Materials Response to High Power Energy Lasers
(A Short Course—Part III)

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Abstract: With recent attention to high power energy and its interaction with materials of different type, both in industry and military application, this paper covers a short review course into subject of materials response in respect to such high power energy lasers. In this paper, we are covering laser interaction with solid and going through steps of phase changes, from solid to liquid and finally vapor stage. As we indicated in this part of short course mainly Part I and Part II, we have started a series of articles on the subject of Materials Responses to High Power Energy Lasers and continue these series by starting to introduce the Laser Light Propagation into materials. In this part namely Part III, we are discussing, one of the most important effects of intense laser irradiation is the conversion of the optical energy in the beam into thermal energy in the material. This is the basis of many applications of lasers, such as welding and cutting. We shall summarize here this thermal response. It is basically a classical problem, namely heat flow, in a usual manner of heat conduction, we show solutions to the equation which governs the flow of heat and discuss change of phases in targeting material from solid to liquid and finally vapor and plasma stages step by step.

Key words: Radiation wave, electromagnetic and electrical field, laser and laser radiation, Hugoniot Limit, heat transfer and heat radiation, laser interaction with matter, phase changes, melt and vaporization.

1. Introduction

In Part I, we introduce the basic concept of radiation by describing what it is and then we move on to start by deriving wave propagation equation using sets of Maxwell’s Equation in classical electrodynamics approach and deriving the simple wave equation. We also covered, holistic approach to basic laser energy, and solved derivation of wave equation in a complex general form, and then we discussed high energy laser characteristics. Along the way we defined, what is skin depth, extinction coefficient, which both have a direct correlation to the property of materials and their behavior in respect to HEL (High Energy Laser) as a target. We also covered how the lasers are working and what are their characteristics, when it comes to their interaction with matter or materials.

In Part II, we covered the following topics such as, laser light propagation, physics of laser absorption in metals, as well as the description of the phenomena, when it comes to the interaction of electromagnetic wave with matter and finally we end our Part II with light propagation in materials as conclusion of this part.

In Part III, we will discuss the theoretical discussion of laser absorption and reflectivity and other aspect of heat conductivity within materials. Thus, in this part of these series of short courses, we are offering here a normal manner of heat conduction within solid and then walk through phase of change of solid to liquid and finally vapor and plasma stage.

When different parts of solid metal of a body are at different temperatures, heat flows from the hotter parts to the cooler as we understand from our basic physics of heat conduction rules by definition, and this is what we will cover in Part IV of these series, which mainly
would be around heat transfers and heat conduction.

2. Theoretical Discussion of Laser Absorption and Reflectivity

This subject was touched upon in Part II, Section 3.0, and we expand it further again. To consider the coupling of the laser energy to a material, we need first to know the optical reflectivity \( R \) and the transmissivity \( T \) for light incident on a surface which divides two semi-infinite media. The reflectivity plus the transmissivity equals unity at a single surface:

\[
R + T = 1 \quad \text{Eq. (77)}
\]

In most practical situations we are dealing with more than one surface; typically, we have a slab of material with light impinging on one surface. Some light is reflected, and the rest is either absorbed or passes completely through the slab. In such a situation we shall describe the net result of all the reflection, after multiple passes inside the slab and appropriate absorption has been accounted for, in terms of the reflectance \( R \), the absorptance \( A \) and the transmittance \( T \):

\[
R + T + A = 1 \quad \text{Eq. (78)}
\]

The absorption of the laser energy takes place through photon interaction with bound and free electrons in the material structure, which raises them to the higher energy levels. Energy conversion takes place through various collision processes involving electrons, lattice phonons, ionized impurities, and defect structures.

If the surface being machined reflects too much light energy, the absorbed energy is decreased, the operation efficiency is lowered, and the reflected light may do harm to the optical systems. Reflection and absorption of laser beams is closely related to laser micromachining or duelning with target surface materials. The value of absorption and reflection is related by Eq. (78).

