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Local Geoid Model Generation Using the Geometrical Approach

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Abstract: The global navigation satellite systems (GNSS) provide an accurate three-dimensional positioning including the geodetic (ellipsoidal) height (*h*), which is, in most cases, must be transformed to the local/regional orthometric height (*H*) to have physically the desired meaning of the elevation above the national vertical datum (e.g., MSL (Mean Sea Level)). Usually, the local orthometric heights are determined based on the value of geoid undulation (*N*), which is obtained by methods of gravimetrical observations or derived by methods of interpolation utilizing the local geoid models (LGM). The current paper highlights one of the methods of generating LGM that is based on the geometrical relationship between the global datum WGS84 and the Iraqi datum Karbala 1979. The DGPS (Differential GPS) method is used to get the geodetic coordinates of about 97 selected points to cover the experimental area, within the boundaries of Erbil municipality, which has been partitioned into four parts. Therefore, four LGM were generated individually for each one of the four parts with an estimated uncertainty equal to ±0.076 m.

Key words: Geoid undulation, LGM, ellipsoidal height, Karbala 1979.

1. Introduction

The determination of the undulation of the geoid (*N*) has always been one of the main goals of researchers in the field of geodesy and surveying sciences. According to the widespread use of global positioning system (GPS) in geodetic applications, great attention is paid to the precise determination of local/regional geoid with an aim to replace the geometric levelling, which is very onerous measurement work, with GPS surveys [1].

Although the global navigation satellite system (GNSS) observations provide the position including the geodetic (or ellipsoidal) height (*h*) according to the reference global datum WGS84, the observed position can be also given as horizontal coordinates with the orthometric height (*H*) referring to the local datum utilizing the seven transformation parameters and the local geoid undulation (*N*). However, the value of the geoid undulation can be determined even from the gravimetric observations or using the geoid model.

Depending on data availability and accuracy requirements, there are two principal approaches for determining the geoid models. These approaches include the gravimetric method and interpolation between geometrically derived geoid heights using the benchmarks of which three-dimensional coordinates and orthometric heights have been determined according to GPS and levelling measurements [1].

2. Geometrical Relationship between Global and Local Datums

With many different realizations of terrestrial reference systems, as well as local or regional datums, it is important for many geodetic applications to know the relationship between these frames [2].

The geometrical relationship between different datums is usually understood according to the elements of geometry of those datums (size & flattening) and the degree of orientation compatibility (parallelism). Therefore, the position's transformation from one datum to another will be based upon this

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geometrical relationship, which is usually known as the seven parameters of transformation. So, the three-dimensional coordinates (longitude, latitude, and ellipsoid height) of the observed points will be changed according to these geometrical relationships that exist between the datums of interest.

However, identifying the seven parameters of the three-dimensional transformation maybe is the best identification of the geometrical relationship between the datums of our interest. Table 1 shows the parameters of transformation between the global datum WGS84 and the local Iraqi datum Karbala 1979 [3].

The data of Table 1 identify the geometrical relationship between the two datums of our interest, from where we can notice that there is a small difference in orientation between them and they have approximately the same size as well. Meanwhile, there is a shifting in origin represented by the values of ΔX, ΔY, & ΔZ [3].

2.1 Geodetic Height's Difference between the Global and Local Ellipsoids

Generally, the differences in geodetic (ellipsoidal) heights (h) lead to identifying the difference between the surfaces of different ellipsoids. So, the difference between the surfaces of the global ellipsoid (WGS84) and the Iraqi local ellipsoid (Clarke1880) was found as below.

$$\Delta h = h_{WGS84} - h_{Karbala\ 1979} \qquad (1)$$

The obtained values of Δh show that the surface of the datum WGS84 is below the surface of the Iraqi local datum by a maximum value equal to -1.751 m and the minimum value equal to -1.137 m over the case study area, and there is a tolerance in value of Δh equal to 0.614 m over the Erbil territory [4].

Indeed, the results above indicate a very small inclination between the two surfaces, which happens due to the small values of angles of rotation over the three axes. So, this conclusion will be considered in the next development of the research.

Fig. 1 shows the obtained differences in the geodetic heights, which are calculated for 97 selected points to cover approximately the territory of the experiment.

The global geodetic heights (h_{WGS84}) of the points are obtained by the DGPS observations, which provide the latitude, longitude, and h. Whereas, the local geodetic

Table 1 Seven transformation parameters between WGS84 & Karbala 1979.

