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Unification of Gravity and Electromagnetism

Mohammed A. El-Lakany
Physics Department, Faculty of Science, Cairo University, Giza 12613, Egypt

Abstract: Gravity and electromagnetism are two sides of the same coin, which is the clue of this unification. Gravity and electromagnetism are represented by two mathematical structures, symmetric and antisymmetric respectively. Einstein gravitational field equation is the symmetric mathematical structure. Electrodynamics Lagrangian is three parts, for electromagnetic field, Dirac field and interaction term. The definition of canonical energy momentum tensor was used for each term in Electrodynamics Lagrangian to construct the antisymmetric mathematical structure; symmetric and antisymmetric gravitational field equations are two sides of the same Lagrangian.

Key words: Gravity, electromagnetism, general theory of relativity, quantum field theory, nuclear and particle physics, astrophysics and cosmology.

1. Gravity and Electromagnetism Are Two Sides of the Same Coin

Gravitational objects have spin and angular momentum; spin and angular momentum of gravitational objects are related to basic quantum properties of elementary particles. The angular momentum for the sun is given by $J_{sun}{=}\mathrm{M}_{sun}\omega_\sigma R^2{}_{sun} \approx 10^{50}$ ergs.s; for solar system it is $J_{solsys}{=}\mathrm{M}_{solsys}\omega_\sigma R^2{}_{solsys} \approx 10^{52}$ ergs.s . In the case of a galaxy the angular momentum is given by $\mathrm{J_{gal}} = \mathrm{M_{gal}}\ \omega_\sigma \mathrm{R}^2{}_{\mathrm{gal}}$ where $\mathrm{M_{gal}} = 10^{45}\mathrm{g};\ \mathrm{R_{gal}}^2 = 10^{47}\ \mathrm{cm}^2; \omega_\sigma = 2\times10^{-18}\,HZ$ and the value of angular momentum is $J_{\mathrm{gal}} \approx 10^{74}$ ergs.s . Similarly for cluster of galaxies, the angular momentum is given by $J_{Clust}{=}\mathrm{M}_{Clust}\omega_\sigma R^2{}_{Clust} \approx 10^{110}\ \hbar$ in Hubble scale and for the universe $J_{univ} \approx 10^{120}\hbar$. Spin density (σ = spin/volume) is the same for a wide range; for an electron, the spin density is given by

$$\sigma_e = \frac{0.5\hbar}{\frac{4}{3}\pi r_e^{\,3}} \sim 10^9 \mathrm{ergs.s}/cc.$$

For proton $\sigma_p \sim 10^9 \mathrm{ergs.s}/cc$ and also for the solar system we have $\sigma_{solsys} \sim 10^9 \mathrm{ergs.s}/cc$; for a galaxy $\sigma_{gal} = \dfrac{10^{100}\hbar}{\frac{4}{3}\pi R_{gal}^{\,3}} \sim 10^9 \mathrm{ergs.s}/cc$, spin density for universe $\sigma_{univ} = \dfrac{10^{120}\hbar}{\frac{4}{3}\pi R_H^{\,3}} \sim 10^9 \mathrm{ergs.s}/cc$ [1]. Not only this, but also magnetic fields seem to be everywhere that we can look in the universe [2]. Magnetic fields are observed to be of the order of 10^{13} G in neutron stars, 10^3 G in solar type stars. Magnetic fields of order a few μG also have been detected in radio galaxies [3]. Magnetic fields are associated with all gravitational objects; gravitational objects are magnetic dipoles; electromagnetism not only tied to charged particles, but the planets, stars, galaxies and clusters.

2. Symmetric and Antisymmetric Mathematical Structures

Unification of gravity and electromagnetism has been pursued by many scientists, like Weyl, Eddington, Einstein, Infeld, Born and Schrodinger.

Corresponding author: Mohammed A. El-Lakany, M.Sc., research fields: astrophysics and cosmology, general relativity and gravity, electromagnetism.

Weyl initiated this unification; Eddington considered connection as the central concept then decomposed its Ricci tensor to symmetric Ricci tensor ($R_{\mu\nu}$) which represents gravity and antisymmetric Ricci tensor ($R_{\nu\sigma}$) represents electromagnetism.