What we really are interested in from the point of view of material response is \( A \), the absorptance of the material. In most materials of interest from the practical aim of using lasers to melt, weld, etc., \( T \) is zero, thus Eq. (78) reduces to:

\[
R + A = 1 \quad \text{Eq. (79)}
\]

In metals, the radiation is predominantly absorbed by free electrons in an “electron gas”. These free electrons are free to oscillate and reradiate without disturbing the solid atomic structure. Both Eqs. (78) and (79) are analogous to Eqs. (41) and (42) for what we found out and discussed in Section 3.0 Part II in above. As an Electromagnetic (EM) wave-front arrives at a surface of the target then all the free electrons in the surface vibrate in phase generating an electric filed 180° out of phase with the incoming beam creating “electron gas”. This “electron gas” within the metal structure means that the radiation is unable to penetrate metals to any significant depth, only one to two atomic diameters (or free paths), thus metals are opaque, and they appear shiny.

![Fig. 15 Reflectivity as a function of wavelength for different metals.](image)

According to Fig. 15 reflectivity decreases as wavelength becomes shorter, while absorption increases when photon energy increases.

If sufficient energy is absorbed then the vibration becomes so intense that the molecular bonding is stretched so far that it is no longer capable of exhibiting mechanical strength and the material is said to have melted. On further heating the bonding is further loosened due to the strong molecular vibrations and the material is said to have evaporated.
The vapor is still capable of absorbing the radiation, but only slightly since it will only have bound electrons. The exception occurs if the gas is sufficiently hot so that electrons are shaken free, and the gas is then said to be in plasma status.

To understand reflectivity, we must use some general results from the theory of electromagnetic waves. Let us summarize these briefly at this point.

The electric field of the electromagnetic wave, from Eq. (7-7), is:

\[
\hat{E}(z,t) = \Re e \left[ \hat{E}_0 e^{-2\pi i z / \lambda} e^{i\omega(t-z/c)} \right]
\]

(Eq. (7) as in Part I)

The relationships we need are those among the index of refraction \( n \), the extinction coefficient \( k \), and the material properties. These relationships can be derived by substituting Eq. (7-7) in the wave equation as:

\[
\frac{\partial^2 \hat{E}(z,t)}{\partial z^2} = \frac{1}{\nu^2} \frac{\partial^2 \hat{E}(z,t)}{\partial t^2}
\]

Eq. (80)

where \( V \) is the phase velocity of the wave and for traveling wave in media of density, the phase velocity is given by:

\[
v = f \lambda = v \lambda
\]

and

\[
\omega = 2\pi f = 2\pi v = \frac{2\pi}{\lambda}
\]

In case media as vacuum \( V = c \) speed of light in vacuum.

If we get twice derivation of Eq. (7) in Part I, and substitute the result in Eq. (58), it results in:

\[
\frac{\partial^2 \hat{E}}{\partial z^2} = \mu \epsilon \frac{\partial^2 \hat{E}}{\partial t^2} + \sigma \frac{\partial \hat{E}}{\partial t}
\]

Eq. (81(a))

Eq. (81(a)) results in the expression of Eq. (81(b)) below as:

\[
\left( \frac{2\pi k}{\lambda} + \frac{i\omega}{c} \right)^2 = \mu \epsilon (-\omega^2) + i\omega \mu \sigma
\]

Eq. (81(b))

Note that we are using rationalized MKS units throughout. The material properties enter through \( \mu \), \( \epsilon \), and \( \sigma \), which are the magnetic permeability, the dielectric function, and the electric conductivity of the medium. Using the usual equations between the field vectors,

\[
\hat{D} = K_c \hat{e}_0 \hat{E}
\]

Eq. (82(a))

\[
\hat{B} = K_m \mu_0 \hat{H}
\]

Eq. (82(b))

\[
\hat{J} = \sigma \hat{E}
\]

Eq. (82(c))

we have:

\[
\epsilon = K_c \epsilon_0
\]

Eq. (82(d))

\[
\mu = K_m \mu_0
\]

Eq. (82(e))