Parameters of Translation (m)			Parameters of Rotation (degree)			Scale factor
$T_1 = \Delta X$	$T_2 = \Delta Y$	$T_3 = \Delta Z$	Ɛx	Ɛy	Ɛz	$(1 + \Delta\mu)$
211.0000002	-207	-13.9999997	$1.68199\ 10^{-14}$	$-4.64628\ 10^{-14}$	$2.81983\ 10^{-18}$	$1 + 1.55431\ 10^{-15}$

Ɛx, Ɛy, & Ɛz are the rotation angles around the main axes X, Y, & Z, respectively.

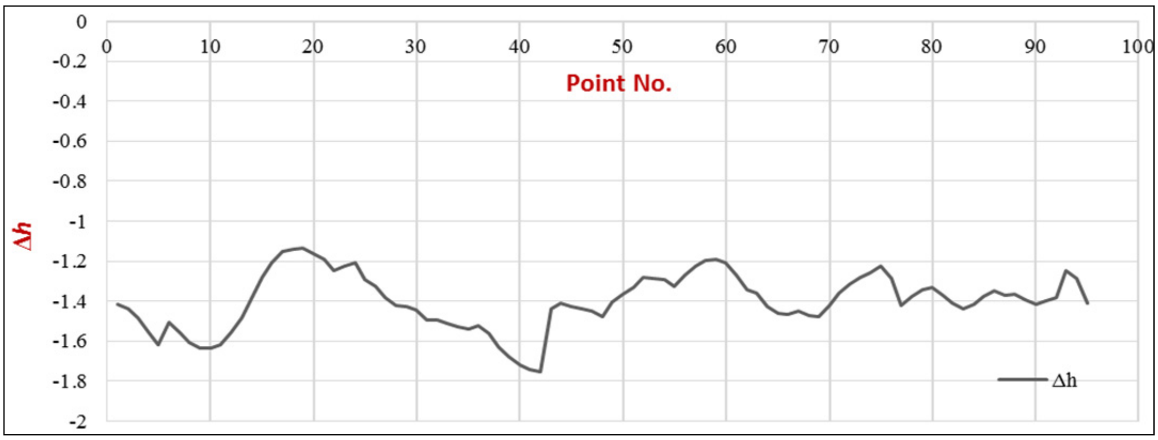

Fig. 1 The values of Δh in Erbil territory.

heights ($h_{Karbala1979}$) have been determined by the transformation of the measured coordinates, of the selected points, from WGS84 into Karbala 1979 using the seven transformation parameters.

3. Undulation and Geoid Model

It is worth reminding that the geodetic (ellipsoidal) height (h) contains the orthometric height and the undulation at a particular position (Fig. 2).

$$h_i = H_i + N_i \quad (2)$$

where:

H_i is the orthometric height of the i^{th} point.

N_i is the undulation (or the geoid separation, which is the ellipsoidal height of geoid) at the i^{th} point.

The geodetic heights over the Iraqi local datum (Clarke 1880) and the global datum WGS84 are:

$$h_{WGS84} = H_{WGS84} + N_{WGS84} \quad (3)$$

$$h_{Clarke1880} = H_{Clarke1880} + N_{Clarke1880} \quad (4)$$

Then, substitution Eqs. (3) and (4) in Eq. (1) gives:

$$\Delta h_i = [H_{i\,(WGS84)} + N_{i\,(WGS84)}] - [H_{i\,(Clarke1880)}\ N_{i\,(Clarke1880)}] \quad (5)$$

But, for a particular position the orthometric height should be kept as a fixed value (i.e. $H_{WGS84} = H_{Clarle\ 1880}$) (Fig. 3), so:

$$\Delta h_i = N_{i\,(WGS84)} - N_{i\,(Clarke1880)} = \Delta N_i \quad (6)$$

where, ΔN_i represents the difference between local and global undulation at the i^{th} point.

3.1 Effect of Datums Non-parallelism

For the standard Cartesian systems, like the realizations of a global datum WGS84 and another local datum, the angles of rotation serve as a basic model for illustrating the parallelism in the orientation of these datums [5].

The case of non-parallelism is illustrated in Fig. 4. The data of Table 1 indicate the degree of the non-parallelism between the ellipsoids Clarke 1880 and WGS84, through the values of the rotation angles around the main axes on the values of Δh_i.