Infeld and Born followed the path of Eddington then derived the Lagrangian $L_{GR} \equiv \sqrt{-\det(g_{\mu\nu}+F_{\nu\sigma})}-\sqrt{-g}$, they considered the asymmetric metric $g_{(\mu\nu)} = g_{\mu\nu} + F_{\nu\sigma}$, its symmetric term $g_{\mu\nu}$ represents gravity and antisymmetric term ($F_{\nu\sigma}$) represents electromagnetism, g is the determinant of the symmetric metric tensor $g_{\mu\nu}$ [4]. Schrodinger generalized Eddington Lagrangian to a new form containing the cosmological constant (Λ) [5]; despite the failure of these previous attempts, they in its entirety refer to something cannot be neglected that gravity and electromagnetism should be represented by two mathematical structures.

3. Curvature Tensor

Riemann tensor in terms of Christoffel's symbols is defined by

$$R^{\delta}{}_{\mu\nu\sigma} = \Gamma^{\lambda}{}_{\mu\sigma}\Gamma^{\delta}{}_{\lambda\nu} - \Gamma^{\lambda}{}_{\mu\nu}\Gamma^{\delta}{}_{\lambda\sigma} + \Gamma^{\delta}{}_{\mu\sigma,\nu} - \Gamma^{\delta}{}_{\mu\nu,\,\sigma} \quad (1)$$

Riemann Christoffel tensor is of rank four, contravariant in δ and covariant in μ, ν, and σ, and also

$$R^{\delta}{}_{\mu\nu\sigma} = 0 \quad (2)$$

Is the necessary condition for the validity of the special theory of Relativity and for the absence of permanent gravitational field or the necessary and sufficient condition that the space time is flat [6].

Lowering the last index in the Riemann Christoffel tensor with the symmetric metric tensor, the lowered tensor $R_{\mu\nu\sigma\varepsilon} = R^{\delta}{}_{\mu\nu\sigma} g_{\delta\varepsilon}$ is symmetric under interchanging of the first and last pair of indices and antisymmetric in μ, ε and in ν, σ. Symmetric and antisymmetric Ricci tensors can be written as follow

$$R_{\mu\nu} = R^{\delta}{}_{\mu\nu\delta} = \Gamma^{\lambda}{}_{\mu\delta}\Gamma^{\delta}{}_{\lambda\nu} - \Gamma^{\lambda}{}_{\mu\nu}\Gamma^{\delta}{}_{\lambda\delta} + \Gamma^{\delta}{}_{\mu\delta,\nu} - \Gamma^{\delta}{}_{\mu\nu,\,\delta} \quad (3)$$

$$R_{\nu\sigma} = R^{\delta}{}_{\delta\nu\sigma} = \Gamma^{\delta}{}_{\delta\sigma,\nu} - \Gamma^{\delta}{}_{\delta\nu,\,\sigma} \quad (4)$$

Symmetric and antisymmetric Ricci tensors give us the opportunity to have symmetric and antisymmetric gravitational field equations.

4. General Theory of Relativity

General relativity is the modern theory of gravity; General theory of relativity relates gravitational field to the curvature of space time. Symmetric stress energy tensor $T_{\mu\nu}$ is the source of gravitational field in general theory of relativity.

In the presence of permanent gravitational field, the symmetric gravitational field equation is

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} \quad (5)$$

R is the Ricci scalar and G is the gravitational constant.

Einstein-Hilbert action for gravity is given by $S = \int L_{GR} d^4V = \int \frac{c^4}{16\pi G}(R-2\Lambda)\sqrt{-g}\, d^4x$, where, $dV = \sqrt{-g}\, d^4x$ is invariant volume element and gravity Lagrangian is defined by

$$\mathcal{L}_{GR} = \frac{c^4}{16\pi G}(R - 2\Lambda) \quad (6)$$

Gravity Lagrangian is a combination of Ricci scalar and cosmological constant.

