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Theoretical Models of Space and Its Philosophical Category

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Diverse concepts of space developed in history of natural philosophy, mathematics, physics, and other natural or cultural studies form theoretical models of spatial relations, given in human's experience. Their diversity is due not only to the multiplicity of philosophical and methodological approaches to the concept of space, but also to the variety of ways, in which spatial relationships can be organized. This variety gives a possibility to distinct autonomous spaces of different types with diverse sets of properties as well as separate spaces with their own ordinal, metrical, and sequential structures. Particularly, various ways of space semiotization in culture generate different types of autonomous and separate spaces: written texts, maps, pictures, chessboards, etc. In the same time, all particular notions of space are included in a general logical class. Its volume and content are covered by the philosophical category of space. Such general category cannot be reduced to mathematical, physical, or other concepts of space elaborated in particular sciences, however, it serves as a philosophical basis for their comparison.

Keywords: spatial relations, autonomous and separate spaces, variety, theoretical model, philosophical category

Distinctions in Theoretical Models of Space

Historical Distinctions

Theoretical ideas about space have been formed over a long history. Still from Hesiod's times, the mythological images of an initial *chaos* arise in the Pythagorean concept of *apeiron* (τό ἄπειρον)—as of something unlimited and non-defined, which opposed to any limit and order (see Aristotle, *Phys.* 203a3, 203b4). Democritus considered absolutely emptiness (κενόν) as an opposition to atoms that, unlike it, have all the fullness of being (see Aristotle, *Met.* A 4. 985 B 4). In Plato's *Timaeus* the amorphous "chora" (χώρα) is opposed to the "eidos" (εἶδος) which forms it (Plato, *Tim.* 50c). Aristotle in *Metaphysics* interprets the same opposition as distinction of "matter" (ὕλη) and "form" (μορφή) formulating in *Physics* the concept of "place" (τόποσ), which, unlike the apeiron, chora, or kenon, is a fully structured and defined "boundary of the enclosing body" (Aristotle, *Phys.* 212a5).

Already in ancient natural philosophy, these concepts were recognized as different aspects of the spatiality. Thus, Epicurus, according to Sextus Empiricus, believed that "the same nature being deprived of any body is called emptiness (κενόν), occupied by a body, it is called place (τόποσ), and it is called space (χώρα), when bodies pass through it" (Empiricus, 1975, p. 316).

In the Early Modern Age, Descartes considered space a necessary attribute of bodily substance, abstracted from it. As places filled with bodies, space is also thought of as extended when considered without bodily

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filling (Descartes, 1989, pp. 353-355). In a similar way, Leibniz (1982) believed that "Space, finally, is what comes out of the sum of all the places" (p. 479), treating a place as a certain way of entering into a set of relationships. Unlike Descartes, Leibniz (1982) did not regard space as an attribute of corporeal substance, but saw in it an "order of possible arrangements" (p. 496) in which coexisting bodies could find themselves. Rejecting both the attributive approach of Descartes and the relativistic view of Leibniz, Newton in the *Mathematical Principles of Natural Philosophy* leaned toward its substantial treatment, believing that in addition to the relative space between moving bodies, there is also the absolute space, which "remains always the same and immobile" (Newton, 1989, p. 30).

Kant (1994, pp. 50, 52) made his "Copernican coup" by treating space not as a beginning inherent in the external world, but as a "necessary a priori representation underlying all external contemplations", "a subjective condition of sensibility", and "the possibility of phenomena", inherent in the transcendental subject. Thereby, the subject no longer thought of as acting in an external space, but conversely space and time began to be treated as forms of contemplation inherent in the subject.

Disciplinary Differences

If the first ideas about space were historically formed in natural philosophy (to which still Newton paid tribute with the title of his book), its scientific description began to form in mathematics, physics, and later, in studies of culture.

Soon after Kant, *mathematicians* showed that Euclidean geometry, which he, following Newton and others, considered the single and universal, is only one of many mathematical models capable of precisely describing spatial relationships (Rosenfeld, 1976).

Physics can choose among them those that are more consistent with its laws. Only in combination with a certain physical theory, a geometric system ceases to be a purely speculative construction and serves as a more or less convenient language for the description of the real world (Einstein, 1966, pp. 85-86; Poincaré, 1983, pp. 63-64, 350-351; Reichenbach, 1985, pp. 55-56).

Other *natural sciences* began to form their own notions of space. Thus, Pasteur, Curie, and then Vernadsky considered different states of space, sharply contrasting the symmetric space of crystals and other inanimate bodies to the dissymmetric and curved space of living bodies that is described better by geometry of Riemann (Vernadsky, 1988, pp. 174-175, 261-264, 273).

In the same time, it turned out that the model of space, which Euclidean geometry provides, is not relevant also for *psychophysiological descriptions* of its perception by the subject. The visual field always includes only a limited "frame", where the space is heterogeneous and anisotropic due to distinguishes between centre and periphery, right and left, width and depth, etc. (Mach, 1991, pp. 87-100). Their own peculiarities were found in visual, tactile, and motor spaces, which also differed from the space of Euclidean geometry (Poincaré, 1983, pp. 42-45).

In the *cultural sciences* developed from the end of the 19th century, the collective views of space began to be studied. *Ethnographers* start to describe mythological representations of it in archaic cultures (Cushing, 1896). *Sociologists* connected the structuring of social space with the formation of logical classifications (Durkheim & Moss, 1996). *Art researchers* began talking about the differences between "optical" and "haptic" ways of perceiving space in different types of art (Hildebrand, 1991), in different historical epochs (Riegl, 1901) and in stylistically different "forms of vision" (Wölfflin, 1915). Historical changes in the ways of depicting space allowed them to consider also perspective as one of the “symbolic forms” (Panofsky, 1927; cf. Florensky, 1993a; 1993b; Mochalow, 1983).