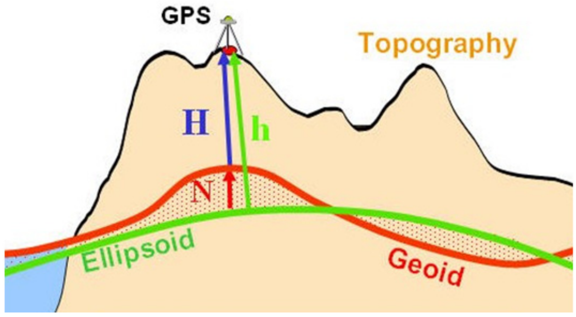

Fig. 2 Geoid undulation & geodetic height.

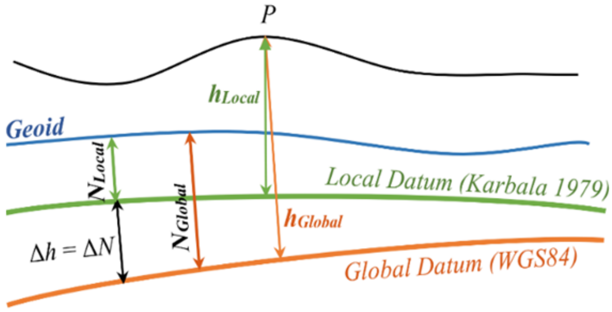

Fig. 3 Geometrical relationships, and undulation.

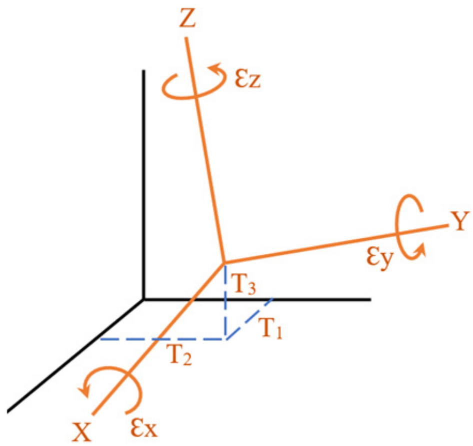

Fig. 4 Datums non-parallelism case.

However, the existence of the small rotational angles around the main axes of the two datums (Ɛx, Ɛy, & Ɛz) indicates a small difference between the normal over the global ellipsoid WGS84 and the normal over the Local ellipsoid Clarke 1880 (Fig. 5).

Let γ be the deflection angle between the normal to a global datum, which contains the undulation (N_G), and the normal to a local datum, which contains N_L.

The difference between the local and the global geoid undulation (ΔN) may have two values according to two directions, the first one is along the direction of normal to a global datum (ΔN_G) and the second one is

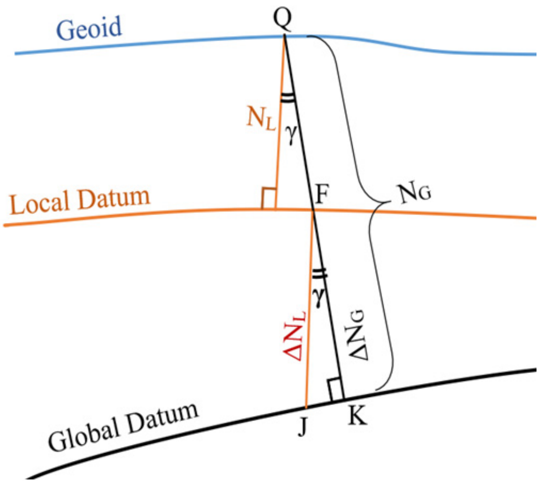

Fig. 5 Effect of datums' non-parallelism on Δ*N*.

along the direction of normal to a local datum (ΔN_L), therefore, the angle between these two directions will be denoted as the deflection angle (γ).

The right triangle (FKJ), in Fig. 5, will serve to define the difference between the values ΔN_G and ΔN_L, which can be represented mathematically as in Eq. (7) below:

$$\Delta N_G = \Delta N_L \cdot \cos\gamma \qquad (7)$$

where:

ΔN_G—the difference between the local and global geoid undulations along the normal to the global datum (WGS84).

ΔN_L—the difference between local and global geoid undulations along the direction of the normal to Karbala 1979.

γ—the deflection angle between the local and global normals.