5. Electrodynamics

Electrodynamics Lagrangian is given by

$$\mathcal{L}_{ED} = -\frac{1}{4} F^{\nu\sigma} F_{\nu\sigma} + \bar{\psi}(i\gamma_{\nu} D^{\nu} - m)\psi \quad (7)$$

where, $F^{\nu\sigma}$ is the electromagnetic field strength tensor, D^{ν} is the gauge contravariant derivative, ψ is matter field, $\bar{\psi} = \gamma_0 \psi^{\dagger}$ is their adjoint, $i = \sqrt{-1}$ and γ_{ν} is the four Dirac matrices with (ν = 0, 1 … 3). The electromagnetic field strength tensor ($F^{\nu\sigma}$) is given by

$$F^{\nu\sigma} = \begin{bmatrix} 0 & -E_1 & -E_2 & -E_3 \\ E_1 & 0 & -B_3 & B_2 \\ E_2 & B_3 & 0 & -B_1 \\ E_3 & -B_2 & B_1 & 0 \end{bmatrix}$$

and

$$F_{\nu\sigma} = \begin{bmatrix} 0 & E_1 & E_2 & E_3 \\ -E_1 & 0 & -B_3 & B_2 \\ -E_2 & B_3 & 0 & -B_1 \\ -E_3 & -B_2 & B_1 & 0 \end{bmatrix}$$

and they lowered index counterpart.

The first term of Electrodynamics Lagrangian for the electromagnetic field is given by

$$\mathcal{L}^{\text{e.m.}} = -\tfrac{1}{4} F^{\nu\sigma} F_{\nu\sigma} \quad (8)$$

Canonical energy momentum tensor for electromagnetic field Lagrangian is

$$\theta_{\nu\sigma}{}^{e.m.} = \frac{\partial \mathcal{L}^{e.m.}}{\partial(\partial^\nu A^\mu)} \partial_\sigma A^\mu - \mathrm{g}_{\nu\sigma} \mathcal{L}^{e.m.} \quad (9)$$

Using the identity $\dfrac{\partial(F^{\nu\sigma}F_{\nu\sigma})}{\partial(\partial^\nu \mathrm{A}^\mu)} = 4F_{\nu\mu}$, we find

$$\theta_{\nu\sigma}{}^{e.m} = -F_{\nu\mu}F_\sigma{}^\mu + \frac{1}{4} g_{\nu\sigma} F_{\delta\lambda} F^{\delta\lambda} \quad (10)$$

Eq. (10) is not antisymmetric due to the asymmetric tensor $(-F_{\nu\mu}F_\sigma{}^\mu)$ [7]; for this, suppose the asymmetric tensor is the sum of symmetric, antisymmetric tensor can be written as follow.

$$-F_{\nu\mu}F_\sigma{}^\mu = \partial^\sigma \chi_{\sigma\nu\mu} - F_{\nu\mu}F_\sigma{}^\mu - \partial^\sigma \chi_{\sigma\nu\mu} \quad (11)$$

The divergence tensor is arbitrary antisymmetric tensor in their first two indices ($\chi_{\sigma\nu\mu} = -\chi_{\nu\sigma\mu}$), it is constructed from electromagnetic field strength tensor ($F_{\nu\sigma}$) and electromagnetic vector potential (A_μ).

Eq. (11) in terms of this definition can be rewritten as

$$-F_{\nu\mu}F_\sigma{}^\mu = \partial^\sigma (F_{\nu\sigma}A_\mu) - F_{\nu\mu}F_\sigma{}^\mu - \partial^\sigma (F_{\nu\sigma}A_\mu) \quad (12)$$

Employing the Maxwell equation, we obtain

$$-F_{\nu\mu}F_\sigma{}^\mu = -\mathrm{j}_\nu A_\mu - F_{\nu\mu}F_\sigma{}^\mu + \mathrm{j}_\nu A_\mu \quad (13)$$

The antisymmetric stress energy tensor for electromagnetic field can be written in the form:

$$T_{\nu\sigma}{}^{e.m} = j_\nu A_\mu + \frac{1}{4} g_{\nu\sigma} F_{\delta\lambda} F^{\delta\lambda} \quad (14)$$

If we multiplied this equation by $\left(\dfrac{8\pi G}{c^4}\right)$, we find

$$\frac{8\pi G}{c^4} T_{\nu\sigma}{}^{e.m.} = \frac{8\pi G}{c^4} j_\nu A_\mu - \frac{8\pi G}{c^4} \mathrm{g}_{\nu\sigma} \mathcal{L}^{e.m.} \quad (15)$$

