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Analysis of Hydrological Simulation Models Using the Parameter Combinatorial Diagram

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Abstract: The aim of this paper is to present graphically the behaviour of a simulation model to the varying parameters and to establish the suitability of this representation as a valid tool for the analysis of the same parameters. In this paper, we define parameter combinatorial diagram as the joint graphical representation of all box plots related to the adjustment between real and simulated data, by setting and/or changing the parameters of the simulation model. To do this, we start with a box plot representing the values of an objective adjustment function, achieving these results when varying all the parameters of the simulation model. Then we draw the box plot when setting all the parameters of the model, for example, using the median or average. Later, we get all the box plots when carrying out simulations combining fixed or variable values of the model parameters. Finally, all box plots obtained are represented neatly in a single graph. It is intended that the new parameter combinatorial diagram is used to examine and analyze simulation models useful in practice. This paper presents combinatorial diagrams of different examples of application as in the case of hydrologic models of one, two, three, and five parameters.

Key words: Parameter calibration, optimization, combinatorial diagram, hydrological simulation models.

1. Introduction

The tables represent information in outline and are ready for subsequent calculations. The graphics convey that information in a more expressive way, and they will allow, with a single glance, to understand that we are talking about, to see their most important features, even to draw conclusions about the behaviour of the sample whose study is being conducted [1]. It is known that when numerical data are available and, before tackling complex analysis or study, a first step is to present this information so that it can be viewed in a more systematic and summarised way. The most common graphics used in the analysis of results are: bar chart, histogram, frequency polygon, pie chart, box plot [2, 3] and others. The application of simulation models involves delivering measurable results of that application or experiment. The clarity of the presentation is of vital importance for the understanding of the results and their interpretation. At the time of representing the results of a simulation analysis in an appropriate manner, there are several publications that we can consult, Unwin et al. [4], Unwin [5], Cole and Eklund [6] and Cimiano et al. [7]. Although the presentation of numerical data is usually done using tables, sometimes a chart or graph can help to represent our data in a more efficient way. In the analysis process of simulation models, it is important to define perfectly the objective function and the optimisation technique [8, 9], as well as determine the number of simulations to be performed [10].

This article will address the graphic representation of the results of the application of hydrological simulation models using the parameter combinatorial diagram, noting its usefulness in the process of data analysis and presentation.

2. Graphical Representation of Simulation Results

Simulation is the process of designing a model of a real system and carries out experiments with it, in

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order to understand their behaviour or to evaluate new strategies for its operation [11]. The real system model can be theoretical, conceptual or systemic and must contain the elements necessary for the simulation (scenario or set of working hypotheses) [12].

The steps to perform a simulation study can be specified in:

- system definition, which is to study the context of the problem, identify the objectives of the project, specify the indices measuring the effectiveness of the system, specify the objectives when defining the system to be modelled;
- model formulation, once precisely defined the results to be achieved from the study, we define and build the model which will yield the desired results. In the formulation of the model, it is necessary to define all variables and parameters that are part of it, their logical relationships and flow charts describing in the whole model;
- data collection. It is important to define clearly and precisely the data that the model will require to produce the desired results;
- implementation of the model in the computer. With the model defined, the next step is to write a computer program to process and obtain results for comparison;
- verification and validation. The verification and validation process is about checking that the simulated model meets the designed requirements for which it was developed. Throughout this stage, we check the differences between the operation of the simulator and the real system through the quantification of an objective function, which is the one that we want to optimize, i.e., maximize or minimize;
- experimentation. Experimentation with the model is done after it has been validated. The experiment is to generate the desired data and to carry out a sensitivity analysis of the levels required;
- interpretation. At this stage of the study, the results shown by the simulation are interpreted and, based on this, we make a decision. The clarity of the representation of the results is of vital importance for the understanding and interpretation of the same.

The parameter combinatorial diagram is defined as the joint graphical representation of all box plots related to the adjustment between real and simulated data, when setting and/or changing the parameters (named generically p) of the simulation model. These are the various stages to specify the combinatorial diagram of n (number of parameters) associated with a simulation model:

- data selection. We define the data that the simulation model is going to require to produce the desired results;
- definition of the objective function. We check the differences between the operation of the simulated system and the real system through the quantification of an objective function, which is the one that we want to optimize, i.e., maximize or minimize;
- application of model simulation varying all parameters. We represent the box plot obtained when varying all the parameters of the simulation model;
- choosing fixed values of the parameters. From the values represented when varying all parameters, we choose a statistical value, such as median or mean, to set the parameters of the model;
- application of the simulation model by setting all parameters. We draw the box plot obtained by the simulation by fixing all model parameters;
- application of the simulation model setting parameters i. We obtain the $n!/i!(n-i)!$ box plots when carrying out simulations combining the i fixed values and the $n-i$ varying values of model parameters;
- parameter combinatorial diagram representation. The final phase is represented neatly in a single graph (Fig. 1), all box plots obtained.

