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State Estimation for Sound and Vibration Emitted from a Machine Based on Sound Measurement Under Existenceof Background Noise

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Abstract: Many studies on the diagnosis for machines have become important recently because of increased use of various complex industrial systems. The correlation information between sound and vibration is very important for machine diagnosis. Usually, vibration pickups are attached directly to the machine in order to measure vibration data. However, in some cases, the sensors can not be attached directly on highly precise devices. In this study, a method to estimate the fluctuation of sound and vibration is proposed based on the measurement data of sound emitted from the machine under existence of background noise. The effectiveness of the proposed theory is experimentally confirmed by applying it to the observed data emitted from a rotational machine driven by an electric motor.

Keywords: Bayes' theorem, background noise and vibration, state estimation, rotational machine.

1. Introduction

The fault diagnose of the machine was examined by skilled workers using the abnormal noise (using their hearings), and/or unusual vibration (based on their feelings). For example, components weariness, degradation and loose screws etc., could be found by hitting a stopping machine with a hammer. However, it gradually becomes a serious problem that the fault diagnosis methods don't keep the pace with the rapid development of modern social infrastructure. Our life is co-existing with industrial systems, and the machine fault leads to the inefficient production.

For example, there are diagnosis imaging methods using machine learning [1, 2]. The accuracy rate is high, however, the tremendous training data are required. Furthremore, there exists the problem that it is difficult to verify the optimum value of parameters setting. Besides, there are diagnosis methods using of either of sound and/or vibration emitted from the machine [3-5]. Previous studies have also shown that it is important to use the relationship between sound and vibration data [6]. Especially, the vibration usually contains important information on causes of the abnormal state. To measure vibration data, vibration pickups are attached directly to the machine. However, the sensors can not be attached directly on highly precise devices in some cases.

In this study, based on measurement data in real environment under existence of background noise, the vibration are estimated using the sound generated from the machine. Specifically, considering the system characteristics of sound and vibration as the conditional probability distribution with unknown parameters, the vibration and the unknown parameters are estimated at the same time from the observation of sound by using Bayes' theorem. The validity of proposed method is verified by experiment using the observed sound from rotational machine.

2. Theory

We consider the random vibration x_k and sound

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y_k emitted from a machine at discrete time k. The observed sound z_k is contaminated by a background noise v_k. Fig. 1 shows the relationship between sound and vibration emitted from a machine.

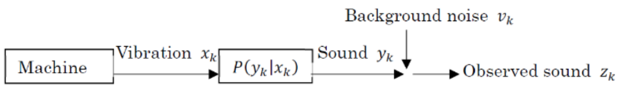

Fig. 1 Observed data model.

When we consider energy variables (e.g., sound intensity) for y_k, v_k and z_k, according to the additive property of energy variables, the following relationship can be established [7].

$$z_k = y_k + v_k \tag{1}$$

In order to derive the statistical relationship between x_k and y_k, the conditional probability distribution of x_k is expressed as

$$\begin{aligned} P(y_k \mid x_k) &= \frac{P(x_k, y_k)}{P(x_k)} \\ &= \frac{P(x_k)P(y_k)\sum_{r=0}^{\infty}\sum_{s=0}^{\infty} B_{rs}\theta_r^{(1)}(x_k)\theta_s^{(2)}(y_k)}{P(x_k)} \\ &= P(y_k)\sum_{r=0}^{\infty}\sum_{s=0}^{\infty} B_{rs}\theta_r^{(1)}(x_k)\theta_s^{(2)}(y_k) \end{aligned} \tag{2}$$

$$B_{rs} = \left\langle \theta_r^{(1)}(x_k)\theta_s^{(2)}(y_k) \right\rangle \tag{3}$$

where, $\langle\ \rangle$ denotes the averaging operation with respect to the random variables. The orthogonal polynomials $\theta_r^{(1)}(x_k)$ and $\theta_s^{(2)}(y_k)$ are obtained by using Schmidt orthogonalization [6].

$$\theta_r^{(1)}(x_k) = \sum_{i=0}^{r} \lambda_{ri}^{(1)} x_k^i \tag{4}$$

$$\theta_s^{(2)}(y_k) = \sum_{i=0}^{s} \lambda_{si}^{(2)} y_k^i \tag{5}$$

Where $\lambda_{ri}^{(1)}$ and $\lambda_{si}^{(2)}$ are coefficients.

Since the instantaneous values of x_k and y_k are unknown, expansion coefficients B_{rs} in Eq. (3) have to be estimated on the basis of the observation Z_k. Let's regard the expansion coefficients B_{rs} as unknown parameters a_k.