The second term in electrodynamics Lagrangian for Dirac field is given by

$$\mathcal{L}^{\text{Dirac}} = \bar{\psi}\left(i\gamma_\nu \partial^\nu - m\right)\psi \quad (16)$$

The canonical energy momentum tensor is defined by

$$\theta_{\nu\sigma}{}^{Dirac} = \frac{\partial \mathcal{L}^{Dirac}}{\partial\left(\partial^\nu \psi\right)} \partial_\sigma \psi + \frac{\partial \mathcal{L}^{Dirac}}{\partial\left(\partial^\nu \psi^+\right)} \partial_\sigma \psi^+ - \mathrm{g}_{\nu\sigma} \mathcal{L}^{Dirac} \quad (17)$$

$$\theta_{\nu\sigma}{}^{Dirac} = \overline{\psi} i\gamma_\nu \partial_\sigma \psi - \mathrm{g}_{\nu\sigma} \bar{\psi}\left(i\gamma_\lambda \partial^\lambda - m\right)\psi \quad (18)$$

The canonical energy momentum tensor that has been presented in this equation is not antisymmetric due to the symmetric term ($\overline{\psi} i\gamma_\nu \partial_\sigma \psi$). For this, the antisymmetric stress energy tensor can be written as the canonical energy momentum tensor minus this symmetric term as follow:

$$T_{\nu\sigma}{}^{Dirac} = \theta_{\nu\sigma}{}^{Dirac} - \overline{\psi} i\gamma_\nu \partial_\sigma \psi \quad (19)$$

$$T_{\nu\sigma}{}^{Dirac} = -\mathrm{g}_{\nu\sigma} \mathcal{L}^{Dirac} \quad (20)$$

Multiplying Eq. (20) by $\left(\dfrac{8\pi G}{c^4}\right)$, we have

$$\frac{8\pi G}{c^4} T_{\nu\sigma}{}^{Dirac} = -\frac{8\pi G}{c^4} \mathrm{g}_{\nu\sigma} \bar{\psi}\left(i\gamma_\lambda \partial^\lambda - m\right)\psi \quad (21)$$

Third term is the interaction Lagrangian and given by

$$\mathcal{L}^{int} = -e\overline{\psi}\gamma_v \psi A^V \quad (22)$$

The canonical energy momentum tensor is given by

$$\theta_{v\sigma}{}^{int} = -g_{v\sigma}\mathcal{L}^{int} \quad (23)$$

And antisymmetric stress energy tensor is

$$T_{v\sigma}{}^{int} = -g_{v\sigma}\mathcal{L}^{int} \quad (24)$$

Antisymmetric stress energy tensor for interaction Lagrangian is the same canonical energy momentum tensor; multiplying the previous equation by $\left(\frac{8\pi G}{c^4}\right)$, we find

$$\frac{8\pi G}{c^4} T_{v\sigma}{}^{int} = -\frac{8\pi G}{c^4} g_{v\sigma}\mathcal{L}^{int} \quad (25)$$

If we added Eqs. (15) and (21) to Eq. (25), we have

$$\frac{8\pi G}{c^4}\left[T_{v\sigma}{}^{e.m} + T_{v\sigma}{}^{int} + T_{v\sigma}{}^{Dirac}\right] = \frac{8\pi G}{c^4} j_v A_\mu - \frac{8\pi G}{c^4} g_{v\sigma}\left[\mathcal{L}^{e.m.} + \mathcal{L}^{Dirac} + \mathcal{L}^{int}\right] \quad (26)$$

$$\frac{8\pi G}{c^4}\left[T_{v\sigma}{}^{e.m} + T_{v\sigma}{}^{int} + T_{v\sigma}{}^{Dirac}\right] = \frac{8\pi G}{c^4} j_v A_\mu + \left[\frac{2\pi G}{c^4} F_{\delta\lambda}F^{\delta\lambda}\right] g_{v\sigma} - \frac{8\pi G}{c^4} g_{v\sigma}\left[\overline{\psi}\left(i\gamma_\lambda \partial^\lambda - m\right)\psi - e\overline{\psi}\gamma_\lambda \psi A^\lambda\right] \quad (27)$$