According to Cassirer (1923-1929, 1985), the transition from the natural philosophical approach in the theoretical description of space to its understanding in the cultural sciences is associated with a shift of interest from the *being* of space to the various ways of its *understanding* by human—from what space *is* to what it *means*.

Since the 1960s, *semiotics of space* began to develop as a discipline that studies not only the meanings of various spatial artefacts, but also the semiotic systems that provide the links between the former and the latter (Eco, 1998; Lagopoulos, 1993; Lotman, 1986; Pellegrino, 1999-2007; 2006; 2020; Tchertov, 2019). Obviously, space is for this discipline one of the key categories that still needs special understanding.

Distinctions of Methods

The variety of space models is related to the tasks they have to solve and to the *methods* used to their creating. In particular, the specificity of pure mathematical modelling of space is that it leaves all material bodies and their visual images outside the theory, being limited only by the requirement of its logical consistency with a certain system of axioms (Hilbert, 1948). Unlike it, physics, and natural sciences in general, must reconcile theoretical models of space with empirical data and relate them to their concepts—mass, energy, velocity, etc.

The sciences of culture have their own empirical basis, and their task is to describe various ways of structuring and comprehending space that emerge in collective consciousness. Whereas exact sciences avoid anthropomorphic representations of space and seek to identify general laws independent of the observer’s position, the cultural sciences, by contrast, analyse subjective human ideas. Instead of a unified continuum of zero-dimensional and qualitatively indistinguishable points, which mathematicians and physicists develop, cultural sciences look for a variety of qualitatively different elements and structures in space, endowed with different meanings.

These differences in the subject of research are also related to the distinctions in the methods of natural sciences and humanities. In particular, the principle of relativity claiming the independence of movement of physical bodies or properties of geometrical figures from their position in space is not relevant for the cultural sciences. On the contrary, they are interested in the distinctions of meanings connected with different positions of spatial objects and in that absolutization of the oppositions of top-bottom, centre-periphery, etc., which people tend to produce when interpreting spatial relations.

Philosophical Problems of the Spatiality

Despite a multitude of studies devoted to it, the concept of space retains Plato’s characteristic of the chora being an “extremely elusive” entity (Plato, *Tim.* 51b). At the same time, for all differences between the approaches to problems of space in the exact sciences and in the humanities, both constantly reproduce certain oppositions.

These are, in particular, different views on the relationship of space to the *amorphous* and the *formed*, the *chaotic* and the *ordered*, the *random* and the *necessary*, the *spatiality* and the *corporeality*, *real* spatial relations and *ideal* representations of them, as well as the opposition of a *single space* to an idea of *multiplicity of spaces* with different properties. Consider them in more detail.

The Amorphous and the Formed

The conceptions of space differ, in particular, in how they relate to the opposition of the *amorphous* and the *formed*. The *chaos* of Hesiod is devoid of any form as well as the *apeiron* of the Pythagoreans or the *kenon* of Democritus. The *chora* of Plato can take a form and lose it as something external. What they all have in common is a lack of qualitative and quantitative definiteness: space is considered as something that not only has no external limit, but also is not delimited internally, is something without any internal structure and quality. Structurality and qualitative certainty are associated with something else, such as shaped *bodies-atoms* (by Democritus) or *eidos-forms* (by Plato).

Unlike them, the *topos* by Aristotle is thought as something formed, bordered, and allowing for internal delimitation (Aristotle, *Phys.* 212a5). The *limit* (πέρας) since Pythagorean times has been thought of as the direct opposite to the *apeiron*. In Euclidian geometry, the boundaries of lines, surfaces, and volumetric bodies also determine their form. Kant (1994, p. 48), oriented to this geometry, believed that the form is brought into the amorphous “matter” of perceived phenomena, which are thereby ordered in subject’s contemplation.

The Spatiality and the Orderliness

The ancient *cosmos* was also thought of as an opposite of the *chaos*. In this no less old notion, the world was seen as something ordered and logically arranged. Opposed to chaos in content, the concept of space is comparable to it in the volume of objects it covers. Indeed, the definition of “a place that contains everything” does not only apply to Hesiod’s chaos—as Sextus Empiricus (1975, p. 318) wrote. At least, from the Pythagorean times, the universe is described by the ancient concept of cosmos (Losev, 1989).

All the more so, Euclidean geometry, introducing scientific knowledge of space instead of mythological notions, banished chaos from its limits and made the order of figures an object of precise mathematical investigation. In modern times, Descartes’ analytic geometry linked this spatial order with a system of numbers, allowing it to be expressed in some system of coordinates. Mathematics thus provided the basis for looking at space as an order of possible relations—as Leibniz (1982, p. 496) has formulated.

However, even in mathematics the idea of space as a non-structural beginning remains in the concept of an amorphous continuum. According to Poincaré, the Euclidean or non-Euclidean spaces can be formed from a spatial continuum in a result of accepting certain agreements about the ways of its measurement—just as a thermometer can be graded in different ways (Poincaré, 1983, pp. 183, 342, 422).

