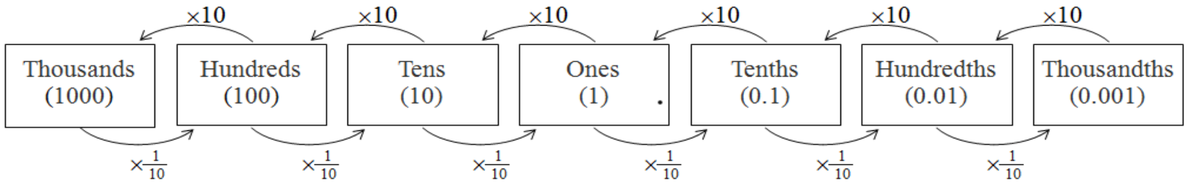


Figure 1. Endless base 10 chain.

Contents of Teaching Materials for Elementary Schools in Taiwan

In elementary school teaching materials in Taiwan, the teaching of decimals comes after students are taught integers and fractions (Ministry of Education, 2008). It can be seen from the ability index of the Grades 1-9 curriculum published by Taiwan's Ministry of Education that students begin learning the decimals for one place value and calculation (addition and subtraction) in their third year. They begin learning decimals for two and three value places as well as their calculation (addition and subtraction) and the calculation (multiplication) for integer multiples in their fourth year. In their fifth year, students begin learning multiple decimal places, calculation (addition and subtraction) for multiple decimal places, multiplication for decimal multiples, division of integer multiples, and division of decimals up to three places. In their sixth year, students learn division of decimal divisors. Among these concepts, the learning contents of one place, two places, three places, and multiple decimal places including the connotation, pronunciation, place value, place name, interchangeability between decimals and fractions, decimal line, decimal values, and other concepts.

Method

This study was a qualitative action research study which is loosely based on Lewin's (1946) framework. The participant was a single student, Alice. The data was collected in the form of original work by student, the teaching videotape, teacher's reflections, and information from student surveys.

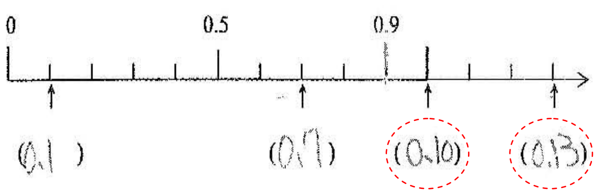
Alice's Unsuccessful Conceptions of the Decimal Number Line

Alice is currently a fifth-year student in elementary school, and has already mastered integers and decimals up to two places, as well as the decimal line up to two places. However, Alice was unable to solve the following decimal line questions (see Tables 1 and 2).

Test A

Table 1

Alice's Unsuccessful Conceptions of Decimals Through Tenths on a Number Line

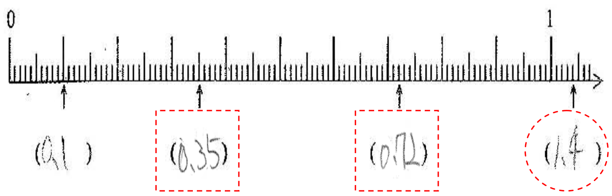
Multiple representation	Teaching approach/interaction
	Teacher: What are the place values as indicated by the arrow? Alice: 0.1, 0.7, 0.10, and 0.13. Teacher: How are the above values derived? Alice: As seen from the four spaces separating 0.5 and 0.9, each space represents 0.1. Thus, each spacing after 0 represents a value of 0.1. Teacher: How about other positions? Alice: Due to the sequence of 5, 6, 7, 8, 9, 10, 11, 12, and 13, the answers are 0.10 and 0.13.

Notes. ◌ represents a wrong answer; ⬚ represents the correct answer with incorrect reasoning.

Test B

Table 2

Alice's Unsuccessful Conceptions of Decimals Through Hundredths on a Number Line

Multiple representation	Teaching approach/interaction
	Teacher: Why are the positions on the number line indicated by the arrow 0.1, 0.35, 0.72, and 1.4 respectively? Alice: As each spacing is divided into 10 further spacing, each small spacing has a value of 0.1. Teacher: What about the rest of the values? How did you obtain them? Alice: This is because (finger pointing from 0 to the position of the second arrow) there are a total of 35 spaces, and thus 35 0.1 s, which makes it 0.35. Then (finger pointing from 0 to the position of the third arrow) there are a total of 72 spaces and thus 72 0.1 s, which makes it 0.72. Teacher: What about the last arrow? What was your reasoning for that? Alice: It is 1.4, because there are four spaces starting from the value 1.

