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The Analysis of Systems of Measurement for the Construction of Geometrical Models of the Product

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Abstract: In the present work, the relevant *components* of the two basic system of measurement are defined through the analysis of existing systems of measurement for geometrical modeling. The major features of such systems related to the ways of formation of the two-dimensional complex drawing of a geometrical image on the basis of laws of projective connections are revealed. *Interrelations* of the elements of different systems of measurement with various projections of a geometrical image are defined. The *relative location of projections* of a geometrical image into the constructed two-dimensional complex drawings for various systems of measurement is discussed. *The* rationale behind a particular *arrangement of views* in the projective drawing of a product in the analyzed systems of measurement is explained.

Key words: Geometry, systems of measurement, image, projection, law, projective connections, drawing, interrelations, arrangement of views.

1. Introduction

In Australia, England, Holland, India, Japan, the United States of America (USA), and some other countries the *American system of an arrangement of projections* in engineering drawing is used. With respect to the front view, the top view is located above, the bottom view is located below, the left-side view — to the left, the right-side view — to the right of the front view, and the rear view — to the right of the product right-side view [1-5].

In Belarus, Germany, Kazakhstan, Kirgizstan, Poland, Russia, Ukraine and other countries the *European system of an arrangement of projections* in engineering drawing is used. Relative the front view, the top view is located below, the bottom view — above, the left-side view — to the right, the right-side view — to the left of the front view, and the rear view — to the right of the product left-side view [1, 2, 6-7].

The Indian Standard Institution (ISI) and the British Standard Institution (BSI) recommend the use of First Angle Projection method (the *European system*) now in all the institutions too [10-11].

As a rule, at various universities of the world one any system of an arrangement of projections in the drawing is taught as applied in design and technological works of the given country [3-5, 12-17, 18-39].

It may happen that an engineer developing a production process comes across a part drawing where the views' arrangement is different than he leans. As a result, he or she may not adequately understand the drawing, and thus part design including tolerances and requirements. As a consequence, an inadequate manufacturing process can be developed and/or a good part can be rejected on its inspection.

Realizable ways of formation of projections and views in the drawing are connected in consciousness of an engineer with a particular system of measurement for construction of geometrical models of a product [2, 7, 40-51].

The American system of measurement for the construction of a geometrical model of a product differs from the *European system of measurement*.

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Thus, the essence of the problem (contradiction) consists in a *necessity of adequate understanding understand the features of formation of projections in the drawing in used system of measurement* by everyone involved in design and product manufacture adequately and *lack* of the systematized knowledge of features of various ways of formation of projections in different systems of measurement in the geometrical modeling.

Therefore, the **purpose** of this work is *to reveal the features* of various systems of the measurement used in the geometrical modeling, and *to provide justifications for the arrangement of projections in* the product drawing.

2. Work Tasks

1) To analyze the existing systems of measurement used in the geometrical modeling defining their relevant components.
2) To reveal the features of the known systems the measurements connected with the ways of formation of the two-dimensional complex drawing of a geometrical image.
3) To define interrelations of elements of the discussed systems of measurement with different projections of a geometrical image.
4) To define relative positioning of projections of a geometrical image in the two-dimensional complex drawing for various systems of measurement.
5) To carry out a logic substantiation of an arrangement of projections in the projective drawing of a product.

3. The Main Part

The *first* mutual *component* of the systems of measurement used for geometrical modeling is a set of *three mutually perpendicular planes* (Fig. 1).

In relation to the subject (to the student, the engineer, the researcher), one of these three planes is located horizontally (H-Π_1) whereas other two planes (F-Π_2, P-Π_3) – vertically.

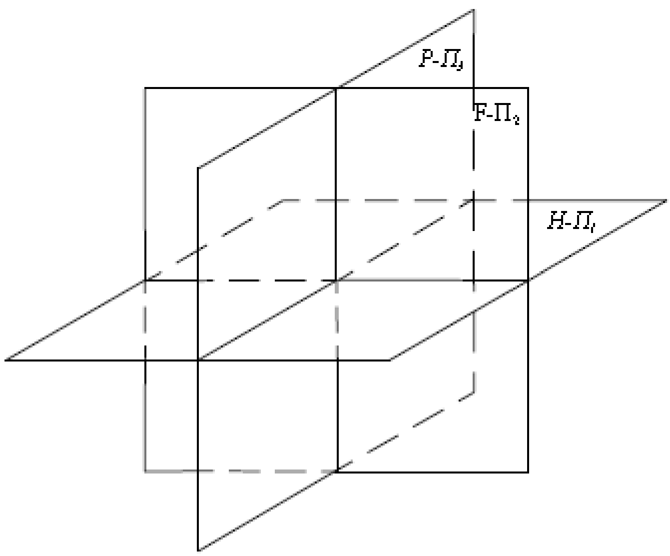

Fig. 1 Three mutually perpendicular planes of systems of measurement.

Horizontally focused plane H-Π_1 is called *as the horizontal plane*. Vertically located plane F-Π_2 is called *as the frontal plane*. Vertically focused plane P-Π_3 is called *as the profile plane*.

For geometrical modeling, the method of orthogonal projections [2, 7, 33] is used. In the method of orthogonal projections, horizontal plane H-Π_1 is called *as the horizontal plane of projections,* frontal plane F-Π_2 is called *as the frontal plane of* *projections*, and profile plane P-Π_3 is called *as the profile plane of projections*.

The *second* mutual *component* of systems of measurement used for geometrical modeling is the *right three-dimensional system of coordinates* OXYZ, developed by Rene Descartes (1596-1650). The beginning of system of coordinates (a point O-Latin, origo) coincides with *the* point of intersection of all three planes.

Thus, the system of measurement for geometrical modeling consists of three mutually perpendicular planes H-Π_1, F-Π_2, P-Π_3 and connected with them right [52, 53] three-dimensional system of coordinates OXYZ.

The allocation of a way of orientation of the right three-dimensional system of coordinates OXYZ relative to the planes of projections H-Π_1, F-Π_2, P-Π_3 can be considered as the *first feature* of systems of measurement for geometrical modeling.

1) In Australia, England, Holland, Japan, the USA and a number of other countries, axes OZ and OX coincide with *horizontal* plane H-Π_1, axes OX and OY are located in *frontal* plane F-Π_2, and axes OY and OZ belong to profile plane P-Π_3 (Fig. 2). All axes of co-ordinates coincide with the lines of intersection of the discussed planes [4, 5].

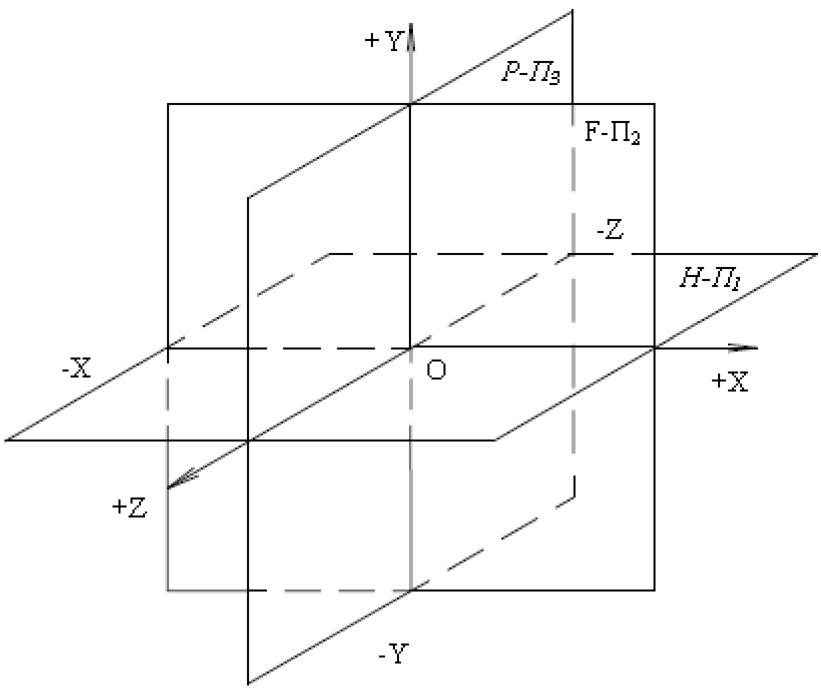

Fig. 2 Linking of the system of coordinates OXYZ with planes of projections H-Π_1, F-Π_2, P-Π_3 in the *American system of measurement*.

Because the positive direction of axis OZ relative to the origin O is directed *to the subject* (observer) for the right system of coordinates OXYZ, the positive coordinates of axis OX settle down *to the right of* the origin O and positive coordinates of axis OY are *above* the origin O.

Thus, axis OY is directed *upwards* concerning the origin O, axis OX is directed *to the right* concerning the origin O, and axis OZ is directed from the origin O *towards the* observer [5].

Accordingly, the portion of axis OX with negative coordinates is *to the left of* profile plane P-Π_3, the portion of axis OY with negative coordinates settles down below horizontal plane H-Π_1, the portion of axis OZ with negative coordinates is *behind* frontal plane F-Π_2 [5].

The measurement system *serves* for placing and the description of geometrical images. *Geometrical images* are understood as abstract elements: a point, a line, a plane, a surface, a body [2, 6-9, 12-30].

