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Effect of Corporate Debt on Firm Value

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"Truth does not do as much good in the world, as its counterfeits do evil."
Francois de La Rochefoucauld, Moral Maxims (64)

We have shown that three classic works considering the effects of corporate debt on the firm value, namely, Modigliani and Miller (1958, 1963), Merton (1974), and Leland (1994), are wrong. Their main mistake is ignoring the business security expenses, BSEs. We suggest the model taking account of BSEs and apply it to the analysis of debt influence on the firm value and survival. Our modeling demonstrates that (1) the debt affects the firm value and its survival, (2) this influence is negative, diminishing the firm value and its chances to survive, (3) the pressure of the negative effect of debt increases as the debt grows, provoking the firm default. The debt *can be* beneficial for the firm if the loan is taken to improve its technology. The model helps estimate the chances to succeed in the technological modernization for various parameters of the firm and its business environment; and by that, to find the technology most suitable for the firm. It is shown that there is a serious problem in reading the market signals concerning a firm and using this information to control this firm.

Keywords: Geometric Brownian model, Extended Merton model, business securing expenses, corporate debt, default probability

Introduction and Critical Review

The questions of corporate debt pricing, effects of debt on the firm development, and the optimal asset structure are among the crucial problems in the theory and practice of financial management. Bitter discussions of the effects of debt on the firm value have got their first answers in Modigliani-Miller Propositions I (1958) and III (1963). MMP1 or, Theorem of irrelevance, proves that there is no relation between the firm's capital structure and its value. MMP3 argues that in the presence of taxes, the value of the levered firm is equal to the value of the unlevered firm, identical to the levered firm in every respect but the asset structure, plus the present value of the tax shield. However, the general opinion is that although providing a solid theoretical basis for future investigations, MMPs have been less effective in practice, suggesting mainly qualitative guidance to the problem (Miller, 1988, Pagano, 2005).

The next important step in corporate debt pricing is made by Merton (1974), who suggests a model for the firm value development taking account of the firm's debt and dividend payments. Assuming the standard conditions of the perfect market and applicability of MMP1 (1958), Merton starts with a portfolio consisting of the firm value, the value of a security issued by the firm, and the riskless cash account. The assets in the portfolio are composed in such a proportion that the total portfolio value is zero. Optimizing this portfolio,

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Merton derives a continuous-time equation “which must be satisfied by any security whose value can be written as a function of the value of the firm and time” (p. 453). While complementing this equation with two boundary conditions and an initial condition, the author remarks that “it is precisely these boundary condition specifications which distinguish one security from the other”. Using this general equation, Merton considers the value of a zero-coupon bond issued by the firm. In this problem, he comes to the geometric Brownian model (GBM) for the firm value V and the option-like equation for the bond value $F(V, t)$, together with the option-like boundary conditions and a condition at the date of debt maturity. No wonder that for the bond value, the author gets a closed-form solution similar to Black-Scholes’ one for the option value (1973). Mark that in Merton’s model, the firm can default on the day of debt maturity only, neglecting the relationship between the firm value and its debt before this date. Summing up, Merton concludes that “while options are highly specialized and relatively unimportant financial instruments [. . .] the same basic approach could be applied in developing a pricing theory for corporate liabilities in general” (1974, p. 449) because of “isomorphic correspondence between almost any corporate liability and options”.

However, the assumption that the firm can default only at debt maturity is far from reality. Black and Cox (1976) have relaxed this assumption introducing a threshold triggering default when the firm’s assets hit the threshold line (more often called the default line). This line has a dramatic effect on the firm development: now the firm has a non-zero probability to default at any moment, and this probability grows on time. From the point of view of physical systems, the model without the default line is a conservative system whose number of Brownian particles remains the same all the time. The model with the default boundary is an open system continuously losing its particles. Therefore, without the inflow from the outside, the open system can exist only for a limited period. Unfortunately, this fact is not recognized by the economists evaluating financial risks. For example, Black and Cox, studying the development of the bond value in the Merton-type model supplied with an absorbing boundary, consider the behavior of a perpetual bond ($t\to\infty$) which just cannot exist in their model. Another effect of the default boundary, missed by the authors, is the development of negative skewness in the probability distribution caused by the continuous loss of particles (for details, see Shemetov, 2020a). Therefore, the GBM-solution presented by Black and Cox is valid only for limited time intervals. Since Black and Cox, (1976), no author using GBM with the default boundary has commented on this fact.

The honor of making the next major step in the theory of debt and its effects on the firm value belongs to Leland (1994). He applies Merton’s general equation supplemented with Black-Cox’s default line to the analysis of the debt value and the optimal capital structure of the firm with corporate taxes and bankruptcy costs. Leland uses all Merton’s assumptions, plus a new one that capital structure decisions, once made, remain static. Leland’s model includes the firm and security representing a claim on the firm which continuously pays for a nonnegative coupon per instant of time when the firm is solvent. At that, the firm finances the net cost of this coupon by selling additional equity from outside of the firm’s portfolio. In these conditions, the asset value is described by GBM. The security value depending on the firm value and time follows Merton’s general equation with boundary conditions determined by payments at debt maturity and by payments in bankruptcy, should it happen before the maturity. Because the closed-form solution of this problem is unknown, Leland looks for the time-independent solution to this problem, tending time to infinity. He comes to a closed-form solution for this marginal case writing explicit equations for the firm’s debt, equity, and the firm value equal to the asset value, plus the tax deduction of coupon payments, less the value of bankruptcy costs. The last expression makes the quantitative basis of the trade-off theory, which Leland uses to determine the optimal asset structure for the exogenous default line. If the firm management can choose the moment of default, they can do it, maximizing the firm's equity at that moment (the endogenous default). Using his powerful formalism, Leland presents a multilateral analysis of various debt covenants and their influence on debt variables. However, his optimal asset structure occurs extremely high (75%-90%), which indicates that the author has missed important phenomena in his analysis of the firm development. The authors of succeeding articles, trying to improve the level of optimal asset structure, add to Leland's model various mechanisms relaxing the perfect market conditions, like different kinds of friction, dynamic borrowing, etc.

Our objective is to reveal the principal error in these three seminal articles and, correspondently, in all succeeding studies based on and developing their ideas. We plan to show that the main mistake comes from ignoring the firm's payments, as in Modigliani-Miller Propositions and Leland (1994), or a mere declaration of taking account of the firm's payoffs without actually doing it (Merton, 1974). We start with revisiting the Merton model (1974):

$$dX = (\mu X - P)dt + CXdW,\ X(0) = X_0, \tag{1.i}$$

$$P = DP + DIV. \tag{1a.i}$$

Here $X(t)$ is the firm value at time t, constant μ is the rate of instantaneous expected returns on the firm per unit time, P is the total dollar payouts by the firm per unit time to either its shareholders or liabilities-holders (dividend DIV or interest DP payments), constant C is the process volatility and C^2 is the instantaneous variance of returns, W is a Gauss-Wiener process. (For the sake of consistency with the further discussion, we use our symbols for the variables and parameters in the model keeping original Merton's interpretation of all symbols.)

Merton includes the income μX and two exterior payments to shareholders and bondholders, but from the accountant's point of view, one must consider *all inflows and outflows* when describing the firm value dynamics. Here the firm has one inflow $\mu_0 X$ from its sales and five types of outflows: the variable costs $VC_i = \delta_i X,\ \delta_i = const(t) > 0$, $i = 1, 2, \ldots, n$, fixed costs FC, debt payments DP, dividends DIV, and corporate taxes TAX. One can take account of the variable costs by adjusting the expected rate of return as $\mu = \mu_0 - \sum_i \delta_i$. The other four outflows compose the *business security expenses* (BSEs), $P = FC + DP + TAX + DIV$, and the failure to pay any of them sooner or later brings the firm to default. So, the revised model is

$$dX = (\mu X - P)dt + CXdW,\ X_0 = X(0), \tag{1.i}$$

$$P = FC + DP + TAX + DIV,$$
$$P(t) = P_0\pi(t),\ P(0) = P_0 > 0,\ \pi(0) = 1. \tag{2.i}$$

P is a *piecewise continuous* function of time; P_0 is a positive constant. The time dependence of FC and DP reflects changes in business conditions; the dollar values of TAX and DIV depend on their rates and year returns. Hereafter we refer to the process (1.i), (2.i) as the Extended Merton model (EMM).

EMM for a stochastic variable $x = \ln(R_0 X/P_0)$ by Ito's Lemma transforms to

$$dx = R_0(1 - \pi(t)e^{-x})dt + CdW, \tag{3.i}$$

$$x(0) = x_0 = \ln(R_0 X_0/P_0),\ R_0 = \mu - C^2/2. \tag{3a.i}$$

This equation represents an ordinary diffusion with a drift whose rate $R(x, t)$ depends on the location of the Brownian particles on the x-axis and time. For the uniform mode of payments, $\pi(t) \equiv 1$, the drift rate is: $R(0) = 0$, $0 < R(x) < R_0$ for $x > 0$, and $-\infty < R(x) < 0$ for $x < 0$. When a part of the distribution of Brownian particles $V(x, t)$ gets below the line $x = 0$, its particles are transported to the negative infinity with an ever-increasing drift rate, creating a deficit of particles below this line. The diffusion force compensates this deficit with particles from the upper part of the distribution, and the process continues until there are no more particles left above the line $x = 0$. For the normal initial distribution:

$$V(x,0) = N(x; H_0, \sigma_0^2),\ H_0 = \langle x_0 \rangle = \langle \ln(R_0 X_0 / P_0) \rangle,\ \sigma_0^2 = \langle (x - H_0)^2 \rangle,$$

one can see the influence of the initial parameters on the process. The increase in the expected rate of return R_0 and/or the initial asset value X_0 rise a location of the distribution center H_0 over the line $x = 0$, while the rise in payments P_0 brings it closer to this line, providing a greater part of the distribution $V(x,0)$ under the line $x = 0$. A similar effect makes the initial variance σ_0^2. The greater the variance, all other parameters equal, the greater part of distribution $V(x,0)$ occurs under the line $x = 0$, the faster grows the negative tail of the distribution. The intrinsic property of EMM is the development of negative skewness even at an initially symmetric distribution. Unfortunately, the closed-form solution for EMM is unknown even in the case of the time-independent drift rate. See Shemetov (2020a) for the details.

The geometric Brownian model (GBM)

$$dX/X = \mu dt + C dW,\quad X(0) = X_0 \tag{4.i}$$

has a closed-form solution named the economic exponent (Samuelson, 1965)

$$X(t) = X_0 \exp(R_0 t + C W_t),\ R_0 = \mu - C^2/2 \tag{4a.i}$$

whose distribution is lognormal

$$U(X,t) = \frac{1}{X\sqrt{2\pi\sigma^2}} exp\left[-\frac{(lnX - H)^2}{2\sigma^2}\right] \tag{4b.i}$$

$$H(t) = H_0 + R_0 t,\ \sigma^2(t) = \sigma_0^2 + C^2 t. \tag{4c.i}$$

This distribution is symmetric about the logarithmic variable $z = \ln(X/X_0)$. Here H_0 and σ_0^2 are the mean and variance of the initial normal distribution $N_0(z; H_0, \sigma_0^2)$. The intrinsic property of the GBM-distribution is to stay normal about z all the time.

Modigliani-Miller Propositions (1958, 1961, 1963)

The idea of proof of Modigliani-Miller Propositions (MMPs) consists of constructing an analog of the Marshallian industry for firms' cash flows and then applying the one price principle to the market of perfect substitutes. Modigliani and Miller (1958) consider firms at the perfect market described with the assumptions:

1. The firm value is determined only by the mean cash flow generated by the firm;

2. All investors have complete information about firms' cash flows; thus, the investors have homogenous expectations on corporate cash flows and their riskiness;

3. There is an "atomistic" competition and no market friction of any kind. That implies, among other things, that at the market of corporate stocks and bonds (a) there are no agency costs, (b) bankruptcy entails no liquidation costs, and (c) all investors, both individuals and institutions, can borrow at the same rate as corporations;

4. The debt of firms and investors is riskless, so the interest rate of all debts is the risk-free rate for all possible amounts of debt;

5. There are no corporate or personal taxes.

The authors argue that all "firms can be divided into "equivalent return classes" such that the return on the shares issued by any firm in any given class is proportional to (and hence perfectly correlated with) the return on shares issued by any other firm in the same class" (1958, p. 266). They insist that "all relevant properties of a share are uniquely characterized by specifying (1) the class to which it belongs and (2) its expected return", and by that, they create "an analog to the industry in which it is the commodity produced by the firms is taken as homogenous" (Ibid., p. 266). In a later paper, Miller confirms that for the "equivalent return class" (here he calls it the "risk class") "the uncertain, underlying future cash flow streams of the individual firms within each class could be assumed perfectly correlated, and hence perfect substitutes", and further: "at the practical level, the risk class could be identified with Marshallian industries" (Miller 1988, p. 103). Modigliani and Miller claim by the method of proof that Propositions are invariant to the firm value distribution and, therefore, Propositions are universal at the perfect market in its state of equilibrium. Now we know that the firm value distribution must meet EMM (1.i)-(2.i). The assumed capability of the two identical levered and unlevered firms to get into and stay long enough in the same risk class (or equivalent return class) for any asset structure means that the firm value distributions are *lognormal*. (The levered firm is identical to the unlevered one in any respect, but the structure of its assets.) Really, *the mean returns of the levered firm must be equal to the mean returns of the unlevered firm at all times*: $H_L(t) = H_{UL}(t), t \geq 0$. (From Assumption A, it follows that all other firm characteristics are of no importance.) It is possible for the lognormal distribution only, for which one has the relation $H(t) = H_0 + R_0 t$, Eq. (4c.i). From the identity of two firms and the distribution lognormality, one has the following conditions $H_0^L = H_0^{UL} = H_0, R_0^L = R_0^{UL} = R_0$. Therefore, the assumption that the levered and unlevered firms have the same mean returns implies that both firms have lognormally distributed values. The lognormal distribution is a GBM-solution neglecting BSEs (P = 0), or using BSEs of a specific form, $P = \delta X$, $\delta = cons(t)$, $0 < \delta < R_0$. We consider these two cases separately.

The condition $P = 0$ means that neither the levered firm nor the unlevered firm pays any BSE, including fixed costs, too. Because there are no dividend payments for both firms (DIV = 0), the dividend policy does not affect the firm value (MMP2, 1961). Because there are no debt payments (DP = 0), the asset structure of the levered firm does not influence the firm value (MMP1, 1958). Because the "levered" firm does not pay for its debt but presumably enjoys the tax shield, its mean after-tax value is higher than the mean after-tax value of the unlevered firm by the present value of the tax shield (MMP3, 1963). However, the following argument shows that MMP3 is a logical error. Because the levered firm does not pay for its debt, it is indistinguishable from the identical unlevered firm, and, therefore, *its tax shield must be zero*! The revised version of MMP3 must run as: in the presence of corporate taxes, the value of the levered firm equals the value of the unlevered firm (MMP1 with corporate taxes). As we demonstrate below, this is the maximum effect of the tax shield on the levered firm value; for all other cases, the debt contribution to the firm value is *negative* even with the tax shield.

Now we address the case of GBM with proportional payments, $P = \delta X$. We consider two GBM-firms (no fixed costs), which BSEs consist of dividends DIV or debt payments DP, Eq. (4.i). For the variable $z = \ln(X/X_0)$, Eq. (4.i) transforms to

$$dz = (R_0 - \delta)dt + CdW, \ P_c = 0. \tag{5.i}$$

Suppose that there are two identical unlevered firms with different dividend policies $P_1 = DIV_1 = \delta_1 X$ and $P_2 = DIV_2 = \delta_2 X$, $R_0 > \delta_2 > \delta_1$ (in this case, the dividend policy is reduced to the choice of δ). The log-value means for these firms are

$$H_1(t) = H_0 + (R_0 - \delta_1)t, \ H_2(t) = H_0 + (R_0 - \delta_2)t, \text{ and } H_1(t) > H_2(t). \tag{6.i}$$

We see that the dividend policy affects the mean returns and value of the firm, and MMP2 (1961) is false for GBM with $P = \delta X$. The same logic can be applied to the case of debt payments only, $P_1 = DP_1 = \delta_1 X$ and $P_2 = DP_2 = \delta_2 X$, $R_0 > \delta_2 > \delta_1$, proving that MMP1 (1958) is also false.

What is the effect of taxes on the values of two identical levered and unlevered firms? The payments of the unlevered firm consist of dividends only, $P_{UL} = DIV = \delta_1 X$, $\delta_1 > 0$, while the payments of the levered firm consist of dividends and debt payments, $P_L = DIV + DP = \delta_2 X$, $R_0 > \delta_2 > \delta_1$. The log-values of both firms, described by Eq. (5.i), can be considered as the log-values of two *unlevered* firms ($P_{cL} = P_{cUL} = 0$) with different effective rates of return. Correspondingly, the returns of the second firm are lesser ($\delta_2 > \delta_1$) than returns of the first firm. As the unlevered firm, the second firm has no right on the tax shield, and it pays a tax of the same rate as the first firm. Therefore, the after-tax mean value of the second firm is *lesser* than the after-tax mean value of the first firm. This conclusion rejects MMP3 (1963). It means that debt does a *negative effect* on the after-tax mean value. So, all kinds of *the trade-off theory,* supposing after MMP3 a positive effect of debt on the after-tax mean value, are wrong for GBM, $P = \delta X$(e. g. Kraus & Litzenberger, 1973; Leland, 1994; Ju et al., 2005; Frank & Goyal, 2007; Strebulaev & Whited, 2012). All papers on the optimal capital structure, using MMP3 and now popular GBM, $P = \delta X$ (e. g. Leland & Toft, 1996; Goldstein et al., 2001), are self-contradictory because GBM, $P = \delta X$, is inconsistent with Modigliani-Miller Propositions.

Now let us consider the EMM case with the levered and unlevered firms identical in all respects, but asset structure, both having a normal initial distribution of the same variance. Suppose that both firms do not pay dividends and taxes, so $P_{UL} = FC \equiv P_0$ and $P_L = FC + DP = P_0 + DP = P_1 > P_0$ and all parameters shown in Eq. (1.i)-(2.i) are the same. Then one has

$$H_0^{UL} = \langle x_0 \rangle = \langle \ln(R_0 X_0 / P_0) \rangle,\ H_0^{L} = \langle x_0 \rangle = \langle \ln[R_0 X_0 / (P_0 + DP)] \rangle,\ H_0^{UL} > H_0^{L}; \qquad (7.i)$$

which means that the levered firm and the unlevered firm belong to different "risk classes" from the very beginning and the deformation of their distributions runs with different rates. The skewness of the levered firm grows faster than the skewness of the unlevered firm, the log-value mean of the unlevered firm is always greater than the log-value mean of the levered firm, and this difference increases over time (see Eq. (3.i)-(3a.i) and comments to them). So, one can conclude that in EMM, the asset structure influences the mean firm value and mean returns rejecting MMP1. The same line of reasoning proves that two identical firms exercising different dividend policies have different mean values. The value of the firm paying more dividends will be lesser than the value of the other firm in the long run if the management of the firms can read market signals correctly and act consistently with this information. Our study (see Section 2) reveals a problem in reading and interpreting the market information. It is clear from the conclusion on MMP3 for GBM, $P = \delta X$, that in EMM, the mean value of the unlevered firm is more than the mean value of the levered firm in the presence of corporate taxes. However, the question of how taxes and dividends affect the mean value of the firm is most interesting for practice, and later we will present EMM-results for this problem.

Proving a theorem is wrong, it is sufficient to demonstrate one counterexample for which this theorem does not hold, and GBM, $P = \delta X$, $\delta = const(t)$, $0 < \delta < R_0$, is such a counterexample for MMPs. In general, all three MMPs are wrong and misleading. However, MMP1 is true for short-term deals ($t << 1$), when the mean firm value remains about constant, and one can neglect payments. The other two Propositions consider the time intervals longer than a year. For such intervals BSEs are essential, GBM always invalid, and MMP2 and MMP3 are never good. The failure of Modigliani-Miller Propositions casts a shadow of profound mistrust towards the *method of their proof* which is now generally accepted (Miller, 1988).

Merton (1974) "On the Pricing of Corporate Debt"

To develop a Black-Scholes pricing model, Merton makes the following assumptions about the market and the firm:

1. There are no transaction costs, taxes, or problems with the indivisibility of assets;
2. There is a sufficient number of investors at the market who can buy or sell as much of an asset as they want;
3. There is an exchange market for borrowing and lending at the same interest rate;
4. Short-sales of all assets with full use of the proceeds are allowed;
5. Trading in assets is continuous in time;
6. The Modigliani-Miller Theorem of invariance (MMP1) that the value of the firm is invariant to its capital structure obtains;
7. The term-structure is "flat" and its riskless rate of interest, r, the same for all time is known with certainty;
8. The dynamics of the firm value, V, is described by the equation

$$dV = (\alpha V - C)dt + \sigma V dz \ , \tag{8a.i}$$

α is the instantaneous rate of return on the firm per unit time, C is the total dollar payouts by the firm per unit of time to either its shareholders or liability-holders if positive, and it is the net dollars received by the firm if negative, σ^2 is the instantaneous variance of the return per unit of time, dz is a standard Gauss-Wiener process. Mark that A.6 (MMP1) is necessary to write A.8 in the form insensitive to the debt structure.

Next, Merton introduces a security whose market value, $Y = F(V, t)$, depends on the firm value and time and follows the equation

$$dY = (\alpha_Y Y - C_Y)dt + \sigma_Y Y dz_Y \tag{8b.i}$$

where α_Y is the instantaneous rate of return on this security per unit time, C_Y is the total dollar payouts by the firm per unit of time to this security, σ_Y^2 is the instantaneous variance of the return per unit of time, dz_Y is a standard Gauss-Wiener process. Then the author forms a portfolio consisting of the firm, the particular security, and the riskless asset taken in such a proportion that the total investment in the portfolio is zero. Using the relations connecting the investments in the optimal portfolio with a riskless asset, Merton comes to the equation

$$F_t + (rV - C)F_V + 0.5\sigma^2 V^2 F_{VV} - rF + C_Y = 0 \tag{9.i}$$

which, as he claims, "must be satisfied by any security whose value can be written as a function of the value of the firm and time". The author insists that this equation, supplied with two boundary conditions and an initial condition, completely specifies each security, distinguishing one security from another. Merton uses this equation for pricing zero-coupon bonds ($C_Y = 0,\ C = 0$) and presents a closed-form solution to the problem. He assumes that the firm can default only at the debt maturity and comes to the option-type condition

$$F(V,T) = \max(0, V - B) \tag{10.i}$$

where B is the debt value, V is the firm value at the debt maturity T. In these settings, the problem of zero-coupon bond pricing is mathematically identical to the option-pricing problem. No wonder that a solution of the bond-pricing problem is similar to the result of Black and Scholes.

