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Hamiltonian Mechanics Systems on Three-Dimensional Almost Kenmotsu Manifolds

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Abstract: In this study, we concluded the Hamiltonian equations on $(M^3, \phi.\xi, \eta, g)$, being a model. Finally introduce, some geometrical and physical results on the related mechanic systems have been discussed.

Key words: Kenmotsu manifolds, three-dimensional Almost Kenmotsu manifolds, Hamiltonian mechanics systems.

1. Introduction

The geometric study of dynamical systems is an important chapter of contemporary mathematics due to its applications in Mechanics, Theoretical Physics.

There are also a large number of studies on this subject, for example, Jun J. and Pathak submitted on Kenmotsu manifolds [1], and De U. C. and Pathak obtained on 3-dimensional Kenmotsu manifolds [2].

It is some important work for examples [3-6]. In this paper we will study Kenmotsu manifolds.

In this paper, we Euler-Lagrange Equations on Three-Dimensional almost Kenmotsu manifolds. After Introduction in Section 1, we consider Historical Background paper basic. Section 2 deals with the study preliminary. Section 3 is devoted to study three-dimensional Almost Kenmotsu manifolds. Section 4 is devoted to study Hamiltonian Mechanics Systems on Three-Dimensional almost Kenmotsu manifolds

2. Preliminary

In this in this preliminary chapter, we recall basic definitions, results and formulas which we shall use in the subsequent chapters of the paper

Definition (Kenmotsu manifolds) 2.1 [1]

Let M^{2n+1} be a $(2n+1)$-dimensional smooth differentiable manifold $(\phi.\xi, \eta, g)$ be an almost contact Riemannian manifold, where $\emptyset$ is a(1-1) tenser field η is a 1-form and the Riemannian metric .It well known that

$$\phi(\xi) = 0$$
$$\eta(\phi(x)) = 0 \; and \; \eta(\xi) = 1$$
$$\phi^2(X) = -X + \eta(X)\xi, \; \phi^2 = -1 + \eta \otimes \xi \quad (1)$$
$$g(X, \xi) = \eta(X)$$
$$g(\phi x, \phi y) = g(x, y) - \eta(x)\eta(y)$$
$$rank\; \phi = n - 1$$

The fundamental 2- form of an almost contact metric manifolds is defined by

$$\phi(x, y) = g(x, \phi y) \quad (2)$$

Lemma 2.2

If f, g and $k-$form be respectively then

$$f \wedge g = -g \wedge f$$
$$(f \wedge g)(x) = f(x)g - g(x)f$$
$$(dx^i \wedge dx^j)\left(\frac{\partial}{\partial x^k}\right) = \frac{\partial x^i}{\partial x^k} dx^j - \frac{\partial x^j}{\partial x^k} dx^i$$
$$\text{(iv)} \quad \left(\frac{\partial x^j}{\partial x^i}\right) = \delta^i_j = \begin{cases} 1 & , i = j \\ 0 & , i \neq j \end{cases}$$

3. Three-Dimensional Almost Kenmotsu Manifolds

Definition 3.1 [4]

Let $(M^3, \phi.\xi, \eta, g)$ be a 3-dimensional almost Kenmotsu manifold. In what follows, we denote by $L = R(.,\xi)\xi$, $h = \frac{1}{2} L\xi\phi$ and $\bar{h} = h \circ \phi$, where L

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denotes the Lie differentiation and R is the Riemannian curvature tensor. From Dileo and Pastore, we see that both h and $\bar{h}$ are symmetric operators and we recall some properties of almost Kenmotsu manifolds as follows:

$$h\xi = I\xi = 0, \qquad tr\ h = tr(\acute{h}) = 0$$
$$h0 = \phi h = 0$$
$$\nabla\xi = h + id - \eta \circ \phi$$

Proposition 3.2

Any 3-dimensional almost Kenmotsu manifold is Kenmotsu if and only if h vanishes.

Definition 3.3 [4]

Let three-dimensional manifold $M = f(x, y, z) \in R^3, z \neq 0$; where (x, y, z) are the standard coordinates in R^3: The vector fields

$$e_1 = \frac{\partial}{\partial x}, \quad e_2 = \frac{\partial}{\partial y} \quad , e_3 = \frac{\partial}{\partial z} \tag{3}$$

are linearly independent at each point of M: Let g be the Riemannian metric defined by

$$g(e_1, e_3) = g(e_2, e_3) = g(e_1, e_2) = 0$$
$$g(e_1, e_1) = g(e_2, e_2) = g(e_3, e_3) = 1$$

Let η be the 1-form defined by $\eta(Z) = g(Z, e_3)$ for any $Z \in \aleph(M)$.

Proposition 3.4

Let ϕ be the (1,1) tensor field defined by

$$\phi(e_1) = -e_2, \phi(e_2) = e_1, \phi(e_3) = 0 \tag{4}$$

Then using the linearity of ϕ and g we have

$$\eta(e_3) = 1\ ;\ \phi^2(z) = -Z + \eta(Z)e_3;$$
$$g(\phi Z; \phi W) = g(Z; W) - \eta(Z)\eta(W);$$

for any $Z, W \in \phi(M)$: Thus for $e_3 = \xi$, (ϕ, ξ, η, g) defines an almost contact metric structure on M: Now, by direct computations we obtain

$$[e_1, e_3] = 0\ ;\ [e_2, e_3] = -e_2, [e_1, e_3] = -e_1$$

Proposition 3.5

The following expressions are given

$$\phi\left(\frac{\partial}{\partial x}\right) = -\frac{\partial}{\partial y}, \quad \phi\left(\frac{\partial}{\partial y}\right) = \frac{\partial}{\partial x}, \quad \phi\left(\frac{\partial}{\partial z}\right) = 0$$