If gauge contravariant derivative ($D^\lambda = \partial^\lambda + ieA^\lambda$) is used in the previous equation, we find

$$\frac{8\pi G}{c^4}\left[T_{v\sigma}{}^{e.m} + T_{v\sigma}{}^{int} + T_{v\sigma}{}^{Dirac}\right] = \frac{8\pi G}{c^4} j_v A_\mu + \left[\frac{2\pi G}{c^4} F_{\delta\lambda}F^{\delta\lambda}\right] g_{v\sigma} - \frac{1}{2} g_{v\sigma}\left[\frac{16\pi G}{c^4}\overline{\psi}\left(i\gamma_\lambda D^\lambda - m\right)\psi\right] \quad (28)$$

$$\frac{8\pi G}{c^4} T_{v\sigma} = R_{v\sigma} + \Lambda g_{v\sigma} - \frac{1}{2} R g_{v\sigma} \quad (29)$$

Antisymmetric gravitational field equation is gauge invariant and antisymmetric stress energy tensor can be written in the form.

$$T_{v\sigma} = \frac{c^4}{8\pi G} R_{v\sigma} + \frac{c^4}{8\pi G}\Lambda g_{v\sigma} - \frac{c^4}{16\pi G} R g_{v\sigma} \quad (30)$$

Ricci scalar is proportional to the sum of Dirac and interaction Lagrangians as follow.

$$R = \frac{16\pi G}{c^4}\overline{\psi}(\, i\ \gamma_\lambda D^\lambda - m)\psi \quad (31)$$

Cosmological constant is a construction from electromagnetic field strength tensor and given by:

$$\Lambda = \frac{2\pi G}{c^4} F^{\delta\lambda} F_{\delta\lambda} \quad (32)$$

Antisymmeric Ricci tensor is given by:

$$R_{v\sigma} = \frac{8\pi G}{c^4} j_v A_\mu \quad (33)$$

Antisymmetric Ricci tensor is the antisymmetric term of Eq. (13) multiplied by $\left(\frac{8\pi G}{c^4}\right)$. Substituting Eqs. (31) and (32) into Eq. (6), we have

$$\mathcal{L}_{GR} = \frac{c^4}{16\pi G}\left[\frac{16\pi G}{c^4}\overline{\psi}\left(i\gamma_\lambda D^\lambda - m\right)\psi\right] - \frac{c^4}{8\pi G}\left[\frac{2\pi G}{c^4} F^{\delta\lambda}F_{\delta\lambda}\right] = \mathcal{L}^{Dirac} + \mathcal{L}^{int} + \mathcal{L}^{e.m.} = \mathcal{L}_{ED} \quad (34)$$

Gravity Lagrangian equal to electrodynamics Lagrangian, but in terms of the secondset of indices.

Electrodynamics Lagrangian and its parts can be written in terms of one of two sets of indices, first set is

$\{\mu,\nu,\sigma\}$ and second set is $\{\varepsilon,\delta,\lambda\}$. If we multiplied Eq. (13) by $\left(\frac{8\pi G}{c^4}\right)$, we find

$$-\frac{8\pi G}{c^4}F_{\nu\mu}F_{\sigma}^{\ \mu}=R_{\mu\nu}+R_{\nu\sigma} \tag{35}$$

$$R_{\mu\nu}=\frac{8\pi G}{c^2}\left[-\mathrm{j}_\nu A_\mu-F_{\nu\mu}F_{\sigma}^{\ \mu}\right] \tag{36}$$

The symmetric Ricci tensor is the symmetric term of Eq. (13) multiplied by $\left(\frac{8\pi G}{c^4}\right)$; it has two parts, first term is a construction of current density and electromagnetic vector potential, and second term is the gravitational field tensor. If we substituted by Eq. (36) into Eq. (3), we find

$$-\frac{8\pi G}{c^2}F_{\nu\mu}F_{\sigma}^{\ \mu}-\frac{8\pi G}{c^2}\mathrm{j}_\nu A_\mu=\Gamma^{\lambda}{}_{\mu\delta}\Gamma^{\delta}{}_{\lambda\nu}-\Gamma^{\lambda}{}_{\mu\nu}\Gamma^{\delta}{}_{\lambda\delta}+\Gamma^{\delta}{}_{\mu\delta,\nu}-\Gamma^{\delta}{}_{\mu\nu,\,\delta} \tag{37}$$