Necessity and Randomness in Space

The questions “how is pure mathematics possible?” and more special: “which must be a representation of space, so that geometry as a priori synthetic knowledge could be possible?” gave an impetus to Kantian reasoning of space aimed at explaining general schemes and laws, but not specific instances of various spatial arrangements. While asserting, “All notions of space are based on a priori (not empirical) contemplation”, Kant (1994, p. 51) does not explain how diverse empirical and given a posteriori configurations of spatial relations are possible, distinguishing one experience from another (cf. Russell on Kant, 1959, p. 732). Meanwhile, the accidental and unpredictable in spatial relations are as intrinsic to them as the regularly and necessary. No game could exist, and no text could carry information, if all the spatial configurations that arise in them were known a priori to those playing or communicating. In a card game, for example, knowing a priori how the spatial relationships between the cards will be distributed would be cheating, forbidden by the rules of the game. In reading a text, prior knowledge of the combination of letters that makes it up would be redundant to obtain information, which, by definition, is a removed uncertainty and cannot be given a priori (Shannon, 1948). Just as knowledge of the alphabet and the rules of grammar does not yet give knowledge of all the texts in which they are used, so knowledge of the general laws of spatial organization does not yet give knowledge of specific combinations of spatial relations (cf. Cassirer, 2009, p. 109).

The Spatiality and the Corporeality

Another aspect of understanding spatiality is the problem of its relationship to corporeality. The sharp opposition of emptiness to atoms by Democritus is softened in Plato, whose *chora* is not alien to corporeality, although it is formless. Aristotle's notion of *topos* eliminates the difference between the body and the place it occupies in relation to being and non-being or shaped and formless; both exist and are shaped, although topos can be filed or emptied.

In Modern time, the Newton's physics revived in another form the old opposition between bodies and absolute space as a universal receptacle independent of them. R. Descartes (1989) thought differently about their relationship, regarding space as an attribute of corporeal substance and after Epicurus writing that "the same extension is the nature of both body and space" (p. 353).

The convergence of spatiality and corporeality has a reverse side: it is possible not only to think of space as an attribute of corporeality, but also, on the contrary, to consider bodies themselves as special states of space, "clots" of energy in relatively small parts of it (Einstein, 1966, p. 243; Einstein & Infeld, 1965, p. 201).

Different relations of spatiality and corporeality can be also found in mathematical concepts. The analytical geometry developed by Descartes brought instead of the "synthetic" geometry of Euclid, and the ancient corporeal view of space was replaced by the New European view of it as a systematically ordered set of "geometrical places"—points. Thanks to this method of attributing all spatial objects to calculable coordinates, it became possible to consider both bodies and all distances between them as differently extended parts of a single ordered space. Thus, Descartes reduced corporeality itself to spatiality.

According to Cassirer (1912), the scientific discovery of analytic geometry performed a philosophical "revolution of the way of thinking" (p. 99). The method of systematical indexation of space by means of numerical series ordered along coordinate axes (cf. Weil, 1989, pp. 61, 195) gave mathematical expression to the idea of a unified and ordered space, which no longer needs to operate with any corporeal figures, nor with the opposition of empty and full.

In this respect, analytic geometry provides essentially the same thing for the theoretical description of space as the system of linear perspective demanded by the same New European consciousness does for its pictorial representation. After all, perspective also subordinates all points of space to a single rule, regardless of whether the spatial relations between bodies or the three-dimensional forms of these bodies themselves are depicted (Panofsky, 1927).

Cultural historians describe a general transition from the static "sculptural" worldview of the ancient Greeks to a dynamic "Faustian" consciousness, which thinks of bodies in a force field acting uniformly in space (Spengler, 1918). This dynamic vision of space theorized in Newton's mechanics is also vividly expressed in the paintings of Rubens and other Baroque artists. Art historians have begun to describe these changes from the Late Roman period to the Baroque and the Impressionists as a transit from an ancient bodily vision to a spatiality that absorbs the corporeal (Riegl, 1901; Wölfflin, 1915). Similarly in the history of architecture, the relation of corporeal masses and space between them has stylistic significance and is treated differently in various periods (Brinkman, 1922; Gabrichevsky, 1923). The question of the relationship between corporeality and spatiality also faces the semiotics of space, which could either be opposed to "semiotics of the body" as a separate field of knowledge or include it as a constituent part. Thus, there are grounds for different approaches to the relationship between the corporeality and spatiality not only in the history of natural philosophy, physics, and mathematics, but in the cultural sciences too.

Space as an Attribute of Corporeal Substance or as an Ideal Schematics

The question of the relation between spatiality and corporeality has more one aspect that is related to traditional epistemological distinctions. In this respect, Descartes' position is opposed to Kant's conception. By Descartes (1989, p. 335), space is an attribute of corporeal substance, just as thinking is an attribute of spiritual substance. However, already Leibniz (1982) believed that as an order of *possible* arrangements, space has an ideal character, because in this order "the spirit comprehends the application of relations" (p. 479). Kant (1994, p. 48) at all viewed any ordering in space and time as an ideal form or schematics given a priori to the subject. The Kantian-oriented mathematicians also thought of space as an ideally represented order of relations. H. Weil regarded space as that which is constructed using the notions of coordinates and the manifold of real numbers (Weil, 1989, p. 61).

If space appears as an ideal order or a Kantian "form of contemplation" introduced by the subject into the "matter" of his sensations and is a *law* that links coexisting objects, then it can no longer be the *place*, *where* bodies reside or a *milieu*, in which their movements take place. To express these differences at least two different concepts are required—like notions of autonomous and separate spaces that will be discussed below.

Uniqueness or Multiplicity of Spaces

Among the various concepts of space, "monistic" and "pluralistic" approaches to this notion are distinguished. The first of them considers space as something unified. In Plato, the chora can take different forms, but it is still a single beginning. In Aristotle, also the relations of form and matter permeate the entire cosmos. In Epicurus and the Stoics, it is said to be possible to describe it differently, however it is "one and the same nature" (Empiricus, 1975, p. 316). Newton distinguished between relative and absolute space, but considered both as different ways of describing the same physical reality. Kant (1994) believed that "space is essentially one" and that all talk of many spaces "only means parts of one and the same single space" (p. 51).

