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Study on the Effects of Mass and Stiffness Variations on Fundamental Periods of Structures

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Abstract: Fundamental period is an important parameter in seismic design and performance assessment of buildings. Hence, comprehensive and detailed investigations of effectiveness as well as affectability of this parameter can result in the design of high-performing earthquake-resistant structures. On this basis, this research intends to evaluate the effects of variations of mass and stiffness on the fundamental periods of two three- and nine-story structures representing low- and high-rise buildings, respectively. To this end, a MATLAB code was developed and validated to determine the fundamental periods of structures with various mass and stiffness characteristics. Numerous case studies were performed to investigate the effects of mass and stiffness variations along the stories of the considered structural models. The objective of this research endeavor is to provide a better understanding of affectability of fundamental period under different design considerations.

Key words: Earthquake engineering, fundamental period, structures, mass, stiffness.

1. Introduction

The ability to accurately establish the natural period of vibration is crucial in determining the behavior of low- and high-story structures during an earthquake. Its establishment is a necessary step in estimating the structural response in both the seismic design and performance assessment. This critical attribute for a building's seismic behavior is determined predominantly by its mass and stiffness. Changes in the mass and stiffness parameters can form a connection to the fundamental period of vibration. Multiple studies have been conducted to estimate the fundamental period based on the code provisions of ASCE-7 [1].

Hafeez et al. [2] evaluated the building code formula for estimating light-frame wood building's fundamental period for seismic analysis and proposed alternative simplified rational approach to seismic analysis of these structures. Mohamed et al. [3] studied the fundamental period of vibrations of moment resisting concrete frames and he examined the interaction between the reduction factor and the reduced period of vibration. The study showed that the values of maximum period of vibration can be used as an alternative method to calculate the inelastic base shear value without taking reduction factors into consideration. S.M Noor et al. [4] determined the displacement demand in irregular reinforced concrete building based on the fundamental period of vibrations, as well as the behavior of the building system and concluded that the displacement increases with the increment of the fundamental period of vibration and that for normalized displacement, the larger displacement occurs at the flexible side of the building. Aboelmaged et al. [5] investigated the effect of lateral load resisting systems and irregularities of building configuration on the fundamental period of vibration for steel structures. The comparison showed that the lateral system and building irregularity have a significant effect on the fundamental period of vibration for the buildings with the same height. Jiang et al. [6] established a simplified theoretical method to predict the fundamental period of masonry infilled reinforced concrete frames and found that the proposed

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method is an advantageous approach compared to other existing methods. Research by Kiani and Emami [7] analyzed the effect of infill panels on the fundamental period of moment resisting frames considering the influence of SSI (soil-structure interaction) and established that the number of stores, infill opening percentage, stiffness of the infill panels and soil type are crucial parameters that influence the fundamental period of steel building frames. Torkia et al. [8] proposed new expressions for the fundamental period of reinforced concrete buildings considering the SFSI (soil foundation and structure interaction) effects as well as their variation of particularly 215 existing buildings located in the United States. The results indicated that the value of vibration changes from one earthquake to another as a principal function due to the soil mass, soil effect has a great influence on the natural period more than that of the foundation, and the height of buildings affects the fundamental period. Last, it was found that the earthquake response reduction effects by SFSI are affected by the number of stories, which has an impact on the fundamental period. Van der Westhuizen et al. [9] focused on the use of machine learning algorithms to obtain a new formula that predicted the fundamental period of steel structures by accounting different geometrical features of superstructures and considering the soil-structure interaction (SSI). The validation resulted in a correlation of 99.71% indicating that the suggested formula exhibits high predictive features for steel structures.

In this research, the fundamental periods of vibration for three- and nine-story structures are predicted using a MATLAB code and investigated. The importance of ascertaining the fundamental period of vibration of a structure is crucial in calculation of the lateral forces arising from a seismic event. The ASCE-7 [1] states: the fundamental period of the structure, *T*, in the direction under consideration shall be established using the structural properties and deformational characteristics of the resisting elements in a properly substantiated analysis. Therefore, analyzing the values of the fundamental period of vibration of buildings can show a distinct relationship between deformational characteristics of the resisting elements such as those of mass and stiffness. This study also intends to enhance the efficiency of existing code provisions (e.g. ASCE-7 [1]) in order to accurately predict the fundamental periods of structures.