The number of box plots that make up the parameter combinatorial diagram is 2^n, which corresponds to the sum of all possible combinations that exist when fixing or varying the n parameters of the simulation model.

Besides being able to represent the box plots associated to the values of an objective function that we want to optimize in the parameter combinatorial diagram, it is also possible to represent the box plots associated to the values of each of the parameters. In this case, we can simplify the representation (Fig. 2).

The number of box plots, in the case of representing the values of a parameter, is 2^{n-1}, which corresponds to the sum of all possible combinations that exist when fixing or varying the $n - 1$ remaining parameters of the simulation model.

3. Parameter Combinatorial Diagram in Hydrological Models

In this paper, we present combinatorial diagrams of different examples of application as in the case of hydrologic models of one, two, three, and five parameters. This section has been structured in steps in order to accurately describe the different parameter combinatorial diagram.

3.1 Description of the Watershed Subject to Study

The hydrological model simulations have been conducted in the watershed of Aixola [13] located in northern Spain in the province of Gipuzkoa. This watershed has a gauging station as part of the hydrometeorological network of Gipuzkoa Provincial Council which records the flow of the river every 10 min. The precipitation data are also recorded every ten minutes by a rain gauge located at the gauging station itself. Aixola watershed is located on the western boundary of the province of Gipuzkoa and is mostly used with forest (> 85% surface), it has an area of 4.70 km^2, with extreme levels of 315 m and 740 m, an average slope of 44.25% and an annual average rainfall of 1,600 mm.

3.2 Description of Events

From the recorded data, we have selected a series of 20 events that could adapt to the hypothesis of unit

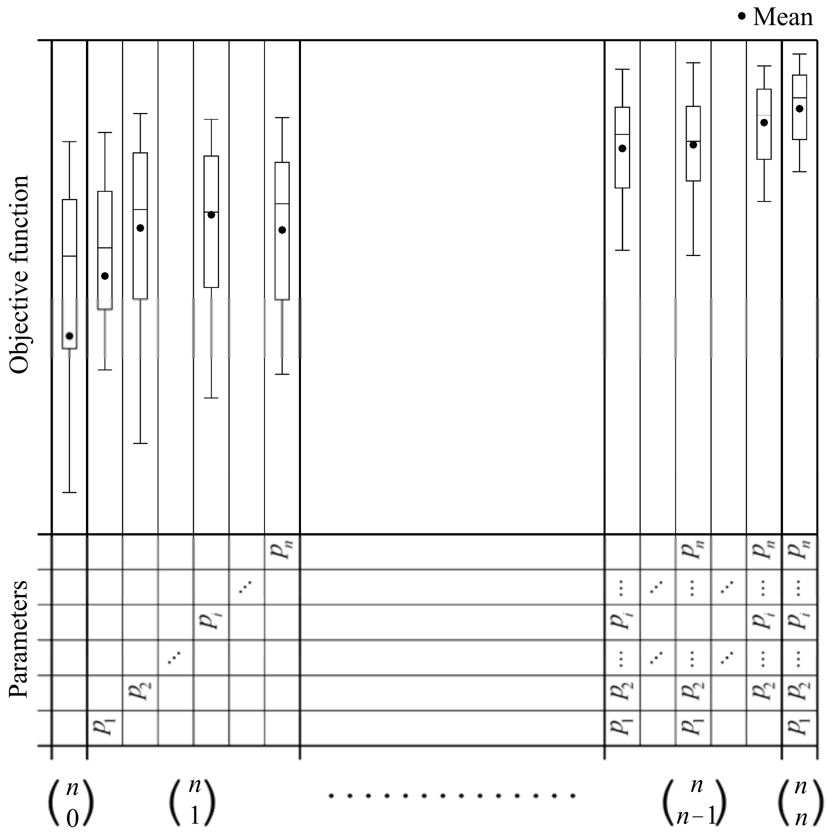

Fig. 1 Parameter combinatorial diagram of an objective function.