Substituting by Eq. (33) into Eq. (4), we find

$$\frac{8\pi G}{c^4}j_\nu A_\mu=\Gamma^{\delta}{}_{\delta\sigma,\nu}-\Gamma^{\delta}{}_{\delta\nu,\,\sigma} \tag{38}$$

This tensor takes the form of curl of vector as follow:

$$\frac{8\pi G}{c^4}j_\nu A_\mu=\partial_\nu\partial_\sigma\log\sqrt{-g}-\partial_\sigma\partial_\nu\log\sqrt{-g} \tag{39}$$

Eq. (37) can be divided into two equations as follow.

$$-\frac{8\pi G}{c^2}F_{\nu\mu}F_{\sigma}^{\ \mu}=\Gamma^{\lambda}{}_{\mu\delta}\Gamma^{\delta}{}_{\lambda\nu}-\Gamma^{\lambda}{}_{\mu\nu}\Gamma^{\delta}{}_{\lambda\delta} \tag{40}$$

$$-\frac{8\pi G}{c^4}j_\nu A_\mu=\Gamma^{\delta}{}_{\mu\delta,\nu}-\Gamma^{\delta}{}_{\mu\nu,\,\delta} \tag{41}$$

Eq. (41) can be rewritten as:

$$-\frac{8\pi G}{c^4}j_\nu A_\mu=\partial_\nu\partial_\mu\log\sqrt{-g}-\partial_\delta\Gamma^{\delta}{}_{\mu\nu} \tag{42}$$

$$\partial_\nu\partial_\mu\log\sqrt{-g}+\frac{8\pi G}{c^4}j_\nu A_\mu=\partial_\delta\Gamma^{\delta}{}_{\mu\nu} \tag{43}$$

$$\Gamma^{\delta}{}_{\mu\nu}=\frac{1}{2}g^{\delta\sigma}\left(\partial_\nu g_{\sigma\mu}+\partial_\mu g_{\sigma\nu}-\partial_\sigma g_{\mu\nu}\right)=\frac{1}{2}g^{\delta\sigma}\left(\partial_\mu g_{\sigma\nu}-\partial_\sigma g_{\mu\nu}\right) \tag{44}$$

$$g_{\mu\sigma}=g_{\sigma\mu}=g_{\sigma}^{\ \mu}=g_{\mu}^{\ \sigma}=g_{\mu\nu}g^{\nu\sigma}=0 \tag{45}$$

$$\partial_\delta \Gamma^\delta{}_{\mu\nu} = \frac{1}{2}\partial_\delta g^{\delta\sigma}\partial_\mu g_{\sigma\nu} - \frac{1}{2}\partial_\delta g^{\delta\sigma}\partial_\sigma g_{\mu\nu} = \frac{1}{2}\partial_\delta \partial_\mu g^\delta{}_\nu - \frac{1}{2}\partial_\delta \partial_\sigma g^{\delta\sigma} g_{\mu\nu} \tag{46}$$

Substitute by Eq. (46) into Eq. (43), we find

$$\partial_\nu \partial_\mu \log\sqrt{-g} + \frac{8\pi G}{c^4} j_\nu A_\mu = \frac{1}{2}\partial_\delta \partial_\mu g^\delta{}_\nu - \frac{1}{2}\partial_\delta \partial_\sigma g^{\delta\sigma} g_{\mu\nu} \tag{47}$$

Equating the first term on the left hand side of the equation with the first term on the right hand side of the equation, we find

$$\partial_\nu \partial_\mu \log\sqrt{-g} = \frac{1}{2}\partial_\delta \partial_\mu g^\delta{}_\nu \tag{48}$$

$$\partial_\nu \log\sqrt{-g} = \frac{1}{2}\partial_\delta g^\delta{}_\nu \tag{49}$$

In Eq. (47) if we equate the second term by the second term, we find

$$\frac{8\pi G}{c^4} j_\nu A_\mu = -\frac{1}{2}\partial_\delta \partial_\sigma g^{\delta\sigma} g_{\mu\nu} \tag{50}$$

Equating Eq. (50) with Eq. (39), we find

$$\partial_\nu \partial_\sigma \log\sqrt{-g} - \partial_\sigma \partial_\nu \log\sqrt{-g} = -\frac{1}{2}\partial_\delta \partial_\sigma g^{\delta\sigma} g_{\mu\nu} \tag{51}$$