The dual form ϕ^* of the above J is as follows

$$\phi^*(dx) = -dy, \quad \phi^*(dy) = dx, \phi^*(dz) = 0$$

Proposition 3.6

The vector fields

$$e_1 = \frac{\partial}{\partial x}, \quad e_2 = \frac{\partial}{\partial y}, e_3 = \frac{\partial}{\partial z} \tag{7}$$

If ϕ^* is defined a complex manifold $\mathcal{M}$ then $\phi^{*2} = \phi^* \circ \phi^* = -1$

Proof.

$$\phi^2(e_1) = \phi(\phi(e_1)) = \phi(-e_2) = -\phi(e_2) = -e_1$$
$$\phi^2(e_2) = \phi(\phi(e_2)) = \phi(e_1) = \phi(e_1) = -e_2$$
$$\phi^2(e_3) = \phi(\phi(e_3)) = \phi(0) = 0$$

As can $\boldsymbol{\phi}^{*2}$ is -1 (complex) or 0

4. Hamiltonian Mechanics Systems

Here, we present Hamiltonian equations on 3-dimensional almost Kenmotsu manifold $(M^3, \phi. \xi, \eta, g)$

such that $\omega = \frac{1}{2}(xdx + ydy + zdz)$ 1-form on

Let ϕ^* be an almost product structure defined by λ Liouville form determined by

$$\lambda = \phi^*(\omega) = \phi^* \left[\frac{1}{2}(Xdx + Ydy + Zdz)\right] \tag{8}$$

$$\lambda = \phi^*(\omega) = \frac{1}{2}\left[X\phi^*(dx) + Y\phi^*(dy) + Z\phi^*(dz)\right]$$

$$\lambda = \phi^*(\omega) = \frac{1}{2}\left[X(-dy) + Y(dx) + z(0)\right]$$

$$\lambda = \phi^*(\omega) = \frac{1}{2}\left[-Xdy + Ydx\right] \tag{9}$$

differential of λ

$$\varphi = -d\lambda = -d\left(\frac{1}{2}\left[-Xdy + Ydx\right]\right)$$

It is known that if φ is a closed 2-form on T^*M^3, then φ_H is also a Symplectic structure on T^*M^3

$$\varphi = -d\lambda = dy \wedge dx \tag{10}$$

Let $(M^3, \phi. \xi, \eta, g)$ 3-dimensional almost Kenmotsu manifold form φ. Suppose that Hamiltonian vector field Z_H associated to Hamiltonian energy H is given by

$$Z_H = X\frac{\partial}{\partial x} + Y\frac{\partial}{\partial y} + Z\frac{\partial}{\partial z} \tag{11}$$

Calculates a value Z_H and φ

$$\varphi(Z_H) = (dy \wedge dx)\left(x\frac{\partial}{\partial x} + y\frac{\partial}{\partial y} + z\frac{\partial}{\partial z}\right)$$

$$\varphi(Z_H) = dy \wedge dx\left(X\frac{\partial}{\partial x}\right) + dy \wedge dx\left(Y\frac{\partial}{\partial y}\right) + dy \wedge dx\left(Z\frac{\partial}{\partial z}\right)$$

$$\varphi(Z_H) = x\frac{\partial}{\partial y} + y\frac{\partial}{\partial y} + y\frac{\partial}{\partial x} + x\frac{\partial}{\partial x} + z\frac{\partial}{\partial x} + z\frac{\partial}{\partial y} \quad (12)$$

Then it follows

$$i_{Z_H}\varphi = \varphi(Z_H) = dH \quad (13)$$

Otherwise, one may calculate the differential of Hamiltonian energy as follows:

$$dH = \frac{\partial H}{\partial x}dx + \frac{\partial H}{\partial y}dy + \frac{\partial H}{\partial z}dz \quad (14)$$

From (12) and (13) with respect to $i_{Z_H}\varphi = dH$, we find Hamiltonian vector field on 3-dimensional almost Kenmotsu manifold space be

$$Z_H = \frac{\partial H}{\partial y}\frac{\partial}{\partial x} - \frac{\partial H}{\partial y}\frac{\partial}{\partial y} \quad (15)$$

Suppose that the curve

$$\alpha: I \subset R \to R$$

be an integral curve of Hamiltonian vector field Z_H, i.e.,

$$Z_H\big(\alpha(t)\big) = \dot{\alpha}, \; t \in I. \quad (16)$$

In the local coordinates we have

$$\alpha(t) = (x(t), y(t), z(t)),$$

$$\dot{\alpha}(t) = \frac{dx}{dt}\frac{\partial}{\partial x} + \frac{dy}{dt}\frac{\partial}{\partial y} + \frac{dz}{dt}\frac{\partial}{\partial z} \quad (17)$$

Now, by means of (13), from (14) and (17), we deduce the equations so-called Hamiltonian equations

$$\frac{dx}{dt} = \frac{\partial H}{\partial y}, \; \frac{dy}{dt} = -\frac{\partial H}{\partial x} \quad (18)$$

In the end, we may say to be mechanical system $(M^3, \phi. \xi, \eta, g)$ triple on 3-dimensional almost Kenmotsu manifold.

5. Conclusions

Thus, the equations of Hamiltonian equations (18) with Three-dimensional almost Kenmotsu manifold. In classical mechanics, a dynamic movement Hamilton equations is found. Symplectic manifolds arise naturally in abstract formulations of classical mechanics and analytical mechanics as the cotangent bundles of manifolds.

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