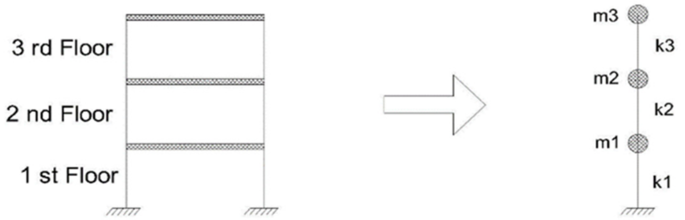
2. Characteristics of Considered Structural Models

In order to understand the characteristics of each structure considered in this study, the side view of a typical 3- and 9-story structures are shown in Fig. 1 along with their mass and stiffness characteristics.

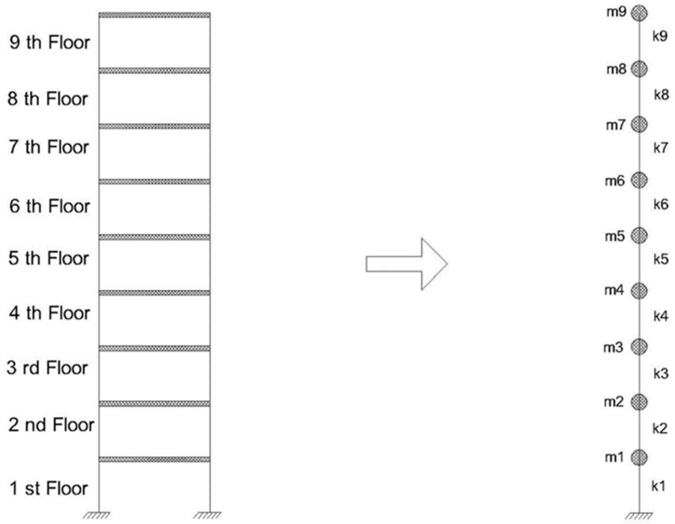
The mass and stiffness variations were chosen based on placing a heavier or a lighter mass than the rest of the floors in each level and similarly a greater or smaller stiffness parameter than the rest of the floors for both the three- and nine-story buildings. In addition, a constant mass and a constant stiffness were considered for each floor, and increasing as well as decreasing patterns were applied to the masses and stiffnesses to observe the differences. The details of the considered cases for the two structural models are summarized in Tables 1~4.

The results of the fundamental periods for the 3-story structure found from the MATLAB Code were gathered and presented in a histogram shown in Fig. 2. Based on the fundamental periods, the trends can now be compared according to each case for the low-rise 3-story structure.

The results of the fundamental periods for the 9-story structure found from the MATLAB Code were gathered and presented in a histogram shown in Fig. 3. Based on the fundamental periods, the trends can now be compared according to each case for the low-rise 9-story structure.



(a) Three-story structure



(b) Nine-story structure

Fig. 1 Considered structural models.

Table 1 Variations of mass with a constant stiffness parameter for a three-story building.

Story #	Stiffness	3M1	3M2	3M3	3M4	3M5	3M6	3M7	3M8	3M9
1	k	m	m	3m	3m	m	m	m/3	m	m
2	k	m	2m	2m	m	3m	m	m	m/3	m
3	k	m	3m	m	m	m	3m	m	m	m/3

Table 2 Variations of stiffness with a constant mass parameter for a three-story building.

Story #	Mass	3K1	3K2	3K3	3K4	3K5	3K6	3K7	3K8	3K9
1	m	k	k	3k	3k	k	k	k/3	k	k
2	m	k	2k	2k	k	3k	k	k	k/3	k
3	m	k	3k	k	k	k	3k	k	k	k/3

Table 3 Variations of mass with a constant stiffness parameter for a nine-story building.

Story #	Stiffness	9M1	9M2	9M3	9M4	9M5	9M6	9M7	9M8	9M9	9M10	9M11	9M12	9M13
1	k	m	m	3m	3m	m	m	m	m	m/3	m	m	m	m
2	k	m	m	3m	m	m	m	m	m	m	m	m	m	m
3	k	m	m	3m	m	3m	m	m	m	m	m/3	m	m	m
4	k	m	2m	2m	m	m	m	m	m	m	m	m	m	m
5	k	m	2m	2m	m	m	3m	m	m	m	m	m/3	m	m
6	k	m	2m	2m	m	m	m	m	m	m	m	m	m	m
7	k	m	3m	m	m	m	m	3m	m	m	m	m	m/3	m
8	k	m	3m	m	m	m	m	m	m	m	m	m	m	m
9	k	m	3m	m	m	m	m	m	3m	m	m	m	m	m/3