The idea of manifold spaces appears in mathematics together with non-Euclidean geometries. Riemann (1956) points to the possibility of an infinite variety of spaces that would differ in measures of constant or inconstant curvature; Klein (1956) relates descriptions of various spaces in distinct systems of geometry to different groups of transformations and their invariants. On the other hand, as said before, differences were found between the spaces of sensory perception and the space described by Euclid's geometry as well as among the spaces constructed in different modalities of sensation (Poincaré, 1983, pp. 42-45).

Art historians also began to talk about different spaces. Tarabukin (1993-1994) suggested distinguishing between the "static" space of ancient art and the "dynamic" space of the art of post-Antique Europe, where "eccentric" space of medieval painting differs from "concentric" space of Renaissance and Baroque painting, etc.

Along with intra-disciplinary concepts of various spaces, inter-disciplinary studies have appeared, in which the concepts of space in different disciplines were compared. This comparison was initiated by representatives of the exact sciences (Helmholtz, 1956; Mach, 1991; Poincaré, 1983) talking about the differences between the space of sensory perception and the space of Euclidean geometry. In the same vein, Carnap (1922) described the differences between formal mathematical space, the space described in physics, and the space that humans perceive with their senses. Russell (1948) in the same key contrasted in general the conceptual space described in sciences as physics or mathematics and the perceptual space of sense experience. Cassirer (1923, pp. 29-31) introduced the notion of *space modes*, considering not only geometrical, physical, and sensually perceived space, but also the space constructed in language, in myth and generally in various "symbolic forms" of culture (Cassirer, 1923-1929; 1985).

The Plurality of Space Models

What Cassirer called "modes of space" is quite legitimate to call its *models*, if each such model will be understood as something functionally related to some other as a means of its representation due to structural similarity to it (Tchertov, 2015). The objects of spatial modelling can depend on the activities of the subject or exist independently of it. The thesis that all spatiality is inherent only in the subject as an a priori form of contemplation of possible phenomena, but not inherent in external objects, refers no longer to critical philosophy, within which Kant sought to remain, but to dogmatic philosophy, from which he distanced himself. It does not follow from the presence of spatial schematics as a form of contemplation that there is no spatiality in what this contemplation is aimed at; the opening of the former does not yet give rise to the closing of the latter. However, the construction of models of space, in which it variously relates with form and formless masses, with order and chaos, with necessity and randomness, with corporeal and immaterial formations, with real objects and ideal images, etc.—is the prerogative of the subject of activity.

It follows from the above that the various aspects of spatiality and diverse approaches to them manifest the *multiplicity of space models* that cannot be reduced to "an only true" one. Various models of a spatial object can be formed on distinct grounds and not canceling each other. One the same object may be modelled in a different way depending on what is important for each model and on ways used for it. A picture at a wall can be represented in physic notions as a body related to other bodies indoors, on Earth or in cosmic space. By the means of geometry, it can be modelled as a rectangle of definite size and proportions. However, for the picture, perceptual ways of its modelling are more important than conceptual methods. At this perceptual level, also at least two ways of modelling are possible: vision of picture surface, where paints are placed, and vision of three-dimensional space of depicted objects. None of these models negates any of the others, and the more, none of them negates a spatiality of the object placed for a viewer. Contrariwise, using them together provides an opportunity to know more on both the object and subject viewing at it.

On Philosophical Concept of Space

Diverse models of spatial objects can enter in various relations—to be separated or combined, excluded or included to each other, etc. At the same time, the connection of all these models with space is justified only under the condition that there is *a general concept of space* common to them all.

Kant (1994, p. 49) is right that such a concept is not given by experience. According to Einstein (1965, pp. 39-40), the "bold notion" of space is useful but not necessary for expressing geometrical relations between bodies. Russell (1997, p. 238), too, thought that the construction of a single space to which all our perceptions are attributed is convenient but cannot claim unconditional truth. However, the notion of space is necessary to establish what the various models have in common.

The comparison of theoretical models of space from different scientific disciplines—“exact” or “inexact”—is the subject of *epistemology*. At the same time, a development of the general concept of space is the task of philosophical *ontology* as a science on universals of being, which alone can afford generalizations of knowledge obtained in particular sciences.

In this respect, even mathematics, by providing ways of modeling spatial objects studied in specific sciences, cannot replace the more general *philosophical concept* of space. This concept does not require figuring out any numerical characteristics of spatial quantities or using such basic geometrical concepts as point, line, plane, angle, etc. Taking into account the concepts of mathematics, physics, or any other specific sciences, philosophy can use its own general notions as “object”, “relation”, “quality” and “quantity”, “part” and “whole”, etc. or as it were “naive” expressions of ordinary language as “closer” or “farther”, “inclusion” and “exclusion”, etc. On this ground, a philosophical concept of space can be formulated in coordination with concepts of the same name in more special sciences.

Spatial Relations

Spatial Relations and Configurations of Objects

In contrast to the “bold concept of space” as a product of theoretical reasoning, *spatial relations* are open to perception as a part of experience and can serve as an empirical basis for theoretical modelling. These relations are formed between coexisting objects and are connected with their mutual arrangement. Qualitative features of such coexistence are characterized by relations of *touching* and *separation*, *joining* and *dissection*, *overlapping* and *isolation*, *inclusion* and *exclusion*, etc. Its quantitative features are manifested through relations of *closer* and *farther*, *longer* and *shorter*, *wider* and *narrower*, etc.

Any members of spatial relations will be called *spatial objects*. Each spatial relation connects at least a pair of such objects. Outside the objects connected by them, there are no spatial relations. Complexes of relations between coexisting objects form their *configurations*, which can be included as parts in new configurations and can themselves be composed of such parts. If one of these objects forms a part of the other, the first is *included* in the second, and the second *includes* the first. If two spatial objects have a common constituent part, then they *intersect* in this part. If none of the objects has common parts, they mutually *exclude* each other. A participation in the qualitative and quantitative spatial relations gives rise to the question “Where is this object?” At the same time, none of these relations is a spatial object, and this question cannot refer to them.