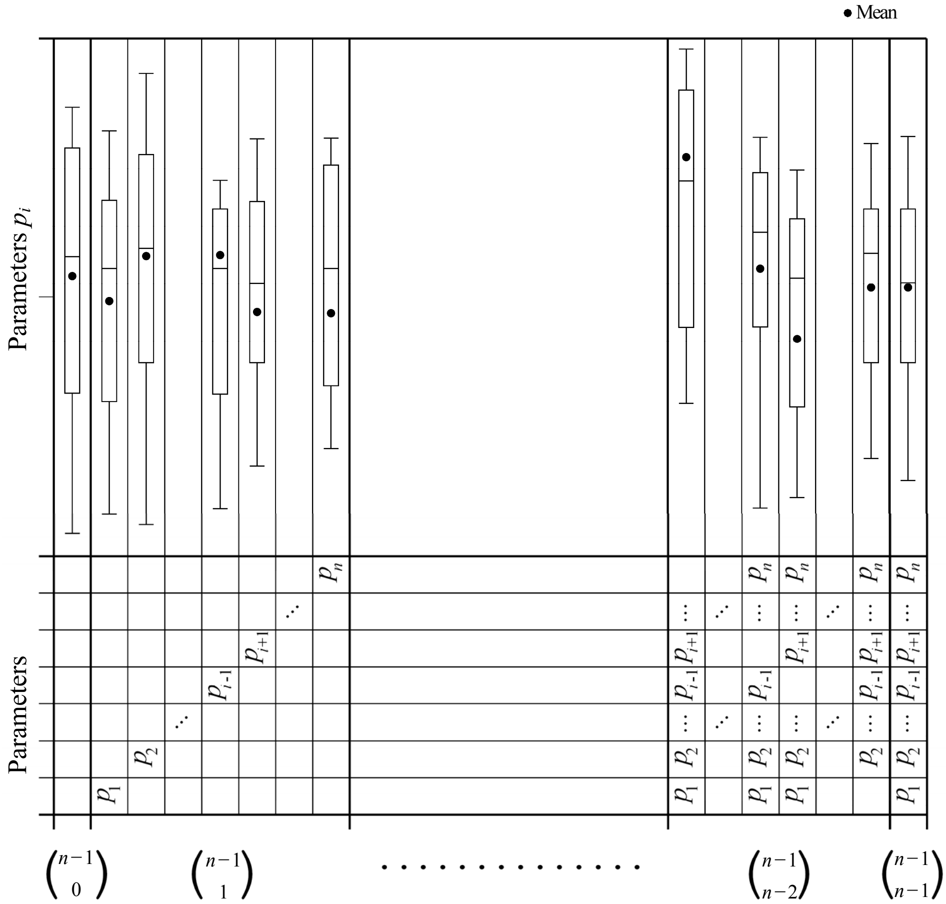

Fig. 2 Parameter combinatorial diagram of the value of a parameter.

hydrograph technique, applicable to hydrologic models of one, two and three parameters subsequently submitted. The base flow has been extracted from each of these events and we have obtained effective precipitation hyetograph. The base flow extraction was performed using a recursive filter, namely the one proposed by Eckhardt [14] of two parameters. After removing the base flow, we have obtained the effective precipitation hyetograph using the curve number method developed by the soil conservation service [15] adjusting the observed direct runoff volume and the beginning of direct runoff hydrograph observed [16]. In addition, we have selected a series of 37 events that could adapt to the application scenarios of the hydrological model of five parameters that it presents. The main features of each of these selected events are on the articles by Goñi et al. [17, 18].

3.3 Definition of the Objective Function

In the application of hydrologic models, the goodness of the adjustment of simulated hydrographs was made regarding the efficiency defined by Nash and Sutcliffe [19]. Thus, the objective function is:

$$E = 1 - \frac{\sum_{j=1}^{m}\left(Q_{ob,j} - Q_{si,j}\right)^2}{\sum_{j=1}^{m}\left(Q_{ob,j} - \bar{Q}_{ob}\right)^2} \tag{1}$$

where, E is the efficiency $Q_{ob,j}$ is the flow which is observed at the time j, $Q_{si,j}$ is the flow simulated at the time j and $\overline{Q_{ob}}$ is the average flow observed.

