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Variable Step Filtered-X Least Mean Square Algorithm Based on Piecewise Logarithmic Function

Zeyi Ding, Jianan Bian, Xinyuan Jiang, and Xi Chen
School of Science, Jiliang University, China

Abstract: In order to improve the problem that the filtered-x least mean square (FxLMS) algorithm cannot take into account the convergence speed, steady-state error during active noise control. A piecewise variable step size FxLMS algorithm based on logarithmic function (PLFxLMS) is proposed, and the genetic algorithm are introduced to optimize the parameters of logarithmic variable step size FxLMS (LFxLMS), improved logarithmic variable step size Films (IFxLMS), and PLFxLMS algorithms. Bandlimited white noise is used as the input signal, FxLMS, LFxLMS, ILFxLMS, and PLFxLMS algorithms are used to conduct active noise control simulation, and the convergence speed and steady-state characteristic of four algorithms are comparatively analyzed. Compared with the other three algorithms, the PLFxLMS algorithm proposed in this paper has the fastest convergence speed, and small steady-state error. The PLFxLMS algorithm can effectively improve the convergence speed and steady-state error of the FxLMS algorithm that cannot be controlled at the same time, and achieve the optimal effect.

Keywords: Active noise control, filtered-x least mean square algorithm, variable step size, genetic algorithm.

1. Introduction

The sounds that typically affect people's learning, work, and rest are collectively referred to as noise. When noise impacts people and the surrounding environment, it forms noise pollution. With the development of modern society, effective control of noise pollution has become a necessary issue that many fields must face [1, 2]. Traditional noise control methods include sound absorption, sound insulation, and the use of silencers to consume sound energy, which are called passive noise control and are more effective for medium and high frequencies. When the noise frequency is low, passive noise control is not ideal, and active noise control emerged, effectively controlling low-frequency noise.

Active noise control is widely applied in fields such as headphones and pipelines [3-5]. Among active noise control algorithms, the filtered-x least mean square (FxLMS) algorithm is extensively used due to its simple structure and easy implementation. When using a fixed step size in this algorithm, the value of the step size affects the convergence speed and steady-state error of the algorithm. When the step size is large, it can accelerate the algorithm's convergence speed; however, the steady-state error also increases simultaneously. When the step size is small, a smaller steady-state error can be obtained upon reaching the steady state, but it results in a slower convergence speed of the algorithm. To address the contradiction in the traditional fixed step size FxLMS algorithm, where it is not possible to improve the algorithm's convergence speed while simultaneously reducing the steady-state error, numerous variable step size FxLMS (VSS-FxLMS) algorithms have been proposed by researchers [6-9]. Widrow B et al. [10] obtained the latest step size by adjusting the step size of the previous moment. Ma et al. [11] proposed a simplified variable step size FxLMS algorithm and optimized the computational complexity. Gao et al. [12] proposed an active control method for in-vehicle noise based on an iterative variable step size algorithm by establishing a nonlinear functional relationship between the step size and the number of iterations. However, when the

Corresponding author: Zeyi Ding, Postgraduate, research fields: active noise control. E-mail: 1448616639@qq.com.

secondary path of the system changes abruptly, the corresponding step size cannot be obtained, thereby affecting the convergence speed of the algorithm. Gomathi et al. [13] introduced the arctangent function as the step size control function of the FxLMS algorithm, achieving faster convergence speed and greater noise reduction. To achieve faster convergence speed or smaller steady-state error, variable step size algorithms based on nonlinear functions such as the Sigmoid function[14], logarithmic function [15], and Lorentzian function [16] have emerged. However, the performance of the above-mentioned nonlinear function variable step size algorithms has limitations, they cannot simultaneously control the algorithm's convergence speed and steady-state error to achieve optimal results [17].

By analyzing the characteristics of the logarithmic function based FxLMS (LFxLMS) algorithm, which has a fast convergence speed, and the improved logarithmic function-based FxLMS (ILFxLMS) algorithm, which has a small steady-state error, this paper proposes a piecewise logarithmic variable step size FxLMS algorithm (Piecewise logarithmic variable step filtered-x least mean square algorithm, abbreviated as PLFxLMS) based on the advantages of both algorithms. The proposed algorithm is theoretically analyzed to achieve a fast convergence speed while obtaining a smaller steady-state error. Furthermore, a genetic algorithm is employed to optimize the parameter selection in the proposed algorithm, thereby improving the algorithm's operational efficiency.