In the case of paying attention to the variables x_k, a_k and Z_k, all the information on mutual correlations among x_k, a_k and Z_k is included in the conditional probability distribution $P(x_k, a_k \mid Z_k)$. By using the well-know Bayes' theorem [8].

$$\begin{aligned} P(x_k, a_k \mid Z_k) &= \frac{P(x_k, a_k, Z_k \mid Z_{k-1})}{P(Z_k \mid Z_{k-1})} \\ &= \frac{P_0(x_k \mid Z_{k\text{-}1})P_0(a_k \mid Z_{k\text{-}1})\sum_{l=0}^{\infty}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty} A_{lmn}\phi_l^{(1)}(x_k)\phi_m^{(2)}(a_k)\phi_n^{(3)}(z_k)}{\sum_{n=0}^{\infty} A_{00n}\phi_n^{(3)}(z_k)} \end{aligned}$$

$$A_{lmn} = \left\langle \phi_l^{(1)}(x_k)\phi_m^{(2)}(a_k)\phi_n^{(3)}(z_k) \mid Z_{k-1} \right\rangle \tag{6}$$

where $Z_k\{= Z_1, Z_2, \ldots, Z_k\}$ is a set of observation variables up to time k. As the fundamental probability density functions $P_0(x_k \mid Z_{k-1})$, $P_0(Z_k \mid Z_{k-1})$, which can be choose as the probability functions describing the dominant part of the actual fluctuation or as the well-known standard probability distributions, the gamma distribution suitable for energy variable is adopted. Furthermore, the functions $\phi_l^{(1)}(x_k)$, $\phi_n^{(3)}(z_k)$ are orthonormal polynomials of degrees l and n with weighting functions $P_0(x_k \mid Z_{k-1})$ and $P_0(Z_k \mid Z_{k-1})$ can be determined as Laguerre polynomials.

$$P_0(x_k \mid Z_{k\text{-}1}) = \frac{x_k^{m_{x_k}^*\text{-}1}}{\Gamma(m_{x_k}^*)s_{x_k}^{*m_{x_k}^*}} e^{-\frac{x_k}{s_{x_k}^*}} \tag{7}$$

$$P_0(z_k \mid Z_{k-1}) = \frac{z_k^{m_z^x - 1}}{\Gamma(m_{z_k}^*)s_{z_k}^{*m_{z_k}^*}} e^{-\frac{z_k}{s_{z_k}^*}} \tag{8}$$

$$\phi_l^{(1)}(x_k) = \sqrt{\frac{\Gamma(m_{x_k}^*)l!}{\Gamma(m_{x_k}^* + l)}} L_l^{(m_{x_k}^* - 1)}\left(\frac{x_k}{s_{x_k}^*}\right) \tag{9}$$

$$\phi_n^{(3)}(z_k) = \sqrt{\frac{\Gamma(m_{z_k}^*)n!}{\Gamma(m_{z_k}^* + n)}} L_n^{(m_{z_k}^* - 1)}\left(\frac{z_k}{s_{z_k}^*}\right) \tag{10}$$

With

$$m_{x_k}^* = \frac{x_k^{*2}}{\Gamma_{x_k}}, s_{x_k}^* = \frac{\Gamma_{x_k}}{x_k^*}$$

$$x_k^* = \langle x_k | Z_{k-1} \rangle, \Gamma_{x_k} = \left\langle \left(x_k - x_k^* \right)^2 | Z_{k-1} \right\rangle$$

$$m_{z_k}^* = \frac{z_k^2}{\Gamma_{z_k}}, s_{z_k}^* = \frac{\Gamma_{z_k}}{z_k^*}$$

$$z_k^* = \langle z_k | Z_{k-1} \rangle, \Gamma_{z_k} = \left\langle \left(z_k - z_k^* \right)^2 | Z_{k-1} \right\rangle \quad (11)$$

As an example of standard probability function $P_0(a_k \mid Z_{k-1})$, Gaussian distribution is adopted.

$$P_0\left(a_k | Z_{k-1}\right) = \frac{1}{\sqrt{2\pi\Gamma_{a_k}}} e^{-\frac{\left(a_k - a_k^*\right)^2}{2\Gamma_{a_k}}} \quad (12)$$

$$a_k^* = \langle a_k | Z_{k-1} \rangle \quad (13)$$

$$\Gamma_{a_k} = \left\langle \left(a_k - a_k^* \right)^2 | Z_{k-1} \right\rangle \quad (14)$$

The orthonormal polynomial with the above weighting probability distribution is then specified as Hermite polynomial [9, 10].

$$\phi_m^{(2)}\left(a_k\right) = \frac{1}{\sqrt{m!}} H_m\left(\frac{a_k - a_k^*}{\sqrt{\Gamma_{a_k}}} \right) \quad (15)$$