Let the difference between the two values (ΔN_L and ΔN_G) be denoted by δ, hence we will have:

$$\delta = \Delta N_L - \Delta N_G \qquad (8)$$

Substituting of Eq. (7) in Eq. (8), yields the following relationship:

$$\delta = \Delta N_L - \Delta N_L \cdot \cos\gamma = \Delta N_L\,(1 - \cos\gamma) \qquad (9)$$

Eq. (9) explains what we called the effect of non-parallelism between the datums on the determined geoid undulation. Obviously, if the value of δ goes to zero the value of deflection angle γ goes to zero as well.

It is worth mentioning that the value of the obtained deflection angle γ is mainly dependent on the rotation angles around the X & Y axes (see Fig. 4), and we should remember that these rotation angles are enough small to be regarded equal to zero (i.e. $\varepsilon x = 0$; and $\varepsilon y = 0$). So, based on this geometrical relationship, the value δ will be regarded as equal to zero for the next steps.

Thus, the desired value of the local geoid undulation will be determined directly as in the equation below:

$$N_{Karbala\ 1979} = N_{WGS84} - \Delta h \qquad (10)$$

where:

$N_{Karbala\ 1979}$ is the local geoid undulation (above the ellipsoid Clarke 1880) at a particular position.

N_{WGS84} is the global geoid undulation (above the ellipsoid WGS84) at the same position.

4. Generation of the Local Geoid Model

Based on the proposed geometrical approach, the desired value of the local geoid undulation will be defined according to the value of the global geoid undulation at a certain position (see Eq. (10)).

So, the online geoid calculations website (GeoidEval) is used to find the value of N_{WGS84} based on the Earth Gravitational Model (EGM 2008) and utilizing the interpolation according to the given point's position. The accuracy of such interpolation is ±1 mm [3].

The interpolated values of global geoid undulation (N_{WGS84}) for the selected points over the whole case study area were determined using the geodetic coordinates of these points. For that purpose, the territory of Erbil city is partitioned into four areas (parts) as is shown in Fig. 6.

However, the geodetic coordinates of about 97 selected points within the boundaries of Erbil city have been precisely observed using the Topcon GR 5 GNSS system and utilizing the DGPS RTK technique according to the available CORS (Continuously Operating Reference Stations) station.

Thus, based on the GNSS observations and the interpolated global geoid undulations the proposed geometrical method has been applied to find the required values of local geoid undulation (N_L) over the experimental area. The values of N_L over the Erbil city territory are found to be between +13.871 m and +12.093 m.

The obtained values of N_L for the selected points are used to generating the local geoid model (LGM) over the four partitioned parts individually using software “Surfer”. The two-dimensional view of the generated LGM is shown in Figs. 7-10, for the four parts (areas) of Erbil city territory.

The 2D and 3D views of the generated LGM that covers the whole Erbil city are illustrated in Figs. 11 and 12, respectively.

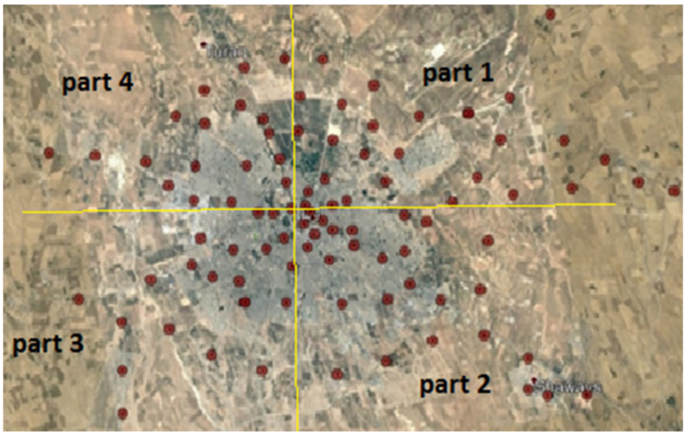

Fig. 6 The four partitioned areas of Erbil city territory.

