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# Probabilistic Failure Analysis of Composite Beams for Optimum Ply Arrangements under Ballistic Impact

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**Abstract:** The probabilistic analysis takes into consideration an effect of scatter in elastic and strength properties of composite beam, and velocity of impactor. The damage model is implemented in the FE (finite element) code by a VUMAT (user-defined subroutine). The inter ply failure is modeled using cohesive surfaces between the plies. Dynamic response is obtained using explicit time domain integration approach. SFEA (stochastic finite element analysis) is used to study the initiation of fiber failure analysis due to ballistic impact. SFEA provided the critical stress input in the limit state which is computationally solved using reliability software. The random variation in these properties is used for determining statistics of stress in the lamina. These are compared to the random strengths in the limit state function and probability failure surface is obtained by using GPRSM (Gaussian process response surface method). GPRSM is used to predict the Pf (probability of failure) for different ply lay-ups arrangement. The Pf of Chang-Chang initiation of fiber failure for simply supported composite beams with symmetric cross ply lay-ups are (88.9%, 1.47% and 58.1%) greater than the anti-symmetric cross ply, symmetric angle ply and anti-symmetric angle ply, respectively. Sensitivity analysis is also carried out for symmetric cross ply arrangements.

**Key words:** Ballistic impact, composite, stochastic finite elements, limit state function, probability of failure.

## 1. Introduction

Laminated composites are increasingly being used as load bearing members in aircraft, spacecraft, missile, aerospace, automobiles, marine and body armor, etc. due to their excellent mechanical properties like high specific strength, specific stiffness, resistance to corrosion, increased fatigue life among others. However, some of these advantages are compromised because these materials experience uncertainties in their elastic properties, strength and load characteristics. An effect of these uncertainties on impact response of composites is very limited although the impact behavior of composites has been widely investigated. Abrate [1] provided a review that focused specifically on composite targets. The review outlined the literature related to impact response of composites, damage behavior and the residual properties of the composite. The damage mechanisms included the matrix cracking, debonding, delamination and fiber breakage. The energy dissipated in these mechanisms during impact has led to use of these in applications such as lightweight body armor. It was also defined the ballistic impact velocity. Analysis of deterministic behavior of composite plates subjected to high velocity impact and resulting damage has drawn much attention lately. Silva et al. [2] used experimental and finite difference method to study ballistic impact of thin laminated composite plates of Kevlar. The numerical model provided an estimate for limit perforation velocity and simulated failure modes and damage. Nishikawa et al. [3] estimated a numerical simulation to address the impact-induced deformation and damage of composite plates subjected to soft-body, high-velocity impacts for application to the bird-strike problem of composite fan blades. A new stabilized contact algorithm was developed based on the Lagrange multiplier method to

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predict appropriate impact forces applied to the plate, in order to solve soft-body impact at high velocity without causing severe numerical instabilities. The bird-strike impact on the composite fan blade was simply modeled by discussing the damage characteristics of a unidirectional composite plate. The fiber failure initiation was modeled using a modified form of Hashin [4] criterion and included the effects of normal stress in the direction of the fiber ($\sigma 11$) and transverse shear stress ($\tau 13$). Numerical analysis of the progressive damage failure model of the laminated composite plate was developed by Yen [5]. In this model, failure initiation and propagation laws were introduced to account for the fiber and matrix failure modes. It also included the effects of material strain rate on composite damage.

The fiber failure initiation was modeled using a modified form of Hashin criterion and included the effects of normal stress in direction of the fiber ($\sigma_{11}$) and transverse shear stress ($\tau_{13}$). The same fiber failure initiation criterion was adopted by Sevkat et al. [6] who combined experimental and 3D dynamic nonlinear FE (finite element) approach to study the damage in composite beams subjected to ballistic impact. The above review papers did not include the uncertainties of material properties and loading condition while considering the failure of material due to the damage initiation. For reliability assessment of the composite the simulations for prediction of the failure under impact, need to incorporate these uncertainties.

The variability in anisotropic composites arises due to the uncertainties in quantities such as volume fractions of matrix and fiber, excess amount of resins in the plies or laminates, curing methods, volume of voids and porosity in the matrix, alignment of fibers, bonding between fibers and matrix, temperature effects, etc.. Sriramula and Chyssanthopoulos [7] discussed the uncertainties in FRP (fiber reinforced plastic) composites and deterministic studies that have attempted to quantify the mechanical behavior of composite materials, and considerable differences are observed between theoretical predictions using micro-scale properties and experimental results at component level. Stochastic studies considered the uncertainties starting at the constituent level, ply level and component level. Response variability in composite structures due to uncertain elastic modulus (Liu et al. [8], Patel et al. [9]), uncertain Poisson's ratio (Noh [10]) and uncertain shear strengths properties (Wu et al. [11]) has been widely studied. Mathematical investigations in stochastic finite element have been followed by Matthies et al. [12]. Lal et al. [13] used higher order shear deformation theory to study the response of composite laminate with scattered material properties and random loading. The results demonstrated the importance of the randomness in the system parameters in the failure response of the plate.