The estimates for mean and variance (i.e., conditional mean and variance) of x_k, a_k, which are the first and second order statistics, can be expressed as follows,

$$\hat{x}_k = \langle x_k | Z_k \rangle = \frac{\sum_{l=0}^{1} \sum_{n=0}^{\infty} A_{l0n} c_{1l} \phi_n^{(3)}\left(z_k\right)}{\sum_{n=0}^{\infty} A_{00n} \phi_n^{(3)}\left(z_k\right)} \quad (16)$$

$$\hat{a}_k = \langle a_k | Z_k \rangle = \frac{\sum_{m=0}^{1} \sum_{n=0}^{\infty} A_{0mn} d_{1m} \phi_n^{(3)}\left(z_k\right)}{\sum_{n=0}^{\infty} A_{00n} \phi_n^{(3)}\left(z_k\right)} \quad (17)$$

$$P_{x_k} = \left\langle \left(x_k - \hat{x}_k \right)^2 | Z_k \right\rangle = \frac{\sum_{l=0}^{2} \sum_{n=0}^{\infty} A_{l0n} c_{2l} \phi_n^{(3)}\left(z_k\right)}{\sum_{n=0}^{\infty} A_{00n} \phi_n^{(3)}\left(z_k\right)} \quad (18)$$

$$P_{a_k} = \left\langle \left(a_k - \hat{a}_k \right)^2 | Z_k \right\rangle = \frac{\sum_{m=0}^{2} \sum_{n=0}^{\infty} A_{0mn} d_{2m} \phi_n^{(3)}\left(z_k\right)}{\sum_{n=0}^{\infty} A_{00n} \phi_n^{(3)}\left(z_k\right)} \quad (19)$$

These estimates are derived by using the orthogonal condition.

$$\int_0^{\infty} P_0\left(x_k | Z_{k-1}\right) \phi_l^{(1)}\left(x_k\right) \phi_{l'}^{(1)}\left(x_k\right) dx_k = \delta_{ll'}$$

$$\int_{-\infty}^{\infty} P_0\left(a_k | Z_{k-1}\right) \phi_m^{(2)}\left(a_k\right) \phi_{m'}^{(2)}\left(a_k\right) da_k = \delta_{mm'} \quad (20)$$

Next, the estimation of sound y_k is derived from the estimates of x_k, a_k by using the correlation information of sound and vibration in Eq. (2).

$$\begin{aligned} \hat{y}_k &\triangleq \langle y_k | Z \rangle_k = \langle \langle y_k | x_k \rangle | Z \rangle_k \\ &= \left\langle \int y_k P\left(y_k | x_k\right) dy_k | Z_k \right\rangle \\ &= \left\langle \int y_k P\left(y_k | x_k\right) \sum_{r=0}^{\infty} \sum_{s=0}^{\infty} B_{rs} \theta_r^{(1)}\left(x_k\right) \theta_s^{(2)}\left(y_k\right) dy_k | Z_k \right\rangle \quad (21) \\ &= \left\langle \sum_{r=0}^{\infty} \sum_{s=0}^{1} B_{rs} e_{1s} \theta_r^{(1)}\left(x_k\right) | Z_k \right\rangle \\ &= e_{10} + \hat{a}_k e_{11} \left(\lambda_{10}^{(1)} + \lambda_{11}^{(1)} \hat{x}_k \right) \end{aligned}$$

$$\begin{aligned} P_{y_k} &\triangleq \left\langle \left(y_k - \hat{y}_k \right)^2 | Z_k \right\rangle \\ &= \left\langle \int \left(y_k - \hat{y}_k \right)^2 P\left(y_k | x_k\right) dy_k | Z_k \right\rangle \\ &= \left\langle \int \left(y_k - \hat{y}_k \right)^2 P\left(y_k | x_k\right) \sum_{r=0}^{\infty} \sum_{s=0}^{\infty} B_{rs} \theta_r^{(1)}\left(x_k\right) \theta_s^{(2)}\left(y_k\right) dy_k | Z_k \right\rangle \\ &= \left\langle \sum_{r=0}^{\infty} \sum_{s=0}^{2} B_{rs} e_{2s} \theta_r^{(1)}\left(x_k\right) | Z_k \right\rangle \\ &= e_{20} + \hat{a}_k e_{21} \left(\lambda_{10}^{(1)} + \lambda_{11}^{(1)} \hat{x}_k \right) \\ &+ \left(P_{a_k} + \hat{a}_k^2 \right) e_{22} \left\{ \lambda_{20}^{(1)} + \lambda_{21}^{(1)} \hat{x}_k + \lambda_{22}^{(1)} \left(P_{x_k} + \hat{x}_k^2 \right) \right\} \quad (22) \end{aligned}$$