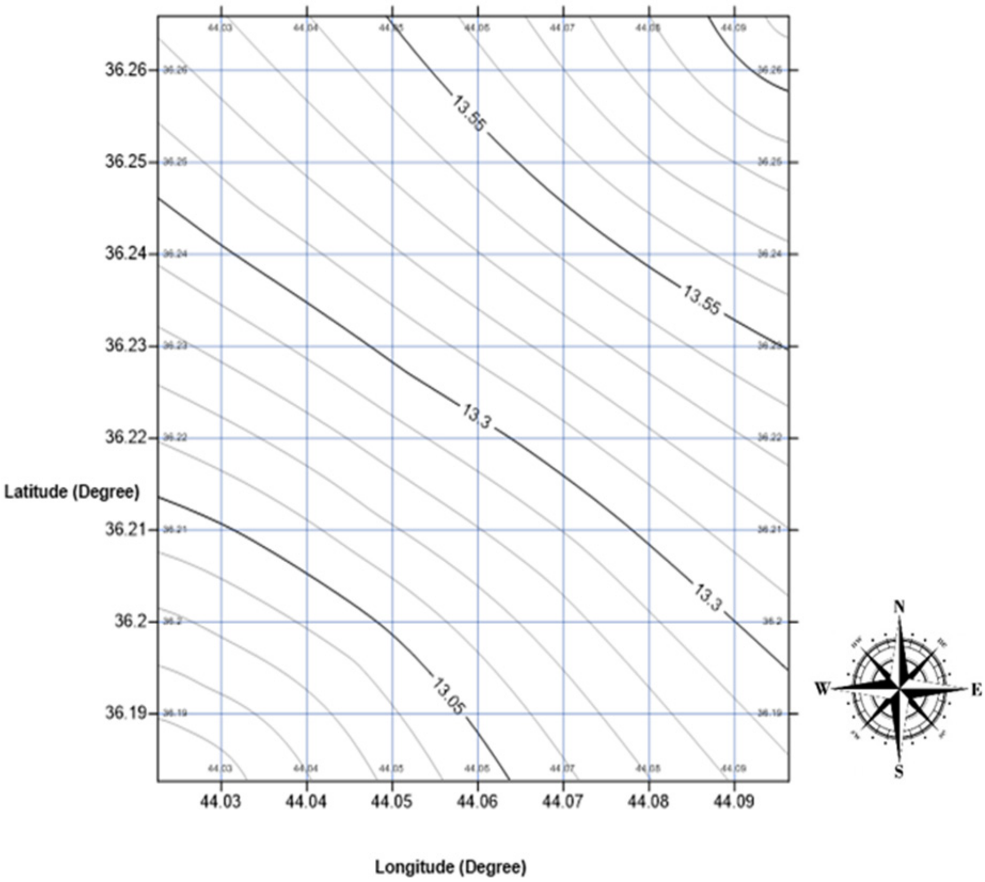

Fig. 7 LGM of part 1—Erbil city.

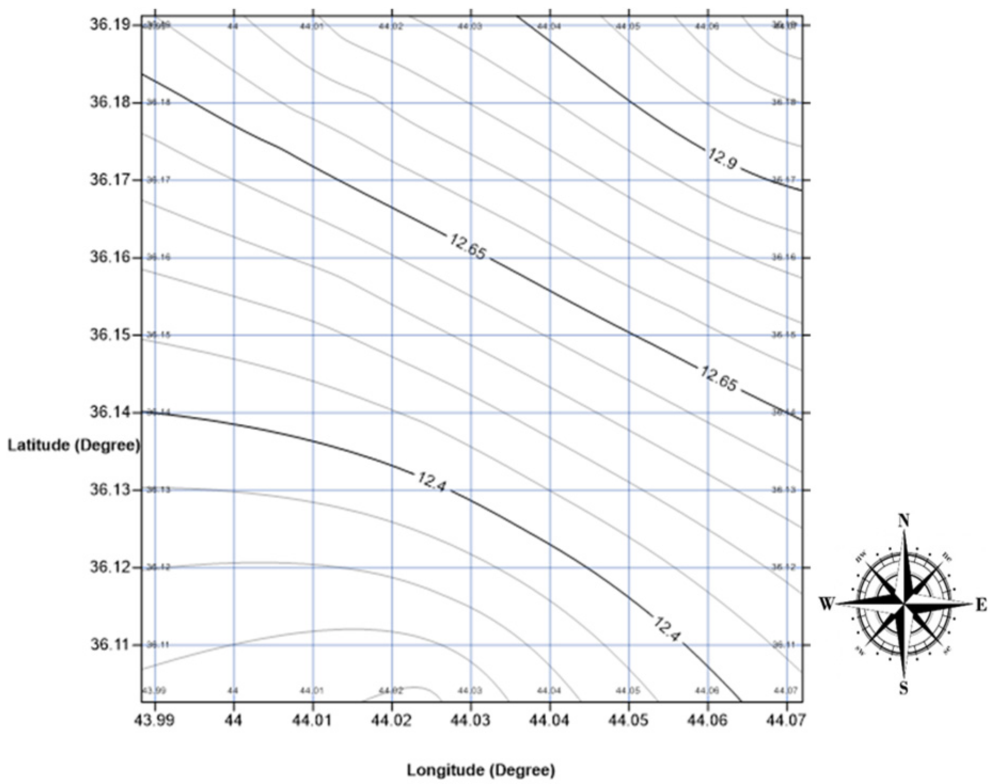

Fig. 8 LGM of part 2—Erbil city.