The interactive failure criteria was proposed by Hashin [4] and used by Chang, F. K., and Chang, K. Y. [14] for ballistic impact, allows the identification of the failure mode. It considers four different failure criteria, namely the tensile fiber and matrix mode and compressive fiber and matrix mode. The maximum stress criterion for fiber tensile failure is a non-interacting maximum allowable stress criterion as proposed by Pinho et al. [15]. These criteria (maximum stress, Chang, F. K., and Chang, K. Y. [14] and Hashin [4] are used in the present study as limit state functions to identify the failure surfaces.

Different reliability methods have been adopted by various authors to predict Pf (probability of failure) for laminated composite plates. These authors use either Tsai-Wu or Tsai-Hill as a limit state function. MCS (monte carlo simulation) has been used by Joeng and Shenoi [16], FORM (first order reliability method) has been used by Boyer et al. [17], and Perturbation technique has been used by Park et al. [18], Onker et al. [19] and Engelstad and Reddy [20]. MCS requires a large number of FE executions for structural analysis making it computationally expensive especially for large and complex structures with high reliability. In order to reduce the computational time to an acceptable level, RSM (response surface methods) have been developed (Bucher and Bourgund [21]). In RSM, the actual limit state function is approximated, usually by a second order polynomial function. Current studies have investigated the limitations of the RSM and have shown that the method fails to estimate the probability of failure accurately in some problems with highly nonlinear limit state functions and in some problems with low probabilities of failure (Rajashekhar and Ellingwood [22]). EGRA (Efficient global reliability analysis) also called GPRSM (Gaussian process response surface method) used by Bichon et al. [23] and Patel et al. [24] has been shown to lead to more efficient estimation of probability of failure.

The reliability of composite beams subjected to impact by projectile has significant application in armor design. However, no noticeable work has been performed to investigate its probabilistic behavior under ballistic impact load considering random scatter in load.

In the current study, we carried out two important studies. First one is deterministic and another one is probabilistic. A progressive damage model is developed and implemented in the FE code ABAQUS. A VUMAT (user-defined subroutine) simulates the post impact progressive damage of the composite target. Numerical results are validated and found to be in good agreement with experimental findings available in the literature. This deterministic progressive damage in different modes is mainly used for validation purpose. The deterministic analysis of an energy balance model is used to for check the accuracy of numerical model. The variation of energy dissipation with time is obtained to demonstrate the total energy balance during impact. Probabilistic study is carried out considering the variability of material properties (elastic modulus, Poisson ratio, shear modulus and strength properties) and initial velocity. The random variation in material properties and initial velocities are used to determine the statistics of critical stresses in the lamina under impact. On substitution of these critical values and the random strength parameters in terms of their statistical characteristics and distribution in the limit state function a joint probability distribution is obtained. This multi dimensional domain due to joint probability distribution is a measure of probability of failure for each ply using reliability software. The GPRSM is used to calculate the probability of failure. This is a computationally efficient method adopted to investigate the probability of failure of composite beams with different ply arrangements. Comparative study of the probability of failure is carried out using different fiber damage initiation criterion. Optimum ply arrangement and sensitivity are carried out for fiber initiation of composite beam for simply supported boundary condition. This new risk based design methodology has the potential to optimize the structural design of the aircraft by reducing or eliminating the “overdesigning” of the aircraft while being a cost effective.

## 2. Numerical Modeling of Composite Beam

Ballistic impact in four composite beams of size 254 mm × 25.4 mm × 6.35 mm made of 24 layers of S2 glass-epoxy is simulated using FE. The plates studied are (a) symmetric cross ply; (b) symmetric angle ply; (c) anti-symmetric cross ply; (d) anti-symmetric angle ply. The SS (simply supported) boundary condition is imposed on the beams and used to study the probability of failure of the laminates. An optimum lay-up amongst these arrangements will be decided using the probability of failure. Each ply of the laminate is assumed to be transversely isotropic. The composite beam and impactor mesh with eight nodes brick elements. Full integration is used to avoid inaccuracies due to hour glassing. The impactor is made of Cu and its dimensions are taken from the literature (Sevkat et al. [6]). The material behavior of the impactor is assumed to be governed by

Johnson-Cook plastic hardening model [25] for ductile materials. Experimental values and respective statistical properties of glass epoxy as shown in Table 1, is taken from the literatures [6, 16]. The contact constraint is used to prevent interpenetration between the impactor and the beam. The coefficient of friction between the impactor and composite beam is taken as 0.3. Cohesive surfaces based on traction and separation law are employed to simulate and predict the extent of damage due to delamination. Cohesive surfaces modeling is used for the elastic stiffness, strength and fracture energy. Damage is assumed to initiate when the maximum contact stress reaches a limiting value. Damage evolution under exponential mixed mode loading is power law fracture energy taken. The isotropic damage variable evolves exponentially and reaches a value of one (composite failure) when sum of energies dissipated in various modes reaches a critical value of fracture energy (Gc). The stress-strain relation between normal stress and normal strain is assumed linear elastic. However, for shear behavior an empirical non-linear shear stress-strain relation suggested by Shi et al. [26] is used. Numerical modeling is carried out using ABAQUS supplemented with a user defined subroutine. The progressive damage modeling in different modes is used only for deterministic validation study. Once a failure has initiated in the plies the modulus is degraded by using a separate damage variable for unidirectional modes [5]. The probabilistic failure analysis carried out later uses fiber damage initiation model. These ideas are implemented in ABAQUS using a user subroutine VUMAT. The progressive damage model given by Matzenmiller et al. [27] and Yen [5] has been incorporated through user subroutine in ABAQUS. The dynamic explicit analysis is carried out to predict the extent of damage for each ply level.