3.4 Combinatorial Diagrams Associated with Reservoir Geomorphological Instantaneous Unit Hydrograph

The hydrological model called the RGIUH

(reservoir geomorphological instantaneous unit hydrograph) has the following expression [20]:

$$h(t) = \frac{\alpha \ e^{-\alpha \frac{t}{\tau}}}{\tau} \sum_{i=1}^{n} \left[\frac{A_i}{(i-1)\ !} \left(\alpha \frac{t}{\tau} \right)^{i-1} \right] \quad (2)$$

where, $h(t)$ is the flow of the instantaneous unit hydrograph at the time t, and A_i is the area of the sub-watersheds.

To determine the geomorphological value $\alpha = \sum_{i=1}^{n} (iA_i) \Big/ \sum_{i=1}^{n} (A_i) = 5.25$ provided by the area relations, the watershed has been divided into sub-watersheds from the permanent drainage system represented in the mapping 1:5,000. The only uncertain parameter τ represents the centre of gravity of the hydrograph. To obtain the direct runoff hydrograph simulated from the instantaneous unit hydrograph, we apply the convolution equation to each effective precipitation hyetograph.

We present in Fig. 3 the parameter combinatorial diagrams associated to the objective function E and to the parameter value τ, obtained by simulating events.

In this case, the combinatorial diagram expresses a relationship with a single variable. You can analyse the strengths arising from the model when fixing or varying its only parameter.

3.5 Combinatorial Diagrams Associated with Instantaneous Unit Hydrograph Represented by the Beta Function of Two Parameters

The hydrologic model called the instantaneous unit hydrograph represented by the statistical function of two parameters *Beta*, β_τ, has the following expression [16]:

$$\beta_\tau\left(t;\alpha_p,\tau\right) = \frac{\Gamma\left[3\left(1+\alpha_p\right)\left(1-\alpha_p\right)\right]\tau^{p_\tau+1}t^{p_\tau-1}}{\Gamma\left[\left(1+2\alpha_p\right)/\left(1-\alpha_p\right)\right]\Gamma\left[\left(2+\alpha_p\right)/\left(1-\alpha_p\right)\right]\left(\tau+t\right)^{2p_\tau+1}} \quad (3)$$

where, t is the time, τ represents the centre of gravity of the hydrograph and α_p is a shape parameter of the hydrograph.

In this model, the two parameters of the function β_τ are α_p and τ. We present in Fig. 4 parameter combinatorial diagrams associated with the objective function E and the values of the parameters α_p and τ, obtained by simulating the events.

In this case, the combinatorial diagram expresses relationships with two variables. These relationships can be represented by a surface drawing.

3.6 Combinatorial Diagrams Associated with the Instantaneous Unit Hydrograph Represented by the Beta Statistical Function of Three Parameters

The hydrological model called the instantaneous unit hydrograph represented by the Beta statistical function of three parameters, $\beta(t;\ \alpha_0,\ \alpha_p,\ \tau)$, has the following expression [16]:

$$\beta\left(t;\ \alpha_0,\alpha_p,\tau\right) = \frac{\Gamma\left[\left(\alpha_0+\alpha_p\right)\left(\alpha_0+2\right)/\alpha_0\left(1-\alpha_p\right)\right]^q t^{p-1}}{\Gamma\left\{\left(\alpha_0+2\alpha_p\right)/\left[\alpha_0\left(1-\alpha_p\right)\right]\right\}\Gamma\left[\left(1+\alpha_0+\alpha_p\right)/\left(1-\alpha_p\right)\right]\left(\alpha_0\tau+t\right)^{p+q}} \quad (4)$$

where, t is the time, τ represents the centre of gravity of the hydrograph, α_p and α_0 are two shape parameters of the hydrograph.

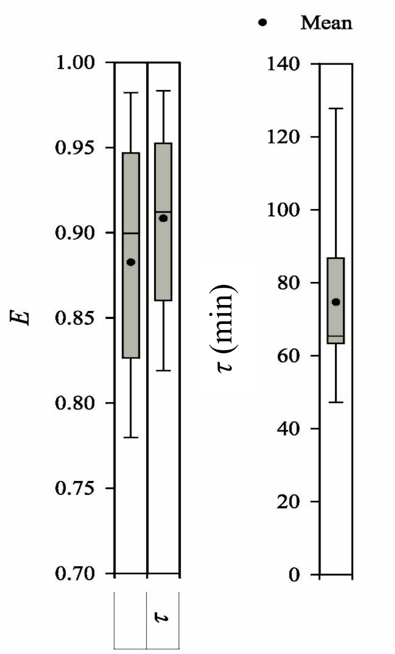

Fig. 3 Parameter combinatorial diagrams of RGIUH.