Ordinal Relations and Structures of Spatial Objects

There are in spatial configurations *asymmetrical relationships*. Each of them involves two opposite and inseparable “halves”—“direct” and “converse” relations. For example, if one object is “longer” than another is, then the conversion of this relation is also true, and the second object is “shorter” than the first. Both relations are here not two single entities, but two mutually converse sides of the same relationship “longer/shorter”.

A configuration, where all members are connected with one the same asymmetric relationship, and one of any three of its members is always *between* them, is an *ordered row* and has therefore a *spatial order*. Each member that is between others in an ordered row dissects it at two parts in a unique way that differs from entering ways in the same row of any other its member. Therefore, each such unique way of entering spatial objects into the ordered row sets a special *ordinal place* in it.

An ordered row can be coordinated with the "*forward*" and "*reverse*" successions of transition from one of its members to another one. This gives grounds to distinguish the *directions* of these series, which are in a mutually inverse relation.

If the same spatial object can be a part of several rows ordered by different independent relations (for example, on anthropomorphic axes "above-lower", "left-right", etc.), then one can say that these rows *intersect* in its location and consider this place as a *node* of their intersection. For example, in the crossword space, a letter included at once in both vertically and horizontally arranged words is located in the node of their intersection.

A set of ordinal places belonging as nodes to more than one order can be called a *network of ordinal relations*. Such a network forms a *unified ordinal structure*, if each of its members belongs to at least one ordered row from it, and each of these rows has at least one common member with another row from this net. For example, a network of subway stations has a unified ordinal structure, if either each of them can be reached from each other directly or by transfers within the same net; this network, at the same time, may not have a common ordinal structure with the network of ordinal relations between metro stations in another city.

Extension and Metric Structure

Along with qualitative asymmetric relations forming a spatial order, there are also quantitative relations between its members. A pure spatial order still does not specify certain quantities, and therefore its establishment is not yet sufficient for determining whether objects of a certain size can or cannot fit in a particular place. The ordinal relationships of nodes on a railroad track map remain the same as between real stations, but no train will fit on this map.

These quantitative spatial relations are generalized by the notion of *extension* and its derivatives. In particular, the notion of *extended place* is derived from it as such a complex of spatial relations, which, unlike an ordinal place, has a certain *magnitude*, and in which configurations of spatial objects may be or not be placed depending on their quantitative characteristics. The latter are determined as a result of *measurements*, i.e. by correlation with some unit of scale assumed unchanged when transferred relative to other quantities (cf. Riemann, 1956, p. 311).

If there is a certain way to measure, then *distances* between spatial objects can be determined, and these objects themselves can be compared by their *sizes*. An establishing of these sizes and distances using a certain number of accepted units is the *metrization* of an objects formation. If all external and internal relations in such formation can be correlated with a unified measure of magnitude, and in this sense, are *commensurable*, this formation has a *unified metric structure*.

The last, like the unified ordinal structure, includes not every formation of spatial relations. The spaces of Riemannian geometry with a variable measure of magnitude are devoid of it—as it is in the case of the space-time continuum in the general relativity theory, which have a "point-to-point variable metric" (Einstein, 1966, p. 158). The difference in metric structure can also be found in various cultural phenomena: geographical maps with different scales, paintings depicting human figures in a reduced or enlarged scale, etc.

Spaces and Their Features

Spaces and Their General Properties

The formations of spatial objects characterized by *orderliness* and *extension* can be called *spaces*, if they also have a number of stable properties for all possible changes of their particular configurations. Unlike these constant configurations, a space can contain moving relations between objects. Meanwhile, it does not change itself, remaining the same milieu of objects' coexistence and motion, as long as a certain set of its invariant properties is preserved.

These general properties include such oppositions as *isotropy* or *anisotropy*, *reversibility* or *irreversibility*, *discreteness* or *continuity*, *finiteness* or *infinity*, as well as *dimensionality*, which can be differed. Although these properties are named in the same way as the known topological concepts and the coincidence of names here is by no means accidental, at the same time, the similarity of names does not mean the identity of concepts and is due only to the commonality of intuitive ideas about the properties of space underlying both of them.

Peculiarities of Ordinal Structure

The mentioned properties are the features of the ordinal structure of space. In particular, this structure has a *dimensionality* that can be understood as a minimum number of independent ordered rows, which are sufficient to determine a whole network of ordinal relations between the objects forming it. For example, it is sufficient for a determination of the letters order in the space of a telegram to consider their relations in one ordered row, and therefore this textual space is one-dimensional not depending on the dimensionality of a physical substratum. To determine the arrangement of pieces on a chessboard, it is enough to consider two dimensions, and therefore chess space is two-dimensional, again, independent of the dimensionality of the physical substratum. However, all three dimensions are necessary to determine the meaningful position of the ball on the soccer field.

Depending on homogeneity or heterogeneity of various dimensions in this ordinal structure, it can be characterized also by the properties of *isotropy* and *anisotropy*. If an ordering in different dimensions does not change any properties of ordered objects, the space is homogeneous regarding these dimensions; that means it is *isotropic*. If it is wrong, and properties of objects depend on dimensions, in which they are ordered, the space is heterogeneous in the same sense, and thereby has a quality of *anisotropy*. For example, unlike isotropic space of Euclidean geometry, the anthropomorphic space of human actions is anisotropic, because body movements in horizontal dimensions are filed as more light, than upward vertical movements.