Coefficients in (16)-(19), (21)-(22) are appropriate constants satisfying the following equalities:

$$x_k = \sum_{l=0}^{1} c_{1l} \phi_l^{(1)}\left(x_k\right)$$

$$\left(x_k - \hat{x}_k \right)^2 = \sum_{l=0}^{2} c_{2l} \phi_l^{(1)}\left(x_k\right)$$

$$a_k = \sum_{m=0}^{1} d_{1m} \phi_m^{(2)} \left(a_k \right)$$

$$\left(a_k - \hat{a}_k \right)^2 = \sum_{m=0}^{2} d_{2m} \phi_m^{(2)} \left(a_k \right)$$

$$y_k = \sum_{s=0}^{1} e_{1s} \theta_s^{(2)} \left(y_k \right)$$

$$\left(y_k - \hat{y}_k \right)^2 = \sum_{s=0}^{2} e_{2s} \theta_s^{(2)} \left(y_k \right) \tag{23}$$

The expansion coefficients representing correlation information between x_k and Z_k, a_k and Z_k are specifically calculated by using Eq. (2).

$$A_{101} = \left\langle \phi_1^{(1)} \left(x_k \right) \phi_1^{(3)} \left(z_k \right) \middle| \, Z_{k-1} \right\rangle$$

$$= \sqrt{\frac{1}{m_{x_k}^* m_{z_k}^*}} \left\langle \left(m_{x_k}^* - \frac{x_k}{s_{x_k}^*} \right) \left(m_{z_k}^* - \frac{z_k}{s_{z_k}^*} \right) \middle| \, Z_{k-1} \right\rangle$$

$$= \sqrt{\frac{1}{m_{x_k}^* m_{z_k}^*}} \left\langle \left(m_{x_k}^* - \frac{x_k}{s_{x_k}^*} \right) \left(m_{z_k}^* - \frac{y_k}{s_{z_k}^*} \right) \right.$$

$$\left. - \left(m_{x_k}^* - \frac{x_k}{s_{x_k}^*} \right) \left(m_{z_k}^* - \frac{v_k}{s_{z_k}^*} \right) \middle| Z_{k-1} \right\rangle$$

$$= \sqrt{\frac{1}{m_{x_k}^* m_{z_k}^*}} \left\langle \left(m_{x_k}^* - \frac{x_k}{s_{x_k}^*} \right) \left(m_{z_k}^* - \frac{1}{s_{z_k}^*} y_k \middle| \, x_k \right) \right.$$

$$\left. - \left(m_{x_k}^* - \frac{x_k}{s_{x_k}^*} \right) \left(m_{z_k}^* - \frac{v_k}{s_{z_k}^*} \right) \middle| Z_{k-1} \right\rangle$$

$$= \sqrt{\frac{1}{m_{x_k}^* m_{z_k}^*}} \left\langle \left(m_{x_k}^* - \frac{x_k}{s_{x_k}^*} \right) \right.$$

$$\left[m_{z_k}^* - \frac{1}{s_{z_k}^*} \left\{ e_{10} + a_k e_{11} \left(\lambda_{10}^{(1)} + \lambda_{11}^{(1)} s_{x_k}^* \frac{x_k}{s_{x_k}^*} \right) \right\} \right]$$

$$\left. - \left(m_{x_k}^* - \frac{x_k}{s_{x_k}^*} \right) \left(m_{z_k}^* - \frac{v_k}{s_{z_k}^*} \right) \middle| Z_{k-1} \right\rangle$$

$$= \sqrt{\frac{m_{x_k}^*}{m_{z_k}^*}} \frac{1}{s_{z_k}^*} a_k^* e_{11} \lambda_{11}^{(1)} s_{x_k}^* \tag{24}$$

$$A_{102} = \left\langle \phi_1^{(1)} \left(x_k \right) \phi_2^{(3)} \left(z_k \right) \middle| \, Z_{k-1} \right\rangle$$

$$= \sqrt{\frac{2}{m_{x_k}^* m_{z_k}^* \left(m_{z_k}^* + 1 \right)}} \left\langle \left(m_{x_k}^* - \frac{x_k}{s_{x_k}^*} \right) \right.$$

$$\left\{ \frac{1}{2} \left(m_{z_k}^* + 1 \right) m_{z_k}^* - \left(m_{z_k}^* + 1 \right) \frac{1}{s_{z_k}^*} \left(y_k \middle| \, x_k + v_k \right) \right.$$

$$\left. \left. + \frac{1}{2 s_{z_k}^{*2}} \left(\left\langle y_k^2 \middle| \, x_k \right\rangle + 2 \left\langle y_k \middle| \, x_k \right\rangle \left\langle v_k \right\rangle + \left\langle v_k^2 \right\rangle \right) \right\} \middle| Z_{k-1} \right\rangle$$

$$= - \sqrt{\frac{2}{m_{x_k}^* m_{z_k}^* \left(m_{z_k}^* + 1 \right)}} m_{x_k}^*$$

$$\left[\left(m_{x_k}^* + 1 \right) \frac{1}{s_{z_k}^{*2}} \left(\Gamma_{a_k} + a_k^{*2} \right) p_{22} \lambda_{22}^{(1)} s_{x_k}^{*2} - \left(m_{z_k}^* + 1 \right) \frac{1}{s_{z_k}^*} a_k^* e_{11} \lambda_{11}^{(1)} \right.$$