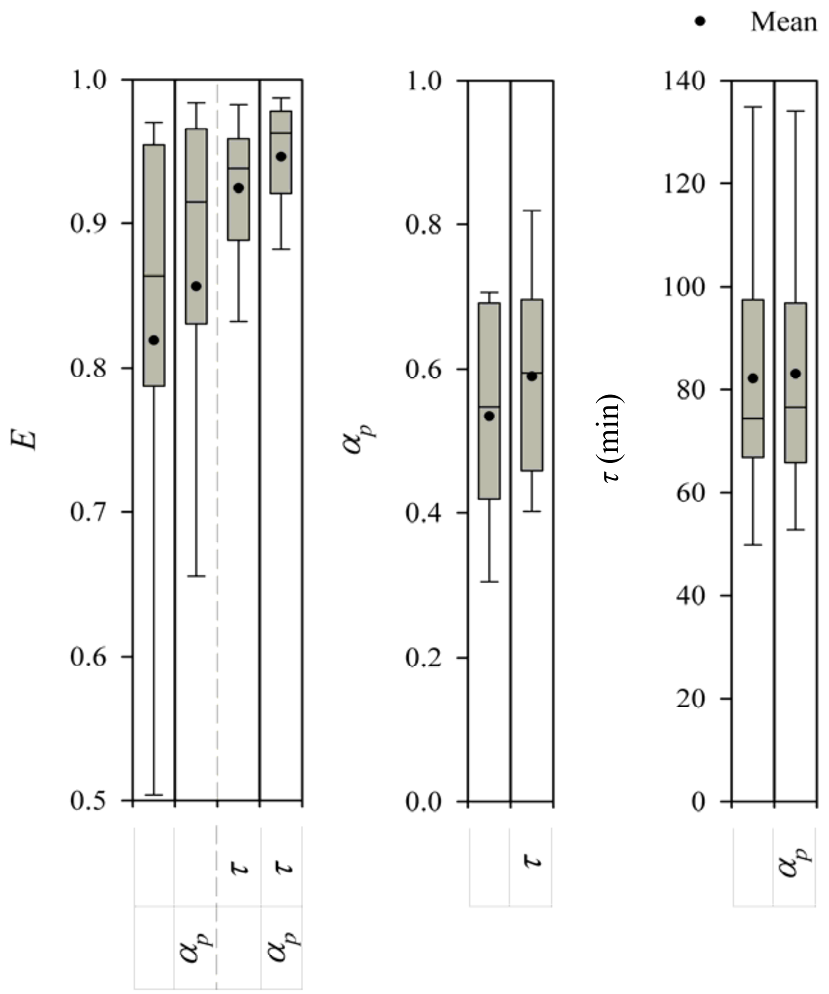

Fig. 4 Beta function combinatorial diagrams of the two parameters.

We present in Fig. 5 the parameter combinatorial diagrams associated with the objective function E and the parameter values α_0, α_p and τ, obtained by simulating the events.

The combinatorial diagram of three parameters expresses, among other things, a volumetric drawing. It clearly displays the different influence of the parameters with respect to the adjustment made.

3.7 Combinatorial Diagram Associated with the Rainfall-Runoff Geomorphological Reservoirs Model of Five Parameters

In Fig. 6, we represent the operation of the hydrological model called reservoir rainfall-runoff geomorphological model R^3GeM [17, 18].

In this model, the five parameters are S_C, S_0/S_C, ϕ, τ_{Sb} and τ_S. We present in Fig. 7 the parameter combinatorial diagram associated with the objective function E obtained by simulating the events.

Analyzing the combinatorial diagram shown in Fig. 8, we can deduce that, varying only the parameter ϕ, an efficiency E is acquired close to that obtained by varying all parameters. This means that the said parameter is the most suited to calibrate the model. In turn, we can see that setting only the parameter S_0/S_C, we can achieve practically the same efficiency than varying it. This means that you can replace this parameter by a fixed value.

We present in Fig. 8 the parameter combinatorial diagram associated to parameter ϕ obtained by simulating the events.