2. FxLMS Algorithm

The feedforward active noise control principle block diagram based on the FxLMS algorithm is shown in Fig. 1, where $x(n)$ is the reference signal, $P(z)$ is the primary path transfer function, $S(z)$ is the secondary path transfer function from $y(n)$ to the error microphone, $\hat{S}(z)$ is the estimated secondary path transfer function, $x'(n)$ is the filtered reference signal,

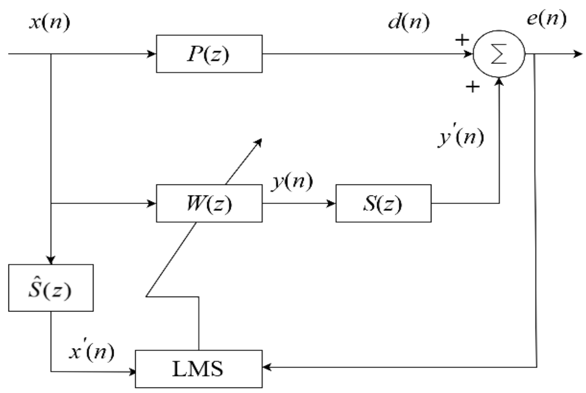


Fig. 1 Block diagram of adaptive feed forward system based on FxLMS algorithm.

$W(z)$ is the adaptive filter, y'(n) is the signal picked up by the error sensor emitted from the secondary sound source, $d(n)$ is the desired signal, and $e(n)$ is the error signal received at the error sensor.

The reference signal $x(n)$ is filtered through an N-thorder adaptive filter $W(z)$ to obtain the adaptive filter output signal $y(n)$,

$$y(n) = \sum_{l=0}^{N-1} w_l x(n-l) \tag{1}$$

where w_l is the coefficient of the adaptive filter $W(z)$, and l takes values of 0, 1, ..., N-1.

The signal $y'(n)$ received by the error sensor from the secondary sound source is obtained by convolving the adaptive filter output signal y(n) with the secondary path impulse response,

$$y'(n) = \sum_{l=0}^{N-1} S(n) y(n-l) \tag{2}$$

where $S(n)$ represents the coefficients of the secondary path impulse response, and for convenience of calculation, its length is set to N. This signal and the desired signal $d(n)$ are superimposed at the error sensor to generate the error signal $e(n)$,

$$e(n) = d(n) + y'(n) = d(n) + \sum_{l=0}^{N-1} S(n) y(n-l) \tag{3}$$

The cost function $J(n)$ is defined as the square of the error signal $e(n)$,

$$J(n) = e^2(n) \tag{4}$$

Taking the partial derivative of $J(n)$ with respect to its adaptive filter coefficients yields:

$$\frac{\partial J(n)}{\partial w} = -2e(n)x'(n) \tag{5}$$

In Eq. (5), $x'(n)$ is the filtered reference signal, which is obtained by convolving the reference signal $x(n)$ with the estimated secondary path impulse response $\hat{S}(n)$,

$$x'(n) = \hat{S}(n) * x(n) = \sum_{l=0}^{N-1} \hat{S}(n)x(n-l) \tag{6}$$

According to the principle of the gradient descent algorithm, the adaptive filter coefficient update formula is:

$$W(n+1) = W(n) - \mu e(n)x'(n) \tag{7}$$

In the equation, μ represents the step size. In the FxLMS algorithm process described by equations (1) to (7), to ensure the stability of the system, the range of values for the step size factor μ is,

$$0 < \mu < \frac{1}{tr(R)} \tag{8}$$

where tr(R) represents the trace of the reference signal autocorrelation matrix R, expressed as,

$$tr(R) = \sum_{l=0}^{N-1} (x(n-l))^2 \tag{9}$$

In Eq. (9), $\sum_{l=0}^{N-1} (x(n-l))^2$ represents the total input power of the signal, which is usually known. In applications, the range of values for μ can be determined by $tr(R)$.

3. Piecewise Logarithmic Variable Step Size FxLMS Algorithm

To improve the influence of step size selection being too large or too small on the steady-state error in the fixed step size algorithm, a nonlinear function is used to establish the relationship between the step size $\mu(n)$ and the error signal $e(n)$. This allows for adaptively selecting a larger step size when the error signal is large, thereby improving the convergence speed of the algorithm, and adaptively selecting a smaller step size when the error signal is small, thereby reducing the steady-state error of the algorithm.

In the LFxLMS algorithm, the logarithmic function is used to control the step size, and its step size control formula is:

$$\mu(n) = a_1 \log(b_1 |e(n)|^{c_1}) \tag{10}$$

In the equation, the selection of parameter a_1 can control the range of step size values, the selection of parameter b_1 can change the shape of the step size function, and the selection of parameter c_1 affects the rate of change of the step size function. When parameters a_1, b_1, and c_1 change, the control variable method is used to fix other parameters, and the influence of parameter changes on the step size function model is obtained, as shown in Fig. 2.