However, this result seems unsatisfactory because it does not explain why the values of two such different financial instruments as options and bonds behave so similarly. The option is a short-living financial instrument whose existence is guaranteed within its expiration period, making typically 60 or 90 days. The short expiration time makes the option insensitive to changes in the value of the firm which has issued the corporate liability. Pricing the option, one can think of the firm value as making no effect on the option price. On the contrary, bonds are long-living financial instruments that theoretically can default with the firm at any moment. Their market position depends strongly upon the state of the firm. Merton expresses this dependence as a function relating the bond value Y and the firm value V, $Y = F(V, t)$.

A more careful analysis of Merton's article shows that his solution could be accepted at best as a GBM-approximation of the bond-pricing problem. First, to write an equation of the firm value dynamics in the form neutral to the asset structure (A.8), the author assumes that MMP1 holds and the firm value does not depend on the asset structure (A.6). However, we have shown that MMP1 is correct in one case only when the firm's payments are zero. Therefore, following Merton's line of reasoning, one must remember that by A.6 the firm payments C are always zero, and A.8 is really GBM (5.i). The same is valid for C_Y and equation (8b.i).

To relate the values of the debt and the firm, the author uses the mean-variance analysis of the optimal portfolio containing a riskless asset (Markowitz, 1952; Sharpe, 1964). The mean-variance portfolio theory is quasi-static (thus, strictly speaking, it does not meet A.5) and considers the assets in the portfolio as uniquely determined by their expected returns and variances. To meet this requirement, the firm's returns must be normal, and the firm's assets must follow GBM. So, while deriving his general equation (9.i) describing any security, Merton remains all the time within the GBM-frames, and we can consider this equation as the GBM-approximation of the firm's security whose value depends on the firm value and time.

To apply the developed technique to a particular security, Merton selects a zero-coupon bond because, within his formalism, this choice reduces the general security equation to the option-pricing equation (Black & Scholes, 1973).

$$F_t + rVF_V + 0.5\sigma^2V^2F_{VV} - rF = 0 \tag{9a.i}$$

The author removes terms with C_Y and C because there are no coupon payments ($C_Y = 0$) and because the firm cannot issue any senior- or equivalent-rank claim on the firm before the debt maturity date ($C = 0$). (As one remembers, $C_Y = C = 0$ automatically because of A.6.) With condition (10.i) at the debt maturity and the assumption that the firm can default at this date only, Merton comes to the option-like solution for the price of zero-coupon bonds. The author mistakes the similarity of his findings and Black-Scholes' results as an argument for the "option hypothesis" that pricing of any firm's liability can be solved using the option-pricing technique. Merton insists that "while options are highly specialized and relatively unimportant financial instruments [. . .] the same basic approach could be applied in developing a pricing theory for corporate liabilities in general" (p. 449). This idea that close kinship between the firm and its liabilities on the one side, and the options on the other side, is helpful for pricing various securities, now prevails among financial economists. In a comprehensive review on the dynamic structural models (the authors call them the contingent claims models), Strebulaev and Whited (2012, pp. 4-5) state that "they (dynamic structural models) start with the acknowledgment that any claims on corporate cash flow streams are derivatives on underlying firm value or firm cash flows. This means that we can apply option pricing methods to value these claims." The authors insist also that GBM can be used for describing the "stock price in the Black-Scholes model; firm value, firm cash flows, or prices of firm output and input in other corporate finance settings." Sundaresan (2013, p. 21) asserts that "since its publication, the seminal structural model of default by Merton (1974) has become the workhorse for gaining insights about how firms choose their capital structure, a "bread-and-butter" topic for financial economists." As one can see now, these praises are a bit excessive.

Leland (1994) "Corporate Debt Value, Bond Covenants, and Optimal Asset Structure"

Leland (1994) starts his analysis of the corporate debt value accepting all Merton's assumptions (A.1-A.8 in the preceding section). As a descriptor of the dynamics of the firm value, *V*, Leland uses GBM

$$dV/V = \mu dt + \sigma dW,\ V(0) = V_0 \tag{4a.i}$$

where μ is the expected rate of return, σ^2 is the instantaneous variance of the return per unit of time, dW is a standard Gauss-Wiener process. As we have shown, GBM is consistent with the assumptions A.1-A.8. To keep Eq.(4a.i) and Eq. (9.i) valid, Leland assumes (A.9) that "any net cash outflows associated with the choice of leverage must be financed by selling additional equity" (p. 1217). At that, this equity must be external to the firm because "bond covenants restrict firms from selling their assets." It makes the whole problem settings quite unreal.

Next, the author considers a claim of the value $F(V, t)$ on the firm that continuously pays a non-negative coupon, *C*, per instant of time when the firm is solvent. The last phrase means that following (Black & Cox, 1976), Leland introduces the default line in his problem settings. The development of the claim value is described by Merton's general equation (9.i) in the form

$$F_t + rVF_V + 0.5\sigma^2 V^2 F_{VV} - rF + C = 0 \tag{9b.i}$$

where *r* is the rate of return on the riskless asset (the interest rate). The author remarks that there is no closed-form solution for Eq. (9b.i) for arbitrary boundary conditions. Thus, he decides to look for the time-independent solution when $F_t = 0$ and the claim value depends explicitly on the firm value only, $F(V)$. In other words, he makes a marginal transition with $t \to \infty$. Black and Cox (1976), making the same transition, speak about finding a solution for the corresponding perpetuity. Unfortunately, this transition in both problems does not exist, and Eq. (9b.i) has no informative time-independent form. The value of the claim $F(V, t)$ exists as far as the firm value exists. The default line is the absorbing boundary; if the firm value touches or crosses this line, the firm ceases to exist. According to properties of the diffusion motion, the probability that a diffusion process starting at the time $t = 0$ from a point *M* in the plane (V, t) will cross an arbitrary straight line, $V = a$, in that plane at a *finite time* $T(M, a)$ is unit almost for sure: $P(T(M, a) < \infty) = 1$ a. s. (Shiryaev, 1998, pp. 302-303). If the line $V = a$ is the default line, then the *firm longevity is finite* almost for sure. Because after time $T(M, a)$, the firm and the claim on the firm do not exist, the time-independent version of Eq. (9b.i) is meaningless. Another detail, making Leland's construction void, is that coupon payments *C must be zero* (because of Merton's assumption A6, see Eq. 8a.i and 8b.i). Equation (9b.i) could be partially excused by its utility provided that it can generate the optimal debt leverage comparable with the levels of debt observed in practice, but it cannot.

For marginal equation (9b.i), Leland derives closed-form solutions for the debt value $D(V, C)$, the bankruptcy costs $BC(V, \alpha)$, the value of tax benefits associated with debt financing $TB(V, \tau C)$, the total value of the firm

$$v(V) = V + TB(V, \tau C) - BC(V, \alpha), \tag{11.i}$$

and the value of the firm's equity $E(V)$. Here α is a share of the firm value lost in bankruptcy, and τ is the corporate tax rate. Equality (11.i) makes the quantitative foundation of the trade-off theory. Using it, Leland finds the optimal asset structure for the exogenous default boundary V_B. He also introduces the idea of the endogenous default and evaluates the optimal value of the default boundary, maximizing the firm's equity at the time of default if the firm management can choose the moment of default. The author gives the most detailed analysis of the behavior of bond prices and optimal debt-equity ratios, the asset value, risk, taxes, interest rates, bond covenants, payout rates, and bankruptcy costs change. Unfortunately, all these results are dubious because they are constructed on a shaky foundation: the equation (9b.i) is not as general as Merton claims, the way of financing the coupon payments of the claim on the firm is unreal, the marginal transition providing for the closed-form solution to the problem is mathematically illegal, and the estimates of firm value, optimal capital structure, and endogenous default are founded on the false Modigliani-Miller Propositions. No wonder that the author comes to a bizarre conclusion that "leverage of about 75 to 90 percent is optimal for firms with low-to-moderate levels of asset value risk and moderate bankruptcy costs. Even firms with high risks and high bankruptcy costs should leverage on the order of 50 to 60 percent when the effective tax rate is 35 percent" (p. 1230). The authors of the succeeding papers non-critically accepting the main construction of Leland's model try to improve his final results by introducing various kinds of friction into his model. However, market friction cannot change the *sign* of the debt effect on the firm value from the positive one (MMP3) to the negative one (EMM; GBM, $P = \delta X$). A series of papers on the optimal capital structure based on GBM (Leland, 1994; Leland & Toft, 1996; Goldstein et al., 2001; Strebulaev, 2007; Titman & Tsyplakov, 2007; Hugonnier et al., 2015, etc.) convincingly demonstrate it. Their authors believe in the positive effect of debt on the firm value. They use different types of friction trying to adjust the optimal asset structure from the extremely high 90% closer to the asset structures observed in practice.

Summing up our review of the three seminal articles that have determined the development of financial economics for a long time, we cannot help noticing their significant methodological difference. Modigliani-Miller Propositions represent a classic example of a thought experiment extensively used by antique Hellenistic philosophers (for example, Democritus and his atomic hypothesis), who introduce a set of hypothetic principles without testing them and, using these principles, try to explain the phenomena of the real world. Modigliani and Miller in their study suppose that the firm market is similar to the commodity (Marshallian) market, and the firm (mean) value is determined by the mean cash flows only. Assuming also that the levered and unlevered firms can get into one risk class and stay there long enough for the equilibrium distribution to settle down over the market, they come to their conclusions coined as the famous Modigliani-Miller Propositions. The esthetically and morally attractive non-arbitraging principle used in their proof made the Propositions looking even more plausible and convincing for the broad circles of economists for more than sixty years.

To study the financial risks of the levered firm, Merton brilliantly introduces an axiomatic equation expressing the asset balance in the firm cash flows in stochastic conditions. The axiomatic approach, verified through extensive scientific observations, experiments, and practice, is, for example, widely used in modern physics. Merton's axiomatic equation, correctly interpreted, could make a solid basis for a new stage in the financial economics development, but, alas, the author understands the firm's payments too narrowly as the payments to debtholders and shareholders only. Second, Merton decides to keep the relationship of his model with the Modigliani-Miller Propositions. This relationship has no logical ground within the frames of his axiomatic approach, and it is absolutely unnecessary methodologically. As we have shown, Merton's equation is consistent with MMPs for zero payments only, which reduces his model to GBM. The undisputable merit of GBM is that it often helps to achieve intuitively clear closed-form solutions, which appeal to many economists studying corporate financial risks.

On the contrary to the bright revolutionary papers of Modigliani and Miller (1958, 1961, 1963) and Merton (1974), Leland's paper (1994) is a compilation by its nature. Following the ideas of most reputable economists (Black, Cox, Merton, Miller, and Modigliani), Leland constructs his model accumulating all the achievements and, alas, errors of his scholar predecessors (the GBM equation for the firm value and the "general" equation for the claim value from Merton, the MMP3 relation between the value of the levered firm and the value of unlevered firm in the presence of taxes, the time-independent equation for the claim value with the default line from Black and Cox). He also makes an unjustifiable assumption that "any net cash outflows associated with the choice of leverage must be financed by selling additional equity". All these errors have brought Leland to his unrealistic conclusions. However, the width and depth of Leland's analysis of the debt effect on the firm paying taxes and bankruptcy costs, his intuitively clear results have deeply impressed the economic community. Many attempts to improve Leland's conclusions by introducing various types of friction into his model have been made since 1994, reporting new optimal debt levels close to the levels observed in practice. Because these papers replicate the errors of Leland's model, their seeming success only increases the confusion and misunderstanding of the problem.

Unfortunately, all three seminal investigations constituting the cornerstone of financial economics in the twentieth century and rewarded with the most prestigious academic prizes (it suffice to mention three Nobel Prizes) occur to be wrong and misleading. Below we consider the debt effect on the firm value, addressed the first time in MMP1 (1958), using a new approach based on EMM.

Model Description

The firm value, X, is described as

$$dX = (\mu X - P)dt + CXdW, X(0) = X_0, \tag{1a.1}$$

$$P = FC + DP + TAX + DIV, \tag{1b.1}$$
$$P(t) = P_0 \pi(t), P(0) = P_0 > 0, \pi(0) = 1,$$

where $X(t)$ is the firm market value at the time t, constant μ is the rate of instantaneous expected returns on the firm per unit time, P is the business securing expenses (BSEs), dollars per unit time, including fixed costs FC, debt payments DP, corporate tax TAX, and dividends DIV; constant C^2 is the instantaneous growth rate of the variance of returns, dW is a Gauss-Wiener process representing a cumulative effect of normal shocks. BSE is a piecewise continuous function of time.

Equations (1.1) for random variable $x = \ln(RX/P_0)$ by Ito's Lemma transform to

$$dx = R(1 - \pi(t)e^{-x})dt + CdW, \tag{2a.1}$$
$$x(0) = x_0 = \ln(RX_0/P_0), R = \mu - C^2/2. \tag{2b.1}$$

Writing a Fokker-Plank equation for Eq. (2.1), one comes to an equation for the probability distribution $V(x, t)$, or x-distribution; V_y is a partial derivative over the variably y:

$$V_t + R(1 - \pi(t)e^{-x})V_x - 0.5C^2V_{xx} + R\pi(t)e^{-x}V = 0. \tag{3.1}$$

The initial condition is

$$V(x, 0) = V_0(x; H_0, \sigma_0^2), \tag{4.1}$$
$$H_0 = \langle x(0) \rangle,\ \sigma_0^2 = \langle (x - H_0)^2 \rangle,$$

where $V_0(x; H_0, \sigma_0^2)$ is a normal distribution. There is also a boundary condition implying that the firm will default when its value falls to X_D ($0 < X_D < X_0$)

$$V(DL, t) = 0,\ \ DL = \ln(RX_D/P_0). \tag{5a.1}$$

If X_D is an outstanding debt as it is in (Black & Cox, 1976), Eq. (5.1) makes an *exogenous* constraint. If the firm is free of debt, there is another constraint. The line $x = 0$ separates the business profitable in average from the business suffering losses in average. In this case, it is reasonable to introduce a soft *endogenous* boundary

$$V(0, t) = 0, \tag{5b.1}$$

and watch the probability of crossing the line $DL = 0$. Under the line, $x < 0$, the firm loses its assets, and its activities are possible only if selling the firm's equity. The nature of this boundary is close to the default line introduced in (Kim et al., 1993), where the firm defaults at this line if it runs out of cash. The boundary conditions (5a.1) and (5b.1) can be joined as

$$V(DL, t) = 0, DL = \max[0, \ln(RX_D/P_0)] \ . \tag{5.1}$$

A solution of the boundary problem (3.1), (4.1), (5.1) is the firm log-value distribution; it is denoted as $\hat{V}(x, t), x \geq DL$. If one knows an open-space solution $V(x, t), -\infty < x < \infty$, then a solution of the boundary problem can be written as

$$\hat{V}(x, t) = V(x, t) - V(2DL - x, t). \tag{6.1}$$

The probability distribution $\hat{V}(x, t)$ is the conditional distribution referring to the firms still active at time t: $x(\tau) > DL$ for all times $0 \leq \tau \leq t$. The intensity of default probability *DPINT* is

$$DPINT(t, DL) = 2\int_{-\infty}^{DL} V(x, t)dx \ . \tag{7.1}$$

The first three conditional moments and the mean assets are calculated along with the distribution $\hat{V}(x, t)$

$$\hat{H}(t) = \int_{DL}^{\infty} x\hat{V}(x, t)dx,\ \hat{V}ar(t) = \int_{DL}^{\infty}(x - \hat{H})^2\hat{V}(x, t)dx \ , \tag{8.1}$$

$$\hat{S}(t) = \int_{DL}^{\infty}(x - \hat{H})^3\hat{V}(x, t)dx \ ,\ \widehat{MX}(t) = \int_{DL}^{\infty} e^x\hat{V}(x, t)dx$$

$\hat{S}(t)$ shows the development of the distribution asymmetry, and $\widehat{MX}(t)$ characterizes the mean value of the firm. One of the central objectives of the credit risk analysis is estimating the default probability over a chosen time interval (e. g. over the debt maturity)

$$DPR(t_0, T) = \int_{t_0}^{t_0+T} DPINT(t)dt, \tag{9.1}$$

here t_0 is the time when the credit is issued, $t_0 + T$ is the moment of credit maturity, and $DPR(t_0, T)$ is the default probability over the credit maturity period T starting at t_0. Here we set $t_0 = 0$.

Effects of Debt on the Firm Development

We consider the influence of debt on the firm development in the following settings. Suppose, there is a business promising the expected annual rate of return of μ percent on the firm's assets, to enter which a firm must have assets of no less than 1000 dollar units. Suppose also that a firm has the necessary assets and joins this business immediately. Because this firm enters the business free of debt, we call it the unlevered firm. The unlevered firm pays its BSEs in the form of fixed costs, $P_U = FC = P_0\pi_U(t)$. We do not include the corporate taxes and dividend payments into consideration at this stage, and we also suppose the continuous mode of BSE payments.

Another firm identical to the first one in all respects but the size of assets has at its disposal the assets of 1000-A units. To enter the business, the firm borrows at the market A units of capital for T_m years at the annual interest rate of *r* percent (hereafter we refer to this firm as the levered one). Thus, the total debt of this firm is

$$X_D = Aexp(rT_m) \ ,$$

and the corresponding debt payments make

$$DP = (A/T_m)exp(rT_m)\pi_D(t) \equiv \ P_D\pi_D(t), \\ \int_0^{T_m} \pi_D(t)dt = T_m, \ P_D = (A/T_m)exp(rT_m) \tag{1.2}$$

where $\pi_D(t)$ is the debt payment schedule. The total BSEs for the levered firm now are

$$P_L = P_0\pi_U(t) + P_D\pi_D(t) = P_0(\pi_U + \beta\pi_D), \tag{2.2}$$

$$\beta = \frac{P_D}{P_0} = \frac{A}{P_0 T_m} exp(rT_m) \ . \tag{3.2}$$

Suppose that fixed costs for both levered and unlevered firms remain the same all the time, $\pi_U(t) \equiv 1$, and the levered firm pays out its debt in equal installments each year, $\pi_D(t) \equiv 1$. BSEs of the unlevered firm make $P_U = P_0$, and the payments of the levered firm are $P_L = P_0(1+\beta)$. The probability distribution equation for the unlevered firm is ($x = \ln(R_0 X/P_0)$)

$$V_t + R_0(1-e^{-x})V_x - 0.5C^2V_{xx} + R_0e^{-x}V = 0, \tag{4u.2}$$

and for the levered firm this equation is

$$U_t + R_0[1-(1+\beta)e^{-x}]U_x - 0.5C^2U_{xx} + R_0(1+\beta)e^{-x}U = 0, \tag{4l.2}$$

$$R_0 = \mu - C^2/2 \ .$$

The initial condition for both equations is

$$V(x,0) = N(x;H_0,\sigma_0^2), \ U(x,0) = N(x;H_0,\sigma_0^2), \tag{5.2}$$

$$H_0 = \langle x_0 \rangle = \langle \ln(R_0X_0/P_0) \rangle, \ \sigma_0^2 = \langle (x-H_0)^2 \rangle,$$

where $N(x;H_0,\sigma_0^2)$ is a normal function.

The boundary conditions are

$$V(DL,t) = 0, \ DL = 0 \ ; \tag{6u.2}$$

$$U(DL(t),t) = 0, \ DL(t) = \max[0, \ln(R_0X_D(t)/P_0)] \ , \tag{6l.2}$$

$$X_D(t) = X_D(0)(1-t/T_m), \ 0 \le t \ \le T_m, \tag{7.2}$$

$$X_D(t) = 0 \ , \ t > T_m \ .$$

Equations (4u.2), (5.2), and (6u.2) describes the development of the unlevered firm, while the equations (4l.2), (5.2), and (6l.2) describes the development of the levered firm. For the analysis of the debt effects we take a good steady firm with H_0 = 2.0, $\sigma^2 = 0.02$; other model parameters: R_0 = 0.10, C^2 = 0.01.

We start our study with two firms taking a small-size loan of *A* = 50 units with the debt maturities T_m = 3 (the initial debt leverage is 0.0576) and T_m = 5 years (the initial debt leverage is 0.0633), and compare the results of these levered firms to each other, and to the results of the unlevered firm. It is easy to see that for maturity times T_m = 3, 5, the default line *DL* = 0. We present the results in Figures 1-11. The slope of the log-value mean *H_OS*(*t*) shows the effective rate of return on the firm's assets after all payments (the suffix *_OS* says that the moment refers to the open-space problem, while the suffix *_BP* shows that the moment refers to the boundary problem). For example, the effective rate of return for the unlevered firm in Fig. 1 makes 0.087. As one can see in Fig. 1, the log-value mean of the levered firm has a lesser slope during the first three years of the firm development with higher payments in this period. Then, with the payments returning to the level of the unlevered firm, the slope of *H_OS*(*t*) of the levered firm rises to the slope of *H_OS*(*t*) of the unlevered firm. The mean *H_OS*(*t*) of the levered firm remains lesser than the *H_OS*(*t*) of the unlevered firm for all the time.

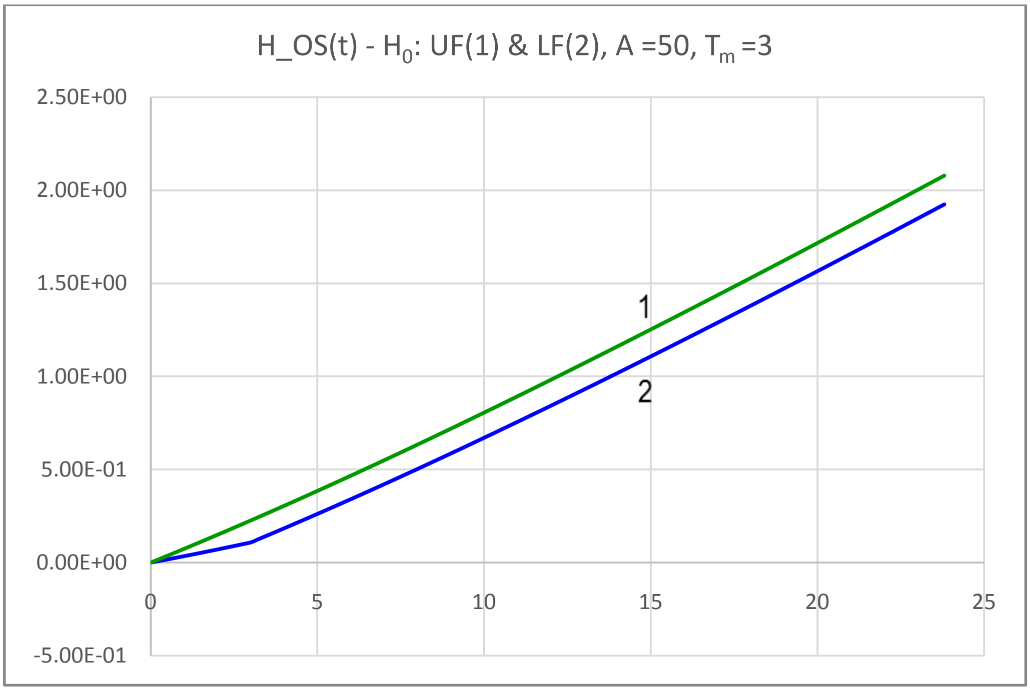

Figure 1. The log-value means $H\_OS(t)\text{-}H_0$ for the unlevered firm (line 1) and for the levered firm (line 2), A = 50, T_m = 3.