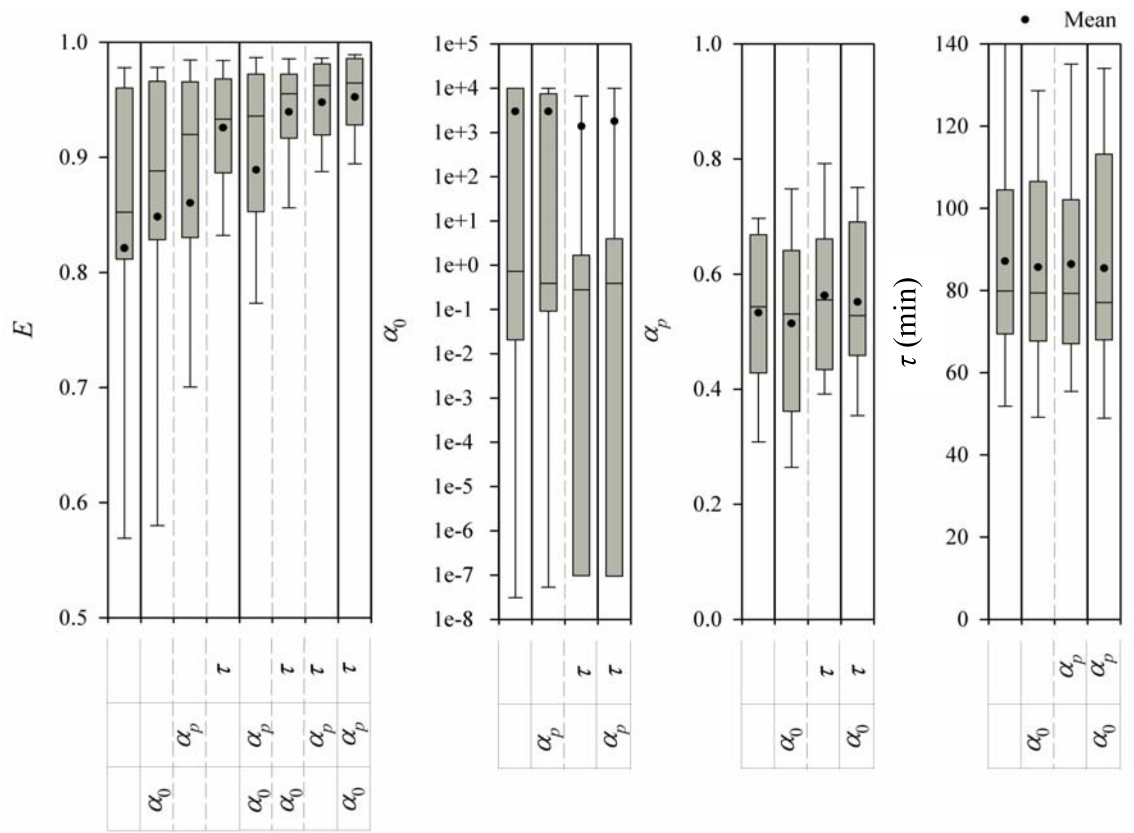

Fig. 5 Beta function combinatorial diagrams of three parameters.