From Fig. 2, it can be seen that the step size control formula curve of the LFxLMS algorithm presents an overall convex curve. The top of the curve is smooth, and a larger step size can be obtained in the early stages of algorithm iteration, resulting in a fast convergence speed. The bottom of the curve is sharp, which leads to a larger step size value in the later stages of algorithm iteration, resulting in a larger steady-state error. Therefore, the characteristic of the LFxLMS algorithm is a fast convergence speed with a larger steady-state error.

The ILFxLMS algorithm uses the logarithmic function to control the step size, and its step size control formula is:

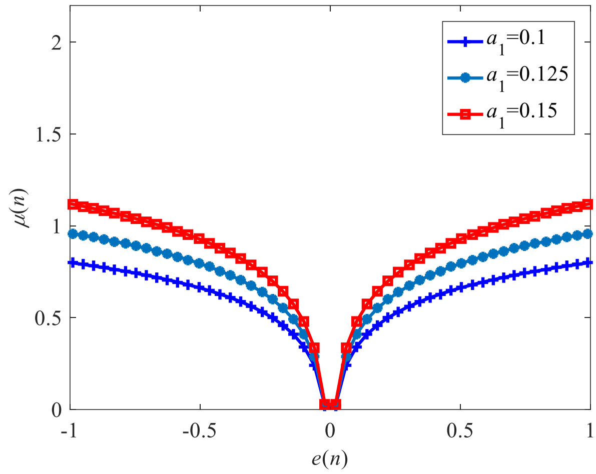
$$\mu(n) = -a_2 \log\left(\frac{1}{1 + b_2 |e(n)|^{c_2}}\right) \tag{11}$$

In the equation, the selection of parameters a_2 and b2 can control the range of the step size function, and the selection of parameter c_2 can control the rate of change of the step size function. When parameters a_2, b_2, and c_2 change, the control variable method is used to fix other parameters, and the influence of parameter changes on the step size function model is obtained, as shown in Fig. 3.

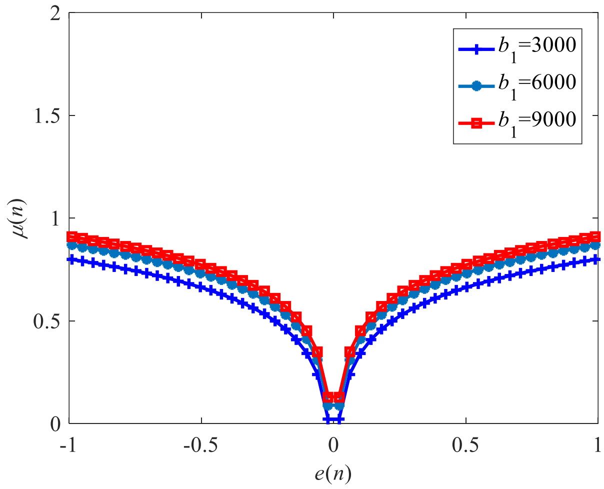
From Fig. 3, it can be seen that the step size control formula curve of the ILFxLMS algorithm presents an overall concave curve. The top of the curve is sharp, and in the middle stages of algorithm iteration, the step size factor rapidly decreases, resulting in a slower convergence speed. The bottom of the curve is smooth, and the curve being too flat leads to the step size factor being reduced to a smaller value too early. Therefore, the characteristic of the ILFxLMS algorithm is a small steady-state error with a relatively slower convergence speed.

Due to the limitations of the performance of a single nonlinear function variable step size algorithm, it is not possible to simultaneously control the convergence speed and steady-state error of the algorithm to achieve optimal results. Considering the advantages of both the LFxLMS algorithm and the ILFxLMS algorithm, this paper proposes a piecewise function variable step size PLFxLMS algorithm. In the early stages of algorithm iteration, based on the fast convergence speed of the LFxLMS algorithm, the LFxLMS algorithm is selected to control the step size factor and accelerate the convergence speed. In the later stages of algorithm iteration, based on the small steady-state error of the ILFxLMS algorithm, the ILFxLMS algorithm is selected to update the step size and reduce the steady-state error. The step size control formula of the PFxLMS algorithm is:

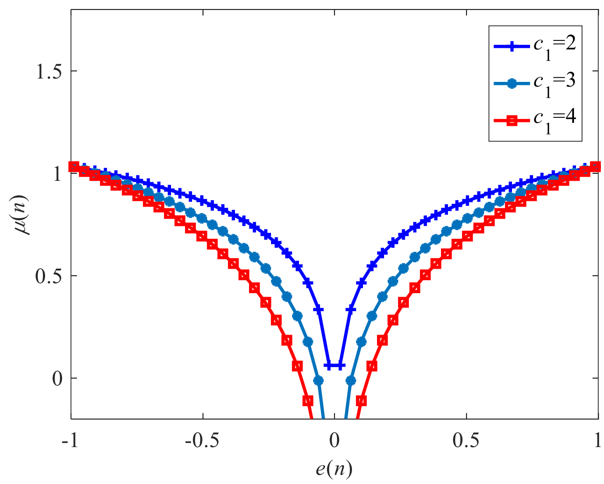
$$\mu(n)=\begin{cases} a_3\log(b_3|e(n)|^{c_3}) & |e(n)|>\tau \\ -a_4\log\left(\dfrac{1}{1+b_4|e(n)|^{c_4}}\right) & |e(n)|\le\tau \end{cases} \tag{12}$$