Notes. represents a wrong answer; represents the correct answer with incorrect reasoning.

The number line in Test A is considered to be a number line for tenths, thus dividing 1 into 10 values for each segment, and thus, resulting in the decimal unit of 0.1 as a count unit. From Alice's answers in Test A, it can be seen that she knows that each segment on the number line is 0.1, and that a number line starts from 0 and increases towards the right with 0.1 as a counting unit. As the counting goes beyond 0.9, however, Alice did not follow the decimal classification principle, instead continuing the counting of the integer behind the decimal point.

The number line in Test B is a number line for hundredths, with two different types of segments. The first segment divides the value of 1 into 10 units, with the decimal unit of 0.1 as the count unit; the second segment divides the value of 1 into 100 units, with the decimal unit of 0.01 as the count unit. It can be seen from Alice's answers in Test B that she was not able to distinguish the different count units used for the two line segments. In addition, even though Alice was able to produce the correct answers of 0.35 and 0.72 in Test B, her reasoning behind those answers was wrong. This incorrect reasoning is similar to that behind her mistake in Test A. Therefore, we can conclude that Alice does not understand the decimal classification concept in the decimal symbol counting system.

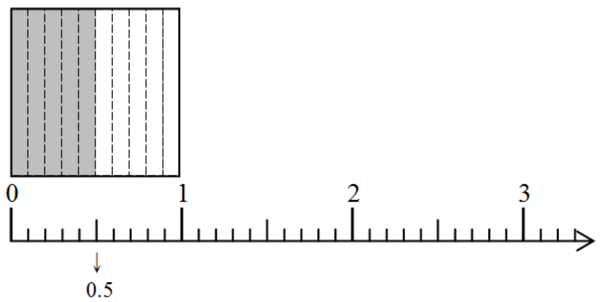
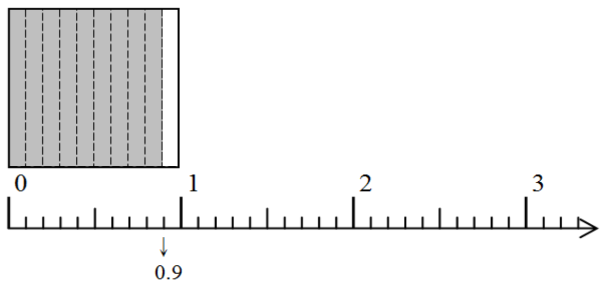
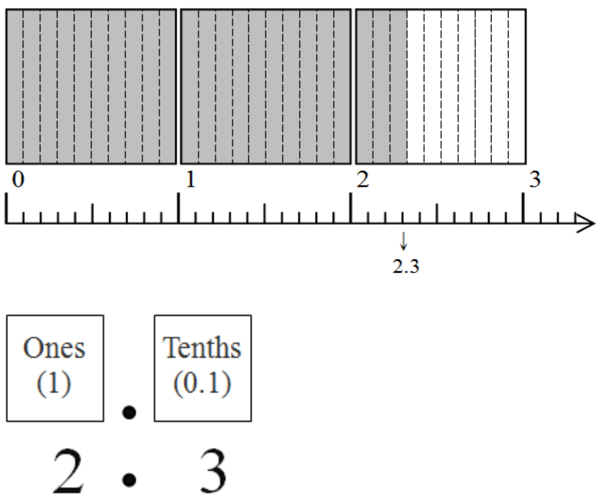
Teaching of Decimal Number Line With Scaffolding Practices

The reason for Alice's mistakes in understanding the number line is her lack of the concept of decimal classification, coupled with her inability to distinguish the count units of the different parts of the number line after segmentation. Martinie and Bay-Williams (2003) pointed out that for students with a weak concept of decimals, teachers should provide learning opportunities involving multiple representations and help strengthen the students' concepts through in-depth learning. After considering the scaffolding teaching principles proposed by Wood, Bruner, and Ross (1976) and Anghileri (2006), this research divided the scaffolding structure used for decimal line teaching into four stages as follows. Examples of teacher's four stages of scaffolding practices are below:

Stage 1: Multiple representations scenario. In Stage 1 of the scaffolding-assisted teaching, the teacher used the presentation of multiple representations to help Alice reconstruct the abstract concept of decimal classification on the decimal line. Teacher's uses of multiple representations and teaching approaches/interactions are showed in Table 3.