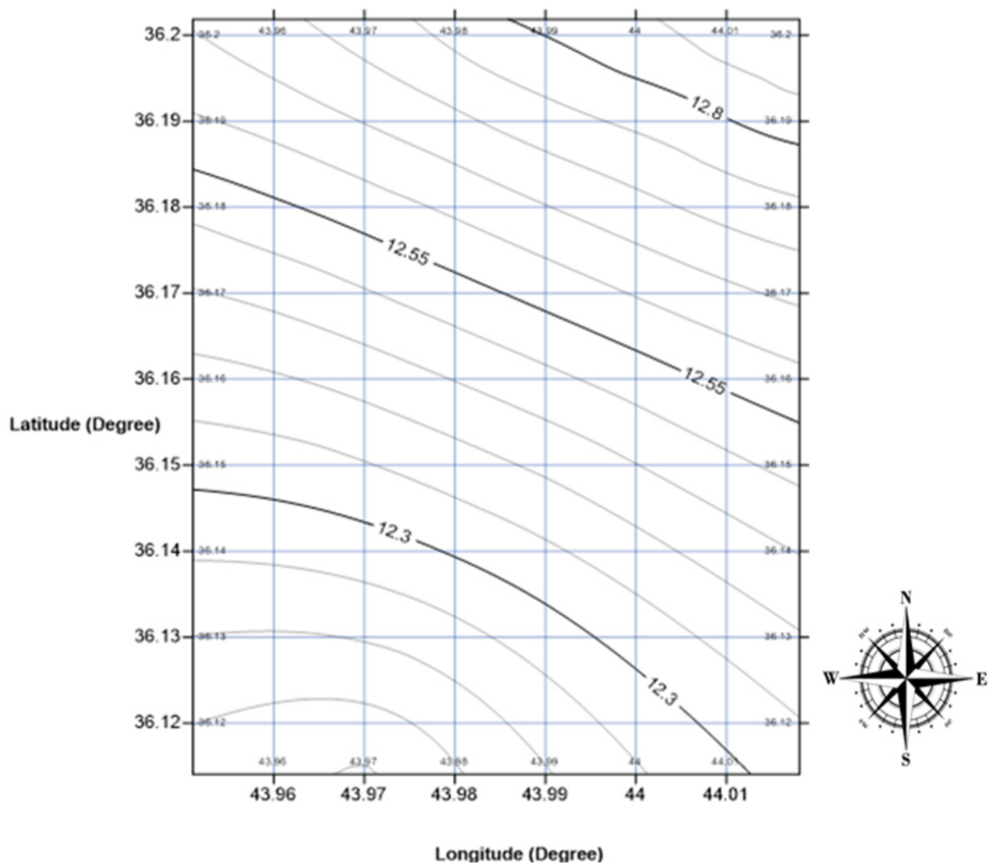

Fig. 9 LGM of part 3—Erbil city.