$$+ \frac{1}{2 s_{z_k}^{*2}} \left(a_k^* p_{21} \lambda_{11}^{(1)} s_{x_k}^* + \left(\Gamma_{a_k} + a_k^{*2} \right) p_{22} \lambda_{21}^{(1)} s_{x_k}^* \right.$$

$$\left. \left. + 2 a_k^* e_{11} \lambda_{11}^{(1)} s_{x_k}^* \left\langle v_k \right\rangle \right) \right] \tag{25}$$

$$A_{201} = \left\langle \phi_2^{(1)} \left(x_k \right) \phi_1^{(3)} \left(z_k \right) \middle| \, Z_{k-1} \right\rangle$$

$$= \sqrt{\frac{2}{m_{x_k}^* \left(m_{x_k}^* + 1 \right) m_{z_k}^*}}$$

$$\left\langle \left\{ \frac{1}{2} \left(m_{x_k}^* + 1 \right) m_{x_k}^* - \left(m_{x_k}^* + 1 \right) \left(\frac{x_k}{s_{x_k}^*} \right) + \frac{1}{2} \left(\frac{x_k}{s_{x_k}^*} \right)^2 \right\} \right.$$

$$\left. \left(m_{z_k}^* - \frac{1}{s_{z_k}^*} \left(y_k \middle| \, x_k + v_k \right) \right) \middle| Z_{k-1} \right\rangle$$

$$= 0 \tag{26}$$

$$A_{202} = \left\langle \phi_2^{(1)} \left(x_k \right) \phi_2^{(3)} \left(z_k \right) \middle| \, Z_{k-1} \right\rangle$$

$$= \sqrt{\frac{4}{m_{x_k}^* \left(m_{x_k}^* + 1 \right) m_{z_k}^* \left(m_{z_k}^* + 1 \right)}}$$

$$\left\langle \left\{ \frac{1}{2} \left(m_{x_k}^* + 1 \right) m_{x_k}^* - \left(m_{x_k}^* + 1 \right) \left(\frac{x_k}{s_{x_k}^*} \right) + \frac{1}{2} \left(\frac{x_k}{s_{x_k}^*} \right)^2 \right\} \right.$$

$$\left\{\frac{1}{2}\left(m_{z_k}^*+1\right)m_{z_k}^*-\left(m_{z_k}^*+1\right)\frac{1}{s_{z_k}^*}\left(y_k|\ x_k+v_k\right)\right.$$

$$\left.\left.+\frac{1}{2s_{z_k}^{*2}}\left(y_k^2|\ x_k+2y_k|\ x_kv_k+v_k^2\right)\right\}\middle|Z_{k-1}\right\rangle$$

$$=\sqrt{\frac{m_{x_k}^*\left(m_{x_k}^*+1\right)}{m_{z_k}^*\left(m_{z_k}^*+1\right)}\frac{1}{s_{z_k}^{*2}}\left(\Gamma_{a_k}+a_k^{*2}\right)p_{22}\lambda_{22}^{(1)}s_{x_k}^{*2}} \quad (27)$$

$$A_{011}=\left\langle\phi_1^{(2)}\left(a_k\right)\phi_1^{(3)}\left(z_k\right)|\ Z_{k-1}\right\rangle$$

$$=\sqrt{\frac{1}{m_{z_k}^*}}\left\langle\left(\frac{a_k-a_k^*}{\sqrt{\Gamma_{a_k}}}\right)\left(m_{z_k}^*-\frac{z_k}{s_{z_k}^*}\right)|\ Z_{k-1}\right\rangle$$

$$=\sqrt{\frac{1}{m_{z_k}^*}}\left\langle\left(\frac{a_k-a_k^*}{\sqrt{\Gamma_{a_k}}}\right)\left\{m_{z_k}^*-\frac{1}{s_{z_k}^*}\left(e_{10}+a_ke_{11}\theta_1^{(1)}\left(x_k\right)+v_k\right)\right\}|\ Z_{k-1}\right\rangle$$

$$=\sqrt{\frac{1}{m_{z_k}^*}}\left\langle\left(\frac{a_k-a_k^*}{\sqrt{\Gamma_{a_k}}}\right)^2\left\{-\frac{1}{s_{z_k}^*}\sqrt{\Gamma_{a_k}}e_{11}\theta_1^{(1)}\left(x_k\right)\right\}\right.$$

$$\left.+\left(\frac{a_k-a_k^*}{\sqrt{\Gamma_{a_k}}}\right)\left\{m_{z_k}^*-\frac{1}{s_{z_k}^*}\left(e_{10}+a_k^*e_{11}\theta_1^{(1)}\left(x_k\right)+v_k\right)\right\}\middle|Z_{k-1}\right\rangle$$