Table 3

Teacher's Teaching Approach With Multiple Representations Scenario

Multiple representation	Teaching approach/interaction
	Teacher: This is a square board, how many equal portions is it divided into? Alice: 10 equal portions. Teacher: What is the value of each portion when represented by decimals? Alice: 0.1. Teacher: What can you observe from this decimal numeral line? Alice: Numbers 0, 1, 2, and 3. Teacher: How many equal portions are there between 0 and 1 on the line? Alice: 10 equal portions. Teacher: What should the value of the decimal representing the gray portion be? Alice: 0.5.
	Teacher: This is a square board which is divided into 10 equal portions. How many of these portions are gray? Alice: 9 equal portions. Teacher: What is the value of the decimal representing the gray portion? Alice: 0.9. Alice: 1.0. Teacher: Adding 1 extra part would result in a size identical to a square board. How do you express the above meaning in decimals? Alice: 1.0.
	Teacher: There are 3 squares here, with each separated into 10 equal parts. Teacher: The first two squares each have 10 equal parts, with the gray portions represented by 2. Teacher: The gray portion of the third square only has 3 equal portions and can be represented by 0.3. Teacher: The value of the decimal used to represent the gray portion is 2.3.

Stage 2: Explanation and demonstration. In Stage 2 of the scaffolding-assisted teaching, the teacher explained and demonstrated the equivalent decimal on the decimal line. Examples are presented in Tables 4 and 5.

Table 4

Teacher's Teaching Approach With Explanation and Demonstration 1

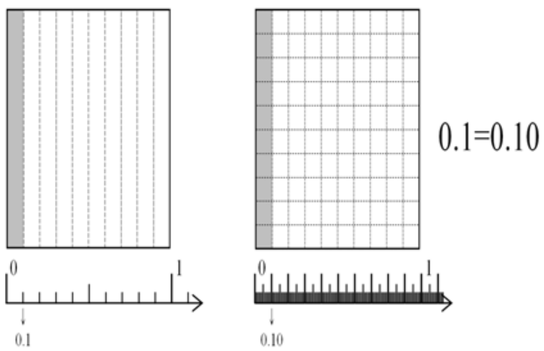
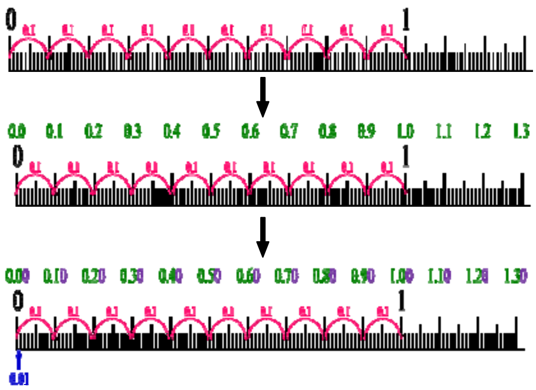
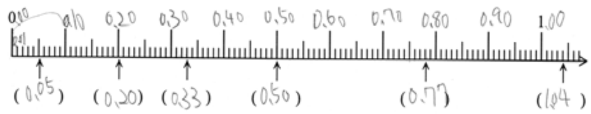
Multiple representation	Teaching approach/interaction
	Teacher: Extract one of the 10 equal parts segmented by the square board and represent it with 0.1. Ten portions would thus be equivalent to 1, also recorded as 1.0, 1 = 1.0, 1.0 is an equivalent decimal of 1. Extract 10 portions out of 100 after segmenting the square board; they can be represented by the decimal 0.10. 0.1 is equivalent in value to 0.10, therefore 0.1 = 0.10. 0.1 and 0.10 are equivalent decimals; 0.10 is the two place equivalent decimal of 0.1. How many portions on the 10-segment square board are equivalent to 0.2? 0.2 is equivalent in value to 20 portions on the square board segmented into 100 portions, therefore 0.2 = 0.20; 0.20 is the two place equivalent decimal of 0.2. Try and write out the two place equivalent decimals for 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1, 1.1… Alice: 0.3 = 0.30; 0.4 = 0.40; 0.5 = 0.50; 0.6 = 0.60; 0.7 = 0.70; 0.8 = 0.80; 0.9 = 0.90; 1 = 1.00; 1.1 = 1.10…