The ordinal structure of space is *reversible*, if all ordinal directions in it are homogeneous and do not affect the properties of objects that are included in them. On the contrary, it is *irreversible* if it has heterogeneous directions, and the placement in a "forward" or "backward" order influences the object's properties. For example, the word "net" read in the reverse order as "ten" will have another meaning and indicate thereby the irreversibility of the writing space. However, ornament formed by the equal figures [NET] or [TEN] remains its decorative properties (dissymmetry, rhythm, figure-background relationship, etc.) independent of change in the order of their discernment, and its space is therefore reversible.

As features of the orders formed by the same kind of "nearer-farther" relationships, one can consider the properties of discreteness or continuity of space. *Discreteness* is a property of the ordinal structure to bind its constituent objects by "nearer-farther" relations so that each of them has at least one nearer object, and there are no other objects between them. *Continuity* is the inverse property that takes place, if for none of the objects included in a given space, such nearest neighbouring objects can be fixed. In other words, for each pair of objects ordered by the "nearer-farther" relation in the continual space, there is a third object of the same row that is between them.

The ordering by the same relationship “nearer-farther” determines the *finiteness* of a space, if in it any row of objects ordered by this relationship has a last member. A space is *infinity*, if there is not a row in it, where such a member could have a place. There is also a possibility that a space can contain finite and infinite rows together and be heterogeneous in this relation.

Features of the Metric Structure

Different spaces and their models can have a variety of quantitative relations between their objects, and depending on the ways of their metrization, they may be combined into different metric structures. This depends on the possibility of repeatedly applying to spatial objects a rigid rod, unchanging in all its motions—as Euclidean geometry and classical physics suggest—or on more complex means calculating of spatial-temporal invariants—as relativity theory states (Einstein & Infeld, 1965). Mathematics elaborates various ways of measure definitions depending on metrical homogeneity or heterogeneity of space, its curvature, convexity or concavity, discreteness or continuality, etc. (Riemann, 1956).

The diversity of metrical structures and ways of their definition characterizes both natural and cultural objects. A chessboard measured not in centimeters or inches, but in the number of cells, has another metrical structure, than its physical carrier, because there are the same eight cells not only in verticals and horizontals, but also in diagonals, and thereby the Pythagoras’ theorem is irrelevant to it. Geographic maps represent the spherical surface of the Earth on the flat in different ways, depending on a cartographic projection system and on a scale chosen. Pictures of different perspective systems have also non-coincident metrical structures. Unlike a map representing a territory at a certain constant scale, a perspective image represents even the same territory at a systematically changing scale—when objects of the same sizes are successively shorted depending on their distances. Therefore, these scale changes themselves become the means using for representation of metrical relations in a depicted space.

It is up to the particular sciences to determine the methods used to metricize the spaces they study. In philosophical discourse, it is legitimate to talk about the presence or absence of a unified metric system in various spaces and about unified or different ways of defining it.

Genetic Connections Between States of Space in Time

Another group of space properties is connected with the relations of stability and variability in time. It is characterized by the *continuation*, or *genetic connection* between the states of space, which is preserved at different moments of time. Unlike duration characterizing quantitative features of pure temporal formations, the continuation is a property of spatial-temporal formations warranting that all changes in them happen in the same spatial and temporal structure.

For example, a chess game contains many positions that related to one the same discrete space as well as to the same discrete (divided into a certain number of moves) time of this game. Other games playing although on the same physical board with the same pieces will occur in other game spaces and times. Each position at the chessboard is a *definite state* of a chess space in a game. This state has a genetic connection with previous and following states, and it does not have such continuity with positions in other chess parties, although it can be taken into account when playing them out.

The example with chess shows especially clear that spatial invariants can be phenomena of various kinds: stable *configurations of objects* moving in space, stable *relations between places*, if they can stay the same by all object’s movements, and *stable conditions* in which changes of spatial states occur. In chess space, the first is the stable shapes of pieces, preserved in all their moves; the second is the stable set of cells as a system of coordinated places; and the third is the set of game rules, unchanged in all states of the chess space.

Similar types of invariants provide the genetic connections between states of space in other cases. The *stable configuration* of moving objects is a ground for concepts of “congruent figures” in geometry and of “hard bodies” in classical physics (Helmholtz, 1956, pp. 368-371, 382; Poincare, 1983, pp. 47-48). Although the Einstein’s theory showed a relativity of synchronicity and therefore of spatial states formed by co-existed objects, this does not cancel a possibility to consider parts of simultaneously visible objects and their spatial arrangement as stable configurations. The stability of object’s forms is no less important for space of practical activity, where constant forms of artefacts are condition of their recognizability, or for space of written text, where repeated forms of letters serve as a condition of their reading.

Unlike stable single forms, the *system of stable places* preserves not internal but external spatial relations of these places with each other and does not depend on how they are filled within. Such notions as “relocation”—the transition of an object from one place to another—or “replacement”—the appearance of another object in place of the previous one—assume that a system of places remains unchanged.

A stable system of places in a given space can be distinguished as a certain *frame of reference* (as in physics from Galileo to Einstein) or a *system of coordinates* (as in mathematics, beginning with Descartes). However, such coordinate systems are used not only in the exact sciences. The anthropomorphic and subjective axes *top-bottom*, *front-back*, *left-right* in the visual field, or in the space of object actions also are important for such systems. The same axes can be projected onto social space or transformed in the mythological cosmic space with a dedicated “world axis”, a distinction between “four sides of the world”, “heavenly” and “underworlds”, etc. (Eliade, 1994; Toporov, 2010). Stable systems of places are formed in theatrical space by the “box” of the stage, in book space by the rectangle of the page, in painting space by the “regular field” of the depiction (cf. A. M. Daniel & S. M. Daniel, 1979; Schapiro, 1969).

