Materials Response to High Power Energy Lasers
(A Short Course—Part I)

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Abstract: With recent attention to high power energy and its interaction with materials of different type, both in industry and military application, this paper covers a short review course into subject of materials response in respect to such high power energy lasers. In this paper, we are covering laser interaction with solid and going through steps of phase changes, from solid to liquid and finally vapor stage. We describe the radiation wave, propagation wave in a complex form solution, utilizing Maxwell’s equation within dielectric materials, then we look at compression of materials, due to melting and boiling driven by heat transfer energy radiation and conduction induced by these high power energy lasers such as Nd:Yag and CO2 lasers with wavelengths anywhere from 1.6 µm to 10.6 µm. We also look at Hugoniot Elastic Limit (HEL) and spall strength of materials, with the energy lasers dueling with targeted material, where also, physics of hydrodynamics effects due to strong shock is involved. We also talk about certain available computer that allows end user to calculate these phenomena in 1-D to 3-D type scenarios. Although covering all these above issues that are very lengthy write-up proposition, we have tried to be very brief, yet to the point presentation in form of a short course in this paper.

Key words: Radiation wave, electromagnetic and electrical field, laser and laser radiation, Hugoniot limit, heat transfer and heat radiation, laser interaction with matter.

1. Introduction

To start with, we introduce the basic concept of radiation by describing what it is and then we move on to start by deriving wave propagation equation using Maxwell’s Equations and to discussion of the response of materials to laser radiation is presented here, with emphasis change on simple, intuitive models. Topics discussed include optical reflectivity of metals at Infra-Red (IR) wavelengths, laser-induced heat flow in materials, the effects of melting and vaporization, the impulse generated in materials by pulsed wave (PW) or continuous wave (CW) radiation, and the influence of the absorption of laser radiation in the blowoff region in front of the irradiated material.

As we have learned from our college physics, the word LASER, of course, is an acronym for “Light Amplification by the Stimulated Emission of Radiation”, but that is not terribly enlightening. More correctly described a laser is a device for producing light that is almost totally coherent. It works in principle like this: An atom emits a photon of light when it decays from an excited energy state to a lower state; the difference in energy between the two states $\Delta E$ determines frequency $\nu$ according to Eq. (1) as:

$$\Delta E = h\nu$$  \hspace{1cm} (1)

where $h$ is Planck’s constant and its value is given as $h = 6.62606957 \times 10^{-34}$ J.s = 4.135667516 $\times 10^{15}$ eV·s. Eq. (1) in term of energy level is depicted below as Fig. 1.
If we assume the wavelength of emitted photon light designated with symbol $\lambda$ and speed of light with $c$, then relationship between wavelength and speed of light can be written as:

$$c = \lambda v$$  

(2)

Substitution of Eq. (2) into Eq. (1) for frequency $v$, we get a new expression for the Plank-Einstein relationship as:

$$\Delta E = \frac{hc}{\lambda}$$  

(3)

The above equation leads to another relationship involving Plank’s constant $h$. Given $p$ for the linear momentum of a particle (not only a photon, but other particles as well), the de Broglie wavelength $\lambda$ of the particle is given by:

$$\lambda = \frac{h}{p}$$  

(4)

In some applications where it makes sense to use the angular frequency, where the frequency is expressed in terms of radians per second instead of rotation per second or Hertz, it is customary to absorb a factor of $2\pi$ into the Planck constant. The resulting constant is called Reduced Plank Constant or Dirac Constant. It is equal to the Plank constant divided by $2\pi$, and is denoted as $\hbar$ and pronounced $h$-bar.

$$\hbar = \frac{h}{2\pi}$$  

(5)

Therefore, the energy of photon with angular frequency $\omega$, where, $\omega = 2\pi v$, is given as

$$\Delta E = h\omega$$  

(6)

The reduced Plank’s constant is the quantum of angular momentum in quantum mechanics.

The numerical value of reduced Plank constant is given as $\hbar = 1.054571726 \times 10^{-34}$ Js $= 6.58211928 \times 10^{-16}$ eV$\cdot$s.

The above conditions and circumstances are the case for any light source, whether laser, flame, incandescent body, etc.

Atoms emit photon for any conventional light source in a random mode, sporadic manner and spontaneously decay in lower energy state of atom energy structure, when is excited by heat or any other heat generated source, such as electric current. On the other hand physics of laser indicates that the photons are emitted in phase and the electromagnetic radiation behavior types are encountered and more or less, we can describe it as a wave propagation of sinusoidal radiation filed takes place and at a microscopic level can be defined by the following mathematical solution of wave equation in conductor and taking the real part of the solution of the wave equation (i.e. we assumed the general solution is complex quantity type that includes both real and imaginary terms as part of solution) under consideration, then the relationship is presented as below:

$$\mathcal{E}(z, t) = \Re \left[ \mathcal{E}_0 e^{-2\kappa z / \lambda} e^{i\omega (t - nz/c)} \right]$$  

(7)

where

$\mathcal{E}$ is the electric field of the radiation;

$\Re$ stands for the real part of the complex quantity in brackets;

$\mathcal{E}_0$ is the maximum amplitude;

$k$ is the extinction coefficient and vacuum, $k = 0$;

$z$ is the direction in which the wave is propagating;

$\lambda$ is the wavelength;

$t$ is time;

$n$ is the index of refraction and in vacuum $n = 1$;

$c$ is the speed of light in vacuum.

Eq. (7) is just Electric Field solution to set of Maxwell’s Equation inside a linear, homogeneous, and isotropic conducting medium that has electric permittivity $\varepsilon$ and magnetic permeability $\mu$. The solution in general form of vector presentation for electric field using complex notation is:

$$\mathbf{E}(z, t) = \mathbf{E}_0 e^{(-kz)} e^{i(kz - \omega t)}$$  

(8)

Readers of this short-course here can refer to Appendix F of Ref. [1], to find how the solution of wave equation in conductor will result in Eq. (7) above, which is nothing more than the standard representation of the electric field of traveling light wave. You can also look at the quick approach to derive Eq. (7) in next
Section under Wave Equation as well.

Note that: Eq. (7) is a standard representation of the electric field of a traveling lightwave. However, if one measures the electric field at some point in space for light from a conventional source, the sinusoidal variation expressed in Eq. (7) does not appear, for the atoms emitting the light are doing so at random, and the sinusoidal variation due to the emission from each atom is averaged to some time-independent value. This is not true of laser emission, where the individual photons are in phase. Measuring the electric field at a point in space for laser light results in the oscillating predicted by Eq. (7).

However, bear in your mind that this coherence (see Section 3 of this article for more detailed information) is created by taking advantage of an atom in an excited state depending on the quantum mechanical selection rules for transition to a lower state, and there are states from which transition to a lower level is extremely improbable. Such states are called metastable, and an atom not disturbed by outside influence will remain in a metastable state for a very long time. If a metastable atom interacts with a photon of frequency such that Eq. (1) of \( \Delta E = \hbar \omega \), where \( \Delta E \) is the energy difference between the atom’s normal and metastable states, stimulated emission will occur. The atom will decay to its normal state by emitting another photon of frequency \( \nu \), so that the net result is two photons, and the second photon will have the same phase temporally and spatially as the first [2-3].

2. Wave Equation Solution

The solution and obtaining Eq. (7) results can be achieved with four sets of macroscopic Maxwell’s Equations that are written in form of Eqs. (9)-(12) as follows:

\[
\nabla \times \mathbf{E} = \frac{\partial \mathbf{B}}{\partial t} \\
\nabla \times \mathbf{H} = \mathbf{j} + \frac{\partial \mathbf{D}}{\partial t}
\]

\[
\nabla \cdot \mathbf{D} = \rho \\
\nabla \cdot \mathbf{B} = 0
\]

The relationship between \( \mathbf{E} \) (Electric Field), \( \mathbf{D} \) (Electric Displacement), \( \mathbf{B} \) (Magnetic Field) and \( \mathbf{H} \) (Magnetic Field Strength) generally speaking is nonlinear, but in our case of interest for high power laser interaction with materials we can approximate them by a linear model and the relationship in general will depend on the frequency of the radiation field. Note that these parameters and relationship between them describe the material behavior. In case of time harmonic fields, the Fourier transformed field quantities are related according to the following sets of equations:

\[
\mathbf{D}(\mathbf{r}, \omega) = \varepsilon_0 \mathbf{E}(\omega) \mathbf{E}(\mathbf{r}, \omega)
\]

\[
\mathbf{B}(\mathbf{r}, \omega) = \mu_0 \mu(\omega) \mathbf{H}(\mathbf{r}, \omega)
\]

With help of Eqs. (9), (10), (13) and (14), the wave equation can be established as Eq. (15) below:

\[
\nabla (\nabla \cdot \mathbf{E}) - \Delta \mathbf{E} = -\mu_0 \varepsilon_0 \epsilon \frac{\partial^2 \mathbf{E}}{\partial t^2}
\]

In homogeneous media and with zero space charge \( \nabla \cdot \mathbf{E} = 0 \), and with \( \mu_0 \varepsilon_0 = 1 / c^2 \), Eq. (15) reduces to the following form:

\[
\Delta \mathbf{E} = \frac{\epsilon}{c^2} \frac{\partial^2 \mathbf{E}}{\partial t^2}
\]

A solution of this Partial Differential Eq. (16) is the equation of plane wave, written as Eq. (17):

\[
\mathbf{E}(\mathbf{r}) = \mathbf{E}_0 e^{j(kz - \omega t)}
\]

with

\[ k — \text{Complex wave number}; \]
\[ \omega — \text{Real angular frequency}. \]

The complex wave number is:
\[
\begin{cases}
\frac{k}{k_0} = \sqrt{\varepsilon} = k_0 n = \\
(k_{\text{real}} + ik_{\text{imaginary}}) = k_r + ik_i
\end{cases}
\]

where \( n \) is the complex index of refraction. The plane wave solution can also be cast into the form of:
\[
\vec{E}(x,y,z,t) = \hat{E}_0 e^{-(k_x x + k_y y)} e^{i(k_z z - \omega t)}
\]

If the imaginary part of the complex wave number \( k_i > 0 \), the wave decays exponentially within the material.

Now that we have established a derivation of Eq. (7) is valid relationship to measure the electric field \( \vec{E} \) in a point in space for a laser emission, where the individual photons are in phase. This equation also allows not only to measure the electric field \( \vec{E} \) at a point in space for laser light, but it will also result in predicting the oscillating of \( \vec{E} \) as well. This is not true for a light from a conventional source, and Eq. (7) does not hold for measuring such electric field at some point in space in order to express the sinusoidal variation, for the atoms emitting the light that are doing so in random, and the sinusoidal variation due to the emission from each atom needs to be averaged out to some, time-dependent value.

The fact is that laser is a very coherent source of light where this coherency is created by taking advantage of stimulated emission in materials in which metastable states can be induced, then by selected rules of quantum mechanics; we know that the lifetime of an atom in an excited energy state depends on these rules for transition to a lower state. Bear in your mind that, there are states from which transition to a lower level is extremely impossible and such states are called metastable states and an atom that is not going to be distributed by outside influence, will remain in a metastable state for a very long time.

If a metastable atom interacts with a photon of frequency so that Eq. (1) holds, whereas we said \( \Delta E \) is the energy difference between the atom’s normal and metastable states, stimulated emission will occur. The atom will decay to its normal state by emitting another photon of frequency \( \nu \), so that the net result is two photons, and the second photon will have the same phase temporally and spatially as the first.

3. High Energy Laser Characteristics

LASER is an acronym for Light Amplification by Stimulated Emission of Radiation. Spontaneous emission is the process by which an excited atom spontaneously emits a photon. Electrons go from excited to a resting state when a photon of energy is released. Photon emission can be stimulated by an external source of energy that will increase the population of excited electrons; a process known as pumping. A laser contains a laser chamber, a lasing medium (solid, liquid, or gas) and an external source of energy. Stimulated emission occurs when the external source of energy causes electrons to be excited in the lasing medium. A cascade reaction is generated when these excited electrons release photons, which then collide with other excited electrons in the lasing medium and cause a release of many identical photons at the same time. Laser light continues to be generated as long as the above cascade perpetuates.

Laser light has the following properties:

1. Coherence: laser beams are both temporally and spatially coherent. This phenomenon results from stimulated emission, and allows laser beams to have a high power density

2. Collimation: laser beams are parallel to each other (i.e., ignoring thermal blooming while traveling through atmospheric environment for a high energy beam), and therefore exhibit collimation. A collimated beam is created in the laser chamber when light is reflected between two mirrors and only the exit of parallel waves is allowed. Collimation allows laser light to travel long distance without loss of intensity. In practice, a lens on a laser focuses the parallel light beam down to the smallest possible spot size, or the diffraction-limited spot, to allow the light to focus on the target.
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(3) Radiometry: the four main concepts in understanding laser light and target interactions are
- Energy;
- Power;
- Fluence, and
- Irradiance.

The amount of light emitted from a laser can be quantified by both energy and power. Energy represents work that is measured in Joules, while power is measured in Watts or Joules per Second which is rate at which energy is expended. The intensity of the laser beam on the target is a function of the area of the target over which it is spread that is known as Spot Size.

**Spot Size** = **Cross-Sectional Area of the Laser Beam**

Fluence which is measured in Joules per Square Centimeter is the energy density of a laser beam.

**Fluence** = Watts × Seconds/Cm² = Joules/Cm² + Laser output × Pulse Duration/Spot Size

Irradiance is measured in Watts per Square Centimeter which refers to the power density of a continuous wave laser beam, and it is inversely proportional to the Square Root of the Radius of the Spot Size.

**Irradiance** = Watts/cm² = Laser Output/Spot Size

Exposure time, fluence, and irradiance of a laser can be altered depending on the particular desired laser dueling target and conditions and circumstances that laser engages the target.

Laser interacts with target in four possible ways as:

1. **Reflection** \( R \): takes place when light “bounces off” the target surface without entry into the target thickness secondary to difference in the refractive index at engagement point and the environment that incoming laser beams travel through. Increasing the angle incidence increases the amount of light reflected. Damages to target surface or target itself occur with particular lasers if adequate reflection of laser beams occurs and there is proper protection employed.

2. **Absorption** \( A \): the absorptivity is the ratio of power that is deposited within the workspace and the power of the incident radiation.

3. **Transmission** \( T \): occurs when the laser beam passes through transparent target without altering either the target surface or the light itself.

4. **Scattering** \( S \): this refers to fragmentation of light after it has entered the target skin, and it results from the interaction of light with varied elements that make up target layers. When scattering occurs, light is dispersed over a larger area within the target, and the depth of penetration (Skin Depth) of the light beam is reduced at the same time.

As we said above, if a metastable atom interacts with a photon of frequency such as \( \Delta E = hν \), where \( \Delta E \) is the energy difference between the atom’s normal and metastable states, stimulated emission will occur. The atom will decay to its normal state by emitting another photon of frequency \( ν \), so that the net result is two photons, and the second photon will have the same phase temporally and spatially as the first.

In laser, then, one establishes a large number of atoms in metastable states and arranges the optics to increase the likelihood of stimulated emission. Schematically, a typical laser oscillator looks like Fig. 2. The pumping radiation (for example, light from a flashlamp) excites the atoms in the lasing medium (for example, Cr⁺⁺⁺ ions in ruby).

In the decay process (if we have a successful laser), a large number of ions are left in an metastable state; this is called a population inversion. As some atoms begin to decay, they stimulate others to decay. But this alone would not provide a laser since the emission would occur in random directions. The role of reflection is very important; the photons moving perpendicular to

![Fig. 2  Schematic representation of a laser.](image)
the reflectors pass through the medium many times and on each pass more and more atoms are caused to emit. This results in the build-up of a very strong coherent light signal that travels in a single direction. Useful light output is obtained by making one of the mirrors a partial reflector [2].

Note that: Skin Depth in electromagnetic wave is defined as that, the skin depth is a measure of the penetration of a plane electromagnetic wave into a material. The magnitude of the field in the material is proportional to \( e^{-x/\delta} \) where \( \delta \) = skin depth, \( x \) = distance into the material from the surface where the wave is incident. Looking at Fig. 3, we see conceptual illustration of distribution of current flow in a cylindrical conductor, shown in cross section.

For alternating current (AC), the current density decreases exponentially from the surface towards the inside. The skin depth, \( \delta \), is defined as the depth where the current density is just 1/e (about 37%) of the value at the surface; it depends on the frequency of the current and the electrical and magnetic properties of the conductor.

In holistic way, skin effect is the tendency of an AC of electric to become distributed within a conductor so that the current density is the largest near the surface of the conductor and decreases exponentially with greater depths in the conductor. The electric current flows mainly at the “skin” of the conductor, between the outer surface and a level called the skin depth. Skin depth depends on the frequency of the AC; as frequency increases, current flow moves to the surface, resulting in less skin depth. Skin effect reduces the effective cross-section of the conductor and thus increases its effective resistance. Skin effect is caused by opposing eddy currents induced by the changing magnetic field resulting from the AC. At 60 Hz in copper, the skin depth is about 8.5 mm. At high frequencies, the skin depth becomes much smaller.

The general formula for the skin depth when there is no dielectric or magnetic loss is given by Eq. (20) as:

\[
\delta = \sqrt{\frac{2\rho}{\omega \mu}} \sqrt{1 + (\frac{\omega \mu}{\rho \omega})^2 + \rho \omega \epsilon} 
\]

where:
- \( \rho \) = resistivity of the conductor;
- \( \omega \) = angular frequency of current = \( 2\pi f \), where \( f \) is the frequency;
- \( \mu \) = permeability of the conductor, \( \mu, \mu_0 \);
- \( \mu_r \) = relative magnetic permeability of the conductor;
- \( \mu_0 \) = the permeability of the free space;
- \( \epsilon \) = permittivity of the conductor, \( \epsilon, \epsilon_0 \);
- \( \epsilon_r \) = relative permittivity of the conductor;
- \( \epsilon_0 \) = the permittivity of free space.

At frequencies much below \( 1/\rho \epsilon \) the quantity inside the large radical is close to unity and the formula is more usually given as Eq. (21):

\[
\delta = \frac{2\rho}{\sqrt{\omega \mu}}
\]

This formula is valid at frequencies away from strong atomic or molecular resonances (where \( \epsilon \) would have a large imaginary part) and at frequencies that are much below both the material’s plasma frequency (dependent on the density of free electrons in the material) and the reciprocal of the mean time between collisions involving the conduction electrons. In good conductors such as metals all of those conditions are ensured at least up to microwave frequencies, justifying this formula’s validity. For example, in the case of copper, this would be true for frequencies much below 1,018 Hz.
However, in very poor conductors, at sufficiently high frequencies, the factor under the large radical increases. At frequencies much higher than \(1/\rho\varepsilon\) it can be shown that the skin depth, rather than continuing to decrease, approaches an asymptotic value:

\[
\delta \approx 2\rho \sqrt{\frac{\omega}{\mu}}
\]  

(22)

This departure from the usual formula only applies for materials of rather low conductivity and at frequencies where the vacuum wavelength is not much larger than the skin depth itself. For instance, bulk silicon (undoped) is a poor conductor and has a skin depth of about 40 meters at 100 kHz (\(\lambda = 3,000\) m). However, as the frequency is increased well into the megahertz range, its skin depth never falls below the asymptotic value of 11 meters. The conclusion is that in poor solid conductors such as undoped silicon, the skin effect does not need to be taken into account in most practical situations: any current is equally distributed throughout the material’s cross-section regardless of its frequency.

It is interesting to look at a few examples of the intensity of laser light. In a typical ruby laser, the concentration \([4]\) of Cr\(^{3+}\) ions is about \(2 \times 10^{19}\) cm\(^{-3}\), and population inversions are of the order of \(3 \times 10^{16}\) cm\(^{-3}\). Crudely speaking, we can think of creating \(3 \times 10^{16}\) quanta/cm\(^3\) in the lasing medium. Since we have arranged the laser, the output is in a single direction, and since photons move with the speed of light, we obtain \(3 \times 10^{16} \times 3 \times 10^{10} = 9 \times 10^{26}\) quanta/cm\(^2\)-sec from the laser. For ruby, the lasing wavelength is 6,943 \(\AA\), and since the energy of each quanta is \(\hbar \nu\), one can readily calculate that the output is about \(2.5 \times 10^{8}\) W/cm\(^2\).

Let us compare this to the power that a hot body, say the sun, emits at the same wavelength with a similar bandwidth. This can be calculated by the use of Planck’s radiation law, and then we can write the following relationship as:

\[
U_w = \frac{\hbar \omega^3}{\pi^2 c^3} \frac{1}{e^{\hbar \omega/kT} - 1}
\]  

(23)

\(U_w\) is the energy, per unit volume and per unit bandwidth, radiated by a blackbody at temperature \(T\); \(k\) is Boltzmann’s constant. The radiation leaves the black-body source at rate \(c\), so the power radiated per unit area of the source, per unit bandwidth, is:

\[
I_w = \frac{cU_w}{4} = \frac{\hbar \omega^3}{\pi^2 c^3} \frac{1/4}{e^{\hbar \omega/kT} - 1}
\]

(24)

If we use the sun’s temperature of 6,000 K, and \(X = 6,943\) \(\AA^0\),

\[
I_w \approx 2 \times 10^{-5}\ \text{erg/cm}^2
\]

(25)

For the ruby laser, a typical line width is 3 \(\AA\), so \(\Delta \omega = 1.2 \times 1,012\) s\(^{-1}\). Thus, the power density at the source is

\[
I \approx 2.5 \times 10^7\ \text{erg/cm}^2\text{s} \approx 2.5\ \text{W/cm}^2
\]

(26)

Thus, the power density for comparable narrow-bandwidth, nearly single-frequency light is much greater at a laser source than at a conventional hot-body source, because laser light is coherent.

Before we go forward to the next step, let us pause and briefly present the Planck’s Energy Density Distribution using Rayleigh’s Energy Distribution by focusing on the understanding the nature of the electromagnetic radiation inside the cavity, by considering the radiation to consist of standing waves having a temperature \(T\) with nodes at the metallic surface. By arguing that, these standing waves are equivalent to harmonic oscillators, for they result from the harmonic oscillations of a large number of electrical charges, electrons that are present in the walls of metallic surface of cavity. When cavity is in thermal equilibrium, the electromagnetic energy density inside the cavity is equal to the energy density of the charged particles in the walls of the cavity, and the average total energy of the radiation leaving cavity for the average energy of the oscillators along with the number of
standing waves or mode of radiation in the frequency interval $\nu$ to $\nu + d\nu$ is written as:

$$N(\nu) = \frac{8\pi \nu^2}{c^3} \tag{27}$$

where $c = 3 \times 10^8$ m/sec is the speed of light and the quantity $\left(\frac{8\pi \nu^2}{c^3}\right) d\nu$ that gives the number of modes of oscillation per unit volume in the frequency range $\nu$ to $\nu + d\nu$ is given by:

$$u(\nu, T) = N(\nu)\langle E \rangle = \frac{8\pi \nu^2}{c^3} \langle E \rangle \tag{28}$$

where $\langle E \rangle$ is the average energy of the oscillators present on the walls of the cavity or of the electromagnetic radiation in that frequency interval and the temperature dependence of $u(\nu, T)$ is buried in $\langle E \rangle$.

Now question is how we can calculate the average energy $\langle E \rangle$? According to classical thermodynamics and equipartition theorem, all oscillators in the cavity have the same mean energy, irrespective of their frequencies.

$$\langle E \rangle = \frac{\int_{0}^{\infty} E e^{-E/(kT)} dE}{\int_{0}^{\infty} e^{-E/(kT)} dE} = kT \tag{29}$$

where $k = 1.3807 \times 10^{-23}$ J/K is the Boltzmann constant. An insertion of Eq. (29) into Eq. (28), leads to the Rayleigh-Jeans formula:

$$u(\nu, T) = \frac{8\pi \nu^2}{c^3} kT \tag{30}$$

Eq. (30) except for low frequencies is in total disagreement with experimental data: $u(\nu, T)$ as given by Eq. (30) diverges for high values of $\nu$, whereas experimentally it must be finite per Fig. 4.

Moreover, integrating Eq. (30) overall frequencies, the integral diverges which is indication of that the cavity contains an infinite amount of energy. Historically, this was called the ultraviolet catastrophe, for Eq. (30) diverges for high frequencies within the ultraviolet range.

Now, studying the Plank’s Energy Density Divergence an interpolation between Wien’s rule and the Rayleigh-Jeans rule—Plank succeeded in avoiding the ultraviolet catastrophe and proposed an accurate description of blackbody radiation. He considered that the energy exchange between radiation and matter must be discrete rather than continuum. His postulation indicates that the energy radiation of frequency $\nu$ emitted by oscillating charges from the walls of the cavity must come only in integer multiples of $h\nu$ as:

$$E = nh\nu \quad n = 0, 1, 2, 3, \ldots \tag{31}$$

where $h$ is a universal Plank’s constant and $h\nu$ is the energy of a “quantum” of radiation, while $\nu$ represents the frequency of the oscillating charge particle in cavity’s wall as well as the frequency of the radiation emitted from the walls. This is because the frequency of the radiation emitted by an oscillating charged particle is equal to the frequency of oscillation of the particle itself [5]. Eq. (31) is known as Plank’s Quantization Rule for energy or Plank’s Postulate.

Therefore, assuming that the energy of an oscillator is quantized, Plank showed that the correct thermodynamic relation for the average energy can be obtained by merely replacing the responding to the discreteness of the oscillator’s energies given as:

$$\langle E \rangle = \sum_{n=0}^{\infty} \frac{nh\nu e^{-nh\nu/kT}}{e^{h\nu/kT} - 1} = \frac{h\nu}{e^{h\nu/kT} - 1} \tag{32}$$

Using a variable change $\beta = 1/(kT)$, we have

$$\langle E \rangle = -\frac{\partial}{\partial \beta} \ln \left( \int e^{-\beta E} dE \right) = -\frac{\partial}{\partial \beta} \ln \left( \frac{1}{\beta} \right) = 1/\beta = kT . \tag{32}$$

To drive Eqs. (7)-(14), one needs: $1/(1 - x) = \sum x^n$ with $x = e^{-h\nu/kT}$. 

\[ \begin{align*}
1/(1 - x) = \sum_{n=0}^{\infty} x^n
\end{align*} \]
and hence by inserting Eq. (32) into Eq. (28), the energy density per unit frequency of the radiation emitted from the hole of a cavity is given by:

\[ u(\nu, T) = \frac{8\pi\nu^2}{c^3} \frac{h\nu}{e^{h\nu/kT} - 1} \]  

(33)

Eq. (33) is known as Plank’s distribution and gives an exact fit to various experimental radiation distribution, as displayed in Fig. 4 and the numerical value of \( h \) obtained by fitting Eq. (33) with the experimental data is

\[ h = 6.626 \times 10^{-34} \text{ J} \cdot \text{sec} \]

We should note that, as shown in Fig. 4, we can rewrite Plank’s energy density Eq. (33) to obtain the energy density per unit wavelength as:

\[ \tilde{u}(\lambda, T) = \frac{8\pihc}{\lambda^5} e^{hc/\lambda kT} \frac{1}{e^{hc/\lambda kT} - 1} \]  

(34)

In above we claimed for ruby laser of wavelength 6,943 Å, the energy of each quanta is \( h\nu \), one can calculate that the output is about \( 2.5 \times 10^8 \) W/cm². If we compare this degree of the power that a black body produces, let us assume sun in this case that emits at the same wavelength with a similar bandwidth, using the Planck radiation law, in term of angular frequency \( \omega = 2\pi \nu = 2\pi c/\lambda \) we can calculate:

\[ U_{\omega} = \frac{h\omega^3}{\pi^2c^3} \frac{1}{e^{h\omega/kT} - 1} \]  

(35)

\( U_{\omega} \) is the energy, per unit volume and per unit bandwidth, radiated by blackbody at temperature \( T \) and \( k \) is Boltzmann’s constant. The radiation leaves the black-body source at rate \( C \), so the power radiated per unit area of the source, per unit bandwidth, is

\[ I_{\omega} = \frac{cU_{\omega}}{4} = \frac{h\omega^3}{\pi^2c^2} \frac{1}{e^{h\omega/kT} - 1} \]  

(36)

If we use the sun’s temperature of 6,000 K, and \( \lambda = 6943 \) Å, then we can show that:

\[ I_{\omega} \approx 2 \times 10^{-5} \text{erg/cm}^2 \]

For the ruby laser, a typical line width is 3 Å, so \( \Delta\omega \approx 2 \times 10^{12} \) sec⁻¹. Thus, the power density at the source is

\[ I \approx 2 \times 10^7 \text{erg/cm}^2 \cdot \text{sec} \approx 2.5 \text{ W/cm}^2 \]

Thus, the power density for comparable
narrow-bandwidth, nearly single-frequency light is much greater at a laser source than at a conventional hot-body source, because laser light is coherent.

4. Conclusion

As we indicated in this part of short course mainly PART I, we have stated series of article on the subject of Materials Responses to High Power Energy Lasers and continue these series by starting to introduce the Laser Light Propagation either in vacuum or through atmosphere by also introducing thermal blooming effects as well, then we cover, subjects such as optical reflectivity, thermal responses of materials by looking at Latent Heat of Fusion as well as Vaporization, No Phase Changes in both Semi-Infinite Solid or Slab of Finite Thickness, Melting and Vaporization and then move on to Effects of Pulsed or Continuous Laser Radiation as well, throughout of next few parts that we report them as further Short Courses content.

This includes Power Levels of Pulsed Lasers, Material Vaporization Effects and Effects from Absorption of Radiation in the Plume and so forth and so on.

References