(a) When parameter a_1 change

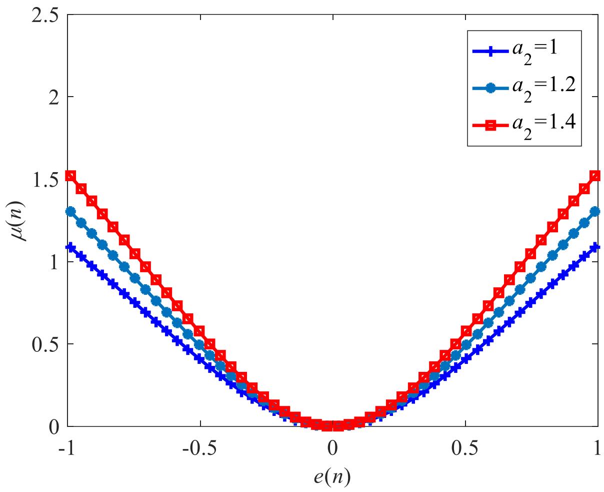


(b) When parameter b_1 change

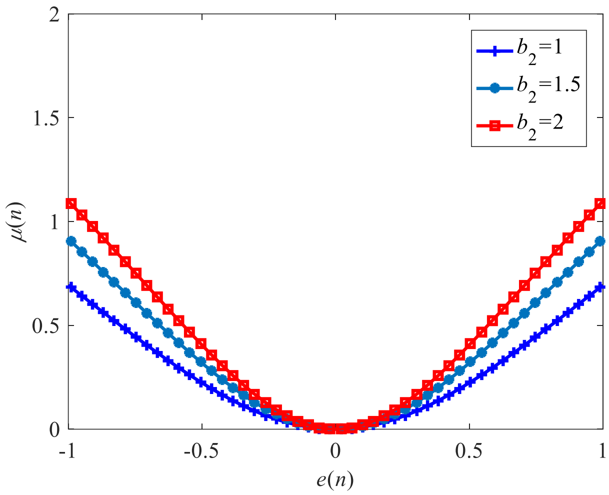


(c) When parameter c_1 change

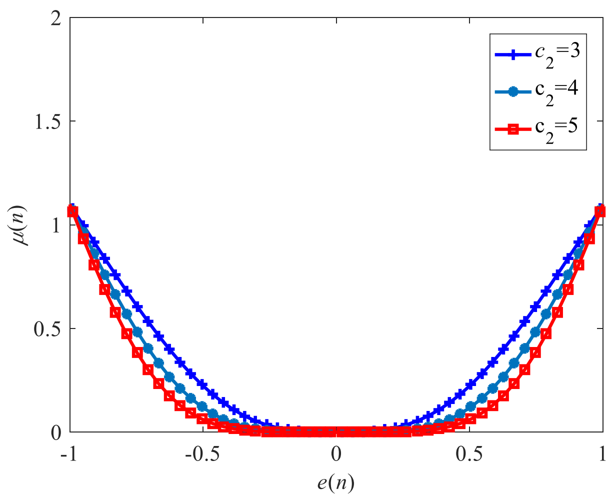
Fig. 2 The impact of parameter changes on the $e(n)$−$\mu(n)$ curve of the LFxLMS algorithm.



(a) When parameter a_2 change



(b) When parameter b_2 change



(c) When parameter c_2 change

Fig. 3 The impact of parameter changes on the *e*(*n*)− *μ*(*n*) curve of the ILFxLMS algorithm.

In Eq. (12), a_3, b_3, c_3, a_4, b_4, c_4 are curve shape control parameters. τ is the function selection threshold, which is determined by controlling the convergence of the logarithmic function used in the initial stage. It prevents the inability to achieve a smooth transition when the threshold is too large and the inability to take advantage of the piecewise function when the threshold is too small.

As shown in Fig. 4, in the early stages of algorithm iteration, the PLFxLMS algorithm can obtain a larger step size than the ILFxLMS algorithm, resulting in faster convergence speed. In the later stages of algorithm iteration, the PLFxLMS algorithm can obtain a smaller step size than the LFxLMS algorithm, resulting in a smaller steady-state error.