Table 4 **Variations of stiffness with a constant mass parameter for a nine-story building.**

Story #	Mass	9K1	9K2	9K3	9K4	9K5	9K6	9K7	9K8	9K9	9K10	9K11	9K12	9K13
1	m	k	k	3k	3k	k	k	k	k	k/3	k	k	k	k
2	m	k	k	3k	k	k	k	k	k	k	k	k	k	k
3	m	k	k	3k	k	3k	k	k	k	k	k/3	k	k	k
4	m	k	2k	2k	k	k	k	k	k	k	k	k	k	k
5	m	k	2k	2k	k	k	3k	k	k	k	k	k/3	k	k
6	m	k	2k	2k	k	k	k	k	k	k	k	k	k	k
7	m	k	3k	k	k	k	k	3k	k	k	k	k	k/3	k
8	m	k	3k	k	k	k	k	k	k	k	k	k	k	k
9	m	k	3k	k	k	k	k	k	3k	k	k	k	k	k/3

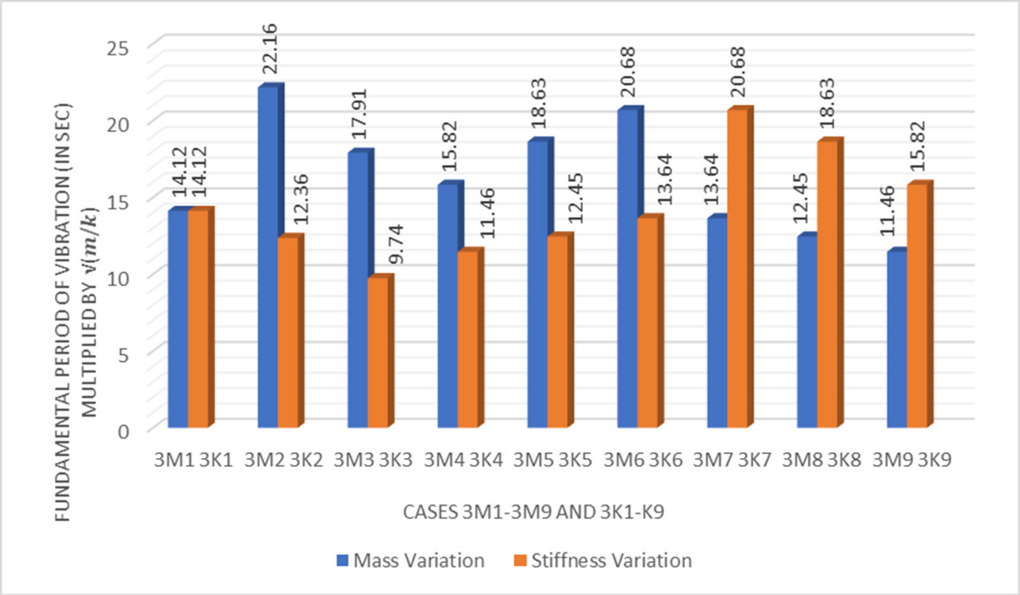


Fig. 2 **Fundamental periods for the mass and stiffness parameters of the 3-story structure.**