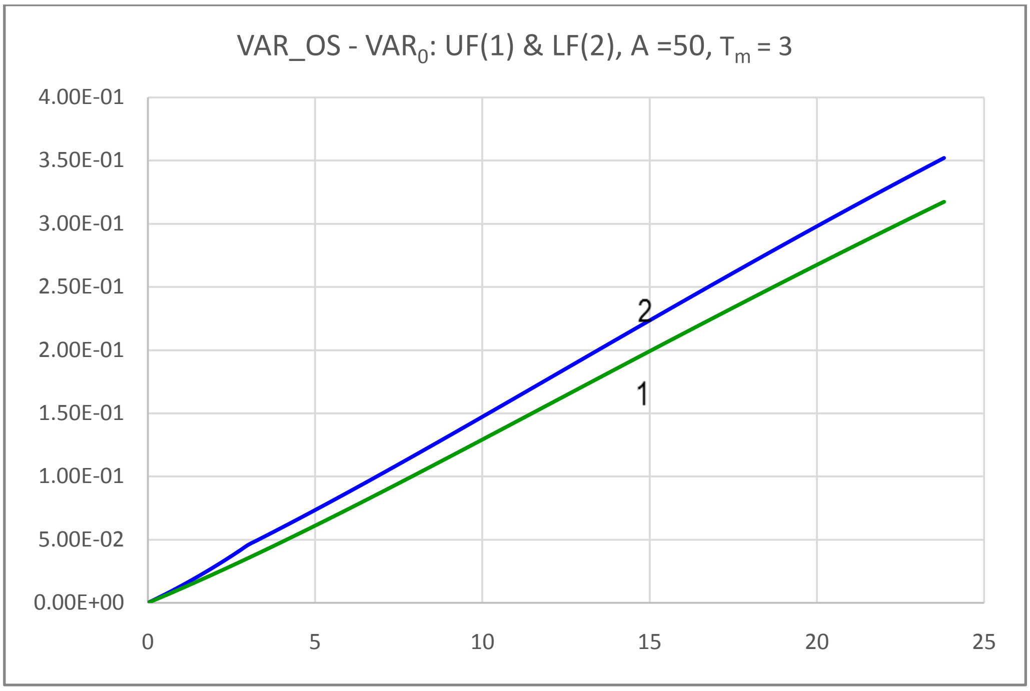

Figure 2. The variances $VAR\_OS(t)\text{-}VAR_0$ for the unlevered firm (line 1) and for the levered firm (line 2), A = 50, T_m = 3.

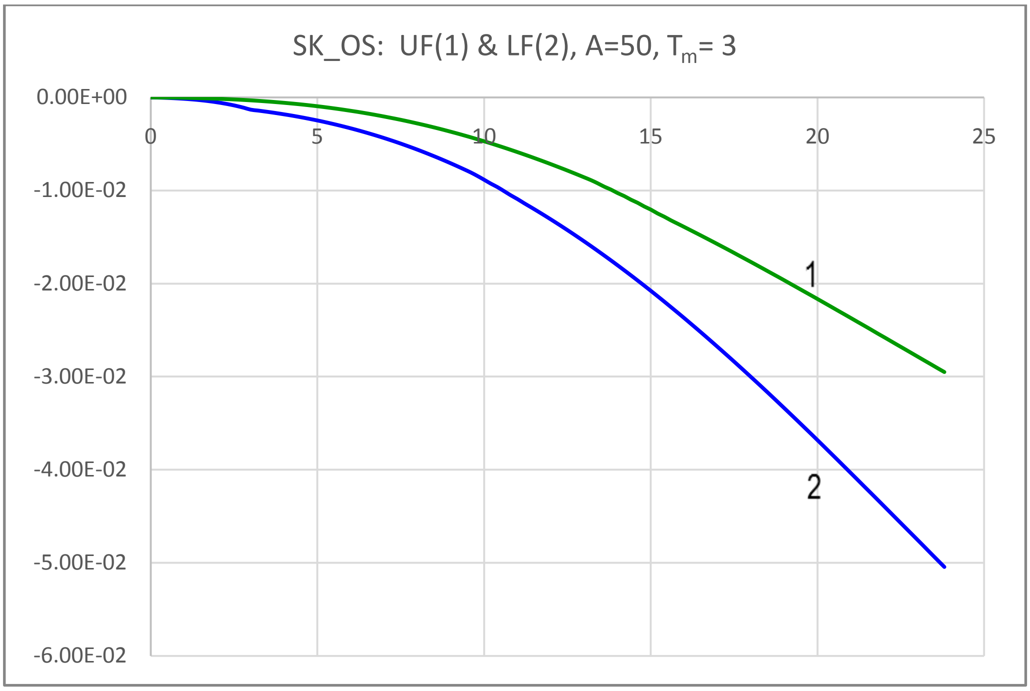

Figure 3. Skewness *SK_OS*(*t*) for the unlevered firm (line 1) and for the levered firm (line 2), A = 50, T_m = 3.

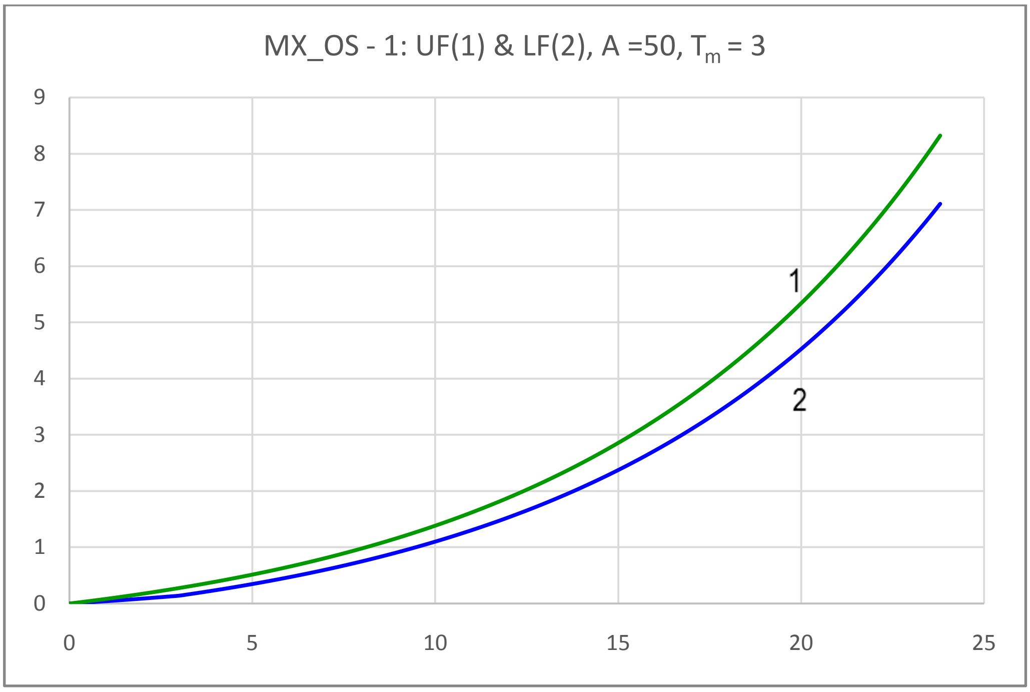

Figure 4. The relative increase in the mean firm's assets *MX_OS*(*t*)-1 for the unlevered firm (line 1) and for the levered firm (line 2), A = 50, T_m = 3.

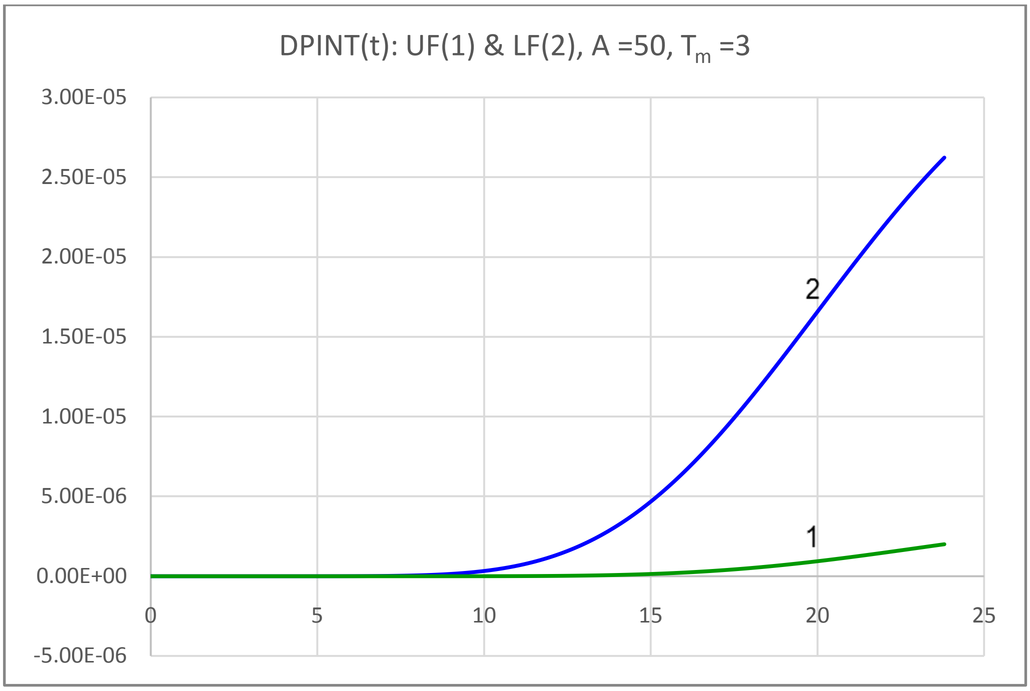

Figure 5. The intensities of default probability *DPINT*(*t*) for the unlevered firm (line 1) and for the levered firm (line 2), A = 50, T_m = 3.

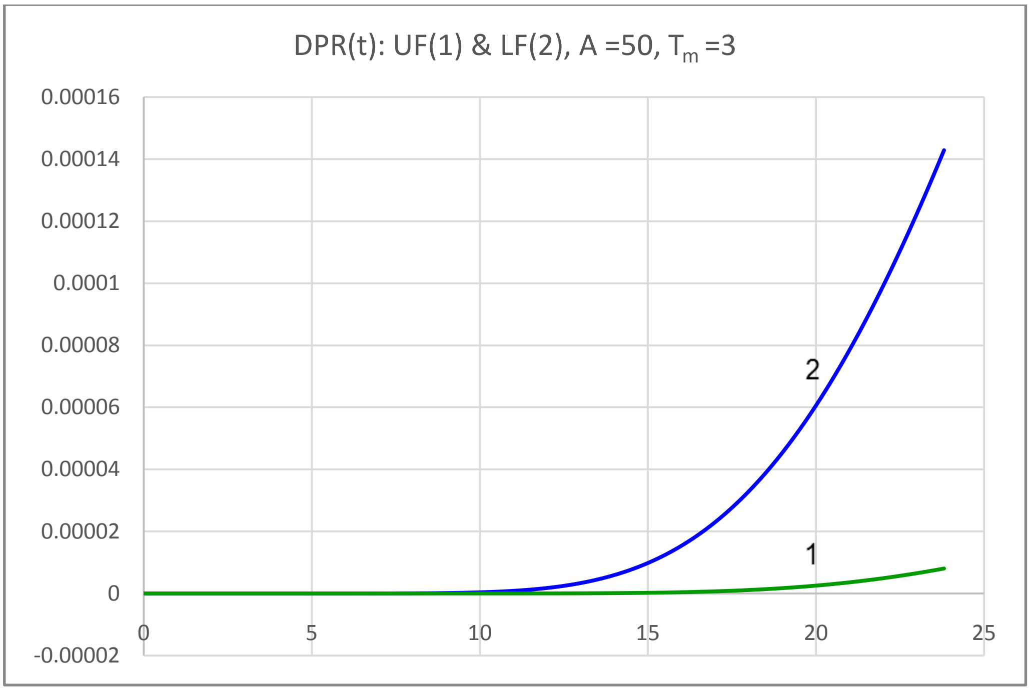

Figure 6. The default probabilities *DPR*(*t*) for the unlevered firm (line 1) and for the levered firm (line 2), A = 50, T_m = 3.

The variance *VAR_OS*(*t*) of the levered firm grows faster in the first three years of debt payment than the variance of the unlevered firm. After discharging the debt, the growth rate of the distribution variance declines, but *VAR_OS*(t) of the levered firm remains greater than *VAR_OS*(*t*) of the unlevered firm. The development of the distribution skewness demonstrates the same behavior: *SK_OS*(*t*) of the levered firm grows faster during the debt payment period than *SK_OS*(*t*) of the unlevered firm. Then the growth rate declines, but *SK_OS*(*t*) of the levered firm remains greater by absolute value than *SK_OS*(*t*) of the unlevered firm.

Due to the choice of the firms (H_0 = 2.0, $\sigma_0^2 = 0.02$, which means good, steady firms), the default probability and its intensity for the levered firm is about 10-15 times greater than the corresponding variables of the unlevered firm, although still low. The general conclusion from the graphs is that the debt does affect the firm development, and its effect is negative. The distribution variance, skewness, default probability and its intensity grow faster than the similar variables of the unlevered firm. On the other side, the log-value mean and mean assets grow slower than the same variables of the unlevered firm.

A series of pictures in Figures 7-12 demonstrate effects of debt maturity for the same debt on firm development. The longer the debt maturity, the heavier the pressure on the firm survival (see Fig. 8, 9, 11, 12). One can explain this phenomenon by a durable exposure of the firm to the elevated BSEs. The log-value mean and the mean assets make an exception because when both firms complete paying out their debts, they develop along the same line (Fig. 7, 10). It comes of the small debt value (or the low debt leverage). As we will see later, the graphs of these variables for bigger loans split for different maturities. The general conclusion about the effect of a small-size debt on the firm development is: (a) the debt makes it possible for the firm to enter the business, which otherwise would be unachievable for the firm, (b) the small-size debt affects slightly negatively the two most essential firm variables (the mean assets and default probability) compared to the same variables of the unlevered firm, (c) the shorter the debt maturity, the better for the firm survival, (d) one should recognize the total effect of small-size debt as beneficial for the firm.

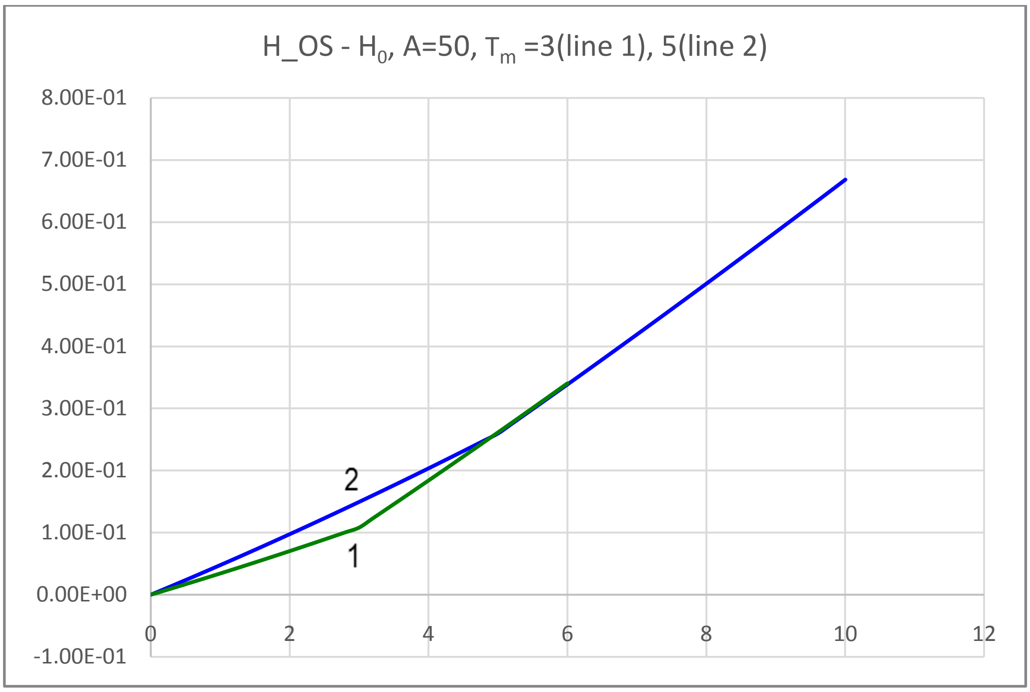

Figure 7. The log-value mean $H\_OS(t)$-H_0 for T_m = 3 (line 1) and 5 years (line 2); *A* = 50.

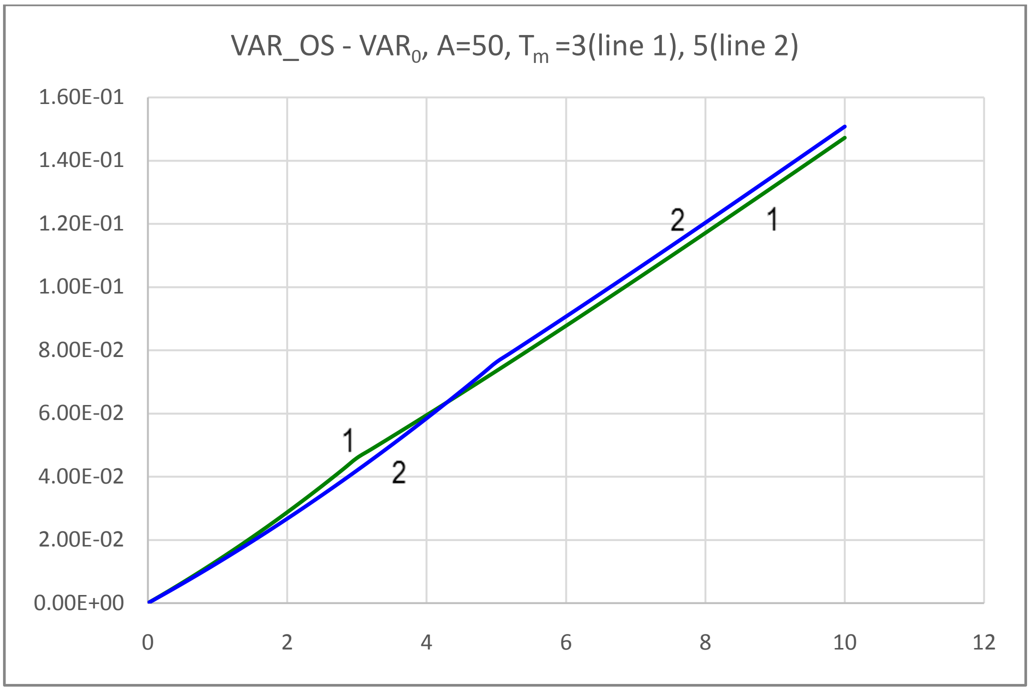

Figure 8. The variance $VAR\_OS(t)$-VAR_0 for maturity T_m = 3 (line 1) and 5 years (line 2); A = 50.

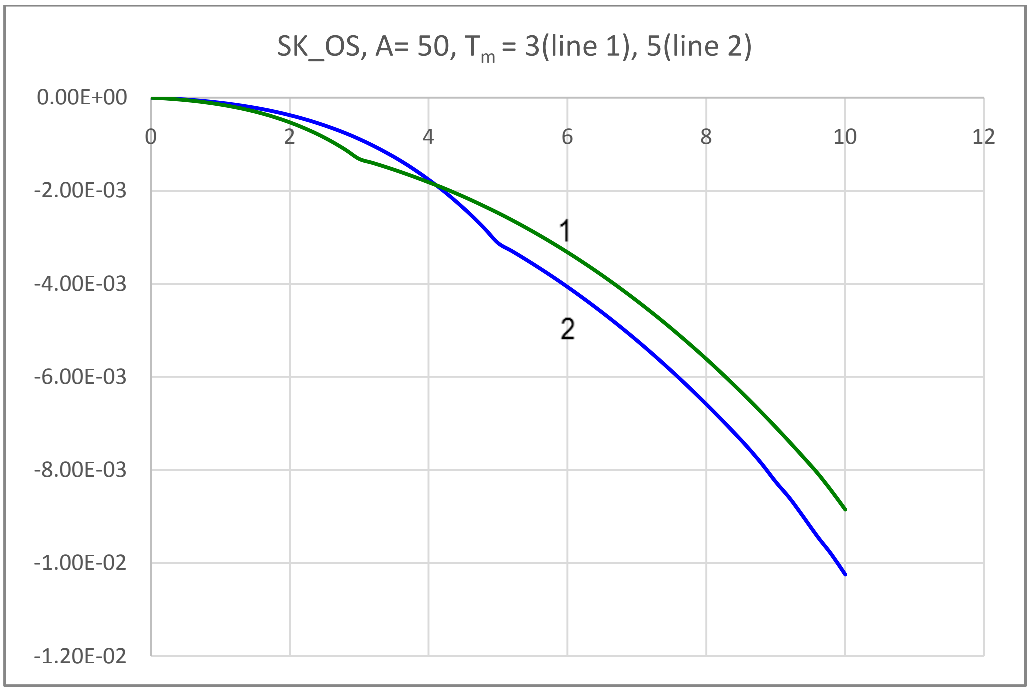

Figure 9. The distribution skewness $SK\_OS(t)$ for maturity T_m = 3 (line 1) and 5 years (line 2); A = 50.

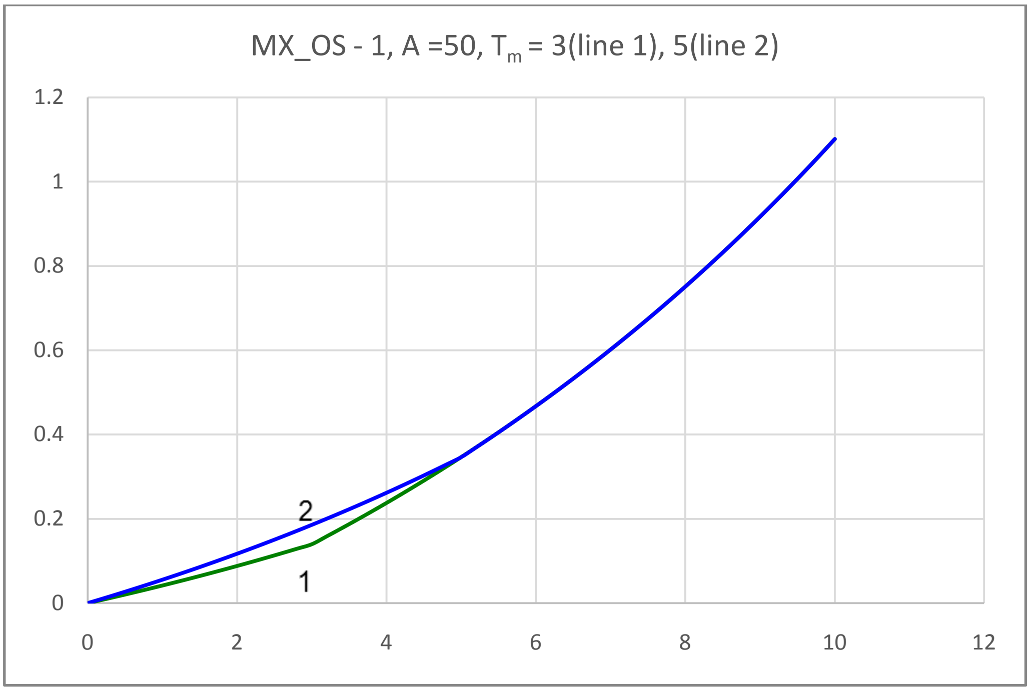

Figure 10. The relative mean *MX_OS*(*t*)-1 for T_m = 3 (line 1) and 5 years (line 2); *A* = 50.