$$=-\sqrt{\frac{1}{m_{z_k}^*}}\left\{\frac{1}{s_{z_k}^*}\sqrt{\Gamma_{a_k}}e_{11}\left(\lambda_{10}^{(1)}+\lambda_{11}^{(1)}x_k^*\right)\right\} \quad (28)$$

$$A_{012}=\left\langle\phi_1^{(2)}\left(a_k\right)\phi_2^{(3)}\left(z_k\right)|\ Z_{k-1}\right\rangle$$

$$=\sqrt{\frac{2}{m_{z_k}^*\left(m_{z_k}^*+1\right)}}\left[-\left(m_{z_k}^*+1\right)\frac{1}{s_{z_k}^*}\sqrt{\Gamma_{a_k}}e_{11}\left\langle\theta_1^{(1)}\left(x_k\right)|\ Z_{k-1}\right\rangle\right.$$

$$+\frac{1}{2s_{z_k}^{*2}}\left\{\sqrt{\Gamma_{a_k}}p_{21}\left\langle\theta_1^{(1)}\left(x_k\right)|\ Z_{k-1}\right\rangle\right.$$

$$+2\sqrt{\Gamma_{a_k}}a_k^*p_{22}\left\langle\theta_2^{(1)}\left(x_k\right)|\ Z_{k-1}\right\rangle$$

$$\left.\left.+2\sqrt{\Gamma_{a_k}}e_{11}\left\langle\theta_1^{(1)}\left(x_k\right)|\ Z_{k-1}\right\rangle\left\langle v_k\right\rangle\right\}\right] \quad (29)$$

$$A_{021}=\left\langle\phi_2^{(2)}\left(a_k\right)\phi_1^{(3)}\left(z_k\right)|\ Z_{k-1}\right\rangle$$

$$=\sqrt{\frac{1}{2m_{z_k}^*}}\left\langle\left\{\left(\frac{a_k-a_k^*}{\sqrt{\Gamma_{a_k}}}\right)^2-1\right\}\right.$$

$$\left.\left\{m_{z_k}^*-\frac{1}{s_{z_k}^*}\left(e_{10}+a_ke_{11}\theta_1^{(1)}\left(x_k\right)+v_k\right)\right\}\middle|Z_{k-1}\right\rangle$$

$$=0 \quad (30)$$

$$A_{022}=\left\langle\phi_2^{(2)}\left(a_k\right)\phi_2^{(3)}\left(z_k\right)|\ Z_{k-1}\right\rangle$$

$$=\sqrt{\frac{2}{2m_{z_k}^*\left(m_{z_k}^*+1\right)}}\left\langle\left\{\left(\frac{a_k-a_k^*}{\sqrt{\Gamma_{a_k}}}\right)^2-1\right\}\right.$$

$$\left.\left\{\frac{1}{2}\left(m_{z_k}^*+1\right)m_{z_k}^*-\left(m_{z_k}^*+1\right)\left(\frac{z_k}{s_{z_k}^*}\right)+\frac{1}{2}\left(\frac{z_k}{s_{z_k}^*}\right)^2\right\}\middle|Z_{k-1}\right\rangle$$

$$=\sqrt{\frac{1}{m_{z_k}^*\left(m_{z_k}^*+1\right)}}\frac{1}{2s_{z_k}^{*2}}\Gamma_{a_k}p_{22}$$

$$\left\langle\left(\frac{a_k-a_k^*}{\sqrt{\Gamma_{a_k}}}\right)^4|\ Z_{k-1}\right\rangle\left\langle\theta_2^{(1)}\left(x_k\right)|\ Z_{k-1}\right\rangle$$

$$=\sqrt{\frac{1}{m_{z_k}^*\left(m_{z_k}^*+1\right)}}\frac{3}{2s_{z_k}^{*2}}\Gamma_{a_k}p_{22}$$

$$\left\{\lambda_{20}^{(1)}+\lambda_{21}^{(1)}x_k^*+\lambda_{22}^{(1)}\left(\Gamma_{x_k}+x_k^{*2}\right)\right\} \quad (31)$$

In order to derive the predicted values of the vibration x_k, the time transition of the vibration x_k is set as follows.

$$x_{k+1}=Fx_k+Gu_k \quad (32)$$

where, u_k is a random input with mean 0 and variance 1. Parameters F and G are calculated from time correlation information between x_k and x_{k+1}:

$$F = \frac{x_{k+1} x_k}{x_k^2}, G = \sqrt{\left(1 - F^2\right) x_k^2} \tag{33}$$

Therefore, x_{k+1}^* and $\Gamma_{x_{k+1}}$ can be expressed as follows:

$$x_{k+1}^* = F\hat{x}_k + Gu_k \tag{34}$$

$$\Gamma_{x_{k+1}} = F^2 P_{x_k} + G^2 u_k - u_k{}^2 \tag{35}$$

Since the parameter a_k is a constant, the following time transition model is introduced for the recursive estimation.

