

Analytic Hierarchy Process (AHP) to Optimize the Service in a Water Supply Network

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Abstract: Decision making in drinking water supply networks is increasingly complex due to the large number of variables involved. In order to make better decisions it is necessary to use adequate and robust methodologies. This paper presents the application of the Analytic Hierarchy Process (AHP) related to the operation of the drinking water supply network of the city of Chihuahua, Mexico, where two possible alternatives are delineated with the objective to optimize the service. The application of AHP was carried out in 24 sectors that have substantial differences in their efficiency but with instrumentation and measurement in all the variables contemplated by the operating agency, with a population of 221,722 inhabitants which represent a 30% of the total population of the city, the results indicate that the best alternative is the one with less criteria to be controlled and fewer repercussions on the cost of operation and investment in the rehabilitation and replacement of the network.

Keywords: Optimize, multicriteria method, decision making, AHP, water supply network.

1. Introduction

The decision making regarding the operation and management of a drinking water supply network for an operating agency involves the analysis and classification of various factors in order to achieve objectives that are part of an optimal service scheme. Among the most important factors is to offer the sufficient volume of water to guarantee the supply to the different sectors, with the adequate quantity to meet the needs of the population and in quality that satisfies the standards for human consumption. The supply offered must consider an additional amount due to leaks, which result into a greater volume of water to be supplied, combined with the growth of urban centers, insufficient investment in infrastructure and lower availability of water resources, this force the managers to make the decision to supply the population through non-continuous service hours.

These variables significantly increase complexity

when analyzing the network and make it necessary to have a tool that allows simplifying and contributing to decision-making regarding the operation of the system. The supply of water through discontinuous service hours can manifest dissatisfaction in the user, incurring non-payment, which provokes lower income collection. Seeking to switch to a continuous supply would be seen as the best solution but it is necessary to consider that as physical and economic scarcity intermingles more, the system becomes more complex, which is not always a viable option. The Analytical Hierarchy Process (AHP), developed by Saaty in 1980, has been widely used for decision making in a large number of fields [1] as it provides the possibility of graphically observing a multicriteria problem, ranking levels (objective or problem, criteria and alternatives).

2. Theoretical Framework

Hierarchical Analysis Process (AHP) was developed by the mathematician Thomas L. Saaty [2], designed to solve complex multi-criteria problems, the process involves and consists in formalizing intuitive

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understanding by building a hierarchical model, combining factors based on experience-intuition, information-reasoning and knowledge-reasoning.

Multicriteria methods are used to make decisions of problems composed of intangible aspects or with qualitative variables to evaluate [3]. These methods do not consider the possibility of finding an optimal solution to a problem, but the solution depends on the predefined objective (s) [4]. The AHP provides the feasibility of including quantitative and qualitative data that in a “normal” analysis are left out because of their difficulty in measuring but relevant in obtaining the objective [5].

The prioritization of actions, is essential for the operating agencies that provide the service of supply and distribution of drinking water, which is imperative to have methodologies that allow an objective evaluation, even considering the subjectivity (experience-intuition) to make an adequate and informed decision on the alternatives of greater benefit for society in water resources management, particularly in drinking water networks [6, 7]. This methodology allows the decision maker to observe a multicriteria problem graphically, ranking levels (objective or goal, criteria and alternatives), comparisons are also made between pairs of elements (criteria and alternatives) and numerical values are assigned to the preferences indicated by the analysts, where the combination of experience and intuition, generation of data or information and reasoning is integrated, finally the knowledge acquired by the solution of similar problems with the information and experience acquired under the reasoning. The process results are in a synthesis by aggregating these partial judgments.

Since 2008, the city of Chihuahua began with analysis processes that aim to optimize the supply and distribution of drinking water in the region, the problem of supply and distribution of the city in more than 3 decades, has been “solved” by drilling wells and incorporating higher water flows for the supply,

but without a significant improvement in the quality of service; considering that more and more areas of the city have had a restriction in their service hours, and an increased volume of water leaked, higher operating costs and a decreased income collection since users do not agree with the tariffs due to the few service hours and the low water pressure deliver to their homes.

That is why a diagnosis of the city’s drinking water supply and distribution was made, generating information through a measurement campaign in the water supply system to know and generate “reliable” data [8]. Later, a hydraulic modeling and simulation was carried out in order to “divide” the problem to face it and solve it through the hydraulic sectorization [9].