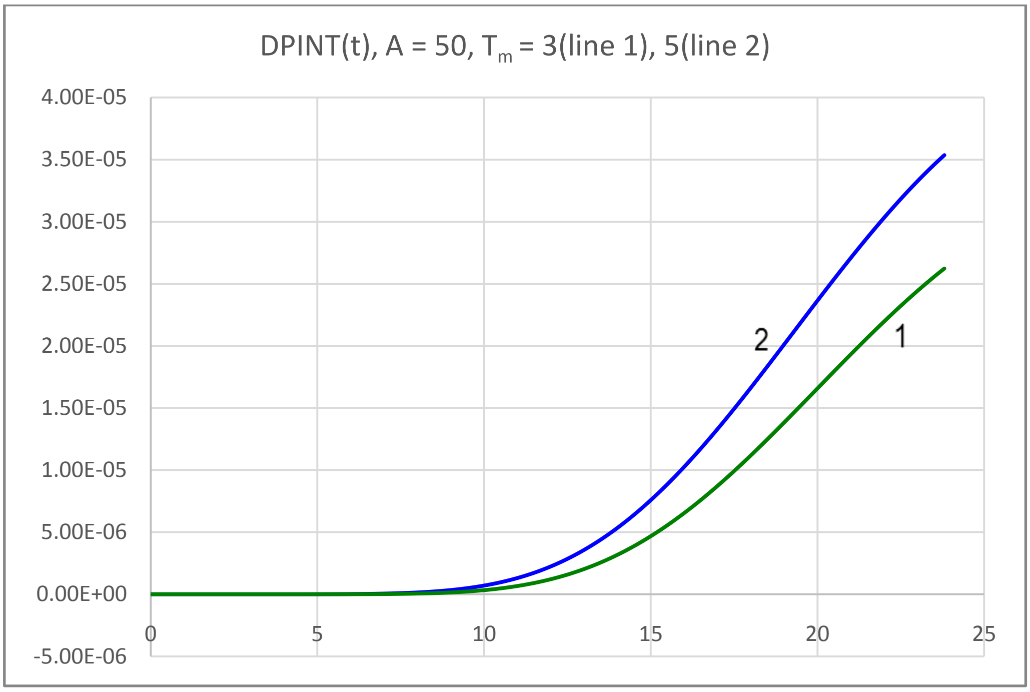

Figure 11. The intensity of default probability DPINT(t) for T_m = 3 (line 1) and 5 years (line 2); A = 50.

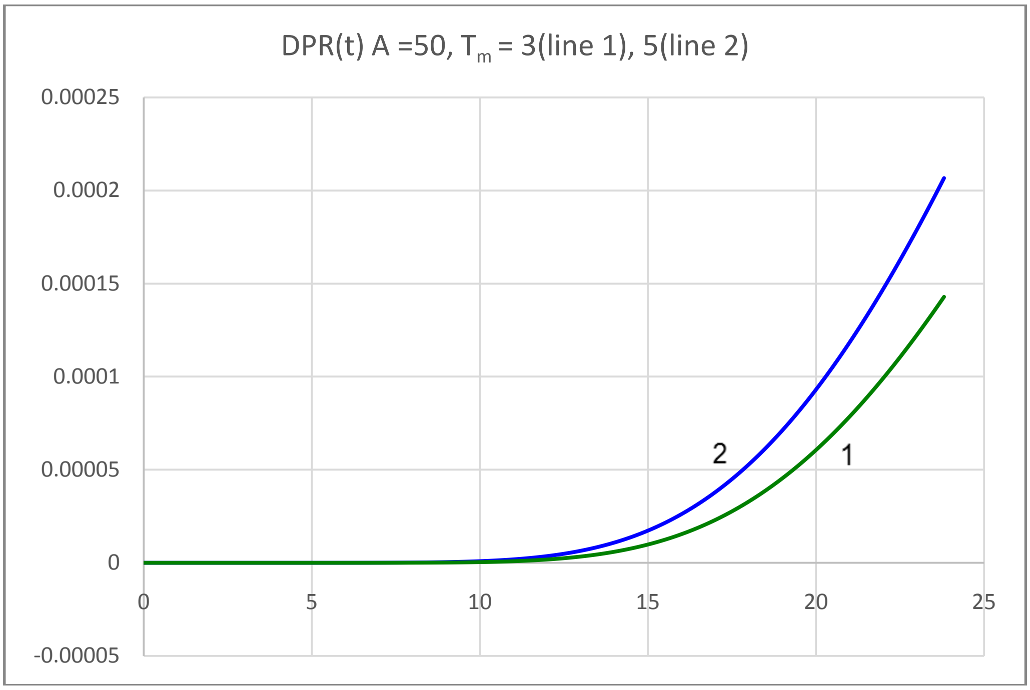

Figure 12. The default probability *DPR*(*t*) for T_m = 3 (line 1) and 5 years (line 2); *A* = 50.

Now we turn to consideration of the effects of medium-size loans *A* = 150, T_m = 10, 15 years (the initial debt leverage makes 0.2254 and 0.2720). Other problem parameters remain the same: R_0 = 0.10, C^2 = 0.01, $\sigma^2 = 0.02$. The first distinction between the cases of small-size debt and medium-size debt is a non-zero default line, the graph of which one can see in Fig. 13. For the case T_m = 15, the time, when the default line turns zero, *tDL0* = 8.671, and β = 1.580 at the debt maturity interval. The last means that the payments of the levered firm are 2.580 times greater than the payments of the unlevered firm at the interval of the debt maturity, and both firms pay the same BSEs outside this interval. These elevated payments explain the low slope of the log-value mean in Fig. 14, the fast growth of the variance in Fig. 15, and the rise of the absolute value of skewness in Fig. 16 at the interval [0, 15]. When the payments decline to the levels specific for the unlevered firm, then the slope of the function *H_OS*(*t*) returns to its unlevered value (Fig. 14), and the growth rates of the variance and skewness diminish.

Figures 18-23 show how debt maturity influences the firm. The general conclusion on that influence is this: the shorter the debt maturity, the better for survival, supporting the similar inference on small-size debt. Considering the functions *H_OS*(*t*) and mean assets *MS_OS*(*t*), one can see a slight splitting of the graphs for T_m = 10 and T_m = 15.

Another difference between the small-size loan and medium-size debt is the slower development of the mean assets (compare Fig. 10 and 21) and the higher risk of the default generated by medium-size debt (see Fig. 12 and 23). Again, we have the evidence that the debt affects the firm, that influence is negative, and the greater the debt, the lesser the commercial achievements of the project (leaving aside that without this debt, the firm will not be able to enter the business). The medium-size debt can be recognized as conditionally beneficial for the firm because of the higher default probabilities intrinsic to this debt level.

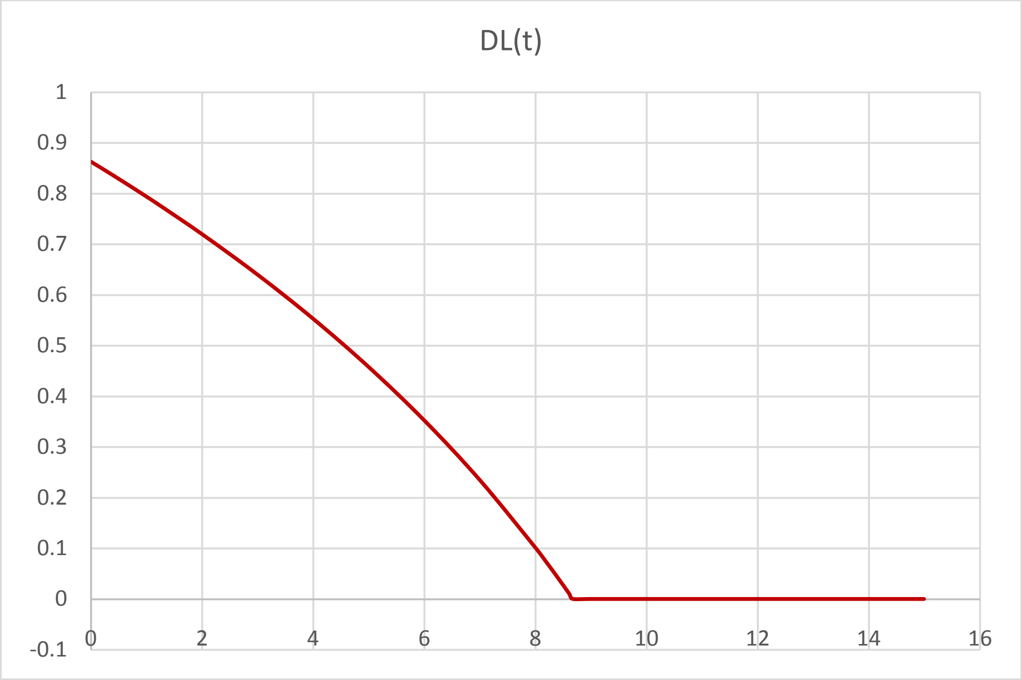

Figure 13. The default line $DL(t)$ of the levered firm with parameters A = 150, T_m = 15, $tDL0$ = 8.671.

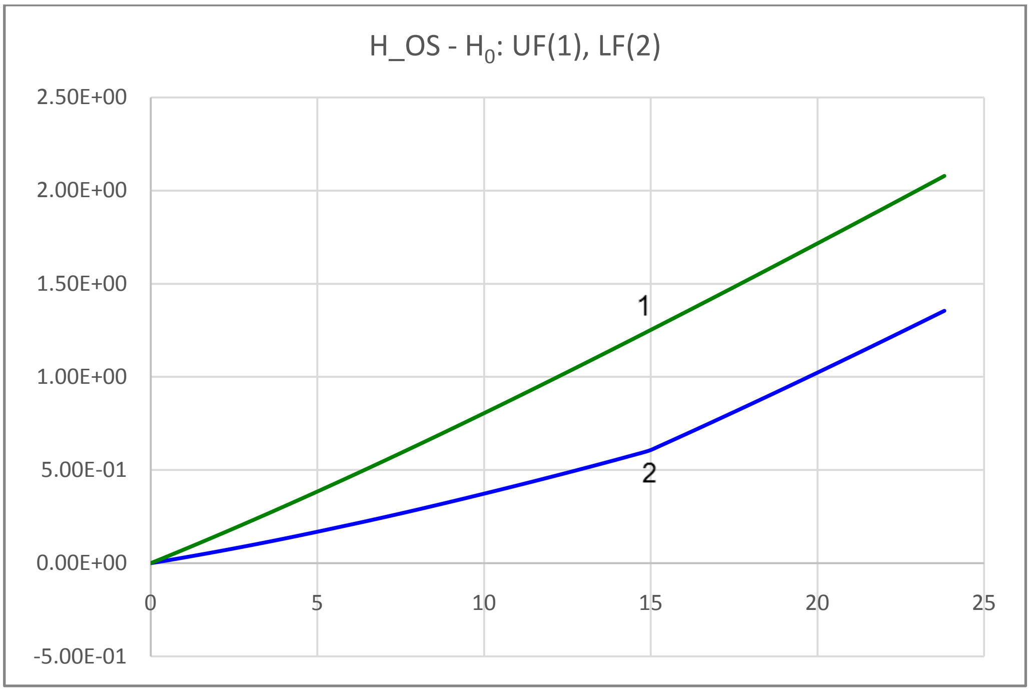

Figure 14. The log-value mean $H\_OS(t)$-H_0 for the unlevered firm (line 1) and for the levered firm (line 1) and for the levered firm (line 2), A = 150, T_m = 15.

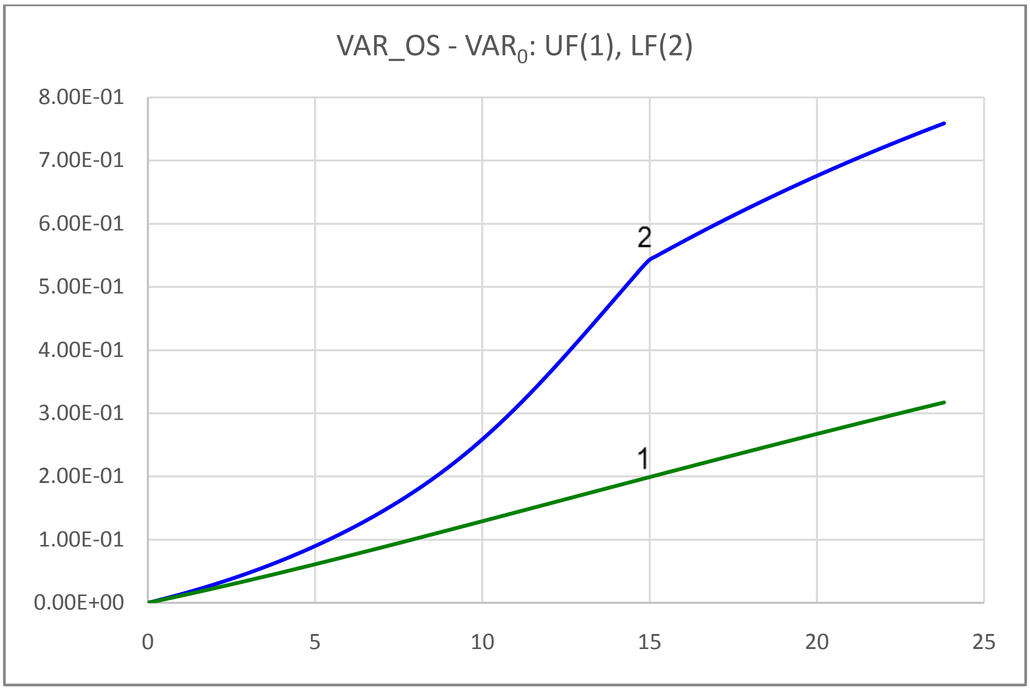

Figure 15. The variance *VAR_OS*(*t*)-VAR_0 for the unlevered firm (line 1) and for the levered firm (line 2), *A* = 150, T_m = 15.

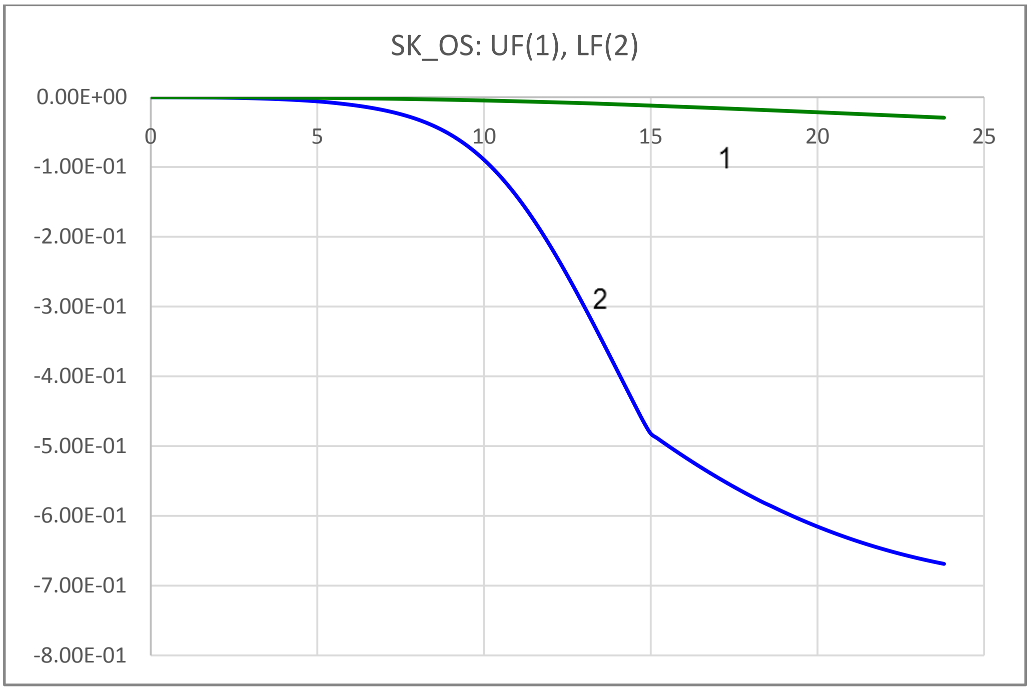

Figure 16. Skewness *SK_OS*(*t*) for the unlevered firm (line 1) and for the levered firm (line 2), *A* = 150, T_m = 15.

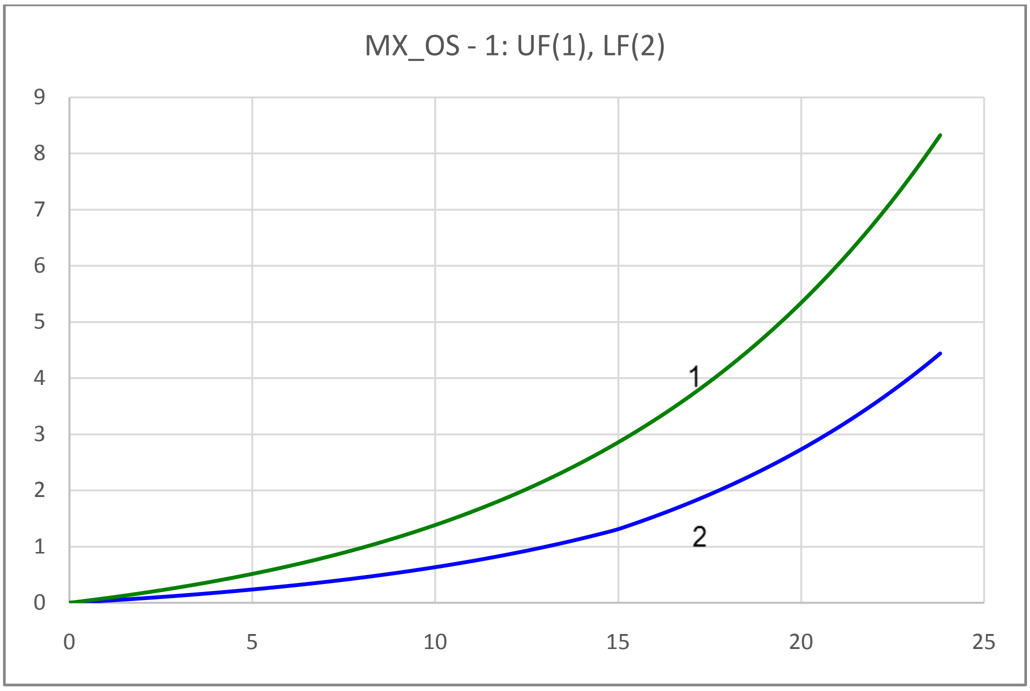

Figure 17. The firm's mean relative assets *MX_OS*(*t*)-1 for the unlevered firm (line 1) and for the levered firm (line 2), A = 150, T_m = 15.

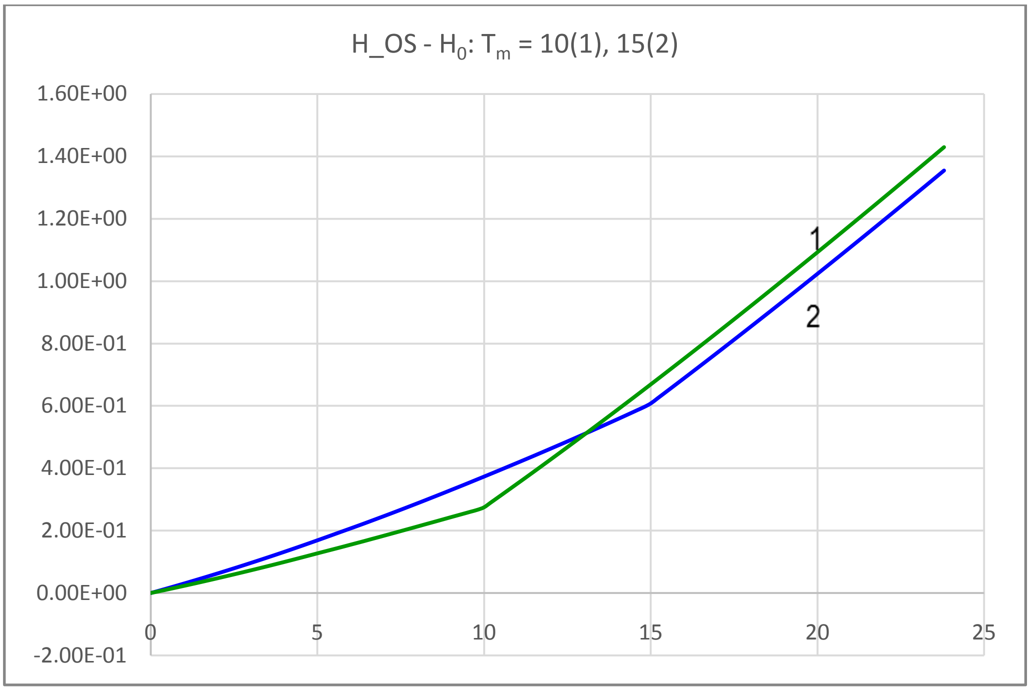

Figure 18. The log-value mean $H\_OS(t)$-H_0 for T_m = 10 (line 1)and 15 years (line 2); A = 150.