$$a_{k+1} = a_k \tag{36}$$

By using the above relationship, the predictions are given as follows,

$$a_{k+1}^* = \hat{a}_k \tag{37}$$

$$\Gamma_{a_{k+1}} = P_{a_k} \tag{38}$$

2. Experiment

The proposed method was applied to estimate simultaneously sound and vibration emitted from rotating machine by observing the noisy sound contaminated by background noise. The observation sound z⍰ were measured by use of a microphone. We experimented on the estimation by dividing 3000 observation data into 12 data sets with 250 observation data in normal situation without faults. In order to confirm the effectiveness of the proposed method involving a nonlinear model, we compared it with extended Kalman filter (EKF) [11]. The EKF is an extension of the well-known Kalman filter (KF), applicable to nonlinear models.

Figures 1 and 4 show the estimated results of vibration for data sets 1 and 2. The EKF cannot estimate the first part of fluctuation. However the proposed method can estimate through the whole of

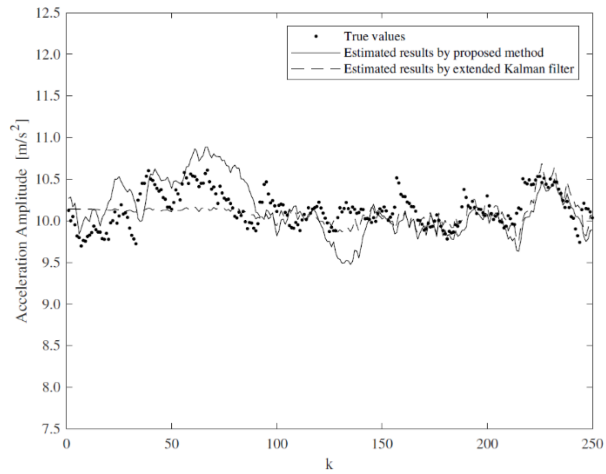

Fig. 1 Estimation results of vibration in for data set 1.

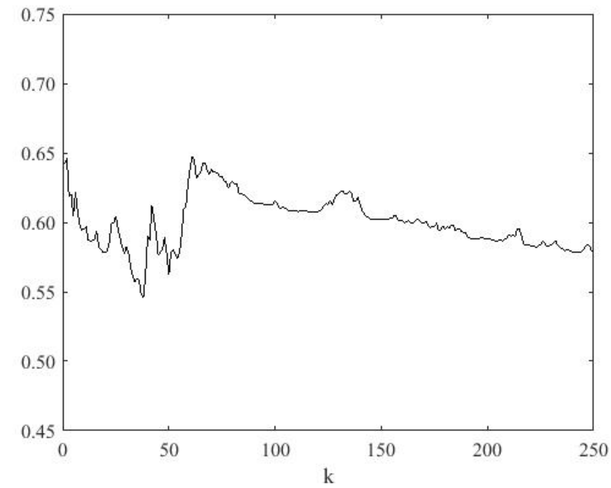

Fig. 2 Estimation results of parameter ak in for data set 1.

fluctuation data. The Root Mean Square (RMS) errors the proposed method and EKF are shown in Tables 1 and 2.

Next, the sound y_k emitted from machine was estimated by use of estimates of x_k and a_k. The estimated results are shown in Figures 3 and 6. The proposed method precisely estimated the whole fluctuation wave without diffusion.

The RMS errors were calculated using true vibration values and estimated values by Eqs. (16) and (21).

Figures 2 and 5 show the estimation process of the unknown parameter a_k of the proposed method for data sets 1 and 2. From the above estimated results, it is clearly obvious that the proposed method is more effective than the well-known EKF.

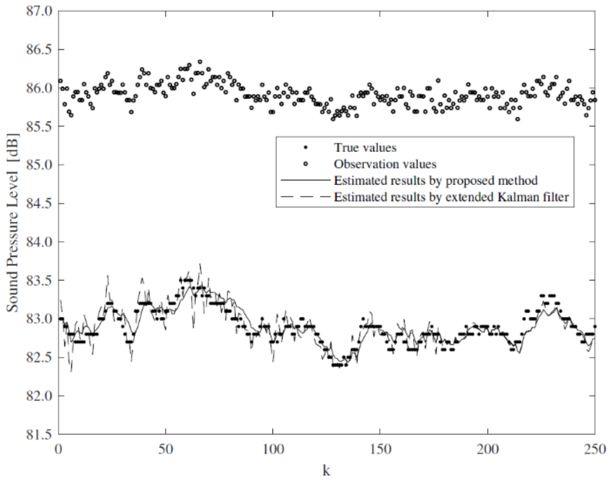

Fig. 3 Estimation results of sound in for data set 1.