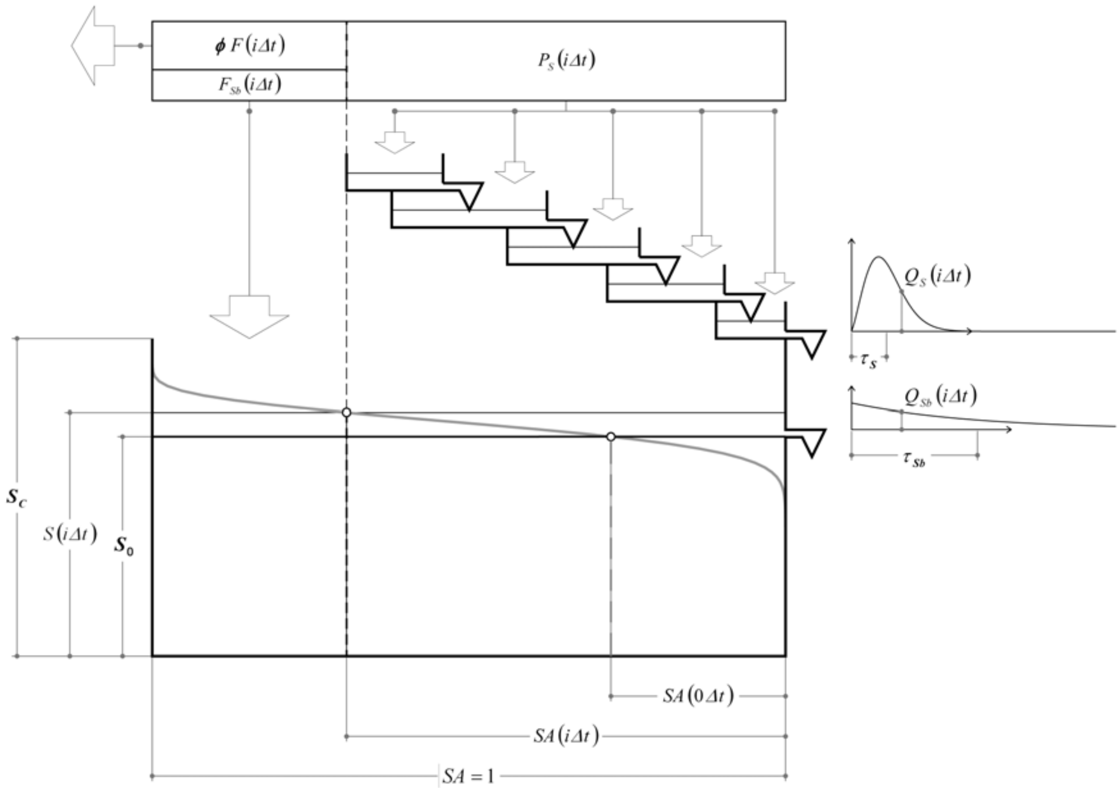

Fig. 6 Reservoirs rainfall-runoff geomorphological model, R³GeM.

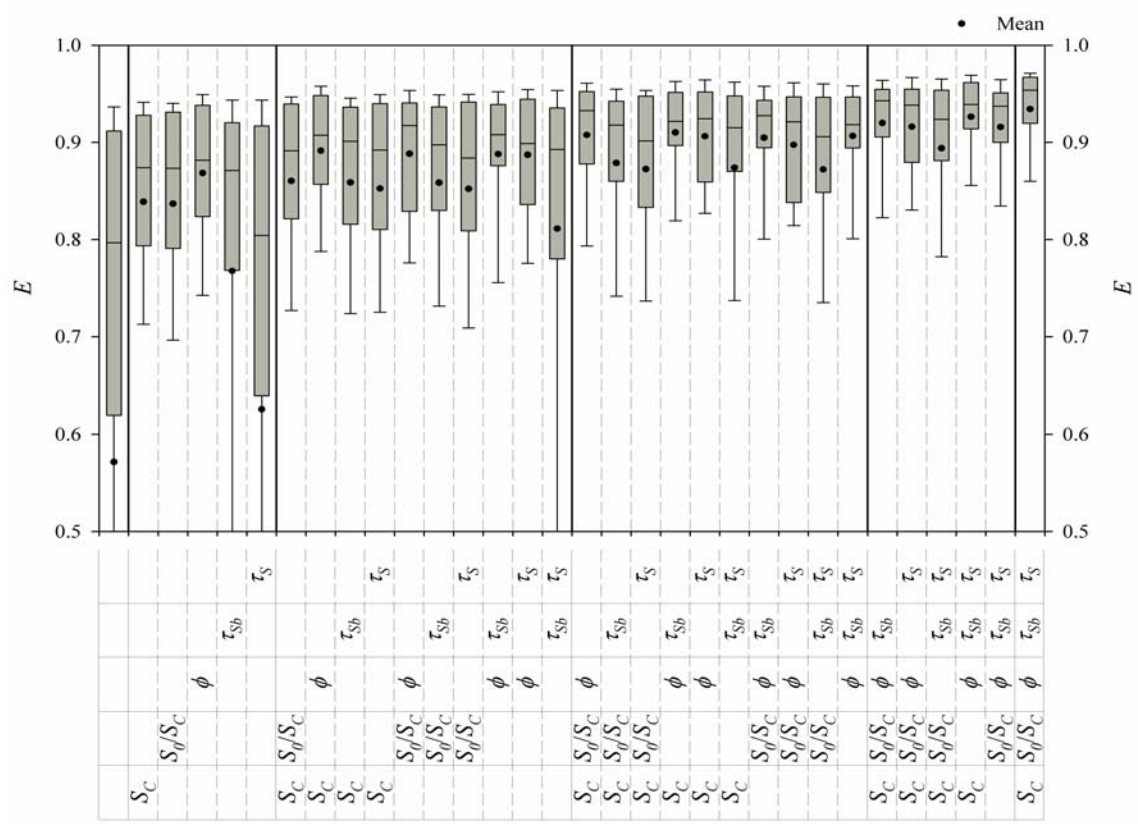

Fig. 7 Parameter combinatorial diagram of model R³GeM of objective function *E*.