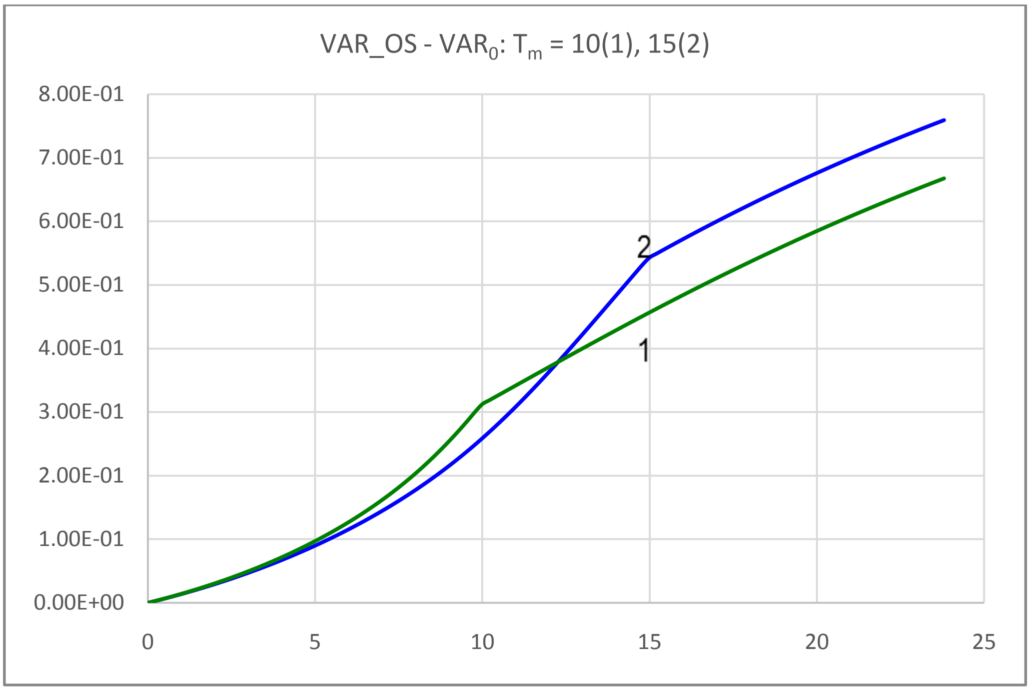

Figure 19. The variance *VAR_OS*(*t*)-VAR_0 for T_m = 10 (line 1) and 15 years (line 2); *A* = 150.

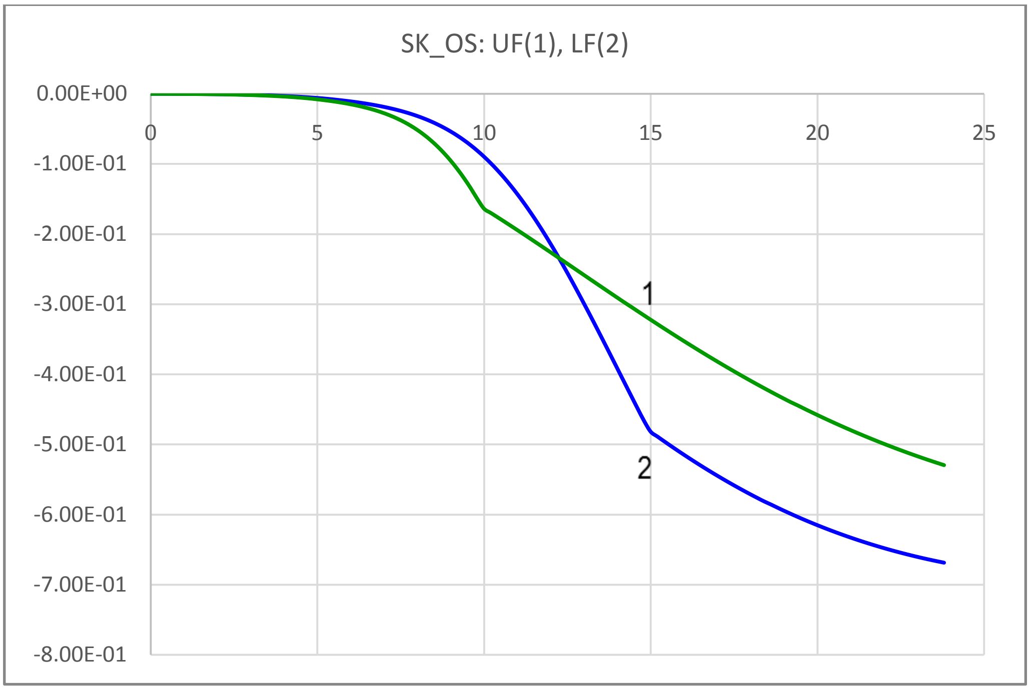

Figure 20. The skewness *SK_OS*(*t*) for T_m = 10 (line 1) and 15 years (line 2); *A* = 150.

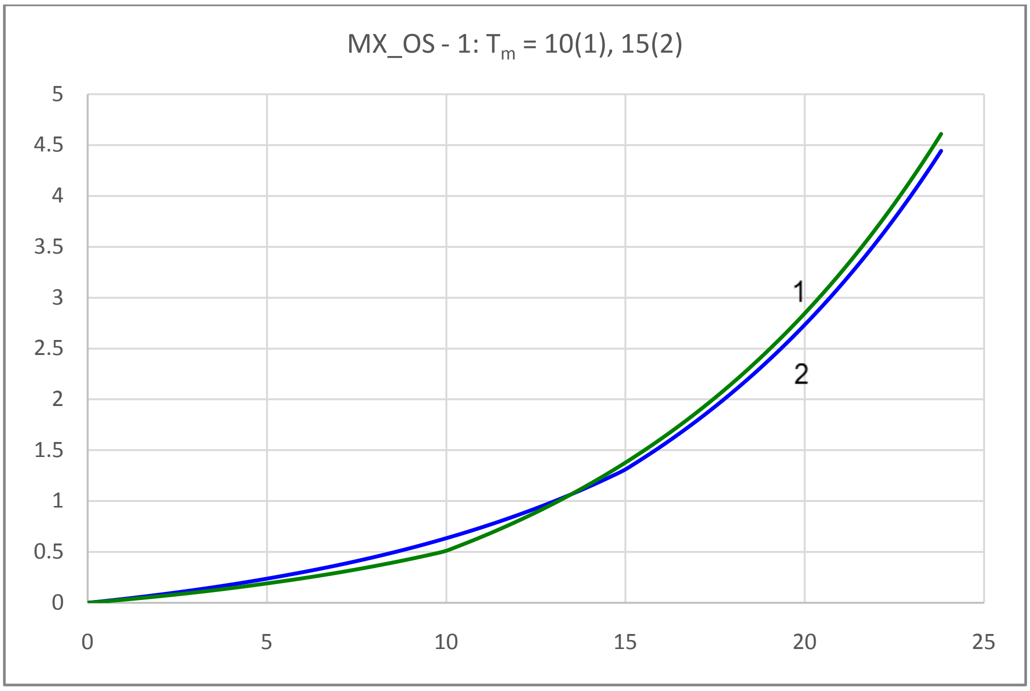

Figure 21. The mean relative assets *MX_OS*(t)-1 as a function of time for T_m = 10 (line 1) and 15 years (line 2); A = 150.

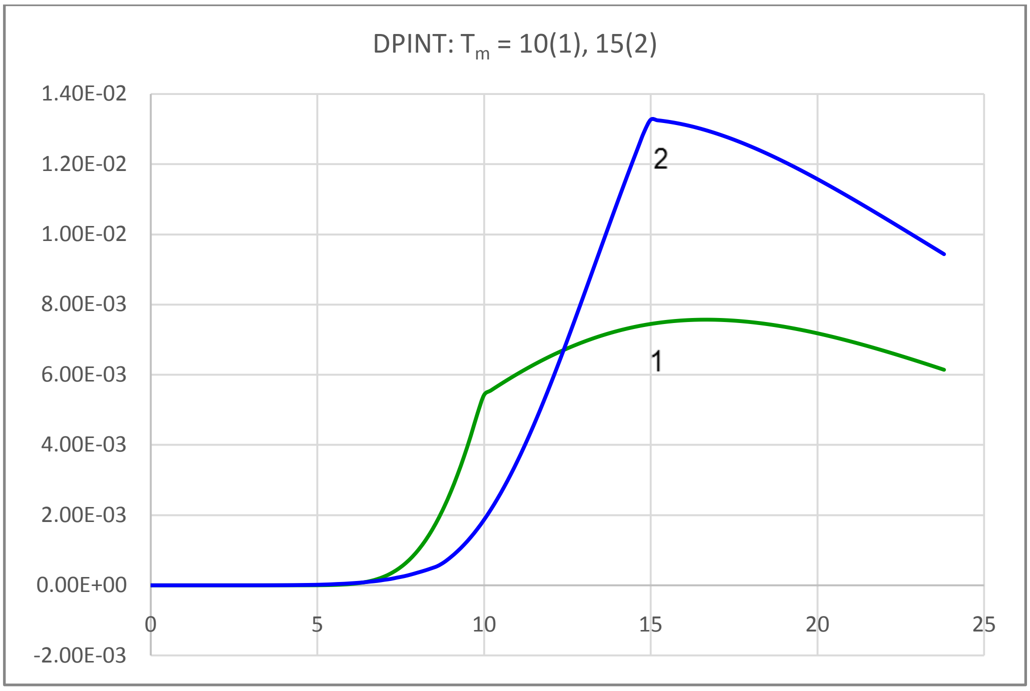

Figure 22. The intensity of default probability *DPINT*(t) for T_m= 10 (line 1) and 15 years (line 2); A = 150.

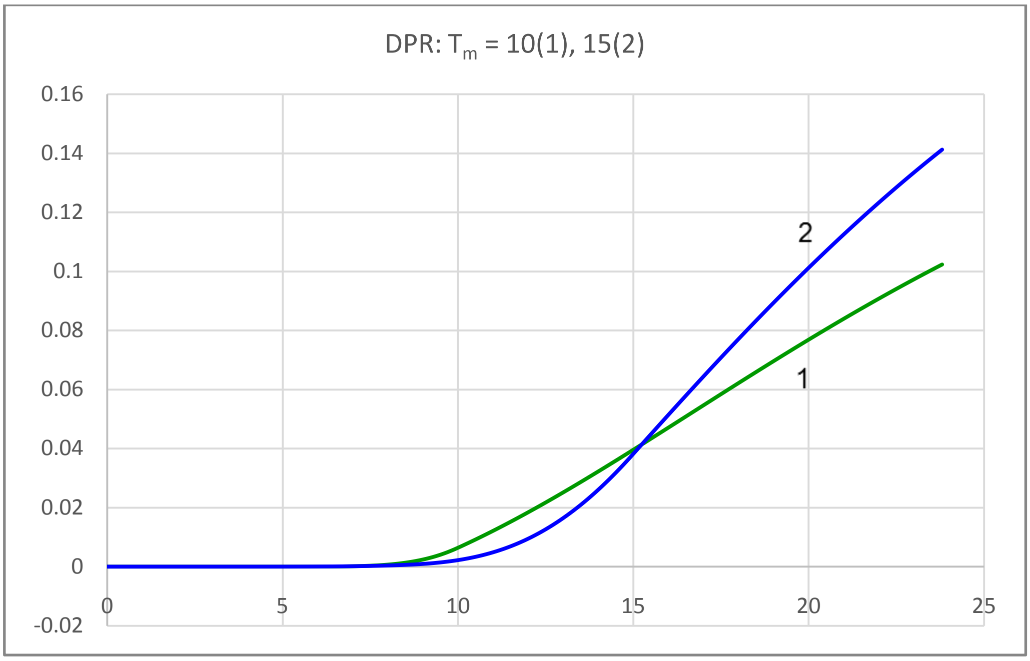

Figure 23. The default probability $DPR(t)$ for T_m = 10 (line 1) and 15 years (line 2); A = 150.

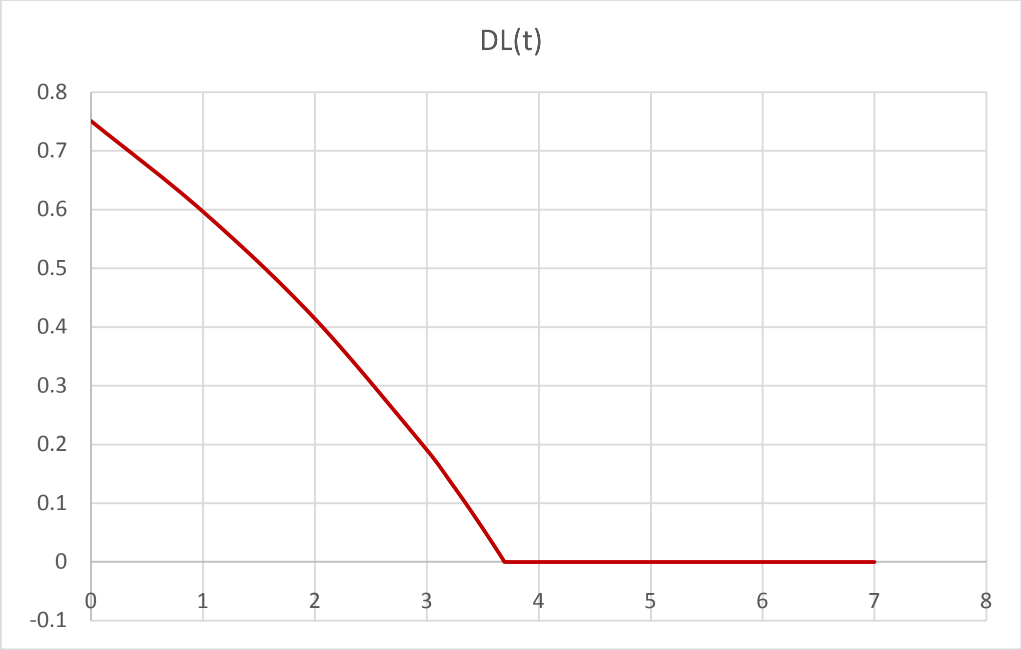

Figure 24. The time-dependence of the default line DL(t) for the levered firm with parameters A = 200, T_m= 7, $tDL0$ = 3.695. Compare this graph with the graph in Fig. 13 describing the levered firm with parameters A = 150, T_m = 15, $tDL0$ = 8.671.

Now let us compare two cases with close initial debt leverages and initial locations of the default line, but drastically distinguishing in debt payments β: A = 150, T_m = 15 (XD_0 = 317.55, β = 1.580, DL_0 = 0.8629, the initial leverage is 0.2720, we refer to this case as Case A for short) and A = 200, T_m = 7 (XD_0= 283.21, β = 3.026, DL_0 = 0.7506, the initial leverage is 0.2619, Case B). It is seen from Fig. 25 that the debt maturity is too short for the firm in Case B: due to the high rate of debt payments, the effective return during the first seven years is negative, and the firm loses its assets. The effect of the integral loss one can see in comparison with the gains of the firm in case A (Fig. 27). After completing the debt payments and returning to the rate of BSE payments specific for the unlevered firm, the effective return assumes the value of 0.083. One can conclude from Figures 24-29 that it is not debt size (or the debt leverage) that makes the principal effect on firm survival, but its BSEs: the greater BSEs, the lesser the survivability, all other parameters equal. It is easy to see that project B (A = 200, T_m = 7) is commercially ineffective because the risk of default and the loss of control over the firm is unacceptably high (40%, Fig. 29).

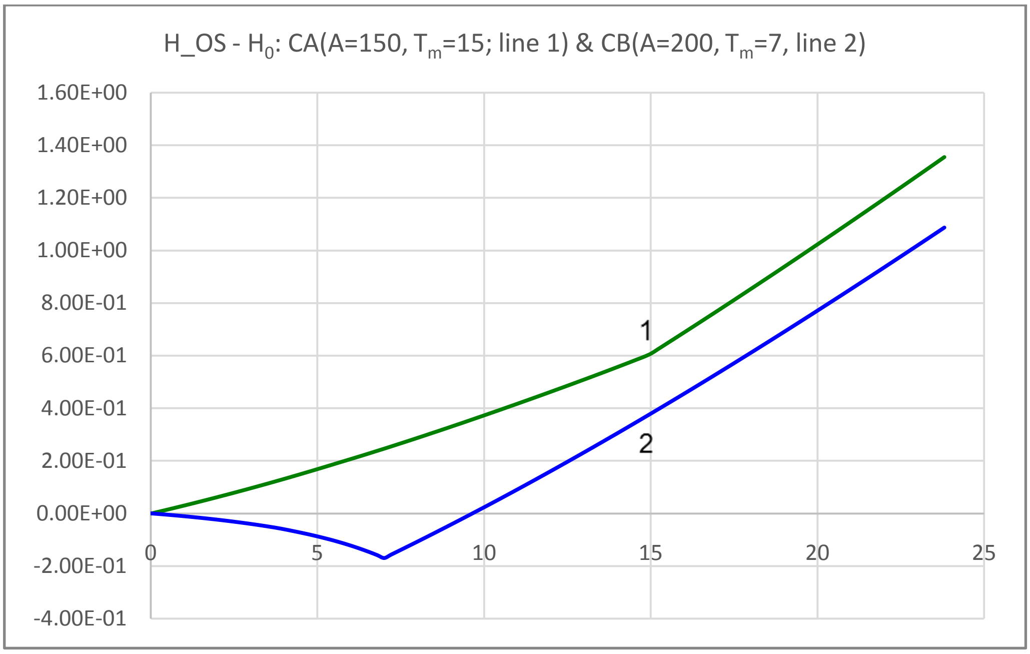

Figure 25. The log-value mean$H_OS\text{-}H_0$ for Case A (line 1) and Case B (line 2).

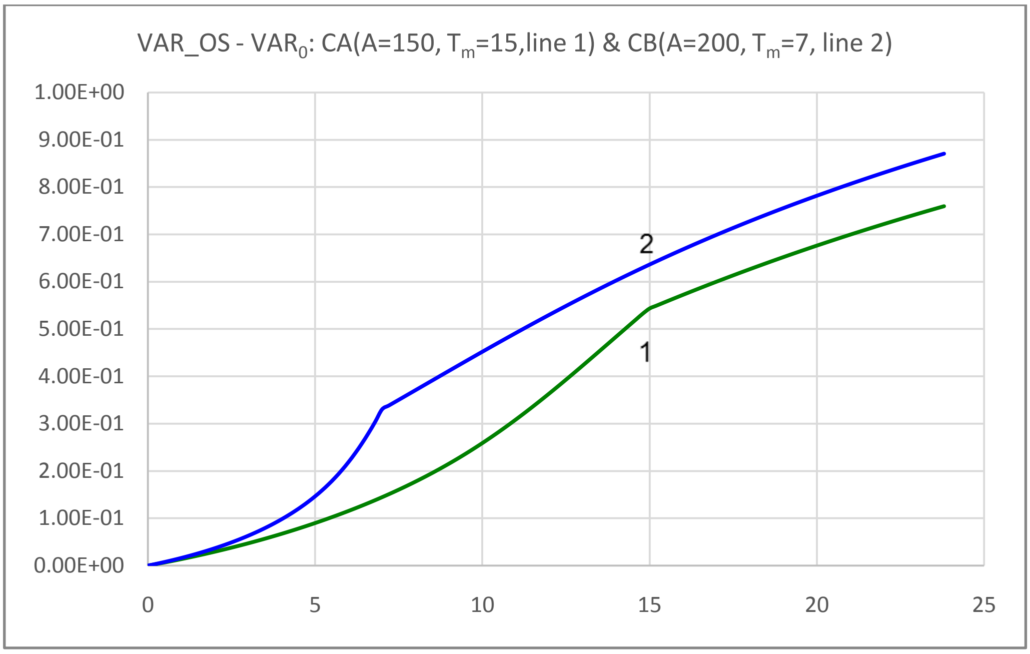

Figure 26. The variance$VAR_OS\text{-}VAR_0$ for Case A (line 1) and Case B (line 2).

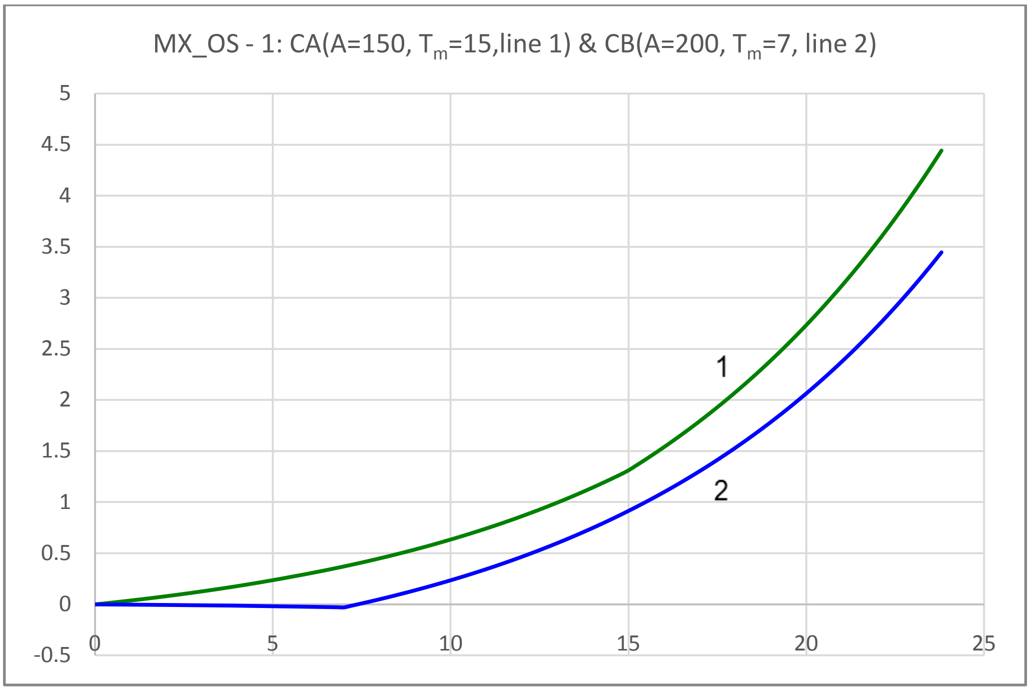

Figure 27. The mean relative assets *MX_OS*(t)-1 for Case A (line 1) and Case B (line 2).

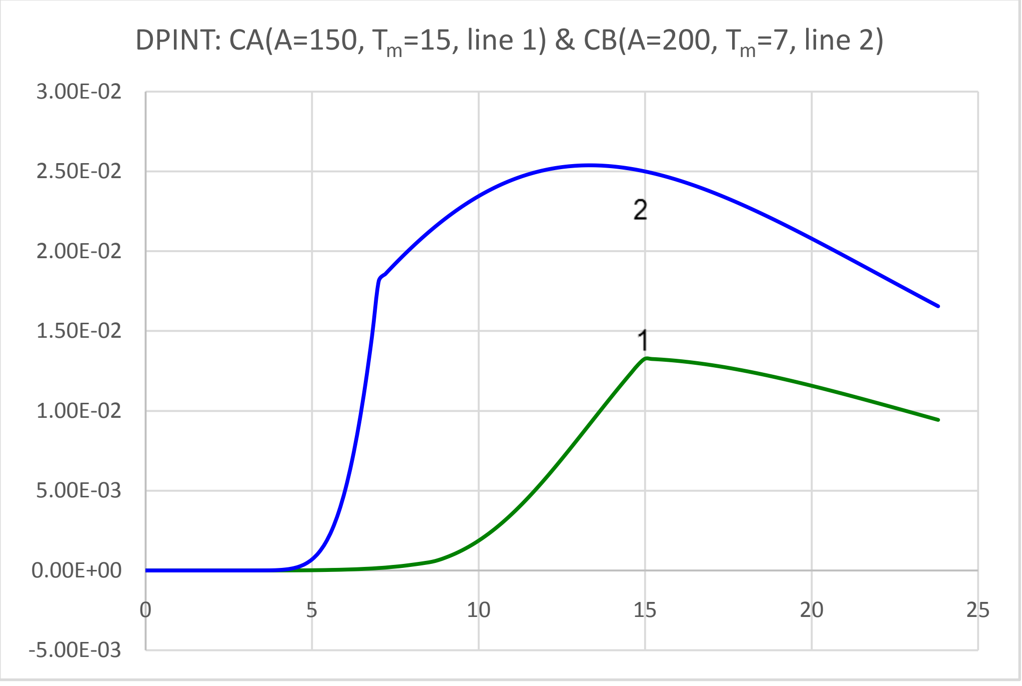

Figure 28. The intensities of default probability *DPINT*(*t*) for Case A (line 1) and Case B (line 2).