In equations (82 series), \( \epsilon_0 \) and \( \mu_0 \) are the electric permittivity and magnetic permeability of a vacuum. \( K_c \) is the dielectric constant and \( K_m \) the magnetic permeability of the material. By substituting Eqs. (82(d)) and (82(e)) into Eq. (81(b)) and using \( (2\pi/\lambda) = (\omega/c) \), we obtain that:

\[
(k + in)^2 = -K_m K_c \epsilon_0 \epsilon_c^2 + iK_m \mu_0 \sigma \left( \frac{\epsilon^2}{\omega} \right)
\]

Eq. (83)

Finally, if we introduce \( c^2 = (\epsilon_0 \mu_0)^{-1} \) and do some algebra, we get:

\[
n = \sqrt{K_m} \sqrt{K_c - i \frac{\sigma}{\epsilon_0 \omega}}
\]

Eq. (84)

This equation relates the material parameters \( K_c \), \( K_m \), and \( \sigma \), which in general may be complex, to index of refraction \( n \) and extinction coefficient \( k \). To describe the propagation of the light wave, it requires knowledge of \( K_c \), \( K_m \), and \( \sigma \). Before we describe these, let us look at two more general properties of our propagating electromagnetic wave.

The first of these is absorption. If the medium is absorbing, the intensity will fall off to \( 1/e \) of its initial value in a distance \( \delta \), obtained by setting \( \delta^2 \) of Eq. (7) of Part I equal to \( (1/e)\delta^2_{\text{max}} \), or

\[
\frac{4\pi k \delta}{\lambda} = 1
\]

Eq. (85(a))

or
\[ \delta = \frac{\lambda}{4\pi k} \quad \text{Eq. (85(b))} \]

This shows why \( k \) is called the extinction coefficient, for it determines skin depth \( \delta \). Eq. 7-63(b) is fairly general in that once \( k \) is known, \( \delta \) can be calculated. As noted, knowledge of the material properties is required to calculate \( k \).

The second general property we wish to derive is the expression for reflectivity, in terms of \( n \) and \( k \). To do this, consider light impinging normally onto an ideal solid surface, as shown in Fig. 16. Here we have illustrated the incident (\( \vec{E}_i \)), reflected (\( \vec{E}_r \)), and transmitted (\( \vec{E}_t \)) electric waves at a vacuum-material interface. For the present, we limit our discussion to the case of normal incidence. We now consider the boundary condition. We electric field, we have:

\[ \vec{E}_i + \vec{E}_r = \vec{E}_t \quad \text{Eq. (86)} \]

For the magnetic field \( \vec{B} \), we can right:

\[ B_i - B_r = B_t \quad \text{Eq. (87)} \]

The minus sign is before \( B_r \) because \( \vec{E} \times \vec{B} \) is positive in the direction of propagation of the wave. Now, the relationship between \( \vec{B} \) and \( \vec{E} \), or, since \( \vec{B} = \mu \vec{H} \), between \( \vec{H} \) and \( \vec{E} \), is required in order to proceed further.

This follows directly from Maxwell’s equations as:

\[ \Delta \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t} \quad \text{Eq. (88)} \]

\[ \Delta \times \vec{H} = \sigma \frac{\partial \vec{E}}{\partial t} + e \vec{E} \quad \text{Eq. (89)} \]

It is convenient to rewrite Eq. (7) or Part I and introduce \( \omega \lambda = 2\pi c \), to have \( \vec{H} \) explicitly in terms of \( \omega \) instead of both \( \omega \) and \( \lambda \). Recall that \( \vec{H} \) is a vector and take it as being along the \( x \)-direction. Thus, we can write:

\[ \vec{E}_r = \vec{E}_i e^{i\omega t} e^{-i\omega (x-ik)} \quad \text{Eq. 7-(90)} \]

Here we have dropped the “Re” notation and shall simply note that we always mean the real part when we write the wave in exponential form. We shall use unit vectors \( \hat{x}, \hat{y}, \) and \( \hat{z} \).

Fig. 16 Incident, transmitted, and reflected electric vectors at an interface.