The weight coefficient update formula for the variable step size algorithm is:

$$W((n+1)=W(n)+2\mu(n)e(n)x'(n) \tag{13}$$

The above derivation process, which includes Eqs. (1), (6), (12), and (13), is referred to as the piecewise logarithmic function variable step size FxLMS algorithm.

4. Optimization and Selection of Algorithm Parameters

The parameters of the step size factor control formula have a significant impact on the

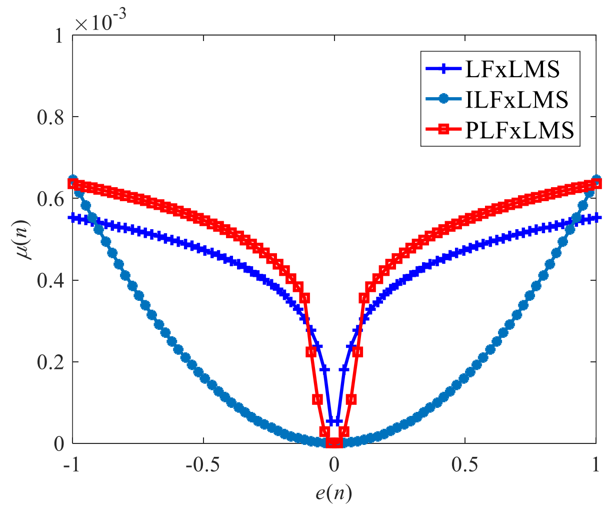


Fig. 4 Comparison diagram of the *e*(*n*)−*μ*(*n*) curve of three variable step size algorithms.

performance of the algorithm. Before conducting simulations, it is necessary to continuously adjust the parameters of the algorithm to achieve optimal performance. Adjusting parameters using the control variable method or trial-and-error method is difficult to ensure that the algorithm's performance reaches the optimum. These parameters are usually nonlinear with respect to the step size; therefore, selecting a nonlinear optimization algorithm is needed to solve this problem. Based on the excellent nonlinear optimization capability of genetic algorithms, this algorithm is chosen to optimize the parameters of the LFxLMS, ILFxLMS, and PLFxLMS algorithms.

The genetic algorithm (GA) is a random probability search algorithm designed to simulate the "survival of the fittest" mechanism in the process of biological evolution, which is suitable for dealing with complex system problems. It was proposed by Professor Holland in 1975 as an optimization method based on natural selection and natural genetics mechanisms [20]. Fig. 5 shows the flowchart of the genetic algorithm.

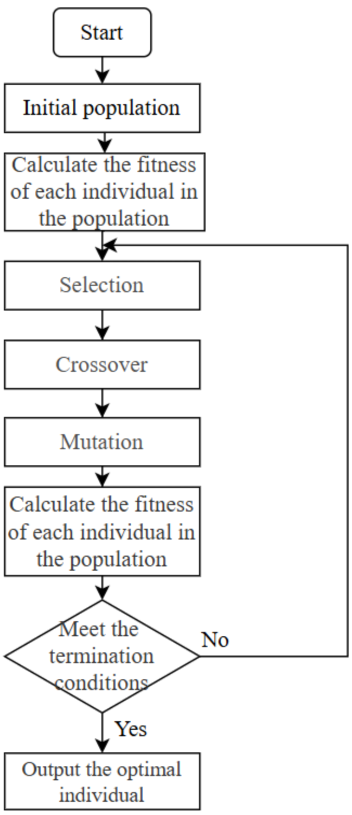


Fig. 5 Flow chart of genetic algorithm.

The initial population is usually generated through random generation, where each individual forms a chromosome through encoding and participates in subsequent computation processes. Based on the principle of survival of the fittest in evolutionary theory, each chromosome is evaluated for its fitness. Based on these evaluation results, individuals with higher fitness are selected from the population and then undergo reproduction, crossover, and mutation operations to produce offspring. As generations of genetic evolution progress, the fittest chromosome is ultimately formed, and the approximate optimal solution to the problem can be obtained through the decoding process.

To evaluate the noise reduction level of the ANC system, the error signal at the moment reflects the noise reduction level at that moment, and the objective function adopts the absolute value of the error signal. To obtain the parameter values that optimize the noise reduction effect of the algorithm, the step size factor control formulas of the LFxLMS, ILFxLMS, and PLFxLMS algorithms are analyzed to derive the optimization objective function.