Both stable *configurations* of moving objects and *coordinations* of stable places between them in all movements are invariant spatial structures and can be described as systems of relations between spatial objects that do not change, when these objects are displaced or replaced. Various combinations of changing and not-changing spatial relations in different dimensions form diverse types of spatial *symmetry* understood as invariants correlated with certain sets of transformations (Weil, 1952).

This structural stability of spatial configurations and coordinations is preserved in the flow of events continuing through time, if the *genetic connection* between different states of the space is preserved. Such temporal connection is a relation between the spatial relations themselves—a “second-order” relation found at more deep level of analysis.

Variety of Spaces

Autonomous Spaces

A set of invariant properties forming the law of a space can be defined in different ways and be related to various spatial formations. Therefore, diverse particular spaces can be modelled that may differ both in the set of invariant properties and in the content of their elements. Accordingly, one can speak on autonomous and separate spaces.

The *autonomous spaces* can be distinguished from one another to the extent that the structure of spatial relations in each of them has its own set of invariant properties and thereby obey to own law (cf. Greek *νόμος* “law”). Where spatial relations are built according to other laws, spaces have autonomies of other types. Each space with a certain type of autonomy has a special combination of the general properties and differs from spaces with other conditions of coexistence and change of the objects forming it. The different types of autonomy are described, for example, in elementary, projective, affine, and other systems of geometry, each of which constructs a theoretical model of some homogeneous space in its own special way, finding properties in it staying invariant by a certain group of transformations (Klein, 1956).

However, the notion of autonomous space has a universal character and is applicable not only to theoretical models, but also to sensually given images of space, and to ideas about it, formed in various spheres of culture. For example, the one-dimensional, discrete, anisotropic, and irreversible space of a written text, the two-dimensional, continual, anisotropic, and reversible space of a geographic map or the three-dimensional, discrete, anisotropic, and reversible space of a compartment car obeys to different laws and have different types of autonomy.

Separate Spaces

Even spaces with the same type of autonomy can be isolated from each other, if they do not have ordinal, metrical, or genetic unity. For example, neighbouring boards in a chess tournament are not connected by a unite system of order relations, and a knight from one game cannot take a pawn from another; distinct games played on the same chess board, as it told, do not have a continuity of states. At the same time, any chess space has the same type of autonomy distinguishing from spaces of other game kinds, spaces of written texts, of maps, of pictures, etc. Meanwhile, each chess game forms its own space and time separated from spaces and times of other games and similar processes. Another example of such separate space and time could be found in a theatrical piece formed by the rules of Classicism requiring “the unity of place, time, and action”. Other pieces playing even by the same actors at the same stage create already other separate spaces and times.

Thus, a particular space can be thought as not only an abstract type of autonomy with a set of invariant properties, but also as a bordered area, where different spatial objects could interact between each other in a unified field. Along with some external borders, such a particular space has also definite internal features: the *ordinal*, *metrical*, and *genetic unity*. As stated above, the *unified ordinal structure* is understood as the possibility to determine the ordinal relations between any places included in a given space; *the unified measure of extension* is an opportunity to correlate spatial objects with each other in some unified way by sizes and distances; and the *continuation of states* is a genetic connection of subsequent states with previous ones. These features are consistent with each other, and their existence characterizes *separate spaces* with diverse ordinal, metrical, and processional structures.

On Autonomy and Separateness of Semiotized Spaces

The concepts of autonomous and separate spaces cover various types of semiotized spaces. The space is *semiotized* as far as some meaningful objects and their relations in it are selected, structured, and interpreted according to one or several semiotic systems. Thereby, a certain semiotic form is introduced into a formation of spatial objects. In particular, the spaces of writing texts, paintings, architectural constructions, urban territories, and many other cultural objects are semiotized using spatial codes of various types, due to which appropriate spatial texts are created in these spaces.

A semiotized space will be separate insofar as it retains a unified order, a metrical structure of meaningful spatial relations, and a genetic connection of states. It will also be autonomous insofar as the rules of organization of these meaningful relations can be described by such laws, which differ from the laws of organization of other spaces. For example, the paintings in the gallery can be considered as semiotized spaces, separated from each other and from their nameplates; the latter will differ from them besides in another ways of semiotization in written texts and thereby—in other type of space autonomy (for more on the concept of semiotized space see in: Tchertov, 2019, pp. 250-282).

Distinctions and Interrelations of Spatial Modes

Various separate and autonomous spaces are not united in a single space not only because they obey to diverse laws and have differences in the structure of ordinal or metrical relations, but also because they can have different ontological statuses (cf. the mentioned above idea of different space modes in: Cassirer, 1923, pp. 29-31). A body moving in physical space cannot fly into the abstract mathematical space, where immaterial conceivable objects perform the function of "bodies", or into the space depicted in a painting, where "bodies" are represented by artificially stimulating certain perceptual images in the viewer.

The concepts of autonomous and separate space apply both to material physical objects and to their ideal images. The last can be constructed as theoretical models, for example, in mathematics or as sensual impressions, such as visual phenomena. These concepts extend to different modes of space, and they cannot be reducible to physical objects, on the one hand, nor to their ideal images, on the other.

The diversity of separate spaces with distinct types of autonomy and different ontological statuses does not prevent their connection and interaction, but rather creates conditions for this. It is clear that in order to change a position in a separate and autonomous chess space, one must move pieces on the board as physical bodies. In a similar way, a creating of a picture, a written text, or any other semiotized space needs a certain organization of physical space. However, this dependence on physical embodiment does not eliminate the autonomy of semiotized spaces or the possibility of their separation from each other. The chessboard with pieces can be considered as a part of physical space and as a subject of its laws only abstracting from the laws of the game. Increasing the weight or size of figures in physical space does not affect their properties in game space, just as, conversely, increasing their game "weight" does not affect the physical properties of their bodily carriers. The same board, without changing physically, can sharply change the structure of autonomous game space, if it is reinterpreted as a checkerboard, from the structure of which, all white squares are dropped, and only movements along black diagonals are allowed. On the other hand, the same position as a state of chess space can be repeatedly reproduced on diagrams in print or in the memory of chess players.