## 3. Stochastic Finite Element Analysis

The probabilistic response under impact, essentially accounts for the most severe fiber damage initiation, occurring at the worst possible location in the composite beam while it is subjected to the highest loads conceivable. A probabilistic approach is a realistic solution that considers the stochastic variability and distribution of characteristic data of materials. In the deterministic study a beam under impact can not be guaranteed as absolutely safe because of the unpredictability of the loading, uncertainties in the material properties, the use of simplified assumptions in the analysis (which include limitations of the numerical methods used), and human factors (errors and omissions). Nevertheless, the probability of failure is usually required to be within a specified acceptable range for the analysis, design and optimization of a component. Now in this paper we are considering the uncertainty of the following composite material properties: $E_1$, $E_2$, $E_3$, $v_{12}$, $v_{23}$, $v_{31}$, $G_{12}$, $G_{23}$, $G_{13}$, $S_{22}$, $S_{12}$ $S_{13}$ and impactor velocity ($V$).

The relevant loads and resistance parameters, essentially random in nature, $X_i$ and the functional relationship between the response variable $Z(x)$ (e.g., stress at a point, deflection, etc.) and the random variables ($X_1$, $X_2$, $X_3$…, $X_N$) are described as:

$$Z(x) = Z(X_1, X_2, X_3 \ldots\ldots\ldots\ldots X_N) \quad (1)$$

A limit state function/performance function is hence defined as:

$$g(x) = Z(x) - Z(0) \quad (2)$$

$Z(0)$ is a limiting value of $Z(x)$, and an implicit or explicit function of random variables $g(x) = 0$ is a boundary region, $g(x) \leq 0$ is a failure region and safe region $g(x) < 0$. The Pf is estimated by the joint probability distribution in $f(X_1, X_2\ldots, X_N)$ and the integration is performed over the failure region $X$ where $g(x) < 0$.

$$P_f = \iiint f(X_1, X_2, X_3 \ldots\ldots\ldots\ldots X_N)\, dX_1 dX_2 dX_3 .. dX_N \quad (3)$$

The vector $X$ consists of material properties ($E_1$, $E_2$, $E_3$, $v_{12}$, $v_{23}$, $v_{31}$, $G_{12}$, $G_{23}$ and $G_{13}$), strength properties

**Table 1 Statistical characteristic of material properties and design variables [6, 16].**

| Material properties | Symbols | Mean values | Standard deviations | Distribution types |
|---|---|---|---|---|
| Modulus of elasticity in longitudinal direction 1 | $E_1$ | 44 GPa | 2.2 GPa | Normal |
| Modulus of elasticity in longitudinal direction 2&3 | $E_2 = E_3$ | 13 GPa | 0.65 GPa | Normal |
| Modulus of rigidity in 1-2 or 1-3 direction | $G_{12}= G_{13}$ | 3.15 GPa | 0.157 GPa | Normal |
| Modulus of rigidity in 2-3 direction | $G_{23}$ | 4.71 GPa | 0.236 GPa | Normal |
| Poisson's ratio 1-2 or 1-3 direction | $v_{12} = v_{13}$ | 0.057 | 0.0029 | Normal |
| Poisson's ratio 2-3 direction | $v_{23}$ | 0.36 | 0.018 | Normal |
| Strength in 1 direction under tension | $S_{11t}$ | 988 MPa | 122 MPa | Normal |
| Strength in tension 2 and 2 or 3 and 3direction | $S_{22t} = S_{33t}$ | 44 MPa | 5.5 MPa | Normal |
| Strength in 2 or 3direction under compression | $S_{22c} = S_{33c}$ | 285 MPa | 35.6 MPa | Normal |
| Strength in shear 2 and 3 direction | $S_{23}$ | 2.2 MPa | 0.28 MPa | Normal |
| Strength in shear 1and 2or 3 direction | $S_{12} = S_{13}$ | 6.06 MPa | 0.76 MPa | Normal |
| Impactor velocity | $V$ | 120 m/s | 10 m/s | Normal |