After taking the reciprocal transformation of Eq. (10), the objective function for parameter optimization of the LFxLMS algorithm is obtained, as shown in Eq. (14),

$$|e(n)| = \sqrt[c_1]{\frac{\exp(\frac{\mu(n)}{a_1})}{b_1}} \tag{14}$$

After taking the reciprocal transformation of Eq. (11), the objective function for parameter optimization of the ILFxLMS algorithm is obtained, as shown in Eq. (15),

$$|e(n)| = \sqrt[c_2]{\frac{1-\exp(-\frac{\mu(n)}{a_2})}{b_2}} \tag{15}$$

After taking the reciprocal transformation of Eq. (12), the objective function for parameter optimization of the PLFxLMS algorithm is obtained, as shown in Eq. (16),

$$|e(n)| = \begin{cases} c_3 \sqrt{\dfrac{\exp(\dfrac{\mu(n)}{a_3})}{b_3}} & |e(n)| > \tau \\ c_4 \sqrt{\dfrac{1-\exp(-\dfrac{\mu(n)}{a_4})}{b_4}} & |e(n)| \le \tau \end{cases} \tag{16}$$

5. Simulation Results Analysis

In the simulation experiment, the simulated sound field environment is a square straight duct with a diameter of 17 cm at the port. One end is closed, and the other end is an absorbing port. The primary loudspeaker is located upstream of the secondary loudspeaker in the duct, and the error sensor is located approximately 34 cm downstream of the secondary loudspeaker. In the simulation, the primary path impulse response and the secondary path impulse response are shown in Fig. 6 and Fig. 7, respectively, both with a length of 256. The sampling frequency is 2400 Hz, and bandlimited white noise in the frequency range of [100 500] Hz is selected as the reference signal. To verify the performance of the PFxLMS algorithm, it is compared with the FxLMS, LFxLMS, ILFxLMS, and PLFxLMS algorithms.

The objective functions of the LFxLMS, ILFxLMS, and PLFxLMS algorithms are shown in Eqs. (13)-(15), respectively. The genetic algorithm is used to optimize the parameters 10 times, and the minimum fitness value among the 10 times is selected to obtain the optimal parameter values for the three algorithms. The parameter value ranges and optimal values for the three algorithms are shown in Table 1 and Table 2.

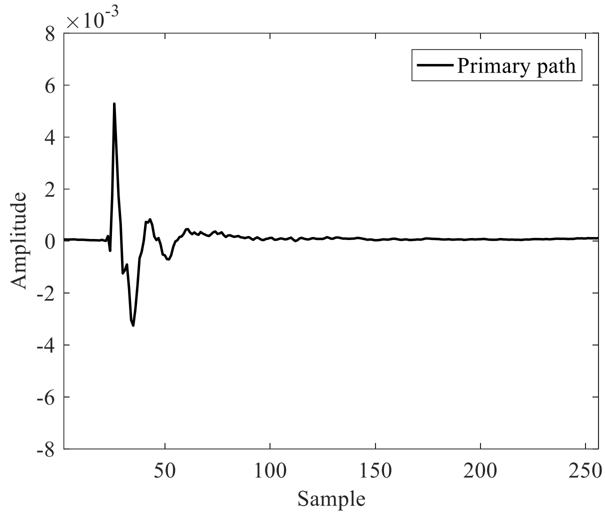


Fig. 6 Primary path impulse response plot.

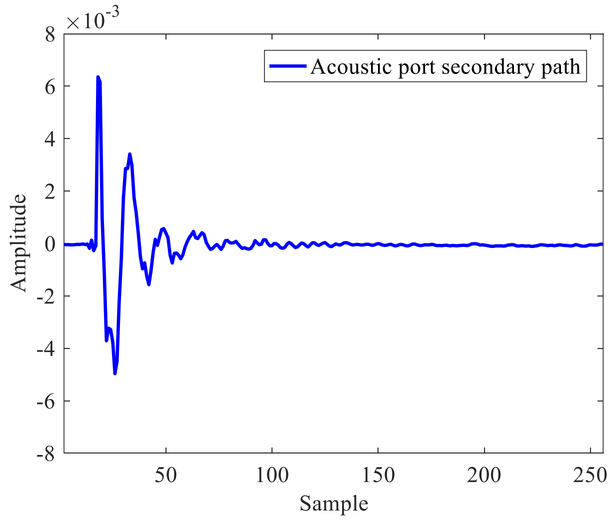


Fig. 7 Acoustic port secondary path impulse response diagram.

Table 1 Pameter value range and optimal value of LFxLMS, ILFxLMS algorithm

	LFxLMS			ILFxLMS		
parameter	a_1	b_1	c_1	a_2	b_2	c_2
lower limit	0	0	0	0	0	0
upper limit	100	100	3	100	100	3
optimal value	2.4×10^{-4}	10	0.3	4×10^{-4}	5	2

Table 2 Pameter value range and optimal value of PLFxLMS algorithm.