Now the curl expression reduces to:

\[ \Delta \times \vec{E} = \hat{y} \frac{\partial \vec{E}_y}{\partial z} \quad \text{Eq. (91)} \]

which, with Eq. (88), tells us that \( \vec{H} \) has only a \( y \)-component, so, we have:

\[ \Delta \times \vec{H} = -\hat{x} \frac{\partial \vec{H}_x}{\partial t} \quad \text{Eq. (92)} \]

Thus Eqs. (88) and (89) become in the following form:

\[ \frac{\partial \vec{E}_y}{\partial z} = -\mu \frac{\partial \vec{H}_x}{\partial t} \quad \text{Eq. (93)} \]

and

\[ \frac{\partial \vec{H}_x}{\partial z} = \sigma \vec{E}_y + e \frac{\partial \vec{E}_y}{\partial t} \quad \text{Eq. (94)} \]

and, of course, \( \vec{E}_y = \vec{H}_x = H_y = H_z = 0 \). Putting the expression for \( \vec{E}_y \) from Eq. (90) into Eq. (93) leads to:

\[ H_y = \frac{n - ik}{\mu c} e^{i\omega t} e^{-i(x-ik)} e^{\omega t} \quad \text{Eq. (95)} \]

This is desired relationship as:

\[ H_y = \left( \frac{n - ik}{\mu c} \right) \vec{E}_y \quad \text{Eq. (96)} \]

At this point we note in passing that Eqs. (94) or (87) could be used to yield the relationship of \( n \) and \( k \) to \( \mu \), \( e \), and \( \sigma \). If the reader is unfamiliar with these relationships, it is instructive to carry out the algebra.
Returning to our consideration of the reflected electric and magnetic fields, we rewrite Eqs. (86) and (87) with the help of the relationship between $H$ and $\varepsilon$, from Eq. (96).

$$\varepsilon_i + \varepsilon_q = \delta_i$$

and

$$\mu_i H_i - \mu \varepsilon_q = \mu_0 H_i$$

become

$$\varepsilon_i - \varepsilon_q = \left(\frac{n_1^2 - ik_1}{n_1 - ik_1}\right) \varepsilon_q$$

Solving for $\varepsilon_i/\varepsilon_q$ by eliminating $\varepsilon_i$, we get:

$$\frac{\varepsilon_i}{\varepsilon_q} = \frac{n_1 - n_2 - i(k_1 - k_2)}{n_1 + n_2 + i(k_1 + k_2)}$$

Finally, the reflectivity $R$ at the surface is:

$$R = \left|\frac{\varepsilon_i}{\varepsilon_q}\right|^2 = \frac{(n_1 - n_2)^2 + (k_1 - k_2)^2}{(n_1 + n_2)^2 + (k_1 + k_2)^2} \quad \text{Eq. (97)}$$

Take medium 1 as a vacuum and drop the subscript 2, since in a vacuum $n_1 = 1$ and $k_1 = 0$, then Eq. (97) reduces to:

$$R = \frac{(n - 1)^2 + k^2}{(n + 1)^2 + k^2} \quad \text{Eq. (98)}$$

Eq. (98) is the second relationship we will find useful in discussing the coupling of optical radiation with metals. Note that it is derived for the special case of normal incidence and is applicable to a vacuum-material interface.

3. Reflectivity of Materials at Infrared Wavelength

We turn now to a derivation of the optical reflectivity of metals for infrared wavelengths, where experiment has shown that the free-electron theory (sometimes called the Drude-Lorentz theory) of metals is adequate. This theory rests on three assumptions.

The first is that electromagnetic radiation interacts only with the free electrons in the metal.

The second is that the free electrons obey Ohm’s law, or, more specifically, that

$$m^* \frac{d\nu}{dt} + \frac{m^*}{\tau} \nu = -e\phi \quad \text{Eq. (99)}$$

where $m^*$ is the effective mass of the electron, $\nu$ the drift velocity of electro, $\tau$ the relaxation time due to collisions (i.e., mean free time between ionic collisions. See Fig. 17), and $-e\phi$, the force on the electron due to the electromagnetic field.

The third assumption is that the free electrons of a metal can be described in terms of a single effective mass, carrier concentration, and relaxation time. There has been a good deal of discussion about the validity of these assumptions in the literature.