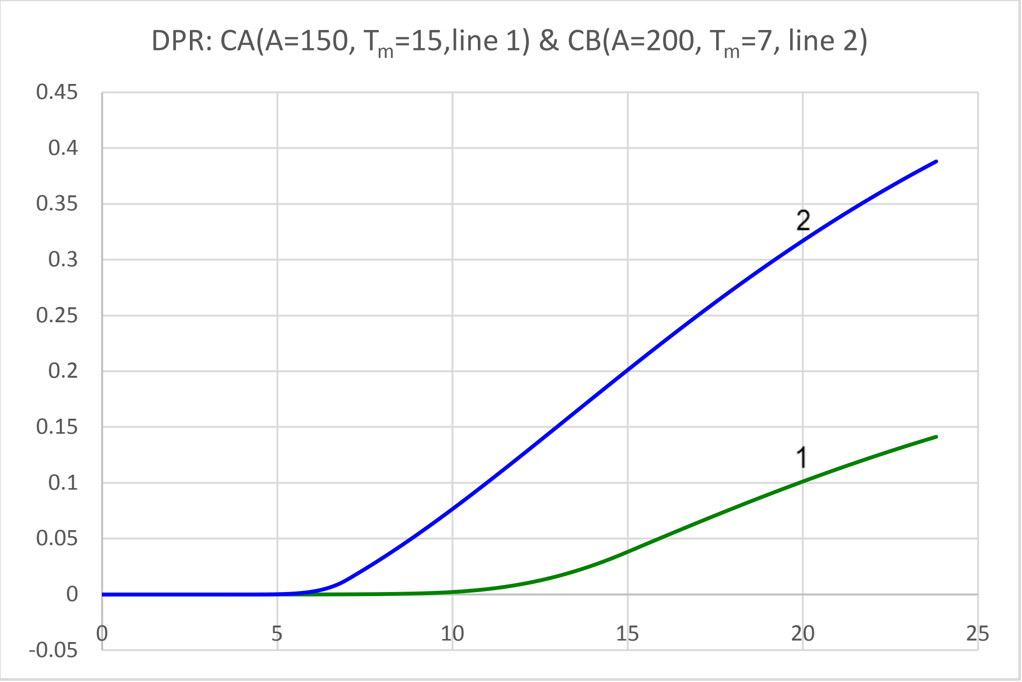

Figure 29. The default probability *DPR*(*t*) for Case A (line 1) and Case B (line 2).

Now we consider effects of big-size loans A = 250, T_m = 10 and 15 years; the initial debt leverage makes 0.3547 and 0.4137, correspondingly. The other problem parameters remain the same: R_0 = 0.10, C^2 = 0.01, $\sigma^2 = 0.02$.

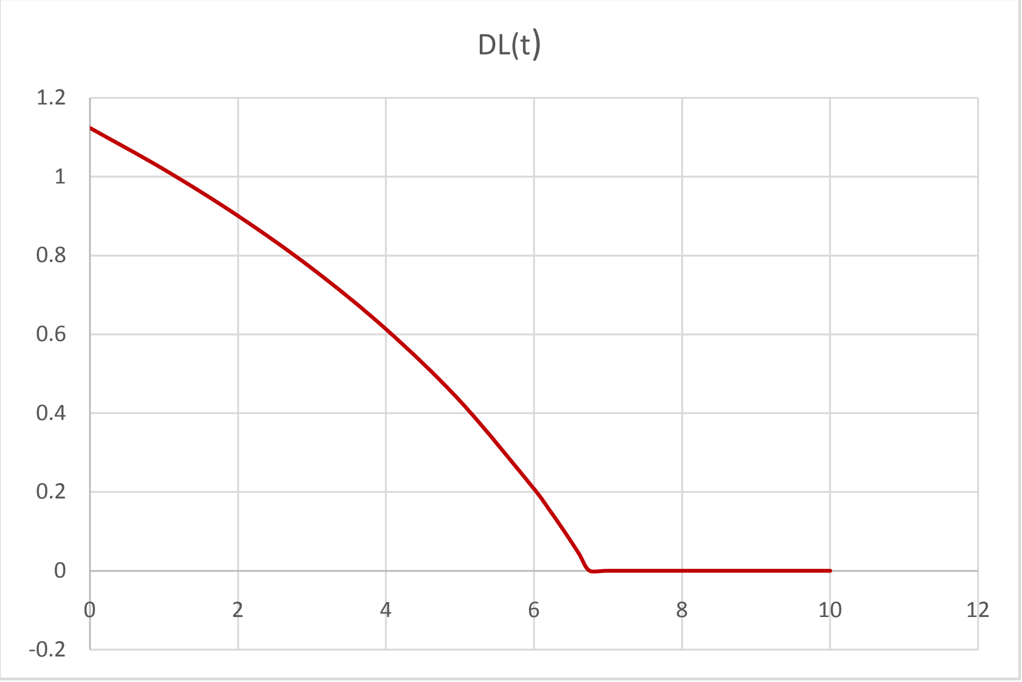

Figure 30. The default line *DL*(*t*)for the levered firm with parameters A = 250, T_m = 10, *tDL0* = 6.749.

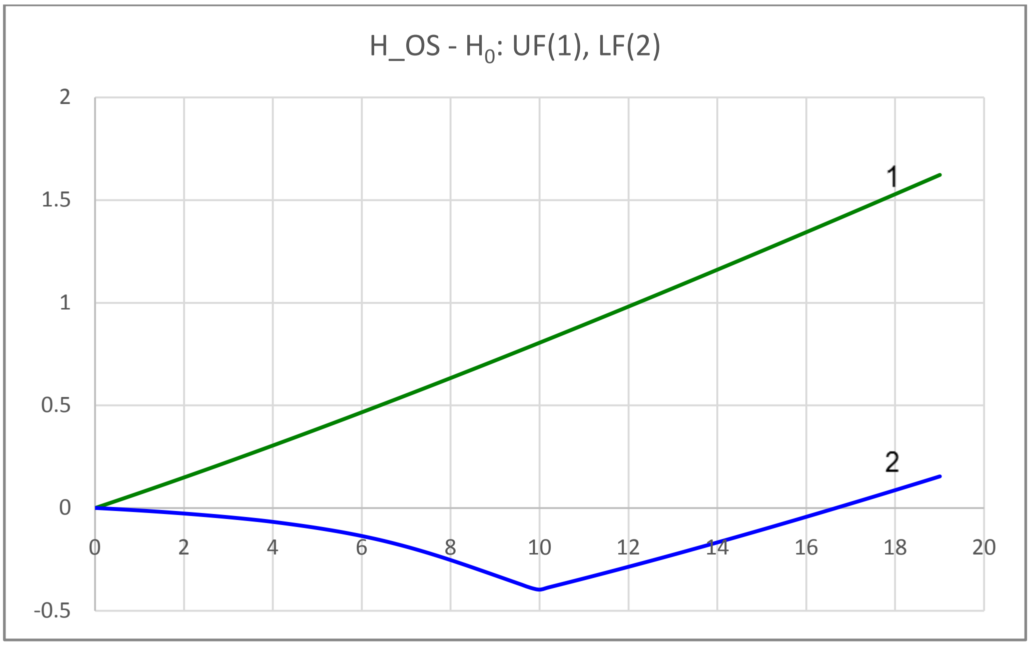

Figure 31. The log-value mean $H\_OS(t)$-H_0 for the unlevered firm (line 1) and for the levered firm (line 2), A = 250, T_m = 10.

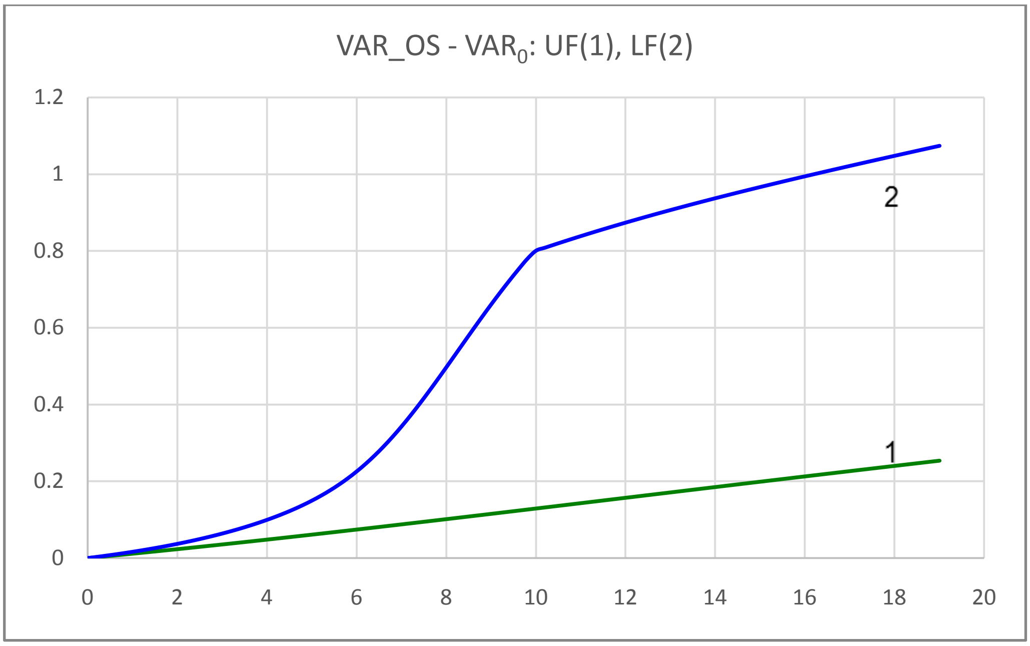

Figure 32. The variance $VAR\_OS(t)$-VAR_0 for the unlevered firm (line 1) and for the levered firm (line 2), A = 250, T_m = 10.

The first conclusion from Fig. 31 is that the debt maturity is short for such debt and the debt payments make the firm lose its assets: the effective return is negative at the interval of debt maturity. When the levered firm has paid out the debt, the effective rate of return does not restore the effective rate of return of the unlevered firm (0.087) as it was in the case of the small- and medium-size debts; but it descends to a lesser value of about 0.061. This loss in the rate of return is due to the longer negative tail of the distribution, reflected in the variance and skewness accumulated in the debt maturity period (see Fig. 32 and 33). The result of this project is about doubling the firm's assets in 20 years (Fig. 34) under the risk of default of 90% (Fig. 35) which is, of course, absolutely unacceptable. One must recognize the project with the initial debt leverage of 35.5% as a commercial failure. The pictures in Figures 35-37 represent the comparative development of the firms with the net debt A = 250 and two debt maturities T_m = 10 and 15 years. As before in the cases of small- and medium-size debts, it is seen that the longer the debt maturity, the worse for the firm survival, and the project with T_m = 15 (the initial debt leverage 41.4%) is also a failure.

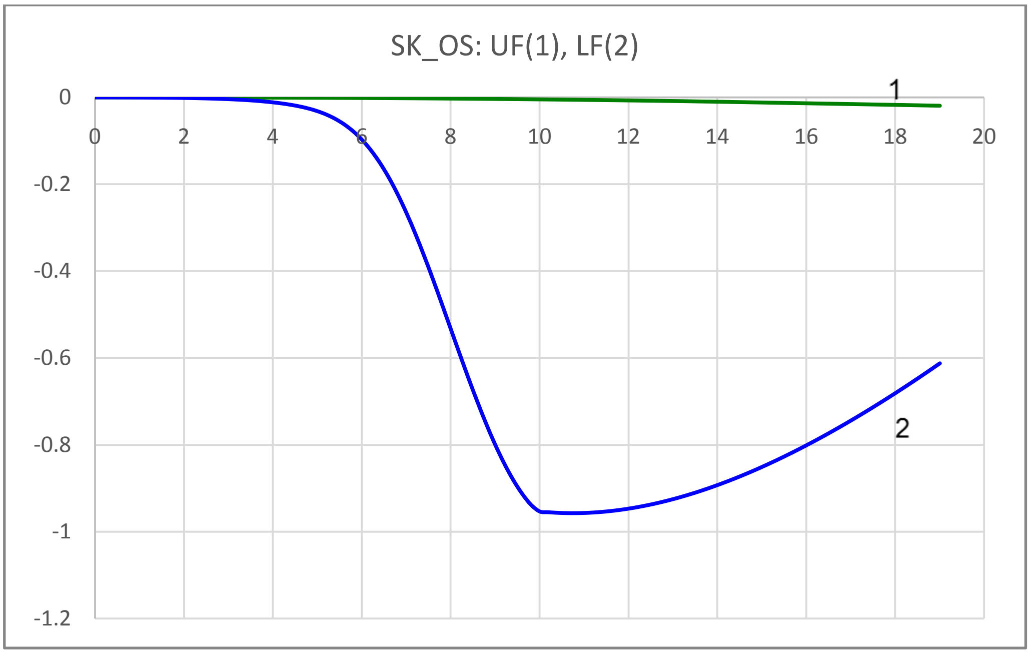

Figure 33. The skewness *SK_OS*(t) for the unlevered firm (line 1) and for the levered firm (line 2), A = 250, T_m = 10.

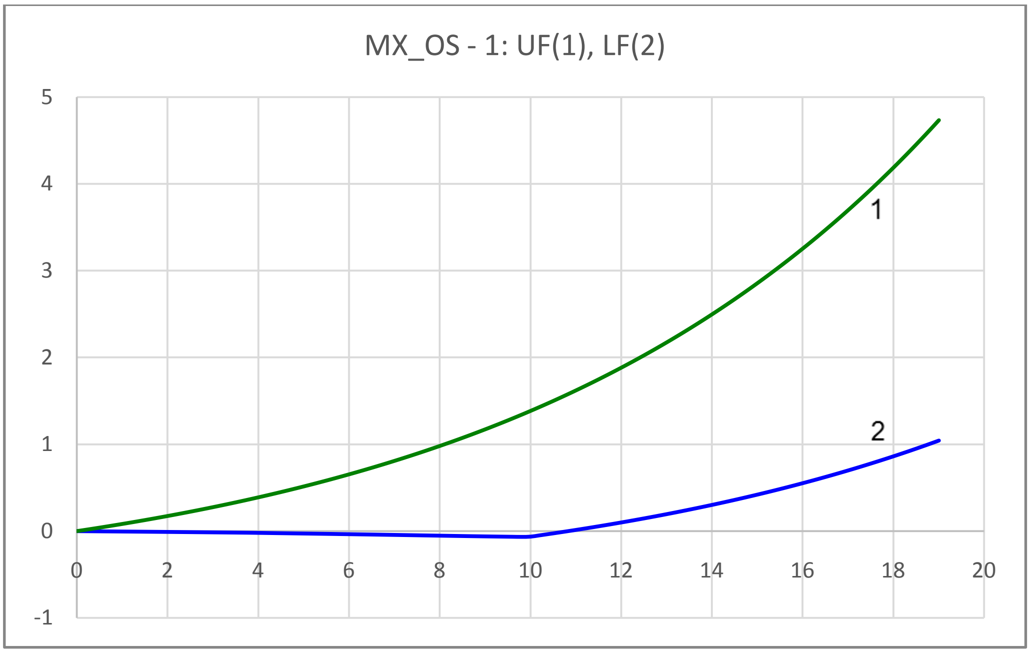

Figure 34. The mean assets *MX_OS*(t)-1 for the unlevered firm (line 1) and for the levered firm (line 2), A = 250, T_m = 10.

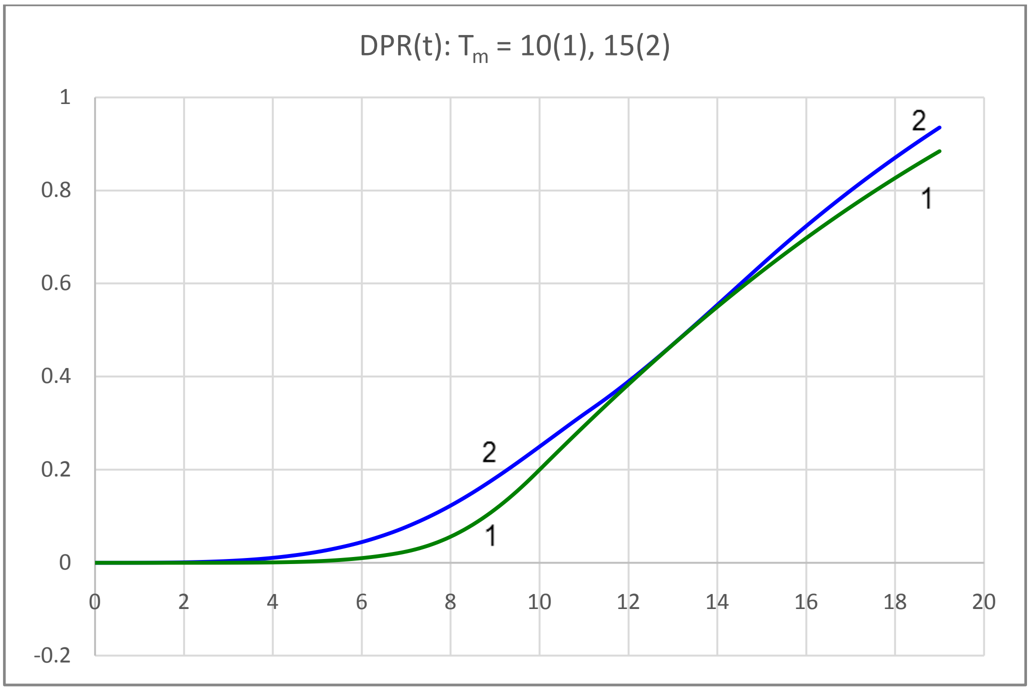

Figure 35. The default probability *DPR*(*t*) for T_m = 10 (line 1) and 15 years (line 2); *A* = 250.

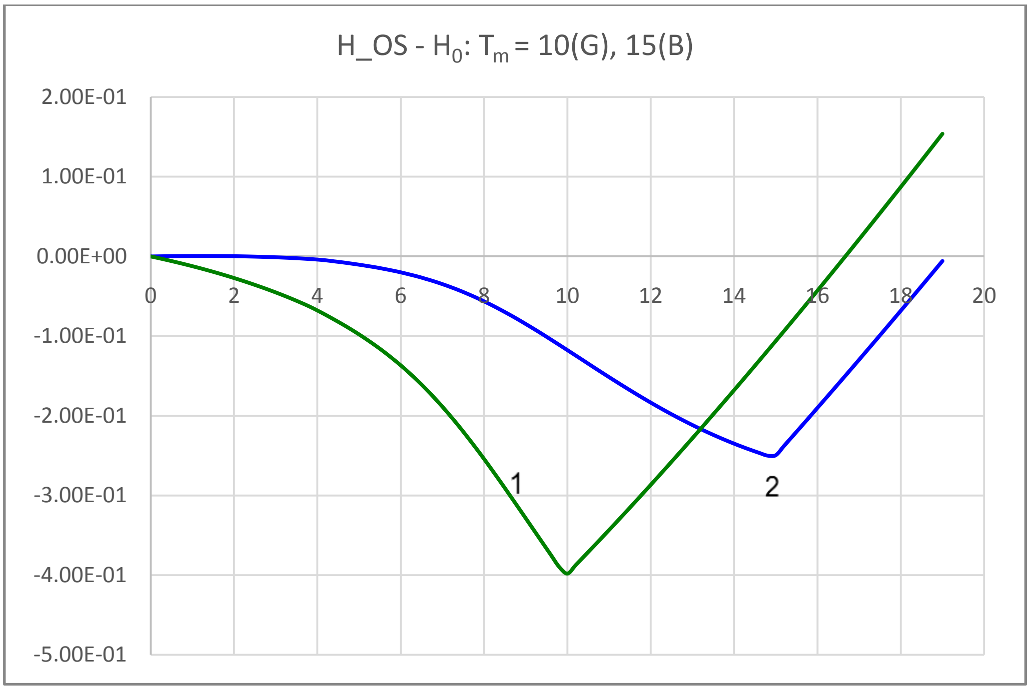

Figure 36. The log-value means *H_OS*(*t*)-H_0 for T_m = 10 (line 1) and 15 years (line 2); *A* = 250.

If the debt leverage surpasses 35%, all other conditions are the same, default and loss of control of the present proprietors over the firm will come sooner, say, in ten years instead of twenty years, depending on the parameters of the firm and business environment. One can say that the debt affects the firm survival like slow poisoning and the bigger the dose, the sooner the fatal end comes. Compare our results with the results of (Leland, 1994), who recommends the debt of 75%-90% as the optimal one for the firm with infinite debt maturity and paying corporate taxes. Because of the relation: the higher the payments, the lesser the firm stability, the inclusion of corporate taxes in EMM will decrease the safe debt leverage. We will consider the effect of taxes and dividends on the levered firm survival later. As one can see, default is the natural consequence of heavy debt. On the contrary, Leland needs the cost of bankruptcy in his model to bring the firm to default because, in his model, debt produces a positive effect on the levered firm development (see Eq. 10i).