To date, efforts have been made with large investments to physically delimit sectors of the city, however it has not been sufficient to improve the service. Therefore, the strategy since 2018 has been the instrumentation for the measurement of variables in an automated way that allows generating information and assessing the actions that must be implemented to obtain the efficiency of the hydraulically delimited sectors in the drinking water supply and distribution network.

That is why a multicriteria analysis was determined and the hierarchical analysis process (AHP) was selected.

3. Methodology

The process enables the assignation of numerical values to the relationships of the factors involved in the solution or in the search for the objective that is to optimize the service. This analysis was applied with the factors and criteria with the greatest impact in the lack of success of the optimization of water supply service in sectors (OS), to assess its result, we studied sectors where other functionality indices are already taken and efficiency metrics, to later apply to the rest of the city.

The steps to follow in order to prepare the hierarchical process analysis (AHP), are basically summarized by the following phases: problem identification, definition of the objective, identification of criteria and identification of alternatives.

The application of AHP was carried out in 24 sectors that have substantial differences in their efficiency but that have instrumentation and measurement in all the variables contemplated in the operating agency, with a user population of 221,722 inhabitants, which represent 30% of the total population of the city. The problem to solve is to choose a policy or alternative of action that allows the efficiency of the water supply service to the sectors. Therefore, the objective is to optimize the water supply service to the sectors (O.S).

As a first step, a hierarchical model was constructed as shown in Fig. 1. The number of criteria or comparative elements required by this method is at most $n = 7 \pm 2$ [10]. In case of exceeding that number, the conglomerate composition is suggested [11].

The evaluation of partial judgments is given by experience and intuition, as well as information, knowledge and reasoning of the analyst is quantified according to the scale developed by Saaty, related to the objective. A matrix is developed where all the criteria are placed in rows (i) and columns for comparison and definition of value in the partial trial. The scale of values for partial judgments is shown in Table 1.

A comparison between pairs of criteria is made. Starting from the scale of values in Table 1, a square matrix $A = [a_{ij}]$ is constructed, where $1 \leq i, j \leq n$ is an element a_{ij} represents the comparison between element i and element j .

The AHP considers that if the evaluation of more than one alternative is required, we proceed to form as many matrices as criteria, to make a comparison between pairs of alternatives considering each of the criteria.

The alternatives that are formulated for the application of the AHP, start from the evaluation of the predecessor solutions.