Recent work indicates that, for wavelengths in the intermediate infrared (a few microns to many tens of microns) and beyond, the free-electron theory does a reasonable job of predicting the reflectivity of metals.

To derive the free-electron optical reflectivity, we try solutions to Eq. (99) of the form:

$$\nu \approx e^{i\omega t}$$

So that Eq. (99) with substitution of above relation results in:

$$\left[ m^*(i\omega) + \frac{m^*}{\tau} \right] \nu = -e\phi \quad \text{Eq. (100)}$$

![Fig. 17 Drude-Lorentz Model electrons (shown here in blue or small dot) constantly bounce between heavier, stationary crystal ions (shown in large red dot).](image-url)
Manipulating Eq. (100), we can find that:
\[ \nu = -\frac{e\tau}{m(1 + i\omega\tau)} \]  
Eq. (101)

Now the current flow obeys:
\[ \vec{J} = \sigma \vec{E} = -N e \vec{v} \]  
Eq. (102)

where \( N \) is the electron concentration (number of electron per unit volume). By comparison of the last two equations,
\[ \frac{\sigma}{Ne} = \frac{e\tau}{m(1 + i\omega\tau)} \]  
Eq. (103)

or
\[ \sigma = \frac{Ne^2 \tau}{m(1 + i\omega\tau)} \]  
Eq. (104)

Now the dc conductivity is
\[ \sigma_0 = \frac{Ne^2 \tau}{m} \]  
Eq. (105)

We see \( \sigma \) is a complex quantity and seek to write it as the sum of a real and imaginary part. Thus,
\[ \sigma = \sigma_1 - i\sigma_2 \]  
Eq. (107)

The result is
\[ \sigma_1 = \frac{\sigma_0}{1 + \omega^2 \tau^2} \]  
Eq. (108(a))

\[ \sigma_2 = \frac{\sigma_0 \omega \tau}{1 + \omega^2 \tau^2} \]  
Eq. (108(b))

To proceed further we need to use the general expression for electromagnetic waves developed in Section 2.0. Recall Eq. (84):
\[ n - ik = \sqrt{K_m K_s} - i \frac{\sigma}{\varepsilon_0 \omega} \]  
(Previous Eq. (84))

and from the complex \( \sigma \) of Eq. (107), we can write:
\[ n - ik = \sqrt{K_m} \sqrt{K_s} - i \frac{\sigma_1 - i\sigma_2}{\varepsilon_0 \omega} \]  
Eq. (109)

If we assume only free-electron optical interactions, the metal does not polarize under the wave, and \( K_s = 1 \). In addition, for metals in the infrared, \( K_m = 1 \).

Thus,
\[ n - ik = \sqrt{1 - i\frac{\sigma_1 - i\sigma_2}{\varepsilon_0 \omega}} \]  
Eq. (110(a))

or
\[ n - ik = \sqrt{1 - \frac{\sigma_2}{\varepsilon_0 \omega}} - i \frac{\sigma_1}{\varepsilon_0 \omega} \]  
Eq. (110(b))

It remains only to separate the real and imaginary parts of Eq. (44), which will yield two equations in \( n \) and \( k \) and thus give \( n \) and \( k \) in terms of the dc conductivity or and the relaxation time \( \tau \). Then we can use our expression for the reflectivity from Eq. 7-76 to generate \( R \) from \( n \) and \( k \).

To carry out the algebra we use the identity:
\[ \sqrt{A + iB} = \frac{R + A}{2} + i \sqrt{\frac{R - A}{2}} \]

where \( R = \sqrt{A^2 + B^2} \).

Letting
\[ A = 1 - \frac{\sigma_2}{\varepsilon_0 \omega} \]

and
\[ B = -\frac{\sigma_1}{\varepsilon_0 \omega} \]

we have
\[ 2n^2 = \left( i - \frac{\sigma_2}{\varepsilon_0 \omega} \right) + \sqrt{1 - \frac{\sigma_2^2}{\varepsilon_0 \omega^2} + \left( \frac{\sigma_1}{\varepsilon_0 \omega} \right)^2} \]  
Eq. (111(a))

\[ 2k^2 = \left( 1 - \frac{\sigma_2}{\varepsilon_0 \omega} \right) + \sqrt{\frac{\sigma_1}{\varepsilon_0 \omega} - \left( \frac{\sigma_1}{\varepsilon_0 \omega} \right)^2} \]  
Eq. (111(b))

Eqs. (111(a)) and (111(b)), together with Eq. (98) give the reflectivity of \( R \). Notice that \( R \) is a function solely of \( \sigma_1, \sigma_2 \) and \( \omega \). Look again at Eq. (108(a, b)) and note that \( \sigma_0 \) can be used to replace \( \tau \) in the expression for \( \sigma_1 \) and \( \sigma_2 \) as:
\[ \sigma_1 = \frac{\sigma_0}{1 + \omega^2 m^2 \sigma_0^2} \]  
Eq. (112(a))
\[
\sigma_2 = \frac{\sigma_0^2 m \omega}{N e^2} \quad \text{Eq. (112(b))}
\]

Eqs. 112(a) and 112(b) show that \( \sigma_1 \), and \( \sigma_2 \), and thus \( R \), depend on frequency \( \omega \), constant \( m/N \), and dc conductivity \( \sigma_0 \). Thus:

\[
R = f(\omega, \sigma_0, m/N) \quad \text{Eq. (113)}
\]

This means that we can use the dc conductivity to predict the reflectivity. Furthermore, if we know the temperature variation of \( \sigma_0 \), we can use this method to calculate \( R \) as a function of temperature. This is a useful result, because it is difficult to measure optical reflectivity as a function of temperature, whereas it is fairly easy to measure \( \sigma_0 \) vs. temperature.

A number of data on electrical conductivity have been amassed for most metals and alloys. Thus, the free-electron model is currently enjoying a great deal of attention as a way of providing reflectivity-vs-temperature information in the study of laser effects.

There is, of course, one problem in using \( \sigma_0(T) \) data to predict \( R \), and that is the parameter \( m/N \). It turns out that \( R \) is fairly insensitive to this parameter at infrared wavelengths. To see this, we show here some numerical illustration. Define

\[
\beta = m/m_0 \quad \text{Eq. (114)}
\]

where \( m_0 \) is the free electron mass, and then the parameter \( \beta/N \) is equivalent to \( m/N \).

Fig. 19 shows a plot of \( \partial R/\partial (\beta/N) \) as a function of \( \beta/N \) for \( \lambda = 10 \mu \text{m} \) and various values of \( \sigma_0 \). Here \( \sigma_0 \) is in units of reciprocal ohm-centimeters. Typical values are, for example, \( \sigma_0 = 10^5 \Omega^{-1}\text{cm}^{-1} \) and \( \beta/N = 10^{-7} \text{cm}^{-1} \) for aluminum. Then the value of \( \partial R/\partial (\beta/N) \) is about \( 7.2 \times 10^{-20} \text{cm}^3 \). If we take a 10% error in \( \beta/N \), we get:

\[
\frac{\partial R}{\partial (\beta/N)} \Delta(\beta/N) = 7.2 \times 10^9 \times 10^{-24}
\]

\[
\Delta R = 0.00072
\]

Since for these values \( R = 0.97366 \), the change in \( R \) is only about 0.1%. We can obtain quite good predictions by the Drude-Lorentz model using the experimental values of \( \sigma_0 \) and the simplest choice for \( \beta/N \), namely one free electron for each valence electron per atom in the metal, and \( \beta = 1 \). For alloys (See Figure 18), it is sufficient to choose the major constituent of the alloy. For example, with stainless steel we choose iron, or two electrons per atom, to compute \( N \) and hence \( \beta/N \).

Figs. 19 and 20 do show the predictions of the free-electrons theory for a variety of metals and some comparison to experimental data [29]. The abrupt change when the metal melts is caused by the abrupt change in the \( \sigma \) conductivity. Notice in the comparison to data that aluminum films give values closet to the theory. This is probably because they prevent the best surfaces. Defects, oxide layers, etc., tend to trap the incident radiation and cause their real surface to absorb more radiation than the ideal surface. These graphs are in terms of absorptance, which is the
Let us return to the expressionism for \( n \) and \( k \) to look at some limiting forms and thus show how these complete expressions reduce to simple relationships. Remember that \( R \) (Eq. (97)) is determined by \( n \) and \( k \) (Equations sets 111), which are in turn obtained from the dc conductivity and \( \mu/N \) (Equations sets 112(a, b)). The variation of \( n \) and \( k \) with wavelength is shown in Fig. 21 for a typical good conductor like aluminum or copper at room temperature. Note that at long wavelengths \( n = k \). We can derive this by using Equations sets (108 (a, b)) for \( \sigma_1 \) and \( \sigma_2 \) but noting that as \( \omega \rightarrow 0 \), \( \sigma_1 \rightarrow \sigma_0 \) and \( \sigma_2 \rightarrow \sigma_0 \omega \tau \).

By substituting these into Equations set (111 (a, b)) for Eq. (111(a)) and region 1, we can readily show that:

\[
n = k = \sqrt{\frac{\sigma_0}{2\varepsilon_0 \omega}} \quad \text{Eq. (114)}
\]

This is called the Hagen-Rubens limit. Note that \( n \) is very large. Under these conditions’ algebra can be used to reduce Eq. (98) to the following form:

\[
R = \frac{n - 1}{n + 1} \quad \text{Eq. (115(a))}
\]
See Fig. 2 in Part I of these series.

We can also comment on the skin depth. We have at long wavelength \((\omega \to 0)\),

\[
\delta = \frac{\lambda}{2\pi} \sqrt{\frac{2\varepsilon_0 \omega}{\sigma_0}} \quad \text{Eq. (117)}
\]

This can be rewritten as:

\[
\delta = \sqrt{\frac{2}{\mu_0 \varepsilon_0 \omega}} \quad \text{Eq. (118)}
\]

Eq. (118) is the common expression for skin depth used at long wavelength.

Finally, we see from Fig. 22 that \(n\) and \(k\) re-converge at short wavelengths. This is called the plasma resonance. To see this, one must look at the behavior of \(n\) and \(k\) over a larger spectrum. We have already discussed the long-wavelength limiting behavior of \(n\) and \(k\). This is the Hagen-Rubens region, where \(N = k\). At short wavelengths, it is easy to show from Equations sets of (112(a, b)) that:

\[
\sigma_1 \to \frac{N^2 e^4}{m^2 \sigma_0 \omega^2} \quad \text{Eq. (119(a))}
\]

\[
\sigma_2 \to \frac{Ne^2}{m \omega} \quad \text{Eq. (119(b))}
\]
Thus Eqs. (111(a) and (111(b)) can be written, for large $\omega$, as:

$$n^2 = 1 - \frac{Ne^2}{\varepsilon_0 \omega^2} \quad \text{Eq. (120(a))}$$

$$k^2 = 0 \quad \text{Eq. (120(b))}$$

Now the plasma frequency is usually defined from Eq. (120(a)) by setting $n = 0$ to yield:

$$\omega_p^2 = \frac{Ne^2}{\varepsilon_0 m} \quad \text{Eq. (121(a))}$$

and thus

$$n^2 = 1 - \frac{\omega^2}{\omega_p^2} \quad \text{Eq. (121(b))}$$

We see, then, that at very high frequencies the free-electron model predicts a transparent behavior ($k = 0$) and the index of refraction approaches that of a vacuum. The transition to this transparent behavior takes place at the plasma frequency, and it is a fairly abrupt transition, as Fig. 22 shows. In fact, some texts call this transition the “ultraviolet catastrophe”.

Note that at $\omega$ near $\omega_p$, equations sets of (119(a, b)) and (120(a, b)) are not valid. For these frequencies we must use the full expression. If we use again the values of $\sigma_0 = 10^7 \Omega^{-1} \text{cm}^{-1}$ and $\beta / N = 10^{-25} \text{cm}^3$, which are appropriate to a good conductor like aluminum at room temperature, the reflectivity looks like Fig. 23. One can see that, in terms of the reflectivity, the transition is very abrupt, indeed.

The optical reflectivity of real metals is, as we have seen, in reasonable accord with the free-electron model at wavelengths in the infrared. The surface, however, must be nearly perfect for the predicted reflectivities to be achieved, and, of course, as the wavelengths approach the visible region band effects become important and the reflectivity shows rapid actuations with frequency. The absorptance of a practical metal surface is still largely an empirical matter. For high-power, continuous-wave radiation by a CO$_2$ laser, some data are available, but very little information on absorptivity as a function of surface temperature under these conditions is available.

Room temperature absorptances are shown in Table 1 for a few materials.

Data on the reflectivity of a metal during actual irradiation by a laser beam are quite difficult to obtain, although this information is central to the problem of laser-material interaction. One classic experiment along these lines was carried out by Bonch-Bruevich, Imas, Romanov, Libenson, and Maitsev in Russia in 1967. They surrounded their specimens with a sphere to monitor the reflected radiation, as shown schematically in Fig. 24. The output of the photodetector is proportional to the reflectance of the specimen.
Table 1  Room-temperature absorptance of aerospace metal and alloys at 10.6 µm for various surface conditions and at normal incidence.

<table>
<thead>
<tr>
<th>Alloy</th>
<th>Metal</th>
<th>Surface condition</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Ideal</td>
<td>Polished</td>
</tr>
<tr>
<td>Al</td>
<td>0.013</td>
<td>0.03</td>
</tr>
<tr>
<td>Au</td>
<td>0.006</td>
<td>0.01</td>
</tr>
<tr>
<td>Cu</td>
<td>0.011</td>
<td>0.016</td>
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<tr>
<td>Ag</td>
<td>0.005</td>
<td>0.011</td>
</tr>
<tr>
<td>2014 Al</td>
<td>0.033</td>
<td></td>
</tr>
<tr>
<td>304 Stainless Steel</td>
<td>0.11</td>
<td></td>
</tr>
<tr>
<td>Ti Alloy (6Al, 4V)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mg Alloy Az-31B</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Fig. 24  Schematic representation of Bonch-Bruevich experiment.

Fig. 25  Reflectance of steel and copper during irradiation by a laser beam [30].

As time passes, of course, the laser pulse heats the surface and the reflectance decreases. An especially interesting feature of these data is the shoulder. The author has suggested that this leveling off is associated with the surface reaching the melting point and pausing at that temperature while the thickness of the molten layer propagates into the solid. In short, order, however, the molten layer begins to heat up and the reflectance continues to decrease. As the power density of the laser pulse reaches its peak and begins to fall, the surface temperature can no longer be maintained, and as the surface cools the reflectance begins to increase again.

In calculation of the laser interacting with metals commonly used material properties are: (for energy balance analysis) density, heat capacity, specific heat ratio, heat conductivity, heat diffusivity, latent heat, melting point, vaporization point; (for stress and momentum analysis): viscosity, modulus of elasticity, shears modulus, Poisson’s ratio, stress-strain constitutive relation.

Some of their results for steel and copper are shown in Fig. 25. The laser pulse (Nd: glass laser, 1.06 µm), with a peak power density of the order of 108 W/cm², is shown as a broken line.
4. Conclusion

In conclusion, we should state that so far we have gathered enough information through first two parts, namely Part I and Part II along Part III here, that allows us to handle Part IV by starting our topics under title of “Mathematical of Laser Absorption in Metals”, where we are going to have a complete understanding of laser interaction with materials which is still a matter of trials and adjustment.

The real physical processes of laser beam interaction (drilling, cutting, welding, or being used as a directed energy weapon application) with materials are very complex. Problem of laser interaction with materials presents many difficulties, both from modeling as well as from experimental sides. One would expect a reasonable description of the main phenomena occurring during laser interaction, but this is complicated because many of physical processes equally contribute to the development of conservation equations, producing draw back because of a great complexity of the equations to be solved. In most instances, this leads to formulation of a model needed to be solved numerically. A lack of pertinent experimental data to compare with, forces one to simplify some equations and use previous analytical and computational work done in this field [31].

References