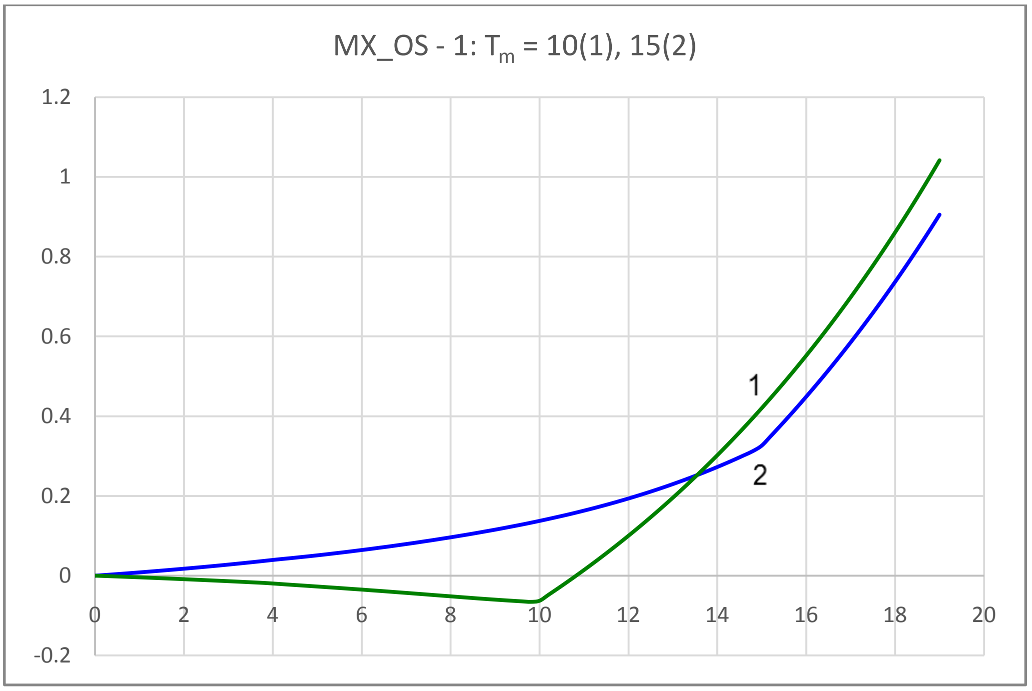

Figure 37. The mean assets $MX\_OS(t)$-1 for T_m = 10 (line 1) and 15 years (line 2); A = 250.

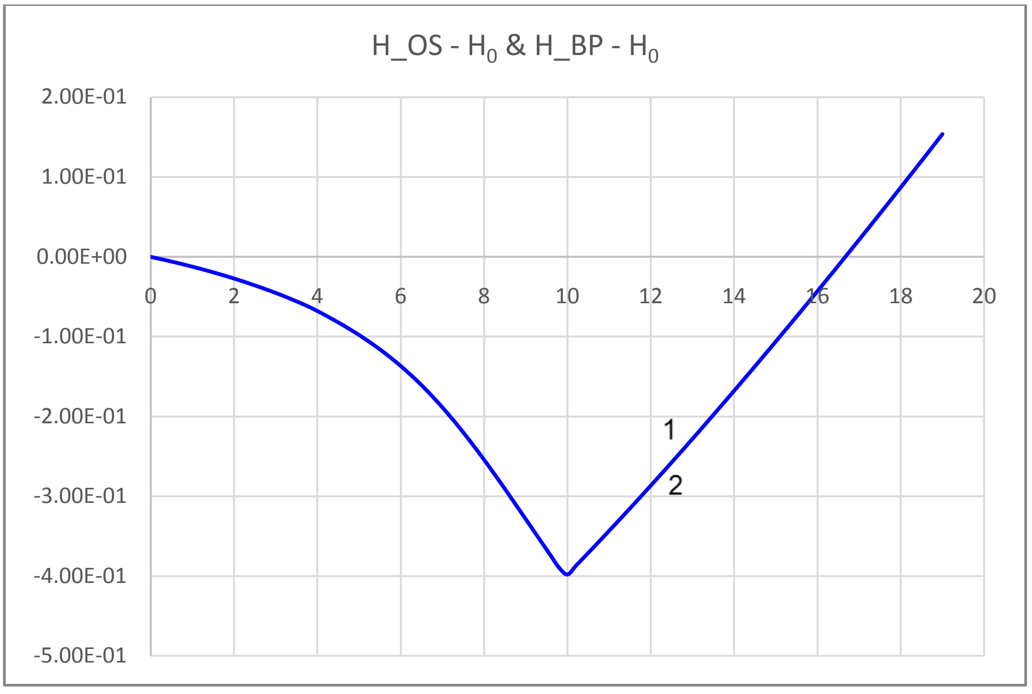

Figure 38. The log-value means $H\_OS(t)$-H_0 (line 1) and $H\_BP(t)$-H_0 (line 2), A = 250, T_m = 10.

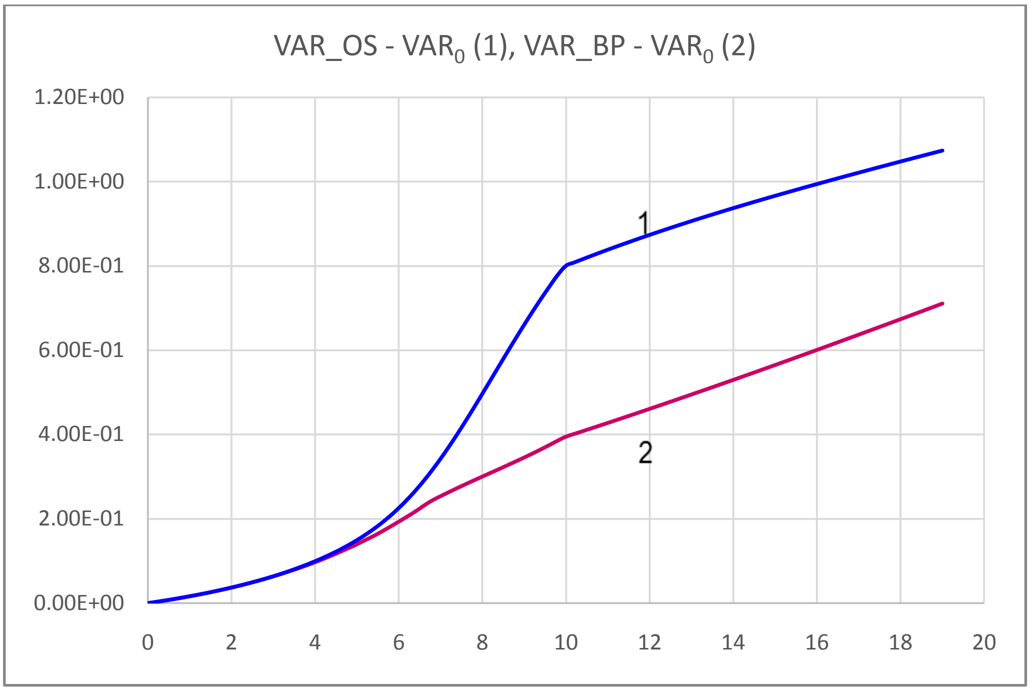

Figure 39. The open-space variance *VAR_OS*(*t*)-VAR_0 (line 1) and the boundary-problem variance *VAR_BP*(*t*)-VAR_0 (line 2), *A* = 250, T_m = 10.

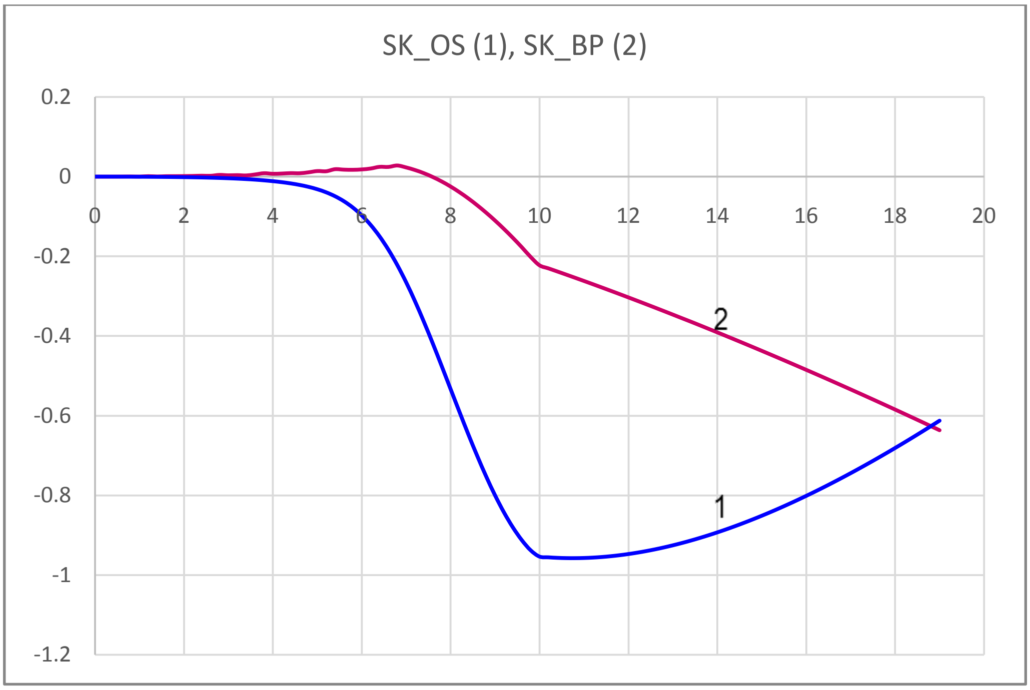

Figure 40. The open-space skewness *SK_OS*(*t*) (line 1) and the boundary-problem skewness *SK_BP*(*t*) (line 2), *A* = 250, T_m = 10.

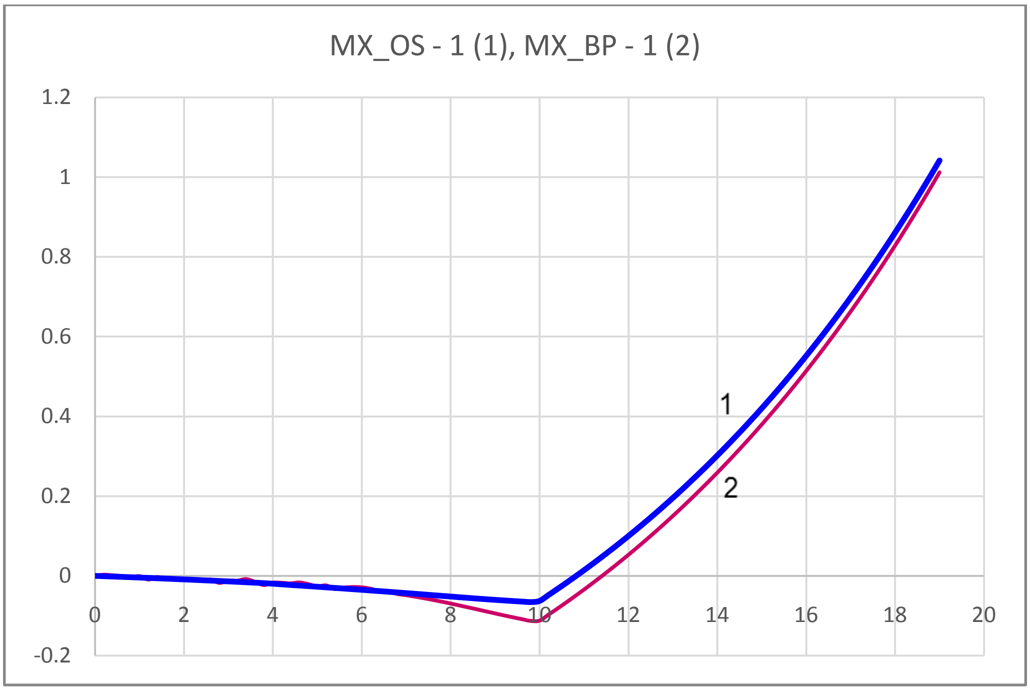

Figure 41. The open-space mean assets *MX_OS*(*t*)-1 (line 1) and the boundary-problem mean assets *MX_BP*(*t*)-1 (line 2), *A* = 250, T_m = 10.

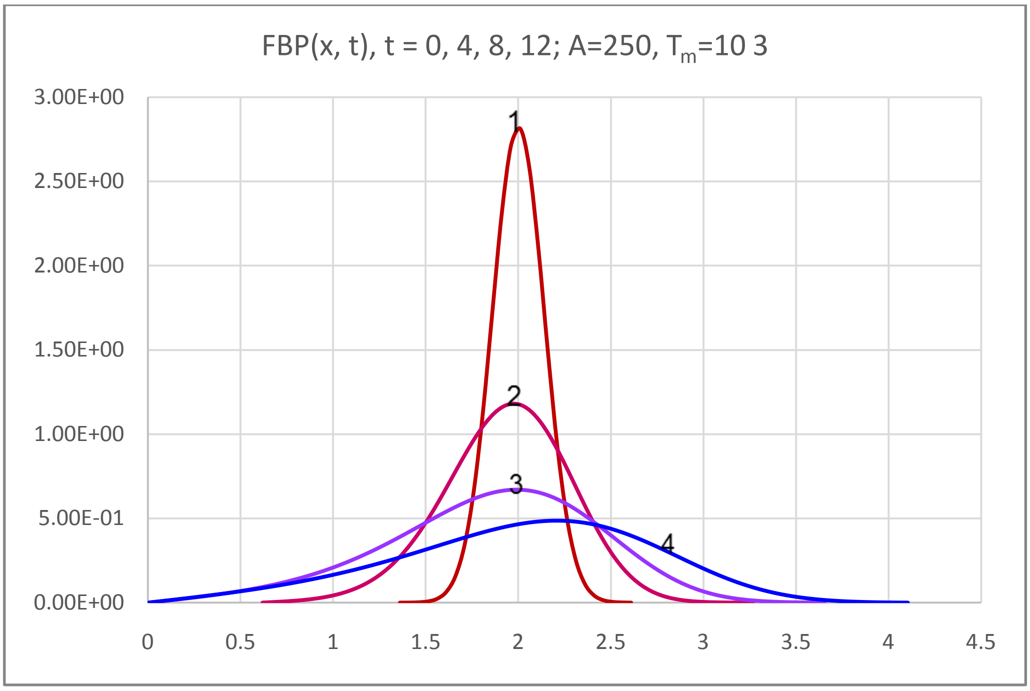

Figure 42a. The boundary-problem distribution *FBP*(*x*, *t*), *t* = 0 (1), 4 (2), 8 (3), 12 (4) for *A* = 250, T_m = 10.

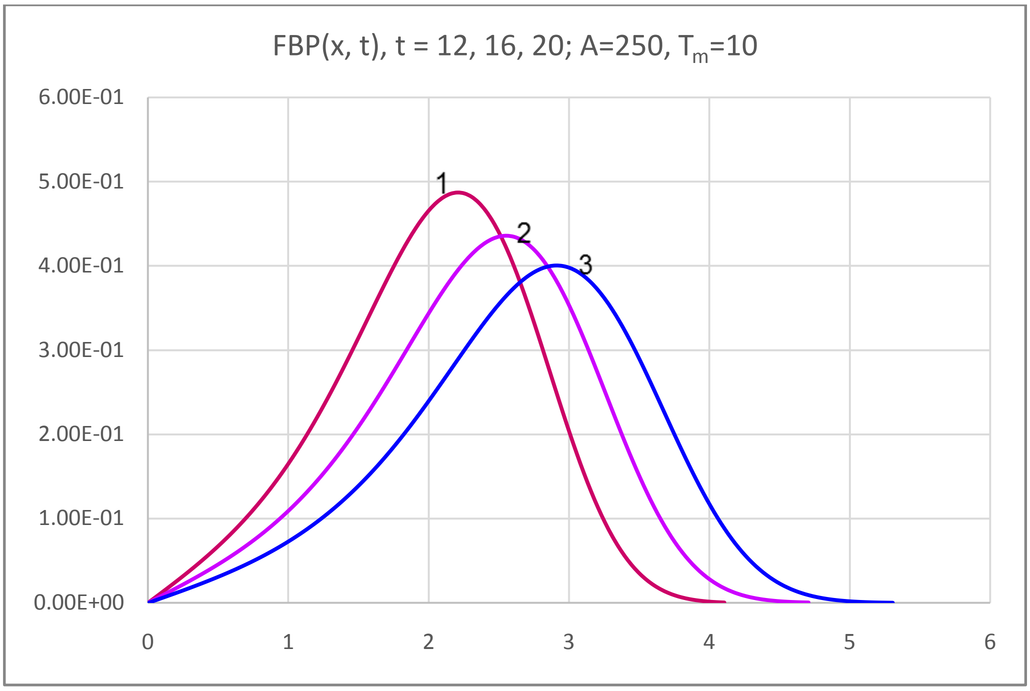

Figure 42b. The boundary-problem distribution *FBP*(*x*, *t*),*t* = 12 (1), 16 (2), 20 (3) for *A* = 250, T_m = 10.

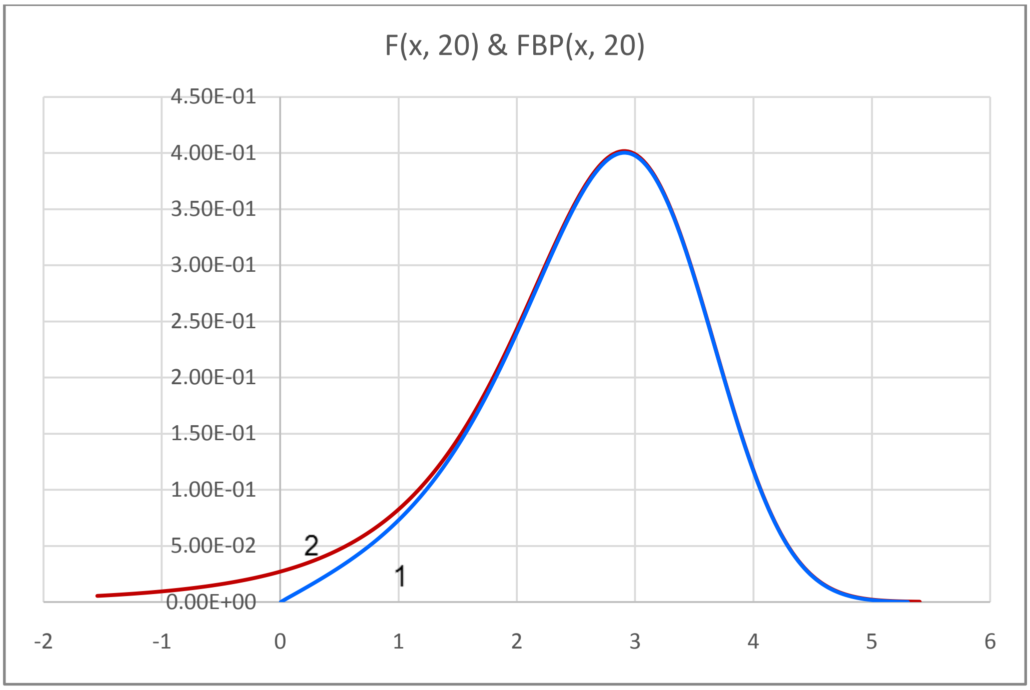

Figure 42c. Comparison of the boundary-problem distribution *FBP*(*x*, *t*) (line 1) and the open-space distribution *F*(*x*, *t*) (line 2) for *A* = 250, T_m= 10, *t* = 20.

Figures 38-41 compare statistical moments of the open-space problem characterizing the firm with the same statistical moments of the boundary problem. We watch the first three statistical moments (the log-value mean, variance, and skewness), the mean assets, and the default probability and its intensity. For small- and medium-size debt, the first three moments and the mean assets of the open-space problem and of the boundary problem numerically coincide with each other. Considering big-size debt, one can see (Fig. 38 and 41) that the log-value means *H_OS*(*t*) and *H_BP*(*t*) and mean assets *MX_OS*(*t*) and *MX_BP*(*t*) coincide in both models and, therefore, are interchangeable. On the contrary, the variances *VAR_OS*(*t*) and *VAR_BP*(*t*) and skewness *SK_OS*(*t*) and *SK_BP*(*t*) differ significantly from each other. To recognize the meaning of this fact, one must remember that the open-space statistical moments are not observable, one can observe and measure only the boundary problem moments characterizing the ensemble of firms active at the time of observation. On the other hand, the boundary problem distribution does not show the default probability and its intensity directly. To find this probability, one must address the open-space problem conjugated with this boundary problem. The whole procedure of finding the intensity of the firm default probability could be like this. At the time *t*, the firm management estimates by direct measurement four statistical moments for the firm assets: the log-value mean, mean assets, variance, and skewness. Then they compare these measured variables with the same variables calculated for the boundary problem. The calculated statistical moments, close enough to the measured moments, indicate the theoretical boundary problem distribution at the time *t*. Addressing the corresponding open-space distribution, the management gets the intensity of default probability they want to know.

The probability distributions for the boundary problem and the open space problem are presented in Fig 42a, b, c. The described procedure produces the scientifically substantiated default probabilities, which casts a shadow of deep mistrust on the current methods of estimating the firm state and its perspectives. So, we see that the market sends objective signals on the state of the firms, but the traders, mislead by the GBM-based theories, cannot read and interpret those signals correctly.

Buying New Technology on Credit

So far we consider the case when the firm takes a loan to join a business with given parameters such as the minimal asset value, expected return, fixed costs, and volatility. From the examples studied above, one can conclude that (1) the debt does affect the firm development, (2) this influence is negative, (3) the greater the debt leverage, the worse for the firm. At that, we have learned that the factor most frustrating the firm development is debt payments. A natural question arises if it is possible for the debt to make a positive effect on the firm development.

Here we consider the case when a debt is used to modernize the firm's technology, and that technology increases the firm's expected returns. Suppose that the initial rate of the firm's return is μ_0 percent, and, after introducing the new technology, it becomes $\mu = \mu_0 + \Delta\mu,\ \Delta\mu > 0$. To introduce the new technology, the firm must have at least 1000 dollar units of assets, but the firm's assets make only 1000-*A* units, therefore, the firm borrows at the market *A* units of capital for T_m years at the annual rate of *r* percent (we refer to this firm as the levered one). Suppose also that the new technology does not increase the firm's fixed costs and needs T_r years after taking the loan to become effective and increase the expected rate of return from μ_0 to μ, $T_r < T_m$.

The probability distribution in the first period of the firm development, $0 \le t \le T_r$, follows the equation

$$U_t + R_0[1-(1+\beta)e^{-x}]U_x - 0.5C^2U_{xx} + R_0(1+\beta)e^{-x}U = 0,$$
$$R_0 = \mu_0 - C^2/2. \tag{1.3}$$

The initial condition is

$$V(x,0) = N(x;H_0,\sigma_0^2),\ \ U(x,0) = N(x;H_0,\sigma_0^2),$$
$$H_0 = \langle x_0\rangle = \langle \ln(R_0X_0/P_0)\rangle,\ \ \sigma_0^2 = \langle (x-H_0)^2\rangle, \tag{2.3}$$

where $N(x;H_0,\sigma_0^2)$ is a normal function.

The boundary conditions are

$$U(DL(t),t) = 0,\ DL(t) = \max[0, \ln(R_0 X_D(t)/P_0)]\ , \tag{3a.3}$$

$$X_D(t) = X_D(0)(1 - t/T_m),\ 0 \le t\ \le T_m, \qquad X_D(t) = 0\ ,\ t > T_m. \tag{3b.2}$$

In the second period of the firm development, $T_r < t \le T_m$, the equation changes for

$$U_t + R[1-(1+\beta)e^{-x}]U_x - 0.5C^2 U_{xx} + R(1+\beta)e^{-x}U = 0, \qquad R = \mu - C^2/2\ ,\ \mu = \mu_0 + \Delta\mu, \Delta\mu > 0\ . \tag{4.3}$$

In the third period of the firm development, $T_m < t \le T$, the equation becomes

$$U_t + R(1-e^{-x})U_x - 0.5C^2 U_{xx} + Re^{-x}U = 0, \tag{5.3}$$

where T is the final point in the period of observation [0, T], $0 < T_r \le T_m < T$. The initial condition for the second and third stages of the firm development is the correspondent probability distribution in the end of the previous period.