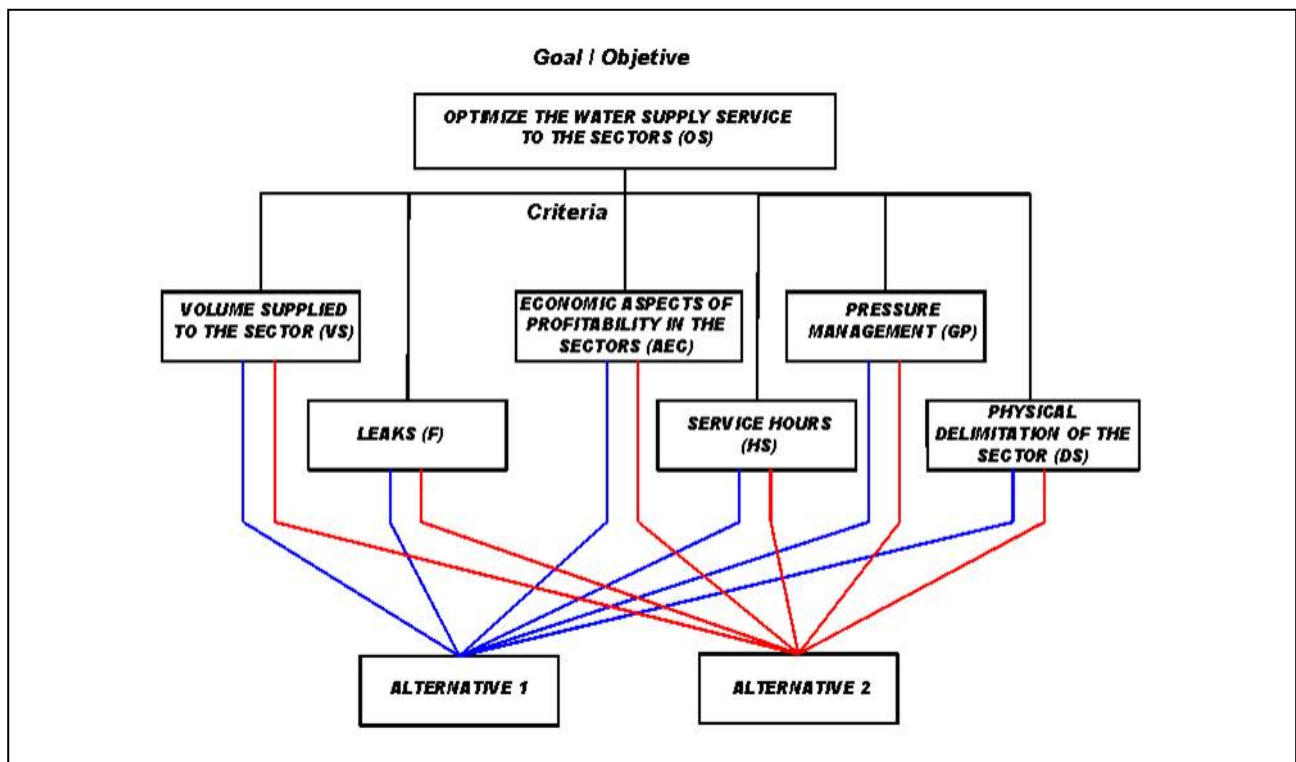


Fig. 1 Hierarchical model.

Table 1 Scale of partial judgment values between criteria [2].

Judgment	Value
Absolute importance of element i over element j	9
Very important importance of element i over element j	7
Marked importance of element i over element j	5
Little importance of element i over element j	3
Indifference or equal importance between i and j	1
Little importance of element j over element i	1/3
Marked importance of element j over element i	1/5
Very important importance of element j over element i	1/7
Absolute importance of element j over element i	1/9

The framework of the application of this methodology considers the following facts that are listed and described chronologically according to the application that has occurred in the city: one of the recurring alternatives in the last 30 years has been to increase the water supply by incorporating new sources, but without a positive impact on a better quality and quantity of hours of service, to identify why it has not been a good or sufficient solution, the automated measurement of the volume supplied (VS) was installed to certain sectors, the analysis showed that there are sectors that have excess volume provision, even considering the storage that each one of the homes has, an average volume of 800 liters.

Another solution in the past 25 years has been to limit the hours of service (HS) in areas of the city, whose purpose includes directing water to downstream areas, controlling the volume supplied to the sector, as well as “mastering” the volume of leaking water.

As a result of the diagnosis [8], actions were initiated for the real and physical delimitation of the sectors (DS) or area of influence by source of supply, resulting in the best management of water transfer between sectors but insufficient to give greater continuity of service hours until 24 h., and maintain a permanence of water in the sector with adequate pressure.

Since 2018 pressure management (GP) began, which consists in instrumentation of the delimited sector, flow and volume measurements through a

controller in the inlet valve through downstream pressure setpoints (upon delivery of the sector) depending on the pressure present upstream from the source of supply to the sector. Measurement is also included at points previously defined by hydraulic modeling, within the sector, which allows us to know the distribution of pressure and water flows; to know the time of permanence of the water in the sector; however, it was not enough, since this method is optimal when there is a 24-hour service, in a “limited” service schedule it is necessary to estimate the minimum pressures that must remain in the network to avoid voids and suction, as well as define the provision that should be given to the sector to satisfy an adequate permanence of the water for the whole sector, considering the intra-residential storage that users have, and that are reluctant to remove them as long as a 24-hour service is not guaranteed.

One of the implications of having a limited service schedule, is the permanence and recurrence of leaks (F) in the sector, whose provision must contemplate the volume escaped, in pressures management this escaped volume can be reduced by controlling flow rates and minimum pressures at night, it has started with an active leak control, which consists in the detection and repair of leaks, but only applies where service hours have been increased.

Based on the described above, two possible alternatives are presented: Alternative 1, consists on run the actions described by the factors or criteria shown in the hierarchy model without considering that a percentage of the sector pays for the service of drinking water, estimating that, by giving a continuous service it is possible to have “control” of the volume supplied and escaped through pressure management and that it is better in the long term not to replace domestic lines or intakes, without excluding that it must be accompanied by a payment culture program on a permanent basis. This alternative is based on a demand pattern per time series that is known as the hourly/daily variation curve (CVH/CVD).