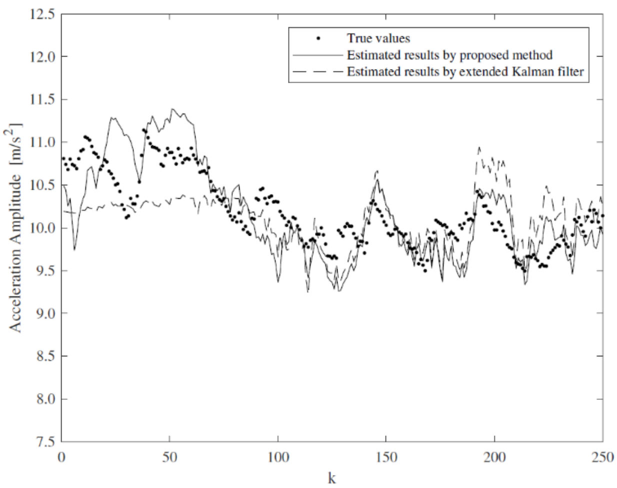

Fig. 4 Estimation results of vibration in for data set 2.

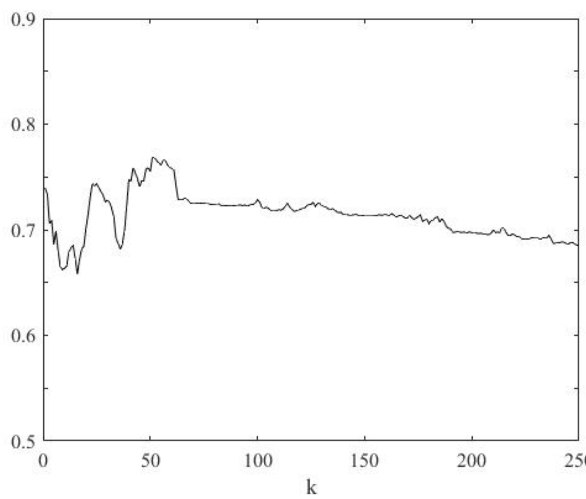

Fig. 5 Estimation results of parameter ak in for data set 2.

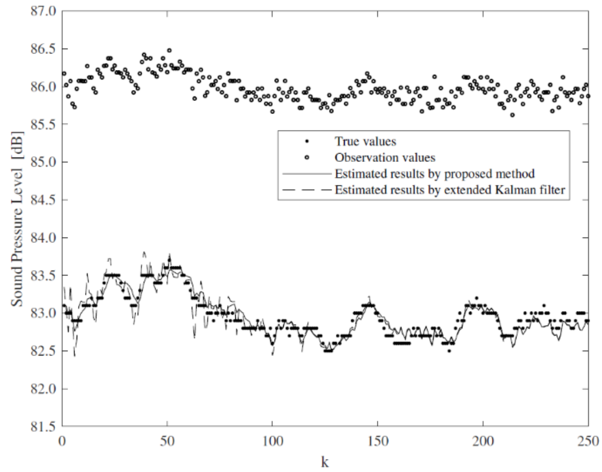

Fig. 6 Estimation results of sound in for data set 2.

Table 1 RMS error of vibration (m/s^2).

	1-250	251-500	501-750	751-1000
Proposed method	0.3198	0.3711	0.2854	0.2449
EKF	0.3549	0.4043	0.2112	0.2956
	1001-1250	1251-1500	1501-1750	1751-2000
Proposed method	0.3446	0.4171	0.3838	0.3322
EKF	0.2417	0.3643	0.4981	0.3919
	2001-2250	2251-2500	2501-2750	2751-3000
Proposed method	0.3530	0.4970	0.4635	0.4268
EKF	0.4345	0.5013	0.2821	0.4111

Table 2 RMS error of sound (dB).

	1-250	251-500	501-750	751-1000
Proposed method	0.1140	0.1006	0.1116	0.1540
EKF	0.1605	0.1369	0.1460	0.1827
	1001-1250	1251-1500	1501-1750	1751-2000
Proposed method	0.1509	0.1325	0.1345	0.1393
EKF	0.1545	0.1429	0.1922	0.1512
	2001-2250	2251-2500	2501-2750	2751-3000
Proposed method	0.2218	0.1849	0.1323	0.1241
EKF	0.1819	0.1793	0.1712	0.1270

3. Conclusion

In this paper, an estimation method of sound and vibration based on the measuring noisy sound data has been proposed the proposed method can get the necessary data to diagnose of the machine without any restriction of measuring instruments. By using the correlation information between sound and vibration, the vibration can be estimated from sound contaminated by background noise. It was confirmed that the proposed method showed more precise estimation than the well-known extended Kalman filter. However, the proposed method is still at the early stage of study. Thus, there are a great number of problems in the future. For example, (1) The practical method should be developed at the actual environment existing background vibration, and (2) The diagnosis method should be proposed by using estimates of sound and vibration.

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