	PLFxLMS						
parameter	a_3	b_3	c_3	a_4	b_4	c_4	τ
lower limit	0	0	0	0	0	0	0.001
upper limit	100	100	3	100	100	3	1
optimal value	3×10^{-4}	10	0.3	7×10^{-4}	100	2	0.1

The step size μ of the FxLMS algorithm is 8×10^{-5}, and the parameters of the other three algorithms are taken from the values obtained through optimization by the genetic algorithm in Table 1 and Table 2. Noise control simulations are performed on bandlimited white noise in Matlab, and the error curves of the FxLMS, LFxLMS, ILFxLMS, and PLFxLMS algorithms after control are obtained, as shown in Fig. 8.

From Fig. 8, it can be seen that the bandlimited white noise control effects of the three variable step size algorithms are better than that of the FxLMS algorithm. The control effects of the LFxLMS and ILFxLMS algorithms are similar, and the PLFxLMS algorithm proposed in this paper has the best control effect.

From Fig. 9, it can be seen that the Mean Square Error (MSE) of the four algorithms is used to compare the convergence speed and steady-state error of the fixed step size FxLMS algorithm, LFxLMS algorithm, ILFxLMS algorithm, and the proposed PLFxLMS algorithm. Among these four algorithms, when comparing the LFxLMS algorithm with the ILFxLMS algorithm, the LFxLMS algorithm has a faster convergence speed, but when reaching the steady state, the LFxLMS algorithm has a larger steady-state error, while the ILFxLMS algorithm has a smaller steady-state error. In the proposed PFxLMS algorithm, since the relationship between the variable step size $\mu(n)$ and the error signal $e(n)$ adopts a piecewise logarithmic function, compared with the ILFxLMS algorithm, it has a faster convergence speed, while compared with the LFxLMS algorithm, it has a smaller steady-state error when reaching the steady state. Thus, it combines the advantages of both the LFxLMS algorithm and the ILFxLMS algorithm, and its convergence speed and steady-state error are comparable to those of the fixed step size FxLMS algorithm with the optimal step size. Comparing the error signal spectra of the four different algorithms, as shown in Fig. 10, it can be seen that the noise reduction effect of the proposed PFxLMS algorithm is comparable to that of the fixed step size FxLMS algorithm.

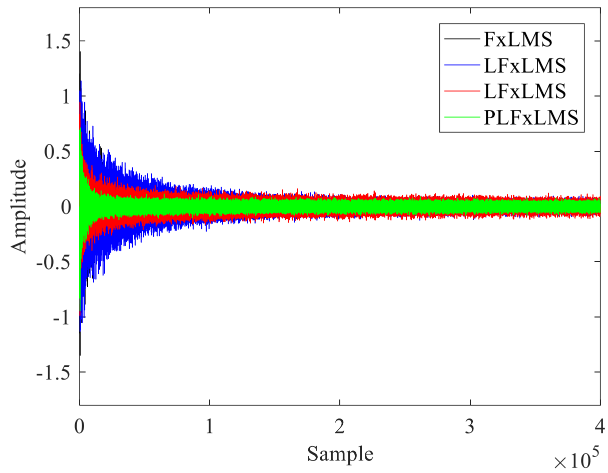


Fig. 8 Time domain error signal diagram of four algorithms.

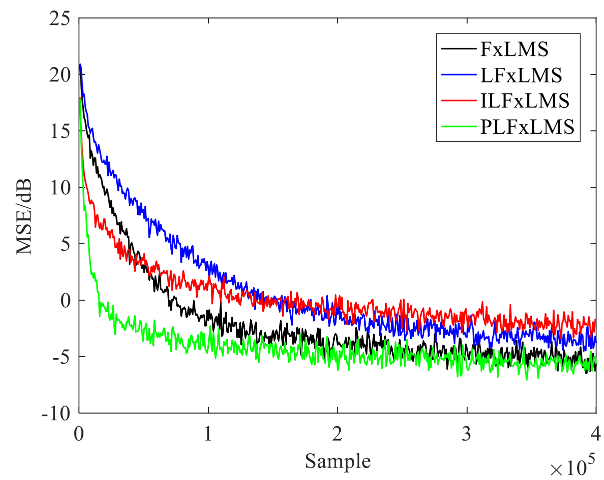


Fig. 9 Mean square error curves of four algorithms.

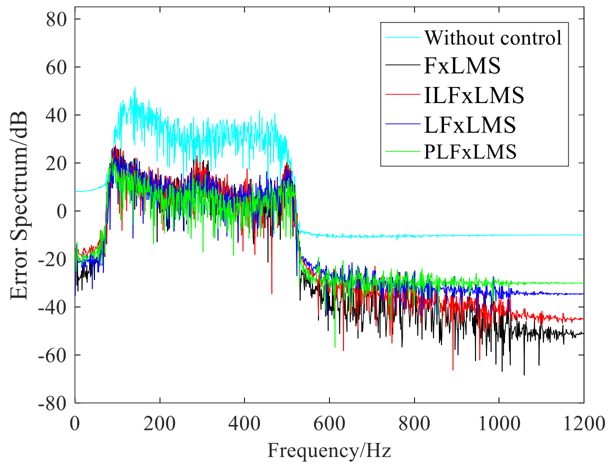


Fig. 10 Error signal spectrogram of four algorithms.