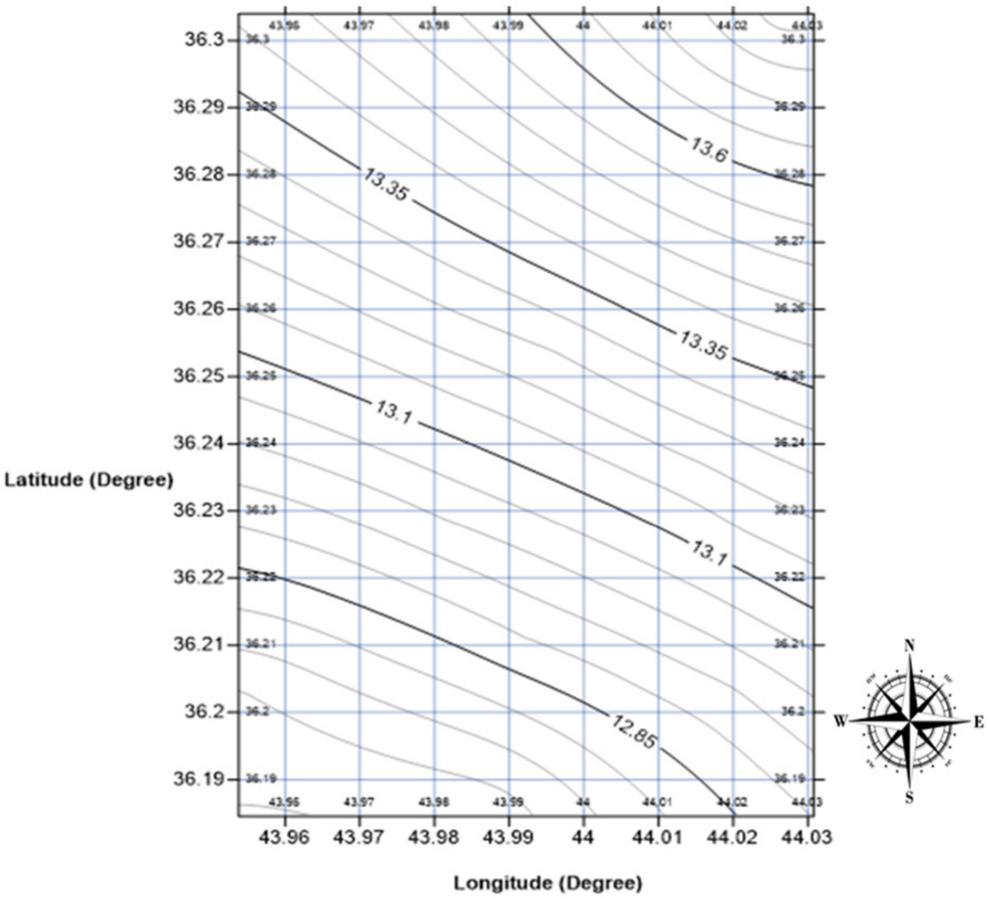

Fig. 10 LGM of part 4—Erbil city.

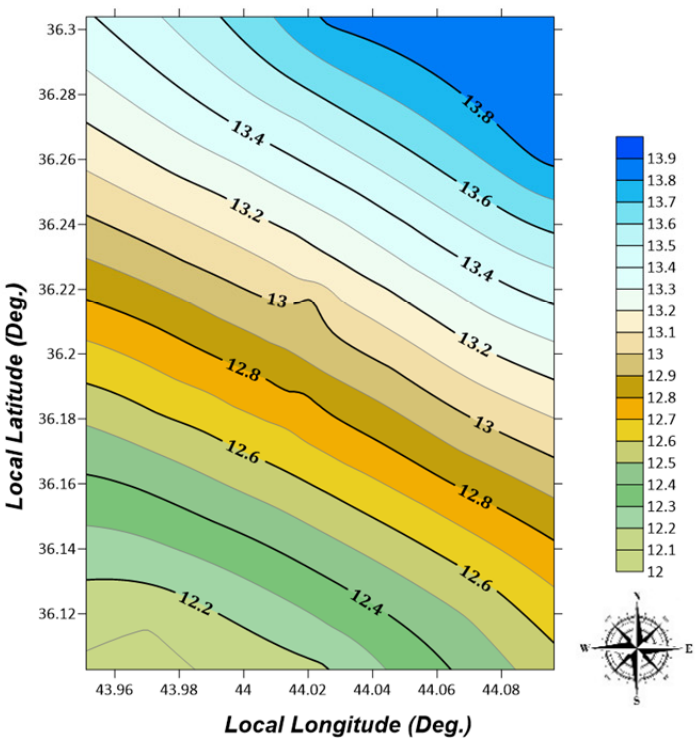

Fig. 11 2D view of LGM for total Erbil city area.