($S_{12}$, $S_{11}$ and $S_{23}$) and initial velocity ($V$). Experimental values and respective statistical properties such as standard deviations and distributions of glass epoxy have been adopted as shown in Table 1. The stresses ($\sigma_{11}$, $\tau_{13}$, $\tau_{23}$) and strains ($\varepsilon_2$, $\varepsilon_3$, $\gamma_{12}$, $\gamma_{23}$, $\gamma_{13}$) are linked with material and strength properties using reliability code as given expression:

$$(\sigma_{11}, \tau_{13}, \tau_{12}, \varepsilon_2, \varepsilon_3, \gamma_{12}, \gamma_{23}, \gamma_{13}) = \text{FE} \times (E_1, E_2, E_3, v_{12}, v_{23}, v_{31}, G_{12}, G_{23}, G_{13}, S_{12}, S_{11}, S_{13}, V) \qquad (4)$$

*3.1 The Performance Function*

Limit state functions/performance functions provide information of composite structures either safe or unsafe region. The limit state of the composite beam under impact is derived from Chang, F. K., and Chang, K. Y. [14] failure model. This is an interacting failure criterion where more than one stress components have been used to evaluate the different failure modes. In order to carry out a comparative probability of failure maximum stress, Hashin [4] and Chang, F. K., and Chang, K. Y. [14] failure models are used. The limit state functions are performed for the comparative study of the composite beam under impact. The out-of-plane fiber damage initiation failure tension and compression are determined from the following equations:

Maximum Stress Failure Criteria:

$$\text{Fib(T)} = \frac{\sigma_{11T}}{S_{11T}} \geq 1 \quad \sigma_{11} \geq 0 \qquad (5)$$

and

$$\text{Fib(C)} = \frac{\sigma_{11C}}{S_{11c}} \geq 1 \quad \sigma_{11} \leq 0 \qquad (6)$$

Hashin Failure Criteria:

$$\text{Fib(T)} = \left(\frac{\sigma_{11T}}{S_{11T}}\right)^2 + \left(\frac{\tau^2_{12} + \tau^2_{13}}{S^2_{13}}\right) \geq 1 \quad \sigma_{11} \geq 0 \qquad (7)$$

and

$$\text{Fib(C)} = \frac{\sigma_{11C}}{S_{11c}} \geq 1 \quad \sigma_{11} \leq 0 \qquad (8)$$

Chang-Chang Failure Criteria:

$$Fib(T) = \left(\frac{\sigma_{11T}}{S_{11T}}\right)^2 + \left(\frac{\tau_{13}}{S_{13}}\right)^2 \geq 1 \quad \sigma_{11} \geq 0 \qquad (9)$$

and

$$\text{Fib(C)} = \frac{\sigma_{11C}}{S_{11c}} \geq 1 \quad \sigma_{11} \leq 0 \qquad (10)$$

where, Fib($T$) and Fib($C$) are fiber failure due to tension and compression, $\sigma_{11T}$ and $S_{11T}$ are the tensile stress and associated strength in the longitudinal (fiber) direction and $\sigma_{11C}$ and $S_{11C}$ are the compressive stress and associated strength in the longitudinal (fiber) direction respectively. $\tau_{12}$, $\tau_{13}$ and $S_{13}$ are the in-plane and out-of-plane shear stresses and strength between fibers and matrix, respectively. Limit states are established according to the above criteria leading to a comparative probabilistic analysis.

This integral is presently computed by the standard Monte Carlo procedure [16]. Although, the method is inherently simple, the large numbers of output sets are generated to build an accurate cumulative distribution function of the output variables. It makes it computationally expensive. Furthermore, the need for a large nonlinear finite element analysis makes the computation prohibitive. For the present problem GPRSM is also used to obtain the probability of failure. The efficiency of this method is compared with the Monte Carlo method. GPRSM adopts the steps as described below.

*3.2 Gaussian Process Response Surface Method*

A probabilistic investigation by GP (Gaussian process) model is dissimilar from other substitute models because they provide not just a predicted value at an un-sampled point. The generalized linear regression model with a mean value and predicted variance is described by Bichon et al. [23] and Patel et al. [24]. The true response function, $g(x)$ is given by Cressie [28] as follows:

$$g(x) = H(x)^T \beta + E(x) \tag{11}$$

where, $H(x)$ is the trend of the model, $\beta$ is the vector of trend coefficients, and $E(x)$ is a stationary Gaussian process with zero mean that describes the departure of the model from its underlying trend. Any function could be assumed by the trend of the model, but taking it to be a constant value is generally sufficient [28]. Reliability code employs a constant trend function and β is determined through a generalized least squares estimate. The covariance between outputs of the Gaussian process $E$ at points $c$ and $d$ is defined as:

$$\mathrm{Cov}[E(c), E(d)] = \sigma_E^2 R(c,d) \tag{12}$$

where, $\sigma_E^2$ is the process variance and $R(c, d)$ is the correlation function. There are several options for the correlation function, but the squared-exponential function is common, and is used here for $R$:

$$R(c,d) = \exp-[\sum_{i=1}^{l} \theta_i (c_i - d_i)^2] \tag{13}$$

where, l represents the dimensionality of the problem, and $\theta_i$ is a scale parameter that governs the degree of correlation between the points in terms of dimension $i$. A large $\theta_i$ is representative of a short correlation length. The mean value $\mu_g(x)$ and variance $\sigma_g^2(x)$ of the Gaussian process model prediction at point $x$ are:

$$\mu_g(x) = H(x)^T \beta + R(x)^T R^{-1}(g - F\beta) \tag{14}$$

$$\sigma_g^2(x) = \sigma_E^2 - [H(x)^T R(x)^T] \begin{bmatrix} 0 & F^T \\ F & R1 \end{bmatrix}^{-1} \begin{bmatrix} h(x) \\ R(x) \end{bmatrix} \tag{15}$$

where, R($x$) is a vector containing the correlations between $x$ and each of the $n$ training points, $R1$ is an $n \times n$ matrix containing the correlation between each pair of training points, $g$ is the vector of response outputs at each of the training points, and F is an $n \times q$ matrix with rows $H(x_i)^T$ (the trend basis function at training point $i$ containing $q$ terms; for a constant trend $q = 1$). This form of the variance accounts for the uncertainty in the trend coefficients $\beta$, but assumes that the parameters governing the covariance function ($\sigma_E^2$ and $\theta$) have known values. The parameters $\sigma_E^2$ and $\theta$ are determined through maximum likelihood estimation. This involves taking the log of the probability of observing the response values $g$ given the covariance parameters, which is given by Sacks et al. [29] as:

$$\log[p(g/R1)] = -\frac{1}{n}\log|R1| - \log(\sigma_E^{*2}) \tag{16}$$

where, $|R1|$ indicates the determinant of R1, and $\sigma_E^{*2}$ is the optimal value of the variance given an estimate of $\theta$ and is defined by:

$$\sigma_E^{*2} = \frac{1}{n}(g - F(B)^T R1^{-1}(g - FB) \tag{17}$$

where, $B$ is the generalized least squares estimate of $\beta$ from:

$$B = [F^T R1^{-1} F]^{-1} F^T R1^{-1} g \qquad (18)$$

Maximizing Eq. (13) gives the maximum likelihood estimate of $\theta$, which in term defined as $\sigma_E^2$. In addition to the CDF (cumulative distribution function) of the response, the GPRSM technique provides additional information regarding the sensitivity of the response to the random variables. The magnitude of the sensitivity factor provides a way to rank the random variables that have the major influence on the uncertainty of the response variable. By controlling the scatter in the more significant variables, the reliability can be improved.

In the present study stresses in an individual lamina are fundamental to control the failure initiation in the laminate. The strength of each individual lamina is assessed separately by considering the stresses acting on it along material axes. The fiber failure initiation of the last ply lamina is prescribed by a failure criterion adopted and given above.

## 4. Numerical Results

For numerical study the data is obtained from the Ref. [6]. The respective statistical data adopted as reported in the Refs. [6, 16]. Explicit time domain analysis has been carried out to obtain the response. The coefficients of variation of Young's modulus, Poisson'S ratio and strengths properties are assumed 10% of the mean. The additional details of beam and the impactor are described in section 2.

### *4.1 Validation Study*

The results used for validation study are produced by Sevkat et al. [6] who performed experiments for an impact test using a projectiles acting at velocities ranging from 120 m/s to 320 m/s. The S2 glass-epoxy laminated composite beams are used. The finite element analysis results, obtained presently are validated by comparing the number of damaged layers with those of Sevkat et al. [6]. Fig. 1 show that the number of damaged plies for cross ply arrangement in a simply supported composite beam obtained presently. It is observed that there is a linear relation between impactor velocities (120 m/s to 300 m/s) and number of damaged layers. The number of damaged plies obtained by the present simulations shows nearly the same trend and values of experimental results produced by literature.

### *4.2 Energy Balance Model*

The energy balance initiation model is basically used for checking the accuracy of numerical modeling of composite beam under impact at velocity of impactor 120 m/s. The energy dissipation prior to fiber damage initiation is investigated. Matrix cracking has already occurred at this time. The time histories of KE (kinetic energy), SE (strain energy), IE (internal energy), TE (total energy) and FE are shown in Fig. 2. It also shows the magnitude of energy balance with time (90 μs). The magnitudes of KE, IE and FE of the target are observed to be significant at damage initiation of composite fiber. However, other energies like DE (damage energy), PE (plastic energy) of the impactor and artificial energy (reduced integration for impactor), etc. are not found to be significant. The overall kinetic energy of the impactor decreases with time as it is converted to KE, SE, and FE of the beam. This state of energy dissipation occurred at the time of damage initiation which is the