6. Conclusions

This paper proposes a variable step size PLFxLMS algorithm based on the logarithmic function, and the parameters of the LPFxLMS algorithm are optimized and selected using a genetic algorithm. With bandlimited white noise as the input signal, the convergence and steady-state characteristics of four algorithms, namely fixed step size FxLMS, LFxLMS, ILFxLMS, and PLFxLMS, are compared. From the simulation results, it can be seen that the proposed PLFxLMS algorithm has the fastest convergence speed and can achieve the smallest steady-state error. When reaching the steady state, the noise reduction amount is comparable to that of the FxLMS algorithm with the optimal step size.

References

[1] Lu, L, Yin, K. L, and De Lamare, R. C. et al. (2021). "A Survey on Active Noise Control in the Past Decade — Part I: Linear Systems". *Signal Processing*, 183: 108039.

[2] Lu L, Yin K L, and De Lamare R. C. et al. (2021). "A Survey on Active Noise Control in the Past Decade — Part II: Nonlinear Systems." *Signal Processing 181*: 10792.

[3] Widrow B., Glover J. R., and Mccool J. M. et al. (1975). "Adaptive Noise Cancelling: Principles and Applications." *Proceedings of the IEEE 63* (12): 1692-1716.

[4] Press C. (2001). "Understanding Active Noise Cancellation." *Schools & Disciplines* (1): 66.

[5] Elliott, S. J., and Nelson P. A. (1993). "Active Control of Sound." *Physics Today 46* (4): 75-76.

[6] Meng H., and Chen S. (2020). "A Modified Adaptive Weight-Constrained FxLMS Algorithm for Feedforward Active Noise Control Systems." *Applied Acoustics 164*: 107227.

[7] Jiang Y., Chen S., and Meng H. et al. (2021). "A Novel Adaptive Step-Size Hybrid Active Noise Control System." *Applied Acoustics 182*: 108285.

[8] George N. V., and Panda G. (2013). "Advances in Active Noise Control: A Survey, With Emphasis on Recent Nonlinear Techniques." *Signal Processing 93* (2): 363-377.

[9] Jiang J., and Li Y. (2018). "Review of Active Noise Control Techniques With Emphasis on Sound Quality Enhancement." *Applied Acoustics 136*: 139-148.

[10] Widrow, B., and Stearns, S. D. (1985). *Adaptive Signal Processing*, Prentice-Hall, Englewood Cliffs, NJ, pp. 362-367.

[11] Ma, Y. P., Xiao, Y. G., and Ma, L. Y. et al. (2021). "Statistical Analysis of Narrowband Active Noise Control Using A Simplified Variable Step-Size FXLMS Algorithm." *Signal Processing 183*: 108012.

[12] Gao, B., Zhang, X. G., and Wang, Y. S. et al. (2017). "Active Noise Control for Vehicle Interior Noise Using Iteration Variable Step-Size LMS Algorithm." *Electronic Sci. & Tech 30* (8): 9-12. (in Chinese)

[13] Gomathi, K., and Saravanan, V. (2016). "Variable Step Size for Improving Convergence of FxLMS Algorithm." *Procedia Technology 25*: 420-426.

[14] Qin Jingfan, and Ouyang Jingzheng (1997). "A Novel Variable Step Size LMS Adaptive Filtering Algorithm Based on Sigmoid Function." *Journal of Data Acquisition & Processing 12* (3): 171-174. (in Chinese)

[15] Ru Guobao, Huang Yan, and Guo Yingjie et al. (2015). "New Variable Step Size LMS Algorithm Based on Logarithmic Function." *Journal of Wuhan University (Natural Science Edition) 61* (3): 295-298. (in Chinese)

[16] Liu Jianfeng, Jiang Zhuoqin, and Li Juan et al. (2009). "A Novel Variable Step-Size LMS Adaptive Filtering Algorithm Based on Lorentzian Function." *Command Control & Simulation 31* (2): 42-44. (in Chinese)

[17] Fei Li, Shuang H., and Hui Guo et al. (2024). "Variable Step Size FxLMS Algorithm Based on Sigmoid-Sinh Piecewise Function." *Journal of Donghua University (Natural Science Edition) 50* (1): 93-100. (in Chinese)

[18] Holland J. H. (1992). *Adaptation in Natural and Artificial Systems: An Introductory Analysis With Applications to Biology, Control, and Artificial Intelligence*, MIT Press.