Table 5

Teacher's Teaching Approach With Explanation and Demonstration 2

Multiple representation	Teaching approach/interaction
	Teacher: The decimal 0.1 is used to represent the value of 1 line portion away from 0, or 0.10 if written in a two-place equivalent decimal form. Similarly, the decimal 0.2 is used to represent the value of 2 line portions away from 0, 0.2 written in a two-place equivalent decimal format is 0.20. The decimal 0.3 is used to present the value of 3 line portions away from 0, 0.3 written in a two-place equivalent decimal format is 0.30. 0.4 written in a two-place equivalent decimal format is 0.40, 0.5… Using the equivalent decimal concept, using decimals to represent the positions on the two-place decimal number line indicated by the arrows. Alice: 

Stage 3: Concept reconstruction. After undergoing scaffolding teaching involving the multiple representations scenario, along with explanation and demonstration, Alice was able to successfully solve the decimal line questions using decimal classification and the equivalent decimal concept (see Table 6).

Stage 4: Concept extension. After solving the single and two places value decimal line problem, Alice was able to use these two concepts as a basis to solve more sophisticated problems (see Table 7).

Table 6

Alice's Successful Conceptions of Decimals Through Tenths on a Number Line

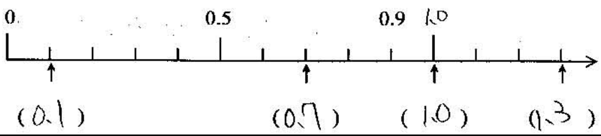
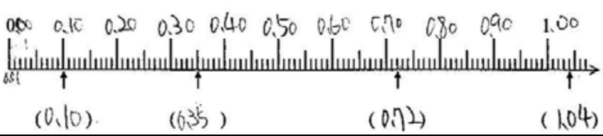
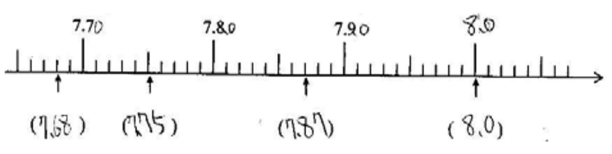
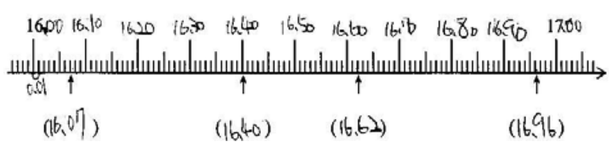
Test A	Test B
	

Table 7

Alice's Successful Conceptions of Decimals Through Hundredths on a Number Line

Test C	Test D
	

Conclusions and Teaching Implications

Irwin (2001) mentioned that the decimal concepts of students are often learned in school and are used as a basis for subsequent learning regarding decimals. Martinie and Bay-Williams (2003) further pointed out that mistakes made in learning about decimals are often caused by pre-experiences in the learning of integers. Therefore, teachers should strive to ensure that students fully understand the similarities and differences between integers and decimals, and conduct teaching through various types of decimal representations in order for students to gain the appropriate conceptual understanding. It can be seen from the responses of the research subject, Alice, that a conceptual understanding of integers affects that of decimals. Under the scaffolding teaching strategy designed for this research, Alice was able to understand the decimal classification concept through the guidance of the teacher and with the assistance of images depicting the concept of place values and representations. Alice was able to understand both the decimal classification and the equivalent decimal concepts, and was even able to solve decimal line problems that did not have a starting point of zero. We can see from Alice's case that she was able to truly understand the structure of a decimal numeral line when she could combine the concepts of integers, decimals, and place values. Therefore, teachers should provide multiple representations and through the concepts of equivalent decimals to help students understand decimal concepts and decimal line structures. This would allow students to understand the decimal symbols and visual representations on the decimal line.

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