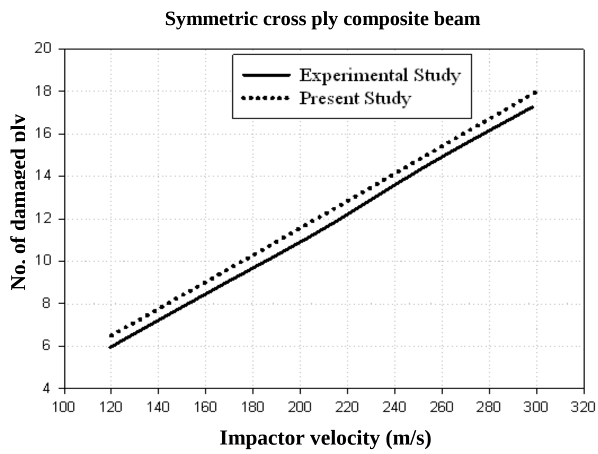


**Fig. 1 Validation study of simply supported symmetric cross ply composite beam.**

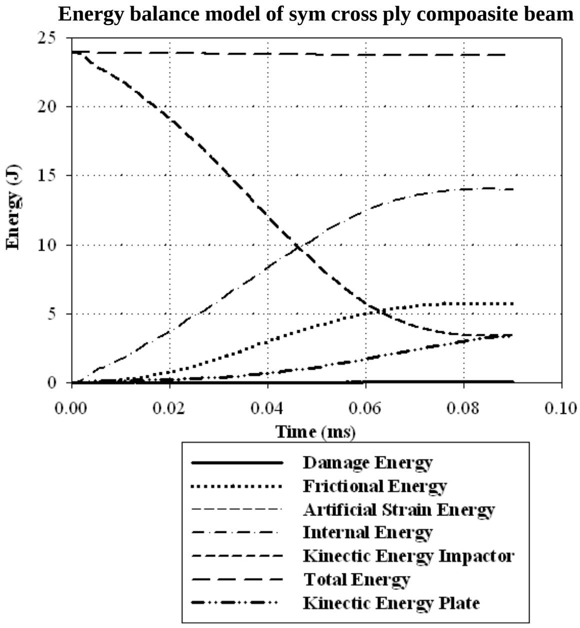


**Fig. 2 Energy time history of fixed composite beam.**

safety limit of the target beam.

*4.3 Computational Efficiency of Reliability Methods*

An impact at a velocity of 120 m/s showed the fiber initiation failure for the bottom most ply of the symmetric cross ply composite beam. Cumulative density function plot (Fig. 3) shows the estimation of cumulative probability of failure against variation of response (Z) using MCS (Monte Carlo simulation) and GPRSM. The comparison shows that the Pf obtained from MCS and GPRSM are very close. However, MCS method required 5,000 cycles to determine Pf and to reach a constant value. It is computationally 10 times expensive in comparison to GPRSM. The GPRSM is able to reduce the time consumed and is computationally efficient while maintaining an acceptable accuracy.

*4.4 Probability of Failure of Composite Beam*

The recommended design value of Pf for the composite application under study by Goh et al. [30] lies between $10^{-3}$ to $10^{-5}$. For Pf ranging from $10^{-5}$ to $10^{-7}$ the system is considered to be conservatively designed, for Pf below $10^{-7}$ it is considered to be over designed. If the Pf is larger than $10^{-3}$ then the system

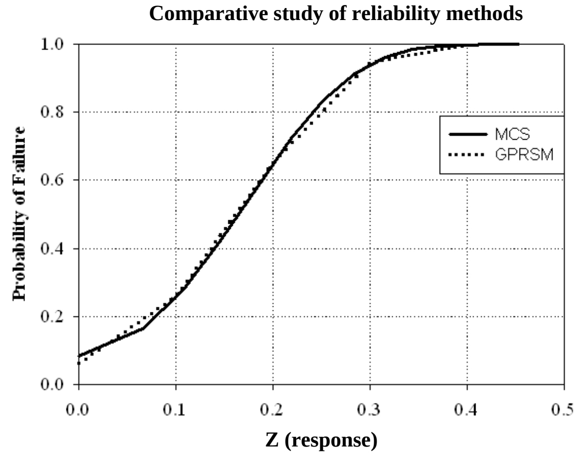


**Fig. 3 Probability of failure for symmetric cross ply composite beam using different reliability methods.**

is considered to be unacceptable design. It is found that the simply supported composite beam the Pf for 22nd ply (anti-symmetric cross ply) lamina is the minimum Pf compare with other ply lay-up arrangements, namely, symmetric cross ply, symmetric angle ply and anti-symmetric angle ply. However, its Pf is not suitable for acceptable probabilistic design Pf (0.001 to 0.0001). Hence, acceptable probabilistic failure is carried out to go through 23rd to 24th ply (bottom ply) lamina to determine the probability of failure.

The probability of fiber failure for bottom most ply predicted by maximum stress, Chang-Chang and Hashin criteria for different ply lay-ups and simply supported boundary conditions are listed in Fig. 4 and

