Nonlinearity in Stock Exchange Markets:  
The Case of Bist 100 Indices

Jamilu Said Babangida
Department of Economics, Ahmadu Bello University, Nigeria; Department of International Finance, Istanbul Commerce University, Turkey; Department of Economics, Ibn Haldun University, Turkey

In this paper, using data for the Bist 100 index, we investigate the presence of nonlinearities by employing several nonlinearity tests. The Brock, Dechert, and Scheinkman (BDS) and runs tests were first applied to the series to show an initial indication of nonlinearity. The findings for the BDS and runs test of randomness were followed by other sets of direct nonlinearity tests developed by White (1989), Terasvirta (1993), Keenan (1985), and Tsay (1986). Also, the Threshold Autoregression (TAR) test is employed as a final test to confirm the existence of nonlinearity in the Turkish stock exchange market. From the results of the nonlinearity test, it is concluded that the Bist 100 index is characterised by the presence of nonlinearities and cycles. This finding is in contrast with the efficient market hypothesis (EMH) implying that the Turkish stock exchange market is inefficient.

Keywords: stock market, nonlinearity, efficient market hypothesis, Bist 100 index

Introduction

In recent time, there have been inconclusive debates on whether stock prices are characterised by randomness or nonlinear dependence. This behaviour could be underpinned on some sources of information lending a stage to the test for efficient market hypothesis (EMH). Numerous empirical works have been conducted to test for the validity of efficient market hypothesis for many stock markets across the globe (Al-Loughani & Chappell, 1997; Dockery & Kavussanos, 1996; Fama, 1970; 1998; Lo & MacKinlay, 1987; Summers & Poterba, 1988, etc.). These debates on the validity of efficient markets or presence of nonlinearity has resulted in a number of empirical studies to investigate whether the stock market is in fact “efficient” or not (Choudhry, Papadimitriou, & Shabi, 2016; Kim, Mollick, & Nam, 2008; McMillan, 2001; 2003; Sarantis, 2001, etc). One common feature of the empirical works is that the presence of nonlinearities and cycles in the market results to stock markets being inefficient (Gozbasi, Kucukkaplan, & Nazlioglu, 2014). In line with the works of Hsieh (1989; 1991) and Scheinkman and Lebaron (1989), nonlinearity in stock indices became a crucial discourse for several studies and a debate that attracted attention from researchers and policy makers (Caraiani, 2012; Hyde & Bredin, 2011; Lim & Liew, 2003, etc.).

Acknowledgement: This paper would not have been possible without the exceptional support of my supervisor, Professor Serkan Çankaya. His enthusiasm, knowledge and exacting attention to detail have been an inspiration for completing this paper.

Jamilu Said Babangida, MSc. Economics, Department of Economics, Ahmadu Bello University, Nigeria; Department of International Finance, Istanbul Commerce University, Turkey; Department of Economics, Ibn Haldun University, Turkey.

Correspondence concerning this article should be addressed to Jamilu Said Babangida, Department of Economics, Ahmadu Bello University, Zaria. PMB 1013, Zaria, Kaduna, Nigeria.
Several studies have been undertaken for the stock market with the adoption of nonlinear approaches across the world. For instance, Sarantis (2001) found the presence of asymmetric cycles in the stock prices for most G7 countries. In the same vein, Kim et al. (2008) argued that the nonlinear model explains the characteristics of the stock returns better than a linear model for G7 economies. In a similar study for Central and Eastern European Countries (CEECs), Caraiani (2012) found clear indication of nonlinear dependence and chaotic dynamics in returns of the CEE stock market indices. Moreover, the presence of inefficient market for the Turkish stock exchange market is confirmed by the study of Gozbasi et al. (2014). They found evidence of no strong form of the EMH in the Turkish which implies that profitable investment cannot be achieved only by assessing the stock market. They argued that any shock to the market have a significant effect on the Turkish market. Similarly, Lim and Liew (2003) tested the nonlinearity in financial markets using data for Association of Southeast Asian Nations (ASEAN) five countries for exchange rate and stock indices. They found strong presence of nonlinear dynamics in the financial markets following a Smooth Transition Autoregression (STAR) process which suggested that financial markets are governed by nonlinear behaviour in the ASEAN countries.

The nonlinear behavior in the stock market is underpinned by numerous theoretical and empirical literature (Sarantis, 2001; Lim & Liew, 2003; Rashid, 2007; Kim et al., 2008; Hyde & Bredin, 2011; Caraiani, 2012, etc.). Therefore, this study tends to fill this gap by investigating the presence of nonlinearity in the stock markets of Turkish economy. The use of nonlinear models in the new strands of literature gives comprehensive information about the behaviour of economic and financial series. Indeed, on theoretical and empirical ground, nonlinear models provide some comprehensive information capable of explaining the behaviours in stock prices. The battery of such in exhaustive information includes transaction cost, heterogeneous agents’ beliefs, noise traders, differences in investment horizon, short sales, diversity in risk profiles, bid-ask spreads, herd behaviour, market frictions, etc. (Sarantis, 2001; Aslanidis, Osborn, & Sensier, 2002; McMillan, 2003; Kim et al., 2008; Hsieh, 1991). Therefore, assuming linearity in stock market could be misleading due to non-accountability for the strong asymmetry and dynamics inherent in the stock exchange market. It is on this basis that this study aims to test for the existence of nonlinear dependence became extremely popular in financial economics.

Surprisingly, despite the economic explanation of the presence of nonlinearities and cycles in stock prices and empirical literature on the developed economies stock markets, nonlinearities in stock indices of emerging economies have not been sufficiently investigated. In light of this, the core purpose of this study focuses on extending the empirical investigating on the presence of nonlinearities in the dynamics of the returns of Turkish stock market. We found evidence of nonlinearities and cycles in the stock market implying that that Turkish stock exchange market is an inefficient market.

The remainder of the paper comprises of methods and empirical results in Section 2 and Section 3 concludes the paper.

**Methods and Empirical Results**

The datasets of this study consist of stock market index for Turkey obtained from Yahoo Finance Database. A total observation of 2808 from 05:01:2009 to 28:02:2020 is used. In this study, the stock indices would be transformed in line with Lim and Liew (2003) to a continuously compounded percentage returns.

\[ r_t = 100 \times \ln \left( \frac{p_t}{p_{t-1}} \right) \]
Where \( p_t \) is the adjusted closing price at time \( t \) and \( p_{t-1} \) the price on the previous time \( t-1 \) or the opening price at time \( t \). The stock return was tested for the presence of stationarity. The analysis on whether shocks to Bist 100 returns have long run effect on their future development across time is tested using Augmented Dickey Fuller (ADF) and Phillips-Perron (PP) unit root tests. ADF and ADF\(_T\) are respectively the test with intercept and no trend; and intercept with a trend. The ADF 1%, 5%, and 10% critical values are -2.567, -2.862, and -3.432 for the case of no trend; and -3.128, -3.411, and -3.961 when there is a trend. Correspondingly, PP and PP\(_T\) are test with intercept and no trend and intercept with trend respectively. The critical values for PP at 1%, 5%, and 10% significant levels are -2.567, -2.862, and -3.432 when there is no trend and -3.128, -3.411, and -3.961 for the case of trend. The result below shows that the null hypothesis of unit root is rejected for the Turkish stock returns at levels. This implies that the stock return is stationary at levels.

<table>
<thead>
<tr>
<th>Country</th>
<th>Index</th>
<th>ADF</th>
<th>ADF(_T)</th>
<th>PP</th>
<th>PP(_T)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Turkey</td>
<td>BIST100</td>
<td>-49.5076</td>
<td>-49.5080</td>
<td>-49.5452</td>
<td>-49.5821</td>
</tr>
</tbody>
</table>

Note. Source: Author’s computation.

The study then proceeds with the set of five nonlinearity test to investigate the presence of nonlinearity in Bist 100 index. Firstly, following Caraiani (2012) and Babangida (2020), we employ the Brock, Dechert, and Scheinkman (BDS) test developed by Brock, Dechert, and Scheinkman (1987). The BDS test as a nonparametric test is used to test for linearity dependence and identically distribution (IID) based on correlation integral with a repeated temporal pattern.

Given the daily time series of Bist 100 stock prices to be \( X_t, t = 1, 2, \ldots n \) with number of autocorrelation history as \( X^n_t = (X_t, X_{t-1}, X_{t-2}, \ldots X_{t-n+1}) \), the correlation integral at the embedding m-dimension would be represented as

\[
C_{m,T}(\varepsilon) = \sum_{\varepsilon < \varepsilon} I_{\varepsilon}(X^n_t, X^n_\tau) \frac{2}{T_n(T_{n-1})}
\]

Where \( T_n = T - (n - 1) \) and \( I_{\varepsilon}(X^n_t, X^n_\tau) \) is an indicator function. The indicator function becomes one if the sup norm \( \| X^n_t - X^n_\tau \| < \varepsilon \) and becomes zero when \( \| X^n_t - X^n_\tau \| > \varepsilon \). The correlation integral helps in estimating the probability that any n-dimensional parts are within a distance of \( \varepsilon \) of each other.

\[
p(|X_t - X_\tau| < \varepsilon, |X_{t-1} - X_{\tau-1}| < \varepsilon, \ldots, |X_{t-n+1} - X_{\tau-n+1}| < \varepsilon)
\]

In the equation above, the \( X_t \) is IID, the probability should be equal to the following function in the limiting case.

\[
C_{m,T}(\varepsilon)^n = p(|X_t - X_\tau| < \varepsilon)^n
\]

Then the BDS statistics would be given as

\[
V_n = \sqrt{T} \frac{C_{m,T}(\varepsilon) - C_{1,T}(\varepsilon)^n}{S_nT}
\]

Where \( S_n, T \) is the standard deviation. The BDS statistics converges in distribution to \( N(0, 1) \) under fairly moderate regularity conditions. Table 2 presents the results for the BDS test. In line with the BDS literature, there are two important parameters in BDS estimation; the choice of embedding dimension \( m \) and the distance \( \varepsilon \). Due to a high number of observations for this study and with the aim of achieving reliability in results, two to seven dimensions are chosen for this study. Also, the distance parameter \( \varepsilon \) is chosen as a factor to depend
on the standard deviation of the series with four different values (i.e., 0.5, 1, 1.5, and 2). As can be seen from Table 2, the BDS statistics for these indices clearly indicate the rejection of the null hypothesis of identically independently distributed (IID). This finding suggests that the BDS test detects strong nonlinear dependence in Turkey stock indices over the period of study. This is not a direct test for nonlinearity. However, rejecting the null hypothesis of independence indicates some form of nonlinear dependency in stock prices. This can be interpreted as an evidence of inefficient market in Turkey stock exchange market. That is, the Bist 100 stock return does not support the efficient market hypothesis.

Table 2
BDS Results for Bist 100

<table>
<thead>
<tr>
<th>σ/m</th>
<th>0.5σ</th>
<th>σ</th>
<th>1.5σ</th>
<th>2σ</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>12.35097</td>
<td>13.54721</td>
<td>13.11933</td>
<td>12.13090</td>
</tr>
<tr>
<td>3</td>
<td>15.90372</td>
<td>16.73676</td>
<td>15.82564</td>
<td>14.06701</td>
</tr>
<tr>
<td>4</td>
<td>19.61887</td>
<td>19.21322</td>
<td>17.46529</td>
<td>15.51974</td>
</tr>
<tr>
<td>5</td>
<td>23.00716</td>
<td>21.38719</td>
<td>18.77014</td>
<td>16.49853</td>
</tr>
<tr>
<td>6</td>
<td>26.84870</td>
<td>23.51644</td>
<td>19.71509</td>
<td>16.97405</td>
</tr>
<tr>
<td>7</td>
<td>32.11396</td>
<td>25.66781</td>
<td>20.21888</td>
<td>16.91522</td>
</tr>
</tbody>
</table>

Note. Source: Author’s computation.

Secondly, the runs test is used to test the randomness of the series, although it does not point directly to nonlinearity. The results for the runs test are presented in Table 3. The null hypothesis of randomness was rejected for turkey. The Turkish stock exchange market is assumed to account for all available information that determines the value of stock in the market. In other words, Bist 100 stock prices are instantaneously adjusted based on information that randomly arrives to the market (Fama, 1970). The finding for Bist 100 index contradicts the presence of “random walk model” due to rejection of null of randomness. In the random walk model, the future stock prices are assumed to be significantly determined by current stock price information. This result could be as a result of the nonlinear and cyclical characteristics in the stock returns in the Turkish stock exchange market. This evidence is in support with previous studies (Gozbasi et al., 2014). Similarly, the runs tests are not a direct test for nonlinearity but the finding also confirms the invalidity of the efficient market hypothesis. Both BDS and runs tests can be regarded as a first indication of nonlinear feature through the independence and randomness of the indices respectively. However, financial theory or economic theory may suggest performing a test for a specific form of nonlinearity and building a nonlinear time series model for the detected form of nonlinearity. This leads to other sets of nonlinearity tests.

Table 3
Runs Test

<table>
<thead>
<tr>
<th>Index</th>
<th>Test Statistics</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>BIST100</td>
<td>-3.0954***</td>
<td>0.002</td>
</tr>
</tbody>
</table>

Note. *** p<0.01, ** p<0.05, * p<0.1

Turning to the third test for this study, we follow the neural network test of White (1989) and modification of Terasvirta (1993) for the case of neglected nonlinearity. In this model, the threshold is reached with the integration of selected inputs. Then, an action is transmitted with a delay to other units of the network once a threshold is reached. Generally, incoming signals from those inputs are summed by the hidden units so that an
output is produced by means of an activation function (Bisaglia & Gerolimetto, 2014). The activation function is specified as follows:

\[ \Phi(\bar{X}', y_j) \]

Where \( \Phi \) is the logistic function and \( \bar{X} = (1, X_1, \ldots, X_k) \). This is passed to the outer layer which is specified as:

\[ f(X, \delta) = \beta_0 + \sum_{j=1}^{q} \beta_j \Phi(\bar{X}', y_j), q \in \mathbb{N} \]

Here \( \beta_0, \beta_1, \ldots, \beta_q \) are hidden to output weights and \( \delta = (\beta_0, \beta_1, \ldots, \beta_q, y_1', \ldots, y_q') \). In line with White (1989) and Terasvirta (1993), the neural network model is augmented by connections from input to output by employing a single hidden layer network. The output in this case is

\[ 0 = \bar{X}' \theta + \sum_{j=1}^{q} \beta_j \Phi(\bar{X}', y_j) \]

The null hypothesis of linearity in this case would be equal zero which is the optimal weight of the network. That is, \( \beta_j^0 = 0 \) for \( j = 1, 2, \ldots, q \) given \( q \) and \( y_j \). Alternatively, the neural network test can be implemented as a Lagrange multiplier test using operations rule:

\[ \begin{align*}
H_0: & E(\Phi_t e_t^2) = 0 \\
H_1: & E(\Phi_t e_t^2) \neq 0 \text{ Hypothesis}
\end{align*} \]

The result shows that the null hypothesis for linearity is rejected for Bist 100 at 5% level of significance. This indicates strong evidence of nonlinearity in Turkish stock indexes. This evidence might be due to asymmetry of information, heterogeneous agents’ belief, herd behaviour, and market frictions in the Turkish stock market. Similarly, the Terasvirta (1993) test reveals evidence of nonlinearity in the stock returns by rejecting the null hypothesis of linearity at 5% level of significance.

**Testing for Nonlinearity: The Tsay and Keenan Tests**

Moving a step further to the fourth test to investigate the presence of nonlinearity in the stock returns of BIST100, The Tsay (1986) and Keenan (1985) tests was used and we found a result in favour of nonlinearity for Tsay (1986) and contrary for Keenan (1985). According to the literature, the Tsay test is a more general form of the Keenan test. From Table 5, the result from the Tsay test indicates that the null hypothesis of linearity is rejected for the stock returns. The Keenan test rather shows quite a strange result contrary to previous tests. The result indicates that we could not reject the null hypothesis of linearity for the stock returns. Under this test, it implies that the Bist 100 stock returns are not characterised by nonlinearities.

**Testing for Nonlinearity: The TAR Tests**

Table 6 presents the result from the TAR test. The null hypothesis of linearity for the stock return is rejected implying the presence of nonlinearity in Bist 100 market. Interestingly, the TAR test results reveal the evidence that emerging markets stock returns are characterised by nonlinearities and cyclical behaviours. The findings support the invalidity of the efficient market hypothesis. The inefficiency and nonlinearity in the market could be explained by the existence of transaction cost, heterogeneous agents’ beliefs, noise traders, differences in investment horizon, short sales, diversity in risk profiles, bid-ask spreads, herd behaviour, market frictions, etc. in the market.
Table 4

**White and Terasvirta Tests**

<table>
<thead>
<tr>
<th>Index</th>
<th>White test</th>
<th>Terasvirta</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>X-squared</td>
<td>P-value</td>
</tr>
<tr>
<td>Bist 100</td>
<td>8.5119***</td>
<td>0.014</td>
</tr>
</tbody>
</table>

*Note. p<0.01, ** p<0.05, * p<0.1*

Table 5

**Tsay and Keenan Tests**

<table>
<thead>
<tr>
<th>Index</th>
<th>Tsay</th>
<th>Keenan</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>T-statistics</td>
<td>P-value</td>
</tr>
<tr>
<td>Bist 100</td>
<td>4.443***</td>
<td>0.004</td>
</tr>
</tbody>
</table>

*Note. *** p<0.01, ** p<0.05, * p<0.1*

Table 6

**TAR Test for Indices**

<table>
<thead>
<tr>
<th>Index</th>
<th>TAR Test</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>T-statistics</td>
</tr>
<tr>
<td>Bist 100</td>
<td>30.432***</td>
</tr>
</tbody>
</table>

*Note. *** p<0.01, ** p<0.05, * p<0.1*

It is interesting to compare the results of the nonlinear tests. The evidence for nonlinearity was much clearer for stock returns in the BDS test, runs test, and TAR test, although these tests provide very strong evidence of nonlinearity in the Turkish stock market. In sum, the TAR test performs better than the other tests of nonlinearity in the study.

**Conclusion**

This paper has investigated whether daily stock returns of Bist 100 is characterised by nonlinearity using different nonlinear tests. We found evidence in favour of nonlinear dynamics and nonlinear dependence in the stock returns for the Turkish economy. The findings suggest that the Bist 100 stock market is not in conformity with efficient market hypothesis. Therefore, it is concluded that Turkish stock exchange market is characterised by the presence of nonlinear and cyclical behaviours as a result of numerous factors in the market, such as asymmetry of information, heterogeneous agents’ belief, herd behaviour, and market frictions.

**References**


Di Statistica, 16(16), 5-32.