The Concept of Space as a Category of Philosophical Ontology

The Logical Class of Spaces

Diverse separate and autonomous spaces of different modes can be included in one *logical class* and falling under the *general concept of space*. Each member of this class is a formation of spatial relations with stable conditions of their coexistence and changes. Each of these members has its own combination of qualities: *isotropy* or *anisotropy*, *continuality* or *discreteness*, *one-*, *two-*, or *three-dimensionality*, etc. At the same time, a certain set of these general qualities as well as invariant properties of *orderliness* and *extension* belong to all members of this logical class.

Having these universal qualities in its content and the totality of all autonomous and separate spaces in its volume, the general concept of space obviously cannot be the environment, where spatial objects coexist and move—as the physical space can. This general concept, of course, is not necessary for the perception or understanding of particular spatial relations, much less for their physical existence. However, it is needed as a generalizing logical category, which can cover spaces of different types and modes. Their mental unification is formed not by physical relations of parts and the whole, but by logical genus-species relations. The basis for it is that each of these spaces has the set of general properties, which together can be regarded as *spatiality*.

The general notions of space and spatiality interpreted in this way reach the level of universal ontological categories (such as quality, quantity, causality, etc.), because they can be used to describe the being of diverse modes of existence and cover not only material bodies, but also ideal images of different levels of cognition or planning. In each of these cases, it is possible to identify some spatial relations, their members and laws or rules of their coexistence and change. Therefore, the universal category "space" can be applied to each such formation—with the condition, however, that their logical unification into one class will not be taken as ontological unification into one environment. It is possible to talk about physical, mathematical, and even chess space, but it is not correct to assume, for example, that the physical bodies are composed of mathematical points, or the moves of chess pieces are subject to the laws of gravity. It is correct at the same time, to believe that physical objects are modelled in mathematical constructions or that movements of some bodies in physical space are a condition of changes in chess spaces. The general concept of space is therefore a prerequisite of mutual modelling and of any comparison of separate spaces at all. As a category of philosophical ontology, it can be applied to phenomena with different ontological statuses, which in principle cannot be reduced to each other.

Relation to the Category of Time

The concept of space interpreted as a universal ontological category is comparable with the equally universal category of *time*. This connection is intrinsic to spatial relations that not only coexist, but also successively replace each other, revealing the dialectic of stability and variability. Indeed, some changes in separate space can exist only if some other relations in it remain unchanged, and v.v., their stability can be only at the background of variable relations.

Similar to space, time can be intrinsic to particular changing formations: a time of certain physical, biological, or sociocultural processes, an internal time of an organism, a musical piece, a game, etc. The intrinsic time of a chess game, for example, is measured by the number of moves made in it, regardless of how many rotations the minute hands was made on the clock face pointing at relation of the game process only to external time. The internal time of certain processes understood in this way is not identical with the more abstract philosophical category of time, as the separate space is not identical with the philosophical category of space. The universal *time* category is applicable to any order of successive and continuing states of real or ideal nature—like universal *space* category is related to any coexisted objects with certain order and extension.

Nevertheless, the connection between space and time does not cancel out their differences. Both of them are characterized by different types of ordinal relations—between coexisting objects in space and between events in time. Types of their magnitudes are also different: extent in space and duration in time. Between space and time, there are also a number of differences in their structural organization.

One of the most important such differences is the *non-one-dimensionality* of space, the possibility of the same object entering in more than one spatial order at once. While the phenomena replacing one another in time are ordered by a single type of "before-later" relations, the space that extends in many dimensions at once gives the objects located in it the possibility to enter into several orders formed by relations of different kinds and not reducible to one another. Whereas the metaphorical “arrow of time” always points only to the future, and the successive moments are never repeated, the arrow of the clock face in its circular motion is pointing in different directions at various moments and regularly returns to the same places.

Thereby, this arrow reveals another difference of space from time—*reversibility*, the ability to return to the same places and to allow movements in both “forward” and “reverse” orders. Thanks to this, it is possible to meet with the same spatial form at different moments of time, to approach it from different sides, to turn it by different sides, etc. There is no such reversibility in time. As a spatial object, the hand of the clock can rotate and return to any place, but as a pointer of time, it ever marks new temporal moments, and the cyclicity of rotation is only a form of spatial modelling of irreversible processes. It is only possible to return to the previous place, but not to the previous time, the *irreversibility* of which is precisely its essence.

Conclusion

Thus, the general concept of space, like the related concept of time and other categories of philosophical ontology, is not reducible to namesake concepts in particular sciences, and it gives a possibility to compare them on a unite basis. A set of theoretical concepts characterizing different aspects of spatiality can be introduced within the framework of philosophical reasoning not depending on the conceptual apparatus of any particular sciences, whether mathematics or physics.

In particular, such universal concepts as autonomous and separate space can be formulated as respectively a set of general properties distinguishing one type of spatial formations from another, and as a bounded milieu with a unified system of order, metric, and processional structures of spatial relations. While the notion of autonomous space defines a logical class of spaces with certain sets of qualities, the notion of separate space makes it possible to distinguish even within the same class its members with different ordinal and metrical systems or with different sequences of states.

These notions can be applied to phenomena of various modes—from physical objects or mathematical constructions researched in exact sciences until diverse semiotized spaces created in different fields of culture—arts, myths, rituals, games, etc. Despite differences of various autonomous and separate spaces, all of them belong to unite logical class represented by general philosophical category of space.

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