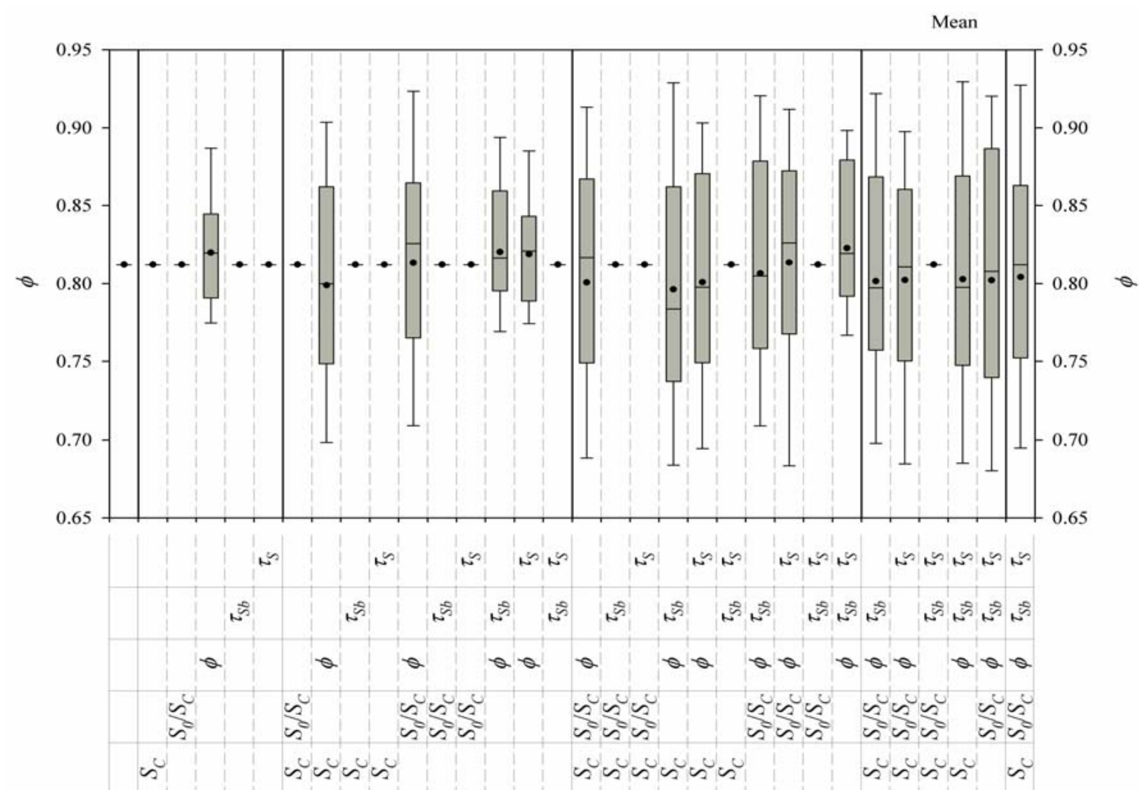

Fig. 8 Parameter combinatorial diagram of model R³GeM of parameter ϕ.

When only the parameter ϕ is varied, this has less deviation than in the case of not setting any parameters. The simplified parameter combinatorial diagram has not been used in Fig. 8, with the aim of better understanding the organization of these diagrams.

4. Conclusions

The parameter combinatorial diagram is an orderly and adequate representation of all box plots related to the adjustment between real and simulated data, by setting and/or changing the parameters of the simulation model. We are able to represent the combinatorial diagrams related to an objective function and the variation of each parameter in order to analyze more deeply the relationship between the parameters of a simulation model.

When the combinatorial diagram of parameters depends on a single variable, it is possible to visualize the benefits and drawbacks of fixing or varying its only parameter. When studying a two-parameter model, the combinatorial diagram allows identifying clearly the parameter that gets better fit to the objective function. Therefore, the use of combinatorial diagrams associated with models of one or two parameters, like the models RGIUH and β_τ, is analogous, in terms of information provided, to the representations of unidimensional and two-dimensional drawings.

The combinatorial diagrams associated to a model of three or more parameters, like the models β and R^3GeM, clearly express the different influence of parameters in the objective function both individually and jointly between parameters.

The use of the parameter combinatorial diagram is advisable for the study and analysis of simulation models.

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