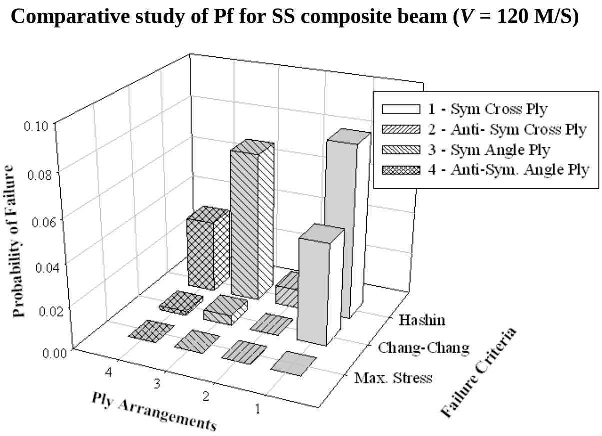


**Fig. 4 Probability of failure for composite beam for SS (simply supported) boundary condition.**

Table 2. Comparative study of different initiation fiber failure criteria is carried out in terms of Pf at an impactor velocity 120 m/s. It is observed that the Pf for maximum stress criterion is not significant influence because Eqs. (5) and (6) show the fiber normal stress does not reach the permissible value of fiber normal strength. Hence, the fiber failure is occurred only due to the in-plane shear stress ($\tau_{12}$) and out of plane stress ($\tau_{13}$). Chang-Chang and Hashin failure criterion for different boundary conditions are discussed as follows.

*4.5 Comparative Study of Probability of Failure*

As shown in Table 2 the probability of failure of the bottom most ply for symmetric cross ply arrangement (Case-I) Chang-Chang criterion is (41.2%) lesser than that for the Hashin criterion (same ply arrangement). Similarly, the probability of failure of symmetric angle ply arrangement (Case-III) Chang-Chang criterion is (94.27%) lesser than that for the Hashin criterion. It is also found that the probability of failure of anti-symmetric cross ply arrangement (Case-II) Chang-Chang criterion is (99.5%) lesser than that for the Hashin failure criterion. Similarly, the Pf of anti-symmetric angle ply arrangement (Case-IV) Chang-Chang criterion is (94.4%) lesser than that for the Hashin failure criterion. The Pf of fiber initiation of anti-symmetric cross ply lay-ups (Case-II) is lesser Pf than the other ply lay-ups namely, symmetric cross ply, symmetric angle ply and anti-symmetric angle ply.

*4.6 Design Optimization for Ply Lay up Arrangements*

The probability of fiber failure initiation for bottom most ply predicted by Chang-Chang criteria for different ply lay-ups and simply supported boundary conditions are listed in Table 2. The ply arrangements for optimum design of simply supported boundary conditions are discussed as follows.

As shown in Table 2, the Pf of the bottom most ply for symmetric cross ply arrangement (Case-I) is (90.3%) more than that for symmetric angle ply laminate arrangement (Case-III). Similarly, the Pf of bottom most ply for an anti-symmetric cross ply arrangement (Case-II) is (97.9%) lesser than that for an anti-symmetric angle ply laminate (Case-IV). It is also observed that the Pf for symmetric cross ply arrangement (Case-I) is (99.9%) more than that for an anti-symmetric cross ply arrangement (Case-II). Similarly, the Pf for symmetric angle ply arrangement (Case-III) is (58.69%) more than that of the Pf for anti-symmetric angle ply arrangement (Case-IV). Anti-symmetric cross ply lay-ups (Case-II) is found that the minimum Pf for other ply lay-ups namely symmetric cross ply, symmetric angle ply and anti-symmetric angle ply. Cumulative probability distribution is an important property of the system to optimize with respect to statistical properties of random variables to achieve the required reliability level.

## 5. Sensitivity Analysis

The Gaussian process response surface method is used for the probabilistic sensitivity analysis that identifies the variables that contribute most to the reliability of the design. Sensitivity analysis provided information on Pf with respect to changes in the mean value or the standard deviation of each random variable. This technique is used for systematically changing the strength and resistance parameters in a model to optimize the design to achieve target reliability. These normalized sensitivities are shown in Fig. 5 and allow the designer to evaluate the effect of the probability of failure due to a change in design parameters.

Sensitivity levels ($\psi 1$ & $\psi 2$) indicate the influence of mean ($\mu$) and standard deviation ($\sigma$) on the probability of failure "$p$" or Pf. The following equations in terms of non dimensionalized parameters $\psi 1$ & $\psi 2$ stated by the following equations:

$$\psi_1 = \left(\frac{\delta p}{\delta \mu}\right)(\sigma / p) \tag{15}$$

**Table 2 Comparative study of Pf of simply supported composite beam for damage initiation using GPRSM.**

| Cases | Ply lay-ups arrangements | Maximum stress (Pf) | Chang-Chang (Pf) | Hashin (Pf ) |
|---|---|---|---|---|
| I | Symmetric cross ply | 0.00 | 0.04790 | 0.0815 |
| II | Anti-symmetric cross ply | 0.00 | 0.00004 | 0.0093 |
| III | Symmetric angle ply | 0.00 | 0.00460 | 0.0803 |
| IV | Anti-symmetric angle ply | 0.00 | 0.00190 | 0.0341 |

$$\psi_2 = \left(\frac{\delta p}{\delta \sigma}\right)(\sigma / p) \tag{16}$$

Eqs. (15) and (16) represent the rate of change of Pf with respect to the mean value of each random variable and rate of change of Pf with respect to the standard deviation of each random variable respectively.