Eq. (40) can be rewritten in the form

$$-\frac{8\pi G}{c^2} F_{\nu\mu} F_\sigma{}^\mu = \Gamma^\lambda{}_{\mu\delta}\Gamma^\delta{}_{\lambda\nu} - \Gamma^\lambda{}_{\mu\nu}\partial_\lambda \log\sqrt{-g} \tag{52}$$

$$\Gamma^\lambda{}_{\mu\delta} = \frac{1}{2} g^{\lambda\sigma}\left(\partial_\delta g_{\sigma\mu} + \partial_\mu g_{\sigma\delta} - \partial_\sigma g_{\mu\delta}\right) = \frac{1}{2} g^{\lambda\sigma}\left(\partial_\mu g_{\sigma\delta} - \partial_\sigma g_{\mu\delta}\right) \tag{53}$$

$$\Gamma^\delta{}_{\lambda\nu} = \frac{1}{2} g^{\delta\sigma}\left(\partial_\nu g_{\sigma\lambda} + \partial_\lambda g_{\sigma\nu} - \partial_\sigma g_{\lambda\nu}\right) \tag{54}$$

$$\Gamma^\lambda{}_{\mu\nu} = \frac{1}{2} g^{\lambda\sigma}\left(\partial_\nu g_{\sigma\mu} + \partial_\mu g_{\sigma\nu} - \partial_\sigma g_{\mu\nu}\right) = \frac{1}{2} g^{\lambda\sigma}\left(\partial_\mu g_{\sigma\nu} - \partial_\sigma g_{\mu\nu}\right) = -\frac{1}{2} g^{\lambda\sigma}\partial_\mu g_{\nu\sigma} - \frac{1}{2} g^{\lambda\sigma}\partial_\sigma g_{\mu\nu} \tag{55}$$

$$\Gamma^{\lambda}{}_{\mu\delta}\Gamma^{\delta}{}_{\lambda\nu} = \left[\frac{1}{2} g^{\lambda\sigma}\left(\partial_{\mu} g_{\sigma\delta} - \partial_{\sigma} g_{\mu\delta}\right)\right]\left[\frac{1}{2} g^{\delta\sigma}\left(\partial_{\nu} g_{\sigma\lambda} + \partial_{\lambda} g_{\sigma\nu} - \partial_{\sigma} g_{\lambda\nu}\right)\right]$$

$$= \frac{1}{4}\left[g^{\lambda\sigma}\partial_{\mu} g_{\sigma\delta} - g^{\lambda\sigma}\partial_{\sigma} g_{\mu\delta}\right]\left[g^{\delta\sigma}\partial_{\nu} g_{\sigma\lambda} + g^{\delta\sigma}\partial_{\lambda} g_{\sigma\nu} - g^{\delta\sigma}\partial_{\sigma} g_{\lambda\nu}\right]$$

$$= \frac{1}{4}\Big[g^{\lambda\sigma}\partial_{\mu} g_{\sigma\delta} g^{\delta\sigma}\partial_{\nu} g_{\sigma\lambda} + g^{\lambda\sigma}\partial_{\mu} g_{\sigma\delta} g^{\delta\sigma}\partial_{\lambda} g_{\sigma\nu} - g^{\lambda\sigma}\partial_{\mu} g_{\sigma\delta} g^{\delta\sigma}\partial_{\sigma} g_{\lambda\nu}$$

$$- g^{\lambda\sigma}\partial_{\sigma} g_{\mu\delta} g^{\delta\sigma}\partial_{\nu} g_{\sigma\lambda} - g^{\lambda\sigma}\partial_{\sigma} g_{\mu\delta} g^{\delta\sigma}\partial_{\lambda} g_{\sigma\nu} + g^{\lambda\sigma}\partial_{\sigma} g_{\mu\delta} g^{\delta\sigma}\partial_{\sigma} g_{\lambda\nu}\Big]$$

$$= \frac{1}{4}\left[g^{\lambda\sigma}\partial_{\mu} g_{\sigma\delta} g^{\delta\sigma}\partial_{\nu} g_{\sigma\lambda} + g^{\lambda\sigma}\partial_{\mu} g_{\sigma\delta} g^{\delta\sigma}\partial_{\lambda} g_{\sigma\nu} - g^{\lambda\sigma}\partial_{\mu} g_{\sigma\delta} g^{\delta\sigma}\partial_{\sigma} g_{\lambda\nu}\right]$$