The *second feature* of systems of measurement for geometrical modeling is the *way of space partition* into semi spaces, quadrants, and octants.

For the description of the location of an object in a particular measurement, space is partitioned by a plane into two *semi spaces*, by two orthogonal planes into four *quadrants* (quarters), by three mutually perpendicular planes into eight *octants* (parts).

2) In the *American* system of measurement, space is partitioned into four *quadrants* by horizontal plane H-Π_1 and *profile* plane P-Π_3 (Fig. 3).

The *third feature* of systems of measurement for geometrical modeling is the *way of numbering* of the allocated parts of space (semi spaces, quadrants, and octants).

3) In the *American* system of measurement, numerical values of coordinates X and *Y* are positive for both axes — 1: +X, +Y. In the second quadrant, numerical values of coordinate *Y* are positive, and coordinates *X* are negative — 2: –X, +Y. In the third quadrant, numerical values of coordinates *X* and Y are negative for both axes — 3: –X, –Y. In the fourth quadrant, numerical values of coordinate *Y* are negative, and coordinates *X* are positive — 4: +X, –Y [5].

Quadrant number increase at consecutive viewing the quadrants from the positive direction +*Z* axes *OZ* counter-clockwise (Fig. 3). Numbers 1, 2, 3, 4 quadrants are specified around the image.

The one eighth part of space, in which all numerical values of coordinates X, Y, Z for three axes OX, OY, OZ are positive +X, +Y, +Z is chosen as the first *octant*.

Numbers of the first four octants correspond to four numbers quadrants (quarters). The fifth, the sixth, the seventh and the eighth octants are bred behind the frontal plane of projections F-Π_2 from the positive direction +Z axis OZ accordingly in the first, in the second, in the third and the fourth quadrant (Fig. 3).

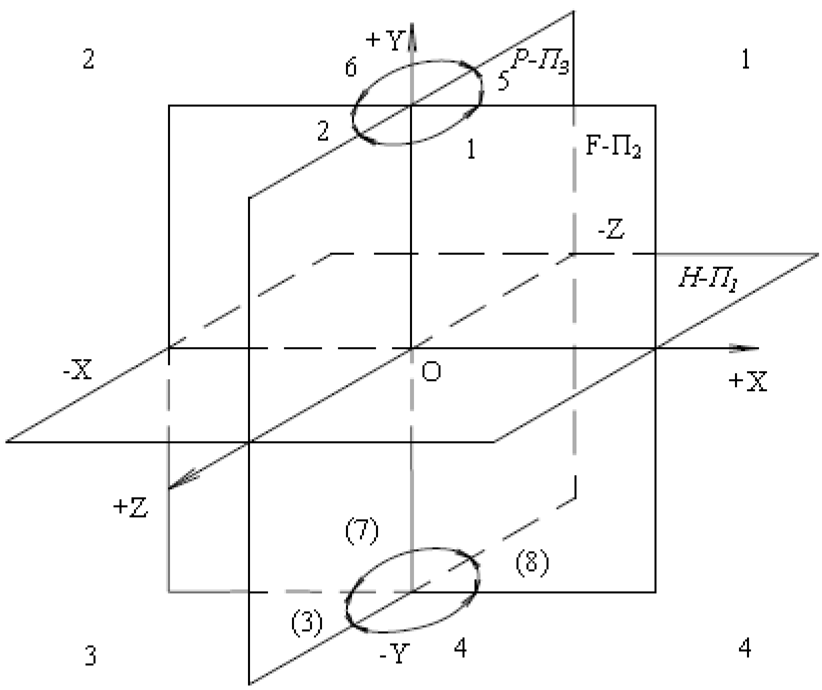

Fig. 3 Quadrants and octants numbering in the *American system of measurement*.

The revealed features of the American system of measurement differ from similar characteristics of the European system of measurement although there are some similarities.

1) In the *European system of measurement*, the right [52, 53] the three-dimensional system of coordinates OXYZ is positioned differently relative to the planes of projections H-Π_1, F-Π_2, P-Π_3 (Fig. 1) as shown in Fig. 4.

Axes OZ and OX coincide with *frontal* plane F-Π_2, axes OX and OY settle down in *horizontal* plane H-Π_1, and axes OY and OZ also belong to profile plane P-Π_3 (Fig. 4). All the axes of coordinates are coincident with the intersection lines of the projection planes [7].

Because the positive direction of axis OZ with respect to the origin O is directed *upwards* for the right system of coordinates OXYZ, the positive coordinates of axis OX are *to the left of* the origin O and positive coordinates of axis OY settle down *before* frontal plane F-Π_2 closer to the observer to the origin O (Fig. 4).

Thus, axis OY is directed from the origin O coordinates *towards the* observer, axis OX is directed *to the left* of the origin O, axis OZ is directed *upwards* relative to the origin O.

Accordingly, the portion of axis OX with negative coordinates is *to the right of* profile plane P-Π_3, the portion of axis OY with negative coordinates settles down *behind* frontal plane F-Π_2, the portion of axis OZ with negative coordinates is *below* horizontal plane H-Π_1 [7].

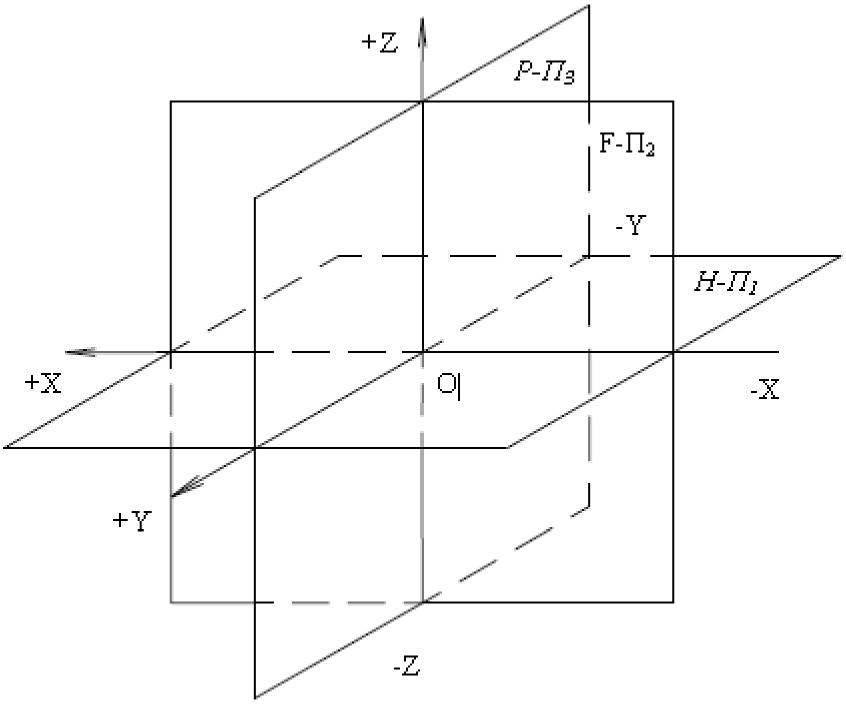

Fig. 4 Linking of system of coordinates OXYZ with planes of projections H-Π1, F-Π2, P-Π3 in the European system of measurement.

2) In the *European* system of measurement, space is partitioned into four quadrants by horizontal plane H-Π_1 and *frontal* plane F-Π_2 (Fig. 5).

3) The way of numbering quadrants in the *European* system of measurement differs from the American system of measurement. In the first, *quadrant* numerical values of coordinates *Z* and Y are positive for both axes — 1: +Z, +Y. In the second quadrant, numerical values of coordinate *Z* are positive, and coordinates *Y* are

negative — 2: +Z, –Y. In the third quadrant, numerical values of coordinates *Z* and Y are negative for both axes — 3: –Z, –Y. In the fourth quadrant, numerical values of coordinate Z are negative, and coordinates Y are positive — 4: –Z, +Y [4]. The quadrant number increases at consecutive viewing quadrants from the positive direction *+X* axes *OX* counter-clockwise (Fig. 5).

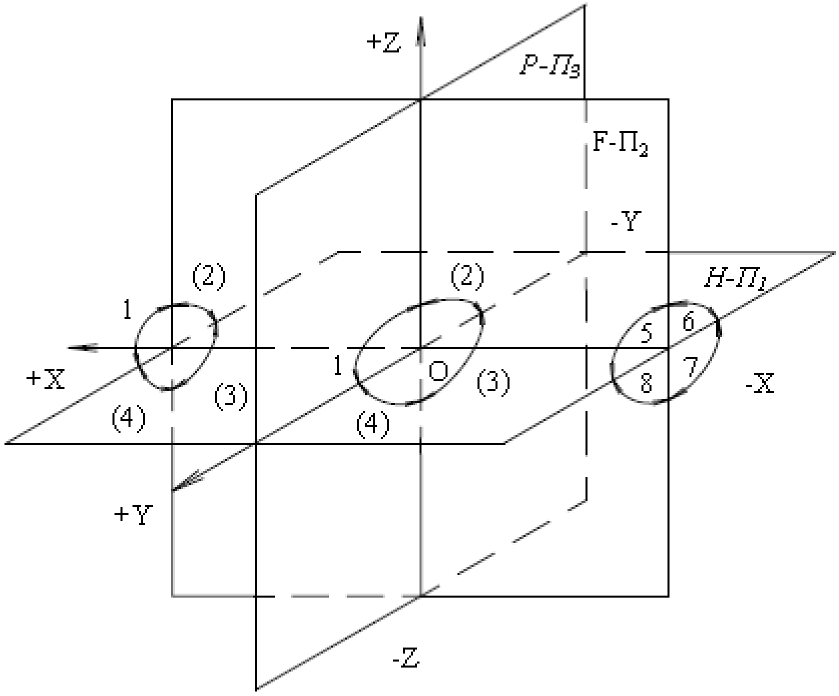

Fig. 5 Quadrants and octants numbering in the European system of measurement.

In the *European* system of measurement, the same as in the American system of measurement, the first *octant is chosen as* one eighth part of space, in which all the numerical values of coordinates X, Y, Z for three axes OX, OY, OZ are positive +X, +Y, +Z is chosen.

The difference for the first octant in the *European system* is that *horizontal* plane H-Π_1 is located between axes OX and OY whereas in the American system frontal plane F-Π_2 settles down, and *frontal* plane F-Π_2 (Fig. 5) is located between axes OX and OZ, instead of horizontal plane H-Π_1 (Fig. 2, 3) in the American system.

Numbering of the first, the second, the third and the fourth octants, located *to the left of* profile plane P-Π_3, coincides with numbering of the first, the second, the third and the fourth quadrants the European system (Fig. 5).

The fifth, the sixth, the seventh and the eighth octants in the European system settles down *to the right of* profile plane P-Π_3 accordingly in the first, the second, the third and the fourth quadrants (Fig. 5).

Let's consider construction of model of a geometrical image in the *American* system of measurement.

The *third octant* in the *third quadrant* is used in the *American* system of measurement for construction of model of a geometrical image (Fig. 3).

Therefore, segment AB of a straight line settles down in the third octant (Fig. 6).

The *method of rectangular (orthogonal) projection* is applied to construct the projections of segment AB [45].

The results of projection of segment AB into three mutually perpendicular planes F-Π_2, H-Π_1, P-Π_3 are *frontal projection* A_fB_f, *horizontal projection* A_hB_h and *profile projection* A_pB_p [46]. The constructed three-dimensional geometrical model of segment AB is shown at the left site in Fig. 6.

For the development *of a two-dimensional complex drawing* from a three-dimensional geometrical model, segment AB and projecting rays AA_f, AA_h, AA_p, BB_f, BB_h, BB_p are mentally removed. Horizontal plane H-Π_1 with projection A_hB_h of the segment are rotated around the axis OX clockwise from the positive direction +X till its full coincidence with frontal plane F-Π_2. Profile plane P-Π_3 with projection A_pB_p of the segment are rotated around the axis OY counter-clockwise from the positive direction +Y till its full coincidence with frontal plane F-Π_2. The constructed two-dimensional geometrical model of segment AB is shown on the right site in Fig. 6.

For the *American* system of measurement in the two-dimensional complex drawing of a geometrical image, the horizontal projection is located *above the* frontal projection, and the profile projection is located to the right of the frontal projection (Fig. 6).

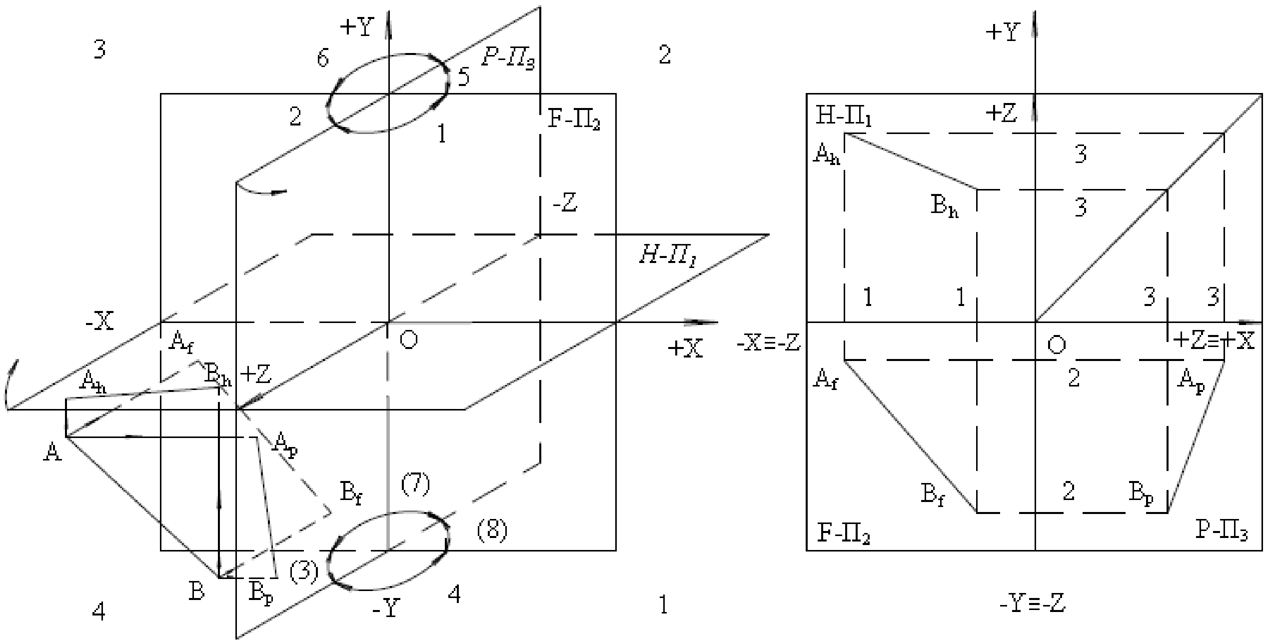

Fig. 6 Geometrical models of segment AB of a straight line in the *American* system of measurement.

Constructed frontal projection A_fB_f, horizontal projection A_hB_h and profile projection A_pB_p of segment AB satisfy *the laws of projective connections* 1-1, 2-2, 3-3 [7, 49].

Elements *of the American* system of measurement are unambiguously connected with the projections of a geometrical image.

a) *Negative* coinciding portions –X≡–Z, –Y≡–Z of axes X and Y and axes of ordinates, applicates OY, OZ limit frontal plane F-Π_2 and the *frontal projection* of a geometrical image.

b) Coinciding *negative* portions –X≡–Z of axes OX, OZ and coinciding *positive* portions +Y≡+Z of axes OY, OZ limit horizontal plane H-Π_1 and the *horizontal projection* of a geometrical image.

c) Coinciding *negative* portions –Y≡–Z of axes OY, OZ and coinciding *positive* portions +X≡+Z of axes OX, OZ limit profile plane P-Π_3 and the *profile projection* of a geometrical image.

For the realized way of construction of the complex drawing (Fig. 6) in the *American* system of measurement, the horizontal projection is the *bottom view*, and the profile projection is the *left-side view*.

Thus, if planes of projections are considered to be opaque in the projective drawing of a product then the *bottom view* settles down *from above the* front view and the *left-side view* settles down *to the right of the* front view. Conversely, in the countries using the *American* system of measurement, the *bottom view* settles down *from below* from the front view, and the *left-side view* settles down *to the left of the* front view. Conditions needed for realization *of such accepted* arrangement of views in the projective drawing of a product will be considered in the separate publication.

Let's consider the construction of model of a geometrical image in the *European* system of measurement.

In the *European* system of measurement, the *first octant* in the *first quadrant* is used for construction of model of a geometrical image as shown in Fig. 5.

That is why segment AB of a straight line settles down in the first octant (Fig. 7).

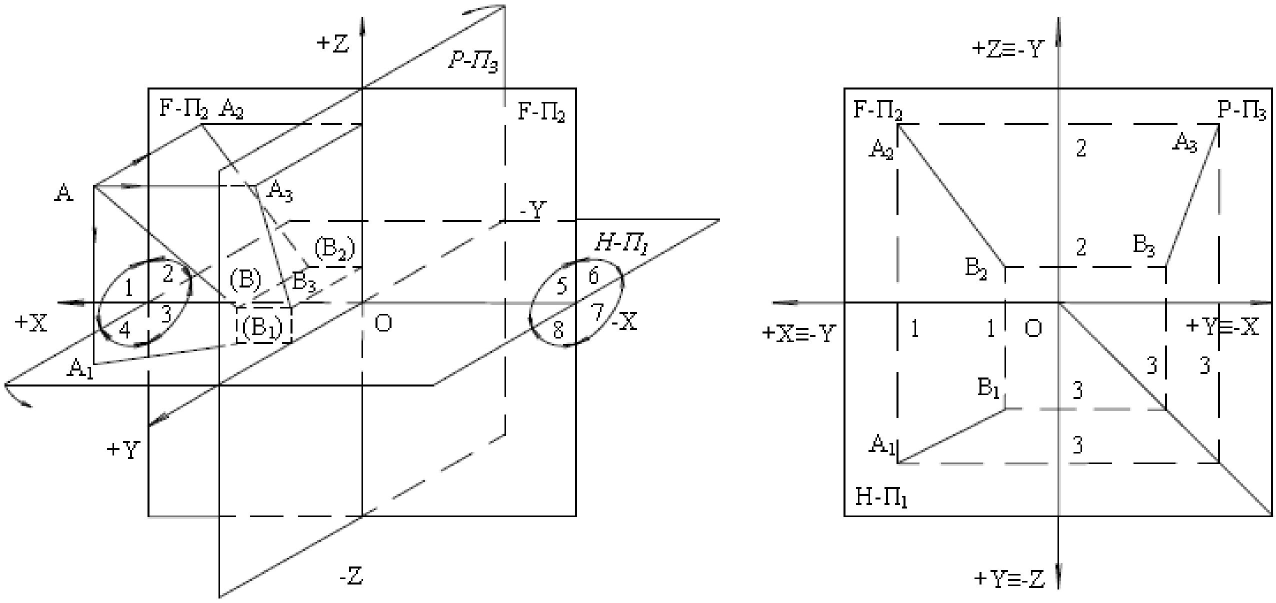

Fig. 7 Geometrical models of segment AB of a straight line in the *European* system of measurement.

The method of orthogonal projecting is also applied to develop projections of segment AB [33].

The results of projections of segment AB into three mutually perpendicular planes $F\text{-}\Pi_2$, $H\text{-}\Pi_1$, and $P\text{-}\Pi_3$ are the frontal projection A_2B_2, horizontal projection A_1B_1 and profile projection A_3B_3 [7]. The constructed three-dimensional geometrical model of segment AB is shown to the left in Fig. 7.

For construction *of the two-dimensional complex drawing* from three-dimensional geometrical model, segment AB and projecting rays AA_2, AA_1, AA_3, BB_2, BB_1, BB_3 are mentally removed.

Horizontal plane $H\text{-}\Pi_1$ with projection A_1B_1 of segment AB is rotated about the axis of abscissas OX clockwise from a positive direction +X till its full coincidence with frontal plane $F\text{-}\Pi_2$. Profile plane $P\text{-}\Pi_3$ with projection A_3B_3 of segment AB is rotated about the axis of applicate OZ counter-clockwise from a positive direction +Z till its full coincidence with frontal plane $F\text{-}\Pi_2$. The constructed two-dimensional geometrical model of segment AB is shown to the right in Fig. 7.

A two-dimensional geometrical model is called *as the complex drawing* of a geometrical image [7, 8, 22-25].

For the *European* system of measurement in the two-dimensional complex drawing of a geometrical image, the horizontal projection is located *below the* frontal projection, and the profile projection is located to the right of the frontal projection (Fig. 7).

Constructed frontal projection A_2B_2, horizontal projection A_1B_1 and profile projection A_3B_3 of segment AB also satisfy *the laws of projective connections* 1-1, 2-2, 3-3 [7, 49].

Elements *of the European* system of measurement also correlated with different projections of a geometrical image as follows:

1) Coinciding (+X≡–Y) the *positive* portion of the axis of abscissas OX *and the negative* portion of the axis of ordinates OY, and also coinciding (+Z≡–Y) the *positive* portion of the axis OZ *and the negative* portion of the axis of ordinates OY limit frontal plane F-Π_2 and the *frontal projection* of a geometrical image.
2) Coinciding (+X≡–Y) the *positive* portion of the axis of abscissas OX *and the negative* portion of the axis of ordinates OY, and also coinciding (+Y≡–Z) the *positive* portion of the axis of ordinates OY and *the negative* portion of the axis OZ limit horizontal plane H-Π_1 and the *horizontal projection* of a geometrical image.
3) Coinciding (+Z≡–Y) the *positive* portion of the applicate axis OZ *and the negative* portion of the axis of ordinates OY, and also coinciding (+Y≡–X) the *positive* portion of the axis of ordinates OY and *the negative* portion of the axis of abscissas OX limit profile plane P-Π_3 and the *profile projection* of a geometrical image.

For the realized way of construction of the complex drawing (Fig. 7) in the *European* system of measurement, the horizontal projection is the *top view*, and the profile projection is the *left-side view*.

Hence, if planes of projections to consider opaque on the projective drawing of a product, the *top view* is located *below* the front view and the *left-side view* is located *to the right of the* front view. Such an arrangement of views corresponds *to the accepted* arrangement of views in the projective drawing of a product in the countries where the *European* system of measurement is used.

4. Conclusions

1) Three mutually perpendicular planes H-Π1, F-Π2, P-Π3 are considered as the components of system of measurement for geometrical modeling. The right three-dimensional system of coordinates OXYZ is allocated to determine the coordinates in these planes.
2) The way of orientation of the right three-dimensional system of coordinates OXYZ with respect to the set planes of projections H-Π1, F-Π2, P-Π3 is the first feature of systems of measurement. In the American system of measurement, the axes OZ and OX coincide with horizontal plane H-Π1, the axes OX and OY settle down in frontal plane F-Π2, and axes OY and OZ belong to profile plane P-Π3 (Fig. 2). In the European system of measurement, the axis OZ and OX coincide with frontal plane F-Π2, axes OX and OY settle down in horizontal plane H-Π1, and axes OY and OZ also belong to profile plane P-Π3 (Fig. 4).
3) The second feature of systems of measurement for geometrical modeling is the way of conditional division of space into parts: semi spaces, quadrants, and octants. In the American system of measurement, the space is divided into four quadrants by horizontal plane H-Π1 and profile plane P-Π3 as shown in Fig. 3. In the European system of measurement, the space is divided into four quadrants by horizontal plane H-Π1 and frontal plane F-Π2 (Fig. 5).
4) The third feature of systems of measurement for geometrical modeling is the order of numbering of the allocated parts of space (semi spaces, quadrants, and octants). In the American system of measurement, this order shown in Fig. 3 differs from that used in the European system of measurement (Fig. 5).

5) Interrelations of elements American (Fig. 6) and European (Fig. 7) systems of measurement with different projections of a geometrical image in the two-dimensional complex drawing are not the same. The differences result in the various arrangements of views in the projective drawing of a product.

A way of elimination *of discrepancy of projections* of an image and *views* of a product for the *American* system we will consider in a separate research [54].

An attempt to reach conformity *of projections* of an image (Fig. 6) and proper visualization of product *views* under the American standard (ISO A of ISO 5456) by simple turn of planes of projections in other direction leads to loss of clearness of the drawing of a geometrical image (Fig. 8).

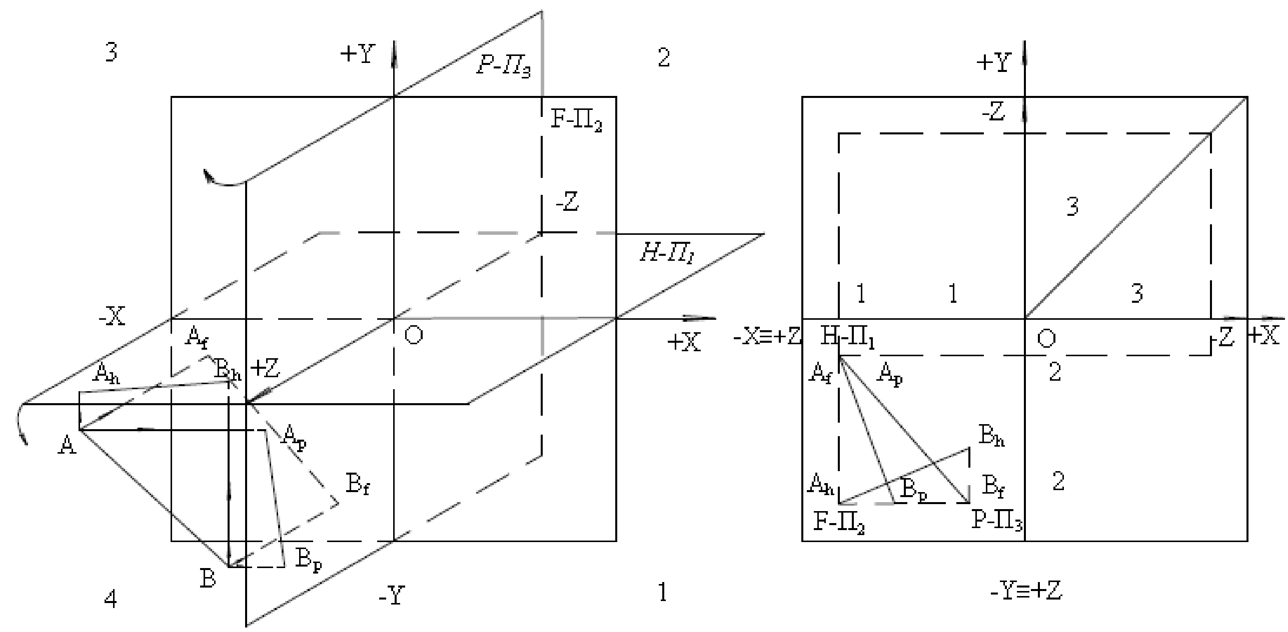

Fig. 8 The rotation of the plane H into the vertical position performed in the other direction (down). The same holds for the profile plane P (Left).

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