### *5.1 Probability of Failure and Sensitivity against Ultimate Limit State*

The Pf of symmetric cross ply arrangement value (0.0479) lies in an unacceptable range as shown in Table 2. The uncertainties associated with random parameters are shown to influence the probability of failure. The mean and standard deviation of the parameters are changed according to sensitivity bar chart in Fig. 5 to achieve the optimized Pf. If the Pf lies from ($10^{-3}$ to $10^{-5}$) then it's indicated acceptable Pf.

The partial derivatives of Pf with respect to the mean value of random parameters ($E_1$, $E_2$, $E_3$, $G_{12}$, $G_{13}$, $G_{23}$, $S_{11T}$, $S_{13}$, $V$, $\nu_{12}$, $\nu_{23}$ and $\nu_{13}$) are shown in the bar chart. The chart shows that the Pf is directly proportional to the above random variables. The scatter (standard deviation) of all the random parameters directly influences the Pf. Likewise, the bars in the chart plotted on the negative side show that the Pf is inversely proportional to the mean value of shear strength $T_{13}$. It is also observed that the mean value of Young's modulus ($E_3$) and initial velocity of impactor are the most sensitive parameters to influence Pf. The most sensitive parameters are modified to achieve the cumulative distribution function shown in Fig. 6.

In the present problem, little sensitivity is observed

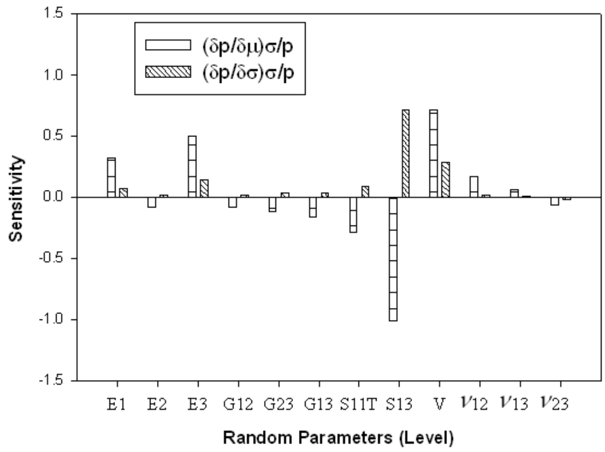


**Fig. 5 Sensitivity behavior of simply supported composite beam for simply supported.**

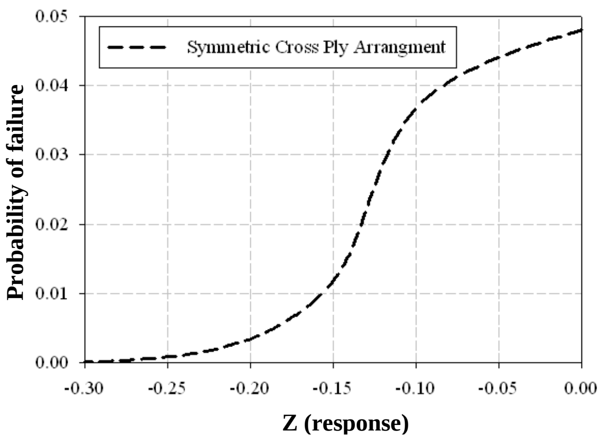


**Fig. 6 Probability of failure of simply supported cross ply composite beam.**

due to material properties such as ($E_1$, $E_2$, $G_{12}$, $G_{13}$, $G_{23}$, $\nu_{12}$, $\nu_{23}$ and $\nu_{13}$) and strength property ($S_{11T}$). Its input parameters may be reviewed for manufacturing process changes that may lead to loosening tolerances and possible cost reduction. The importance of the mean value of the Young's modulus ($E_3$), mean value of shear strength and mean value of initial velocity of impactor ($V$) influence the probability of failure more.

These three input variables are designer control and achieve the target reliability.

## 6. Conclusions

Fiber damage initiation is considered to be the safety criteria for the glass epoxy composite beam under ballistic impact. Linear relation exists between impactor velocities (120 m/s to 300 m/s) and number of damaged layers. This behavior validates the currents finding with respect to published results. An energy balance is found that the magnitude of KE, IE and FE of the target are significant at damage initiation of composite fiber. However, other energies like delamination (damage) energy, PE (plastic energy) of the impactor and artificial energy (reduced integration for impactor), etc. are not found to be significant. MCS takes almost 10 times more computational time than GPRSM does. Comparative study of different fiber failure initiation criteria is studied and found that a maximum stress criterion is most conservative than Chang-Chang and Hashin failure criterion. Hashin and Chang-Chang failure criterion is found that an anti-symmetric cross ply simply supported laminate has minimum Pf than other ply lay-ups namely symmetric cross ply, symmetric angle ply and anti-symmetric angle ply. The fiber initiation (Chang-Chang) of composite beams with symmetric cross ply lay-ups are Pf (88.9%, 1.47% and 58.1%) more than that of anti-symmetric cross ply, symmetric angle ply and anti-symmetric angle ply arrangements. The sensitivity analysis is found that the mean value of the Young's modulus ($E3$), the mean value of shear strength ($S_{13}$) and the mean value of initial velocity of impactor ($V$) have a more sensitive parameter in comparison to other input parameters. This is an important input for the probabilistic design.

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