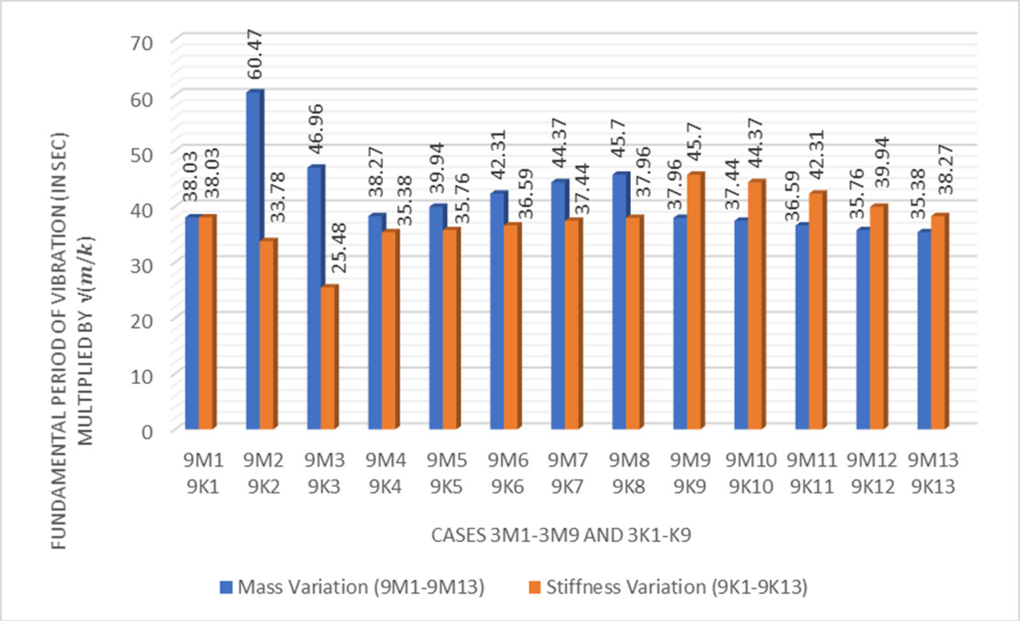


Fig. 3 **Fundamental periods for the mass and stiffness parameters of the 9-story structure.**

Table 5 Validation of MATLAB code with examples taken from various resources.

Resource reference	$\frac{\text{Theoretical Prediction}}{\text{Code Prediction}}$
First example: 3 DOF [10]	For 1st mode $\omega_1 = \frac{0.4450}{0.44504} = 1.00$
	For 2nd mode $\omega_2 = \frac{1.2470}{1.2471} = 1.00$
	For 3rd mode $\omega_3 = \frac{1.8019}{1.8025} = 1.00$
Second example: 2 DOF [11]	For 1st mode $\omega_1 = \frac{0.7071}{0.7071} = 1.00$
	For 2nd mode $\omega_2 = \frac{1.4142}{1.4142} = 1.00$
Third example: 2 DOF [12]	For 1st mode $\omega_1 = \frac{0.5602}{0.56} = 1.00$
	For 2nd mode $\omega_2 = \frac{1.7850}{1.784} = 1.00$

3. MATLAB Code and Validation

Three examples with 2 and 3 DOF (degrees of freedom) were considered from textbooks and a webpage to test the accuracy of the MATLAB code. Table 5 provides the ratio of the source's answer by the value established on MATLAB.

As shown in Table 1, the accuracy of the MATLAB results compared to the theoretical results is very high as all ratios are equal to 1. Therefore, the validation of the code can be defined to be 100% accurate.

4. Parametric Studies and Discussion of Results

4.1 Mass Variations

Fig. 4 illustrates the behavior in the fundamental period for the three- and nine- story structures under the mass variations presented in Tables 1 and 3.

Based on Fig. 4, in the three-story structure plot, the fundamental period rises as the mass increases (see case M2), and it descends as the mass decreases in the higher floors (see case M3). Cases M4-M6 show that when a damped heavy mass is placed in the higher floors, the fundamental period increases. Similarly, cases M7-M9 indicate that when a damped light mass is placed in the higher floors, the fundamental period decreases.

In the nine-story structure plot, the fundamental period rises instantly as the mass increases (see case M2), and it descends at the same rate as the mass decreases in the higher floors (see case M3). Cases M4-M8 show that when a heavy damped mass is placed in the higher floors, the fundamental period increases. Similarly, cases M10-M13 indicate that when a light damped mass is placed in the higher floors, the fundamental period decreases.

By comparing the effects of mass variations in the three- and nine-story structural model plots, there is a distinct similarity between them. Both plots behave the same way, however the effect on the nine-story structure seem to have a bigger effect in the fundamental period than that of the three-story structure. This indicates that the more stories a structure has, the bigger the impact towards the fundamental period. In addition, both plots prove that it is ideal to have lighter masses in the higher floors of the structure, as the fundamental period will be at its lowest value. Therefore, having a small value for the period will lead to a safe design of structures by allowing them to have a stronger resistance against lateral forces.

A comparison can also be made between the base point (M1) and a range of groups, M2, M3, M4-M6, M7-M9 for the 3-story structure and M2, M3, M4-M8, M9-M13 for the 9-story structure. The groups were divided in such a way due to the similarity of mass variations, as well as sharing the same trends in Fig. 4. For example, groups M4-M6 in the 3-story structure present a heavy mass being placed at each floor of the structure and share an increasing trend in the fundamental period. In addition, the base point (M1) was highly considered for this comparison as it represents a typical case to which the mass remains the same in every floor.

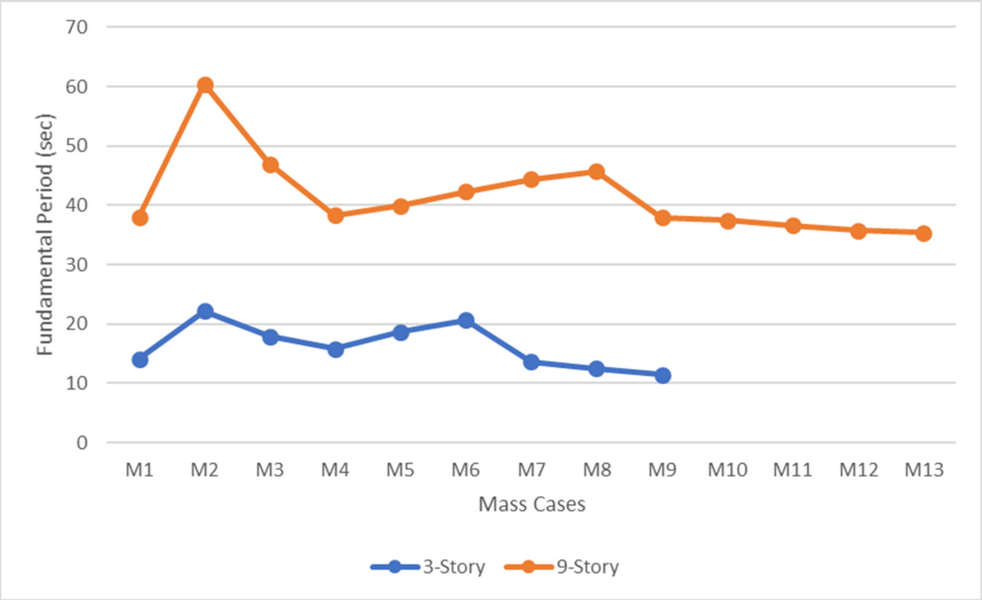


Fig. 4 Effects of mass variations in the three- and nine-story structures.

Beginning with the 3-story structure, group M2 shows an increase in the period by 56.94% compared to the base point M1 which indicates that increasing the mass as the story number increases will result a larger fundamental period. Similarly, group M3 still shows an increase in the period by 26.84% compared to the base point M1, which shows that decreasing the mass as the story number increases will result in less of an increase in the period compared to group M2. In the M4-M6 group, there is a significant percent increase by 46.46% compared to the base point M1, indicating that when placing a heavy mass in the higher floors of the structure, it will result in a larger increase in the period compared to group M3, but a lower increase in the period compared to group M2. Last group M7-M9 shows a slight decrease in the period by 18.84% compared to the base point M1, indicating that placing a low mass in the higher floors of the structure will produce a lower period compared to all groups including base point M1.

Subsequently, for the 9-story structure, group M2 shows an increase in the period by 59.00% compared to the base point M1 which indicates that increasing the mass as the story number increases will result a larger fundamental period. Similarly, group M3 still shows an increase in the period by 23.48% compared to the base point M1, which indicates that decreasing the mass as the story number increases will result a lower increase in the period compared to group M2. In the M4-M8 group, there is a slight increase by 20.17% compared to base point M1, indicating that when placing a heavy mass in the higher floors of the structure, it will result a lower increase in the period compared to groups M2 and M3. Last group M9-M13 shows a slight decrease in the period by 6.97% compared to the base point M1, indicating that placing a low mass in the higher floors of the structure will produce a lower period compared to all groups and base point M1.

4.2 Stiffness Variations

Fig. 5 illustrates the behavior in the fundamental period for the three- and nine- story structures under the stiffness variations presented in Tables 2 and 4.

Based on Fig. 5, in the three-story structure plot, the fundamental period decreases slightly from case K1 (constant stiffness) to case K2 (increasing stiffness in higher floors). However, in case K3 (reducing stiffness in higher floors) the fundamental period decreases even more. This indicates that it is ideal to increase the stiffness in the lower floors and decrease the stiffness

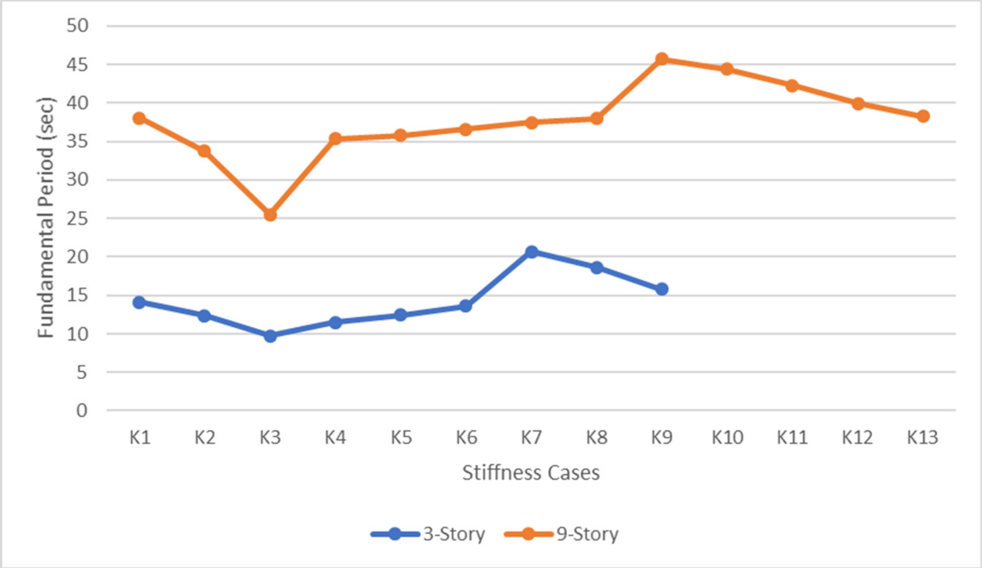


Fig. 5 **Effects of stiffness variations in the three- and nine-story structures.**

in the higher floors as it results a low period. Cases K4-K6 show that having a high stiffness in floors one, two, and three, respectively, the fundamental period increases. This agrees with cases K2 and K3, as they prove that placing a high stiffness in the lower floors will result a lower period. In case K7, a low stiffness is placed on the first floor and greater stiffnesses are placed in floors 2 and 3, therefore as expected, the plot shows an increase in the fundamental period. Similarly, cases K8 and K9 indicate that when a low stiffness is placed in floors two, and three, respectively, the fundamental period decreases. Once again this proves that a lower stiffness placed in the higher floors results a decrease in the fundamental period.

In the nine-story structure plot, the fundamental period still decreases slightly from case K1 (constant stiffness) to case K2 (increasing stiffness in higher floors). The transition from case K2 to case K3 (reducing stiffness in higher floors), reduces the period drastically. This pattern follows the same scenario as in the three-story structure where placing a higher stiffness in the lower floors will result a low fundamental period. Case K4 shows that placing a high stiffness on the first floor results an increase in the period. This indicates that more of the bottom floors need to have high stiffness (similar to case K3), in order to result the lowest period. Similarly for cases K5-K8, placing a high stiffness in one of the higher floors (floors 3, 5, 7, and 9), results an increase in the period. In the case of K9, a low stiffness is placed on the first floor and greater stiffnesses are placed in the rest of the floors, therefore similar to the three-story structure, the plot shows an increase in the fundamental period. Last, cases K10-K13 prove that there is a decrease in the fundamental period as a low stiffness is placed in one of the higher floors (floors 3, 5, 7, and 9, respectively).

By comparing the effects of stiffness variations in the three- and nine-story structural model plots, there is a distinct similarity between them. Both plots behave the same way, however, similarly to the mass variations, the effect on the nine-story structure seems to have a bigger effect in the fundamental period than that of the three-story structure. Again, this indicates that the more stories a structure has, the bigger the impact towards the fundamental period. In addition, both plots prove that it is ideal to have a high stiffness in the lower floors and a low stiffness in the higher floors of the structure. As mentioned before, a low period is recommended to ensure a greater resistance towards lateral forces.

Similar to the mass variations, a comparison can also be made between the base point (K1) and a range of groups, K2, K3, K4-K6, K7-K9 for the 3-story structure and K2, K3, K4-K8, K9-K13 for the 9-story structure. The groups were divided in such a way due to the similarity of stiffness variations, as well as sharing the same trends in Fig. 5. For example, groups K4-K6 in the 3-story structure present a high stiffness being placed at each floor of the structure and share an increasing trend in the fundamental period. In addition, the base point (K1) was highly considered for this comparison as it represents a typical case to which the stiffness remains the same in every floor.

Beginning with the 3-story structure, group K2 shows a slight decrease in the period by 12.46% compared to the base point K1 which indicates that increasing the stiffness as the story number increases will result in a smaller fundamental period. Similarly, group K3 still shows a decrease in the period by 31.02% compared to the base point K1 which shows that decreasing the stiffness as the story number increases will result in a larger decrease in the period compared to group K2. In the K4-K6 group, there is a percent decrease by 18.84% compared to the base point K1, indicating that placing a high stiffness in the lower floors of the structure will result in a decrease in the period between groups K2 and K3. Last group K7-K9 shows an increase in the period by 46.46% compared to the base point K1, indicating that placing a low stiffness in the lower floors of the structure will produce a higher period compared to all groups and base point K1.

Subsequently, for the 9-story structure, group K2 shows a slight decrease in the period by 11.18% compared to the base point K1 which indicates that increasing the stiffness as the story number increases will result in a smaller fundamental period. Similarly, group K3 shows a large decrease in the period by 33.00% compared to the base point K1 which indicates that decreasing the stiffness as the story number increases will result in a larger decrease in the period compared to group K2. In the K4-K8 group, there is a very slight decrease by 6.97% compared to the base point K1, indicating that when placing a high stiffness in the lowest floor of the structure, it will result a smaller decrease in the period compared to groups K2 and K3. Last group K9-K13 shows an increase in the period by 20.17% compared to the base point K1, indicating that placing a low stiffness in the lowest floor of the structure will produce a higher period compared to all groups and base point K1.

4.3 ASCE-7 Code Predictions and Practical Design Recommendations

ASCE-7 [1] provides an approximate method (section 12.8.2.1) to establish the fundamental period of a structure using three empirical equations. The first equation provided (Eq. (12.8-7)) is applied to all structures and is expressed as follows: $T_a = C_u h_n^x$, whereas the other two equations are provided for certain moment frames (Eq. (12.8-8)) and masonry or concrete shear wall structures (Eq. (12.8-9)) and are expressed as follows, respectively: $T_a = 0.1N$ and $T_a = \frac{0.0019}{\sqrt{C_w}} h_n$. It is worth noting that it is permitted to use the approximate method as an alternative approach to calculate the fundamental period. Since this study is correlated with a general case of low rise to high rise structures, Eq. (12.8-7) is applied. Eq. (12.8-7) consists of the period's upper limit (C_u) whose values are provided in Table 12.8-1 multiplied by the structural height (h_n) being raised to the *x* power which in the case of a general structure type the value of *x* given to be 0.75 (see Table 12.8-2). The main purpose of estimating the fundamental period of a structure is to obtain the design base shear and distribution of the shear along the height of the structure. In Eq. (12.8-7) the period's upper limit prevents low equivalent lateral force base shear that are excessively flexible. Therefore, it is ideal to establish the lowest possible fundamental period to satisfy the design criteria by examining the effects in the mass and stiffness in each floor. It is important to state that ASCE does not associate the fundamental period to the mass and stiffness variations of a structure, but rather to the height of the structure.

5. Conclusion

In this study, the effects of variations of mass and stiffness on the fundamental periods of three- and nine-story structures representing low- and high-rise buildings were investigated. The period values were determined using a MATLAB code which was, in turn, validated through comparison with established results.

Based on the results of this study, with invariant stiffness, the largest increase in the fundamental period occurred in those structures having increased masses with story heights. However, the largest decrease in the fundamental period occurred in the structures for which the topmost floor masses were a third of the remaining individual floor masses.

It was also shown that with invariant mass, the largest decrease in the fundamental period occurred in those structures having decreased stiffnesses with story heights. The largest increase in the fundamental period occurred in the structures for which the first floor stiffnesses were a third of the remaining individual floor stiffnesses.

This study showed the effects of variations in mass and stiffness on the fundamental periods of structures illustrating the significance to be taken under consideration in seismic design which, interestingly, the ASCE-7 code neglects such considerations and only associates the fundamental period of the structure as a function of height.

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