Alternative 2, consists in looking for the minimum amount of water volume required by the user and the minimum time required for the water to remain in the sector, with help from pressures management to determine the minimum pressure that does not cause vacuum or suction in the main lines, contemplating a payment culture program permanently. In summary, this alternative seeks to be the bridge to move from a fixed-time service to a continuous service; this fixed or intermittent schedule service that is proposed with a hybrid demand pattern (PHD), which is based on the user's behavior in the use of water when he has an intermittent service, with controlled effects on the pipes; unlike the deterministic supply pattern that occurs in the current operation in all communities where there is a fixed schedule of drinking water service.

To analyze the two alternatives that allow us to optimize the supply and distribution of drinking water in the sectors, the criteria shown in Fig. 1 were considered.

The volume supplied to the sector (VS) is the amount of water delivered in the sector, which must be sufficient to meet the needs of the user and consider losses in the network.

Leakage (F), is the volume of water that is lost due to failures in the pipes and in the domestic outlet as a cause of the intermittent supply operation due to the filling and emptying of the pipes or in the water meter of the user for overpressures at the time of the arrival of the water and the eviction of the air in the outlet.

Economic aspects of profitability in the sector (AEC), represents the relationship between the volume delivered to each user, the volume billed and the number of users who pay for the service.

Pressure management (GP), is the action of implementing and automating the operation of the sector, in order to give the minimum pressure that covers the daytime needs, is monitored in real time in points defined as critical, to see the maximum and minimum pressures that are presented, without jeopardize the water needs of the user in the hours of

greatest demand, but also with the purpose of limiting the pressure in night time to avoid water losses due to leaks

Service hours (HS), time span in which water is supplied to the sector.

Physical delimitation of the sector (DS), are the boundaries that are hydraulically defined for the supply of water in the sector and that entails the installation of valves and line cuts to prevent water from passing from one sector to another without control, these limits were defined by the characteristics of the sector, such as topographic slopes, user density, etc. After defining the criteria, structuring the hierarchy model and the final objective, the next step is the construction of the matrix that contemplates the described criteria and the comparison of judgments between them.

In the criteria comparison matrix four features are taken for the construction of pairs of comparison matrices: positivity, reciprocity, homogeneity and consistency [12]. Positivity in all values (a_{ij}) and (a_{ji}), reciprocity—if $a_{ij} = x$, then $a_{ji} = 1/x$. It is that the comparison of element i with element j corresponds to the value given to the comparison of element j with element i , homogeneity—If the elements i and j are considered equally important or have no interference with each other, then $a_{ij} = a_{ji} = 1$, consistency—It is satisfied that $a_{ij}a_{jk} = a_{ik}$ for all $1 \leq i, j, k \leq n$.

Consistency is defined as the coherence that exists between the elements of a set. Since the judgments are issued with a certain subjectivity, there is a certain degree of inconsistency and disturbance in the matrices (Stewart & Sun, 1990). According to the need to meet a minimum level of consistency, some authors have developed some criteria [5, 13]. Table 2 shows recommended values to determine the degree of inconsistency acceptable according to the size of the matrix in the AHP.

The measure of consistency of a matrix of comparisons, according to Saaty [12], a matrix is consistent when the normalized sum of each row shows us which element dominates the others in

Table 2 Consistency values [2].

Matrix size (n)	Consistency Index (IC)
1	0.00
2	0.00
3	0.52
4	0.89
5	1.11
6	1.25
7	1.35
8	1.40
9	1.45

Table 3 (E0) Values of autovectors by the standardized method.

Criteria	ALTERNATIVE 1		ALTERNATIVE 2	
	Σ Row	Value (E_0)	Σ Row	Value (E_0)
VS	22.07	0.05	137.44	0.13
F	32.03	0.08	143.73	0.14
A.EC	99.49	0.24	171.37	0.16
G.P	56.97	0.14	121.95	0.12
H.S	80.45	0.19	115.24	0.11
D.S	123.91	0.30	369.59	0.35

relative terms. He also considers that the sum of each column says how much each element is dominated by the other elements.

A matrix is considered valid in matters of consistency, as long as when obtaining the consistency rate (TC), the value does not exceed 10%. If this does not happen, we must proceed to modify the judgments issued to achieve acceptable consistency, or to improve consistency through mathematical methods, the consistency rate is obtained from Eq. (1).

$$TC = IC / IA \quad (1)$$

Eq. 1. Consistency rate (TC)

IC, is the consistency index obtained from Table 2; AI is the random consistency index of matrix and depends on the number of elements or criteria to be compare (n) Eq. (2) [14].

$$IA = (1.98 (n-2)) / n \quad (2)$$

Eq. 2. Random consistency index (IA)

4. Results

Below is a summary of what was obtained given the extension of the project. First is the definition of the

elements that integrate the problem to be evaluated Fig.1.