We study the development of a firm with the big-size debt of A = 250 units, maturity time T_m = 15, interest rate 0.05; the total debt XD_0 = 529.25. The firm parameters for this case are: R_0 = 0.1, C^2 = 0.01, $\sigma_0^2 = 0.02$, the initial leverage 0.4137, fixed costs P_0 = 13.400 dollar units a year, the ratio of debt payments to fixed costs β = 2.633. The default line as a function of time is shown in Fig. 43.

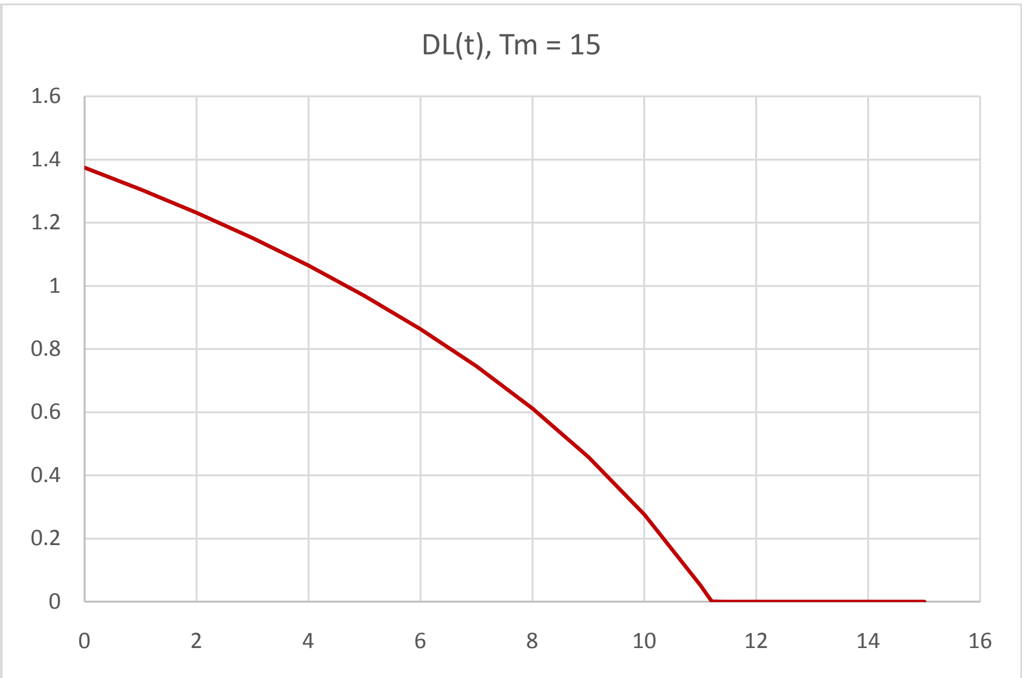

Figure 43. The default line $DL(t)$ for the levered firm with parameters A = 250, T_m = 15, *tDL0* = 11.204.

We consider five cases with different changes in the expected returns, $\Delta\mu = 0, 0.2, 0.4, 0.6, 0.8$. The results of modeling are presented in Figures 44-46.

As one can see in Fig. 44, the firm that borrowed the capital but still uses the old technology has a negative effective return in the interval of debt maturity (line 1). After paying out the debt, the firm's effective return becomes 0.061, which is less than the effective return of the unlevered firm (0.087, UF-line). The difference between the effective returns of the levered and unlevered firms arises because of the slowing down effect of the longer negative tail acquired by the levered firm during the debt payment. This tail brings the firm within twenty years to default with the probability close to unit (Fig. 45, line 1). The default probability of the unlevered firm one can see in Fig. 6. The firm, which expected return increases from 0.10 to 0.12 by introducing new technology, has the effective rate of return close to zero in the interval of debt maturity; after it, its effective rate of return increases to 0.100 (line 2). Line 3 shows the change in the effective rate of return for the firm whose expected rate of return has changed from 0.10 to 0.14. As one can see, the effective return is positive and increasing in the interval between T_r= 5 and T_m = 15 achieving the rate of 0.138 after discharging the debt. This effective rate of return is a good deal more than the effective return of the unlevered firm, proving the effectiveness of the new technology. Unfortunately, financial risks related to the asset borrowing (see Fig. 45, line 3) make this option of the firm development unacceptable. Mark that the difference between the expected rate of return, which does not take the fixed costs into account, and the effective rate of return, which does, is rather small. We shall discuss this phenomenon later. The firm corresponding to line 4 develops more energetically than the firm corresponding to line 3. The effective rate of returns after the debt is paid out makes 0.177, which is more than the expected rate of return (0.16). The effective rate of return after the debt maturity in curve 5 is 0.216 while the expected rate of return makes only 0.18. Compared to the unlevered case, it seems that the firm development in cases 3-5 does not feel the effect of fixed costs at all! Fig. 45 (lines 2-5) shows us that the high expected rate helps soften the negative effect of debt the more, the greater is the rate increment. If one considers the growth of the default probability after debt maturity, one can see that line 5 rises from *DPR*(15) = 0.4076 to *DPR*(24) = 0.4675, increment Δ*DPR* = 0.060, line 4 rises from *DPR*(15) = 0.4560 to *DPR*(24) = 0.5597, incrementΔ *DPR* = 0.1037, line 3 grows from *DPR*(15) = 0.5121 to *DPR*(24) = 0.6925, increment Δ*DPR* = 0.1804, and line 2 grows from *DPR*(15) = 0.5751 to *DPR*(24) = 0.8894, increment Δ*DPR* = 0.3143. So, one concludes that the period, when the firm deals with its debt, makes a demanding test to the firm with a high risk of default, but when the debt is paid out, the increment of default probability decreases rather fast from 0.3143 (line 2) to 0.060 (line 5). Taking into account graphs in Fig. 44, one concludes that cases 4 and 5 presenting some calculated risk, nevertheless, give a good strategy for the firm development. At that, the lesser the debt, the safer the transformation to the new technology, the higher results the firm can achieve for the same time interval. With EMM, the firm management can plan and figure out strategic decisions choosing new technologies for the next steps of the firm development.

It is interesting to note, that a similar beneficial effect of the increasing expected returns with analogous consequences for the firm value and firm survival can be produced by moderate inflation, but, of course, on a much lesser scale. See for details (Shemetov, 2020b).

Now let us consider why the effective rate of return is close or even surpasses the expected rate of return. To answer this question, mark that the effective rate of return is the mean drift rate of the probability distribution. This drift consists of three components: the exogenous drift, determined by the expected rate of return, the backward drift, determined by negative distribution skewness, and the diffusion at the right end of the asymmetric distribution. For the GBM-distribution, the mean drift consists of the exogenous drift only because the distribution remains symmetric all the time. Graphs *DPINT*(*t*), which is proportional to the area of the tail under the default line (see Eq. 7.1), give some ideas about the development of the distribution's negative tail. For small-size debts, when the default line is zero (*DL* = 0), *DPINT*(*t*) and tail grow gradually and monotonically (Fig. 5, 11), creating negative skewness and slowing down the positive drift of the distribution.

Because of the long negative tale of the distribution, the gradient at the distribution's right end produces an additional positive drift to the distribution. This drift compensates to some extent for the negative effect of the tail. The cumulative effect of all those mechanisms explains why the effective return is less than the expected rate of return for small-size debts. The influence of the distribution deformation grows with the firm's payments.

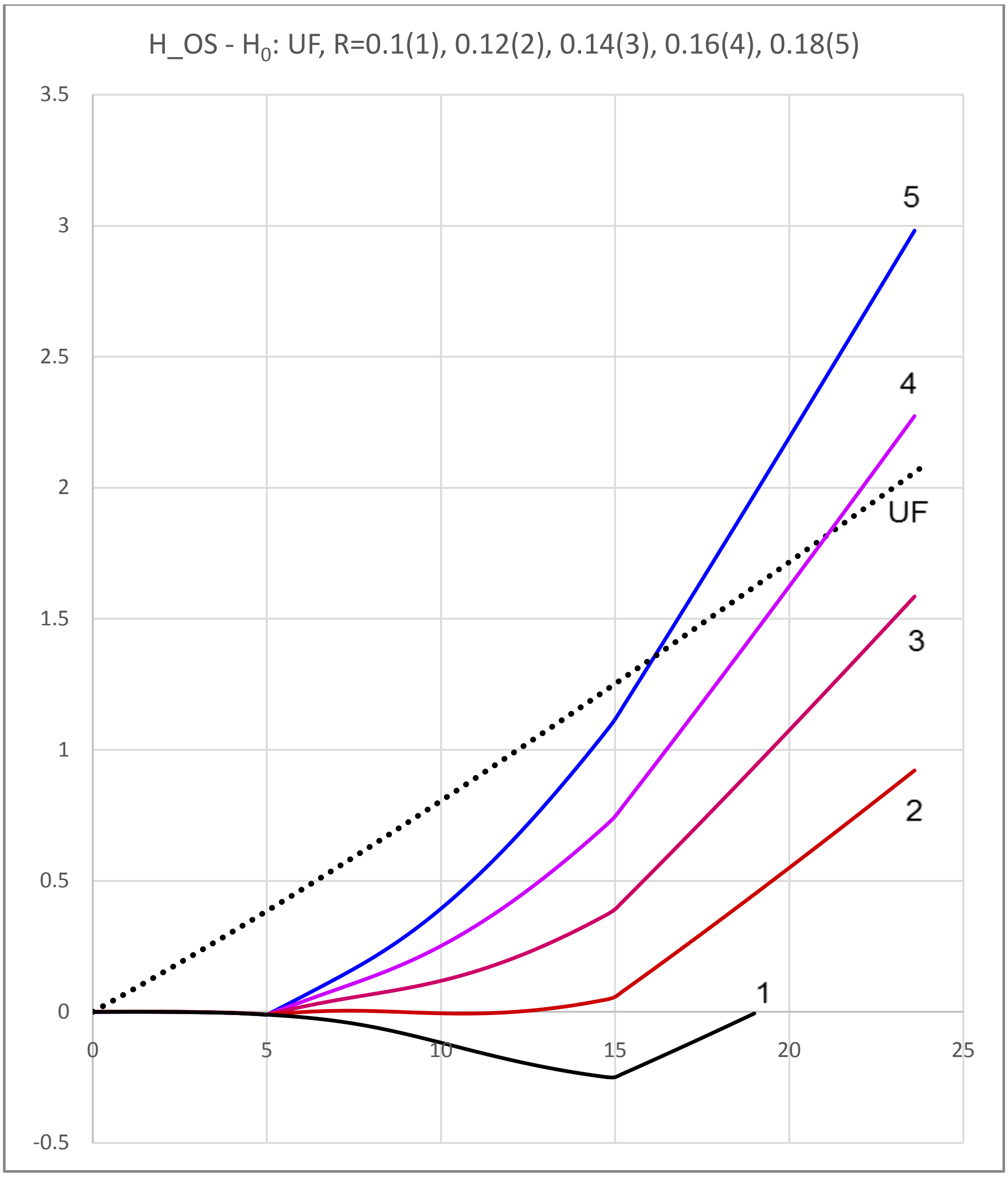

Figure 44. The log-value mean $H\_OS(t)$-H_0. UF-line describes the behavior of the unlevered firm, lines 1-5 describes the levered firms (A = 250, T_m = 15) with different expected returns achieved by introducing the new technology: R = 0.10 (1), 0.12 (2), 0.14 (3), 0.16 (4), 0.18 (5).

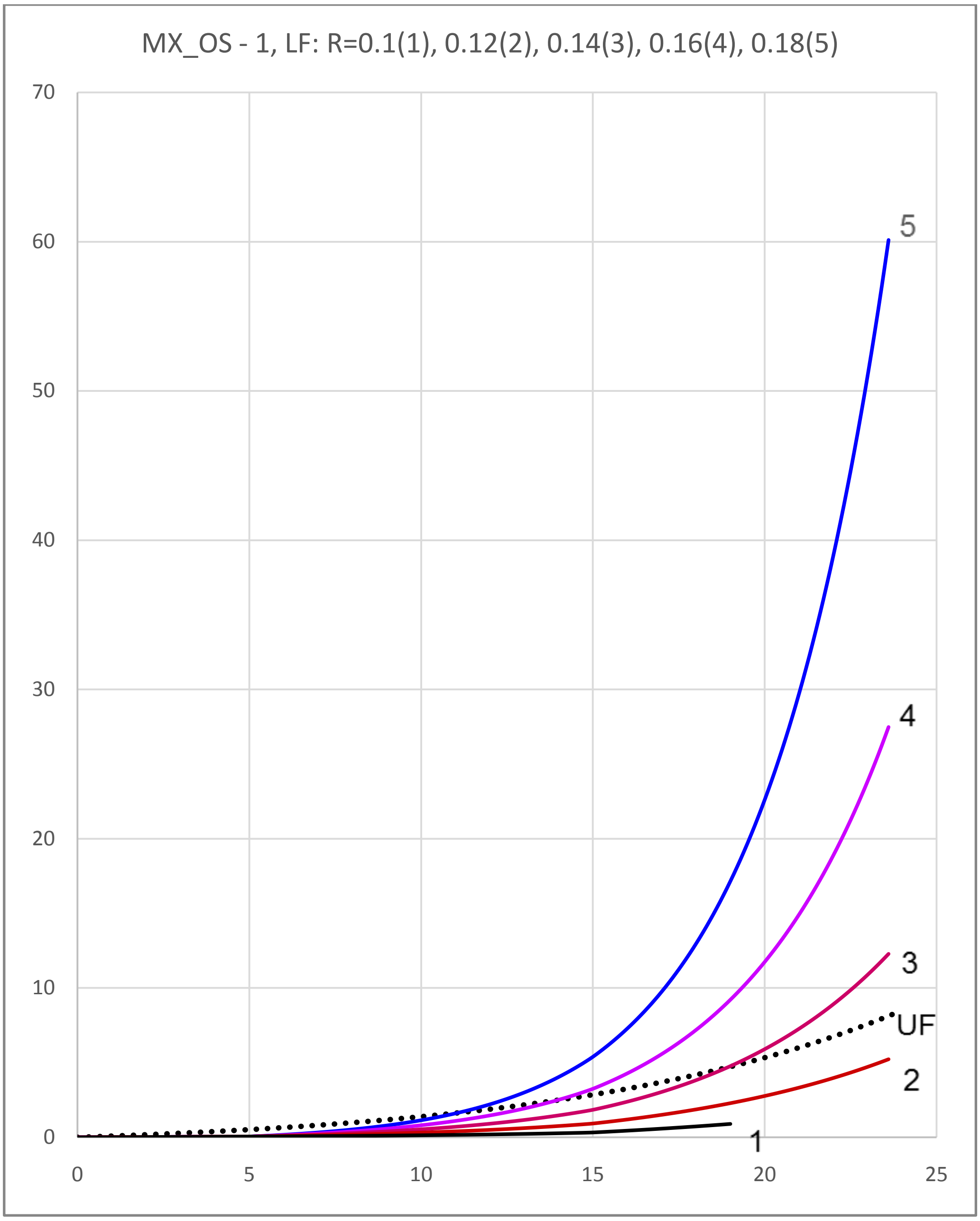

Figure 45. The mean assets *MX_OS*(*t*)-1. Lines 1-5 describes levered firms (A = 250, T_m = 15, H_0 = 2.0) with different expected returns achieved by introducing a new technology: R = 0.10 (1), 0.12 (2), 0.14 (3), 0.16 (4), 0.18 (5). UF-line represents the development of the unlevered firm, R = 0.10, H_0 = 2.0.

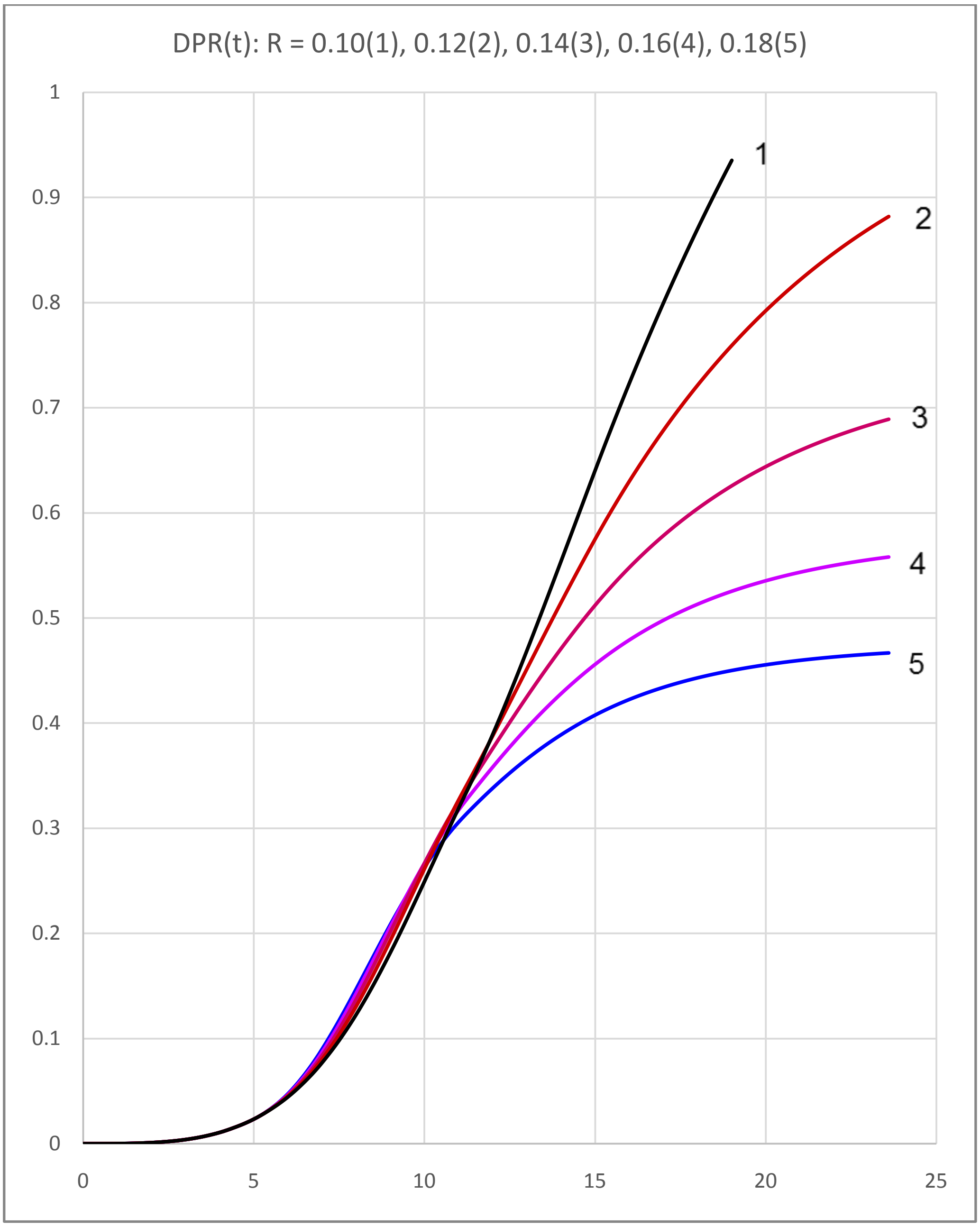

Figure 46. The default probability *DPR*(*t*). Lines 1-5 describes levered firms (*A* = 250, T_m = 15, H_0 = 2.0) with different expected returns achieved by introducing a new technology: R = 0.10 (1), 0.12 (2), 0.14 (3), 0.16 (4), 0.18 (5).

Referring to medium-size debts, one can see a new tendency in the development of the distribution tail (Fig. 22, 28): after a significant growth caused by the closeness of the distribution center to the default line (Fig. 13, 24, 30, 43), *DPINT*(*t*) declines due to the fast shift of the distribution as a whole to the right. Therefore, the backward drag caused by the negative tail gets smaller, and the difference between the effective rate of return and the expected rate of return decreases. The diffusion at the right end of the distribution diminishes this difference even more. When it comes to the firms increasing their expected rate of return by introducing an innovative technology, the right shift of the distribution increases significantly compared to the previous cases with a constant rate of expected returns. This effect also speeds up the right-end diffusion, increasing the effective rate of return. As a consequence, the effective rate of return surpasses the expected rate of return, and the more, the higher the increment of the expected rate of return.

Conclusion

We have shown that three classic works considering the effects of debt on the firm value and firm survival, namely, Modigliani-Miller Propositions (1958, 1961, 1963), Merton (1974), and Leland (1994) are wrong. Their main mistake is ignoring the firm's compulsory payments (business securing expenses, BSEs). We suggest the model taking account of BSEs (the Extended Merton model, EMM) and apply it to the analysis of debt effect on the firm value and its survival. Our conclusions are

(1) the debt does affect the firm value and its survival,

(2) this influence is negative, diminishing the firm value and its chances to survive,

(3) the pressure of the debt increases as the debt grows, and the greater the debt, the sooner the default comes,

(4) the main factors, depressing the levered firm, are its high debt payments added to BSEs of the identical unlevered firm and the length of debt maturity.

The debt effect on firm survival is like slow poisoning: in the beginning, it is negative but tolerable; however, as time runs, the situation becomes more and more critical, even fatal. Therefore, all attempts to find the optimal debt leverage using the trade-off theory, based on the geometric Brownian model (GBM) and believing after MMP3 in the positive effect of debt on the after-tax firm value, are fruitless; they look like the early medieval chasing after the Holy Grail.

Nevertheless, the debt *can be* beneficial for the firm if the firm takes a loan to modernize its technology (a typical example is the industrialization of the developing countries). The model helps to estimate the probability of success in this modernization for different business conditions. With these probabilities, the firm management can choose the technology most suitable for the firm in its specific conditions.

We have also revealed a serious problem for the firm management in proper reading the market signals concerning their firm and using this information to control the firm effectively. Mislead by various forms of GBM-based models, the firm management cannot adequately estimate the state of the firm, which negatively influences their control decisions.

Summing up, one can say that the wrong economic theory is dangerous because it misguides managers in making practical decisions about their firms. Incorrect decisions lead to unjustifiable extra losses up to bankruptcies, impeding the development of the national economy. Wrong theories also mislead students forming a distorted picture of economic relationships in their heads with obvious long-term consequences for the economic theory and business practice.

It is worth noting that economics is, maybe, the only science among the mathematical sciences of the 20th-21st centuries in which erroneous theories live for so long. Modigliani-Miller Propositions are considered as correct over sixty years (since 1958), Merton's theory seems true for about fifty years (since 1974), Leland's theory stays for almost thirty years (1994). Why errors of individual researches are not timely corrected by the scientific community and continue polluting the economic theory and misguiding business for so long? What improvements must be made to the organization of economic studies to prevent repeating such things in the future? We think that these questions must make the subject of broad professional discussion.

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