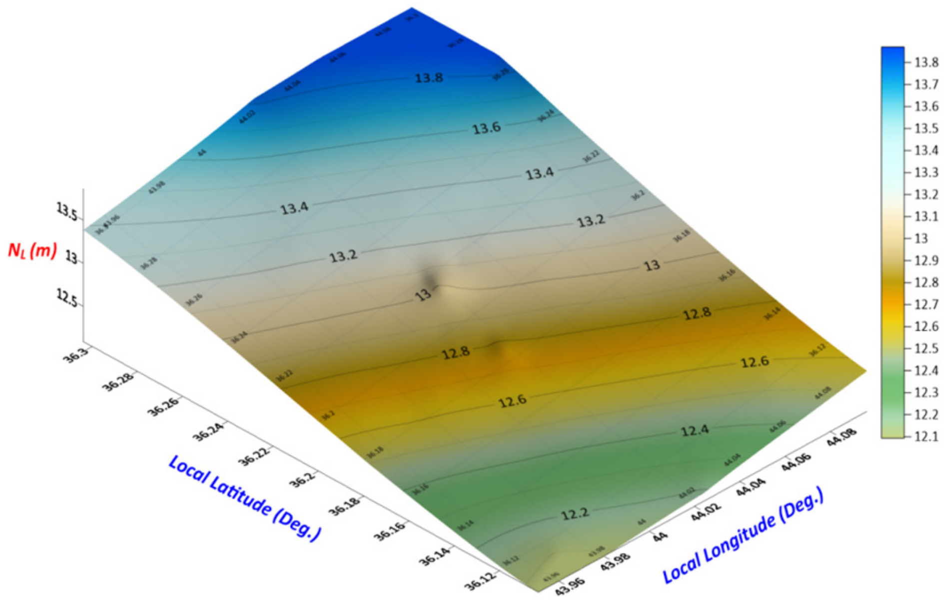

Fig. 12 3D view of LGM for total Erbil city area.

5. Accuracy of the Generated LGM

It is worth mentioning, according to Eq. (10), that the obtained value of local undulation (N_L) is a function of the following two values:

(1) Global geoid undulation (N_{WGS84});

(2) Difference in geodetic height (Δh).

That means, LGM = f (N_{WGS84}, Δh).

So, considering the general law of the error propagation, the uncertainty of the generated LGM will be dependent essentially on the accuracy of the above-mentioned values. Thus, the accuracy of the generated LGM can be estimated based on the following estimated errors:

(a) The standard error of the global geodetic height (σ_{hG}) since it represents the base of the value Δh.

(b) The mean of discrepancies in the EGM 2008 geoid undulations ($\sigma N_{EGM\ 2008}$).

So, the LGM accuracy can be estimated as follow:

$$\sigma N_L = \pm\sqrt{(\sigma h_G)^2 + (\sigma N_{EGM})^2} \qquad (11)$$

where:

σ_{hG}—the average of the standard error in the observed geodetic height over the experiment area.

σN_{EGM}—the mean discrepancy between EGM 2008 and GPS/levelling data, which is in order of ±5 to ±10 cm [6, 7].

So, to estimate the accuracy of the generated LGM we will consider the average value of the standard error in the observed geodetic heights equal to ±0.01 m, and the mean discrepancy of the interpolated value of global geoid undulation is equal to ±0.075 m [7, 8]. Then, according to Eq. (11), the uncertainty of the generated models (LGM) for the four parts, in Erbil city territory, is estimated as equal to ± 0.076 m.

However, the obtained uncertainty of the LGM is calculated essentially based on the estimated discrepancy of the gravitational model EGM2008, and therefore, it can be regarded that the accuracy of any derived value of orthometric heights from this LGM will be of the same order as the discrepancy of the EGM2008 model itself approximately.

From the other side, unfortunately, although most of the documentation is still available, most of the control point monuments of the Iraqi primary network have been damaged [9]. Therefore, the determined uncertainty of the generated LGM can not be validated yet with any independent levelling data, at least in the Erbil city area, because of the damage that has been happened to the control points of the primary network that established according to the Iraqi local datum Karbala 1979.

6. Conclusions

(1) This paper describes the geometrical approach for the generation of a local geoid model (LGM) based on Iraqi local datum Karbala 1979, which is established by the Polish State Enterprise for Geodesy and Cartography (GEOKART).

(2) The local geoid undulation is usually determined by methods of gravity observations, while in the current research the value of the geoid undulation is derived based on the geometrical relationship between the global and local datums.

(3) The GNSS observations according to the DGPS technique based on the CORS station were done for about 97 selected points to cover the whole area of the experiment, which is partitioned into four parts within the boundaries of Erbil city municipality.

(4) The obtained local geoid undulation based on the Iraqi local datum Karbala 1979 has a maximum value equal to +13.871 m and a minimum value equal to +12.093 m over the case study area.

(5) The LGM is generated with accuracy equal to ±0.076 m as an estimated average accuracy for the overall experiment's area.

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