Once the paired trial matrix is constructed, we must check that they comply with the following axioms: A complies that $a_{ij} = 1/a_{ji}$, the elements that are compared are of the same order of magnitude and homogeneity, there is hierarchical dependence on the elements of two consecutive levels, expectations are represented in the structure in terms of criteria and alternatives.

Once the matrices of paired comparisons have been elaborated, the priority is calculated, each of the elements that are compared and known as synthesis, this mathematical process requires the calculation of normalized vectors and auto vectors.

The eigenvector or self-vector, gives hierarchy to the priorities of the evaluated alternatives and the eigenvalue or self-value corresponds to the measure of the consistency of the judgments issued regarding the matrix of paired comparisons.

For the determination of the eigenvector or own value, we proceed to determine the power squared of a matrix A of defined order, later, the summations of the rows of each component of the matrix are normalized and in this way the value of the eigenvector is obtained.

For the determination of self-value or own value, we clear the term $\lambda_{\max}$ corresponding to self-value from the Eq. 3 [15].

$$A \cdot x = \lambda_{\max} \cdot X \quad (3)$$

Eq. 3. Reciprocal matrix of pairwise comparisons where, A is reciprocal matrix of pairwise comparisons; X is own vector; $\lambda_{\max}$ is maximum own value to determine the eigenvector and eigenvalue of a matrix, proceed as follows: determination of the eigenvector "Eigen vector" x , of the comparisons matrices, Table 3 shows the values for the two alternative.

Once the initial matrix values are obtained, for each alternative the necessary iterations are performed to have a CT value by comparing the values of the E_0 and E_1 autovector, through the difference obtained

	Goal	Alt. 1	Alt.2
Criteria 1	P'1	0.06	0.13
Criteria 2	P'2	0.09	0.14
Criteria 3	P'3	0.23	0.16
Criteria 4	P'4	0.12	0.12
Criteria 5	P'5	0.19	0.12
Criteria 6	P'6	0.31	0.34

Fig. 2 Priority autovectors.

between them, the final autovector will be determined of priorities; when the difference between them, is as close as possible to zero. In both alternatives several iterations were necessary.

The following values were obtained for alternative 1: CI = 0.086, IA = 1.32, TC = 0.065; for alternative 2, CI = 0.124 and TC = 0.093. In both cases the consistency rates are acceptable. Next, we proceed to obtain the priority vector of the alternatives (Fig. 2) the two alternatives are analyzed according to each of the criteria. As many own vectors as criteria are obtained, the number of elements of each own vector being equal to the number of alternatives [16].

The result establishes that according to the set of judgments contemplated, in both cases the predominant criterion is to perform a physical delimitation (DS) of the hydraulic sectors, since in the two alternatives it has almost the same weight for the success of the execution regardless of the selected policy.

The selection of alternative 1 is the most appropriate, since it establishes only 3 criteria that should be controlled with greater rigor, being: DS, AEC and HS, however, it cannot be executed without considering a cost recovery of the service, it must be accompanied by a charge for the service efficiently, it shows that it is better decision to have a continuous schedule because of the repercussions that have on the cost of operation and investment in the rehabilitation and replacement of the network, in addition the volume of water supplied and the amount of leaks in the network can be estimated more easily and considered as “stable”.

On the other hand, if the selection is alternative 2, it is essential to have all the actions defined in the criteria almost in equal importance, causing a greater effort for the operating agency.

5. Conclusions

The decision-making process or the selection of an operating policy for an operating agency represents one of the most complicated problems given that for the choice of the most appropriate option, tangible, but also intangible or difficult consideration issues must be included.

Using the AHP as a tool allows us to visualize in numerical form the weights of the criteria considered in a specific problem, without losing the objective, the solution alternatives and the criteria that must be evaluated, giving numerical values to judgments or opinions issued by who should make the decision that focus on a series of square matrices.

When applying the AHP to the operation management, it allows to anticipate requirements and priority actions to be carried out, since it visualizes with greater weight the criteria that most affect the achievement of the desired objective, since the characteristics of the method allow to integrate the evaluation of alternatives all those criteria that involve the problem.

For a correct application of AHP, certain features of the method must be observed which, if not met, runs the risk of not having a coherent perspective during the process and this would lead to making a wrong decision.

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