$$= \frac{1}{4}\left[g^{\lambda\sigma}\partial_{\mu} g_{\sigma}{}^{\sigma}\partial_{\nu} g_{\sigma\lambda} + g^{\lambda\sigma}\partial_{\mu} g_{\sigma}{}^{\sigma}\partial_{\lambda} g_{\sigma\nu} - g^{\lambda\sigma}\partial_{\mu} g_{\sigma}{}^{\sigma}\partial_{\sigma} g_{\lambda\nu}\right]$$

$$= \frac{1}{4}\left[\partial_{\mu} g^{\lambda\sigma} g_{\sigma}{}^{\sigma}\partial_{\nu} g_{\sigma\lambda} + \partial_{\mu} g^{\lambda\sigma} g_{\sigma}{}^{\sigma}\partial_{\lambda} g_{\sigma\nu} - \partial_{\mu} g^{\lambda\sigma} g_{\sigma}{}^{\sigma}\partial_{\sigma} g_{\lambda\nu}\right]$$

$$= \frac{1}{4}\left[\partial_{\mu} g^{\lambda\sigma}\partial_{\nu} g_{\sigma\lambda} + \partial_{\mu} g^{\lambda\sigma}\partial_{\lambda} g_{\sigma\nu} - \partial_{\mu} g^{\lambda\sigma}\partial_{\sigma} g_{\lambda\nu}\right]$$

$$= \frac{1}{4}\left[\partial_{\mu}\partial_{\nu} g^{\lambda\sigma} g_{\sigma\lambda} + \partial_{\mu}\partial_{\lambda} g^{\lambda\sigma} g_{\sigma\nu} - \partial_{\mu}\partial_{\sigma} g^{\lambda\sigma} g_{\lambda\nu}\right]$$

$$= \frac{1}{4}\left[\partial_{\mu}\partial_{\nu} g^{\lambda}{}_{\lambda} + \partial_{\mu}\partial_{\lambda} g^{\lambda}{}_{\nu} - \partial_{\mu}\partial_{\sigma} g^{\sigma\lambda} g_{\lambda\nu}\right]$$

$$= \frac{1}{4}\left[\partial_{\mu}\partial_{\nu} g^{\lambda}{}_{\lambda} + \partial_{\mu}\partial_{\lambda} g^{\lambda}{}_{\nu} - \partial_{\mu}\partial_{\sigma} g^{\sigma}{}_{\nu}\right] \tag{56}$$

Using Eq. (49), we find

$$\Gamma^{\lambda}{}_{\mu\delta}\Gamma^{\delta}{}_{\lambda\nu} = \frac{1}{4}\left[\partial_{\mu}\partial_{\nu} g^{\lambda}{}_{\lambda} + 2\partial_{\mu}\partial_{\nu}\log\sqrt{-g} - 2\partial_{\mu}\partial_{\nu}\log\sqrt{-g}\right] = \frac{1}{4}\partial_{\mu}\partial_{\nu} g^{\lambda}{}_{\lambda} \tag{57}$$

And substituting by this equation into Eq. (52), we have

$$-\frac{8\pi G}{c^2} F_{\nu\mu} F_{\sigma}{}^{\mu} = \frac{1}{4}\partial_{\mu}\partial_{\nu} g^{\lambda}{}_{\lambda} + \frac{1}{2} g^{\lambda\sigma}\partial_{\mu}\partial_{\lambda}\log\sqrt{-g}\, g_{\nu\sigma} + \frac{1}{2} g^{\lambda\sigma}\partial_{\sigma}\partial_{\lambda}\log\sqrt{-g}\, g_{\mu\nu} \tag{58}$$

Now, let's construct the antisymmetric metric tensor; magnetic field in empty space is given by

$$\vec{B}=\vec{B}_{01}e^{i(k_1x_1-\omega_1x_4)}+\vec{B}_{02}e^{i(k_2x_2-\omega_2x_4)}+\vec{B}_{03}e^{i(k_3x_3-\omega_3x_4)} \tag{59}$$

$\omega=\omega_1+\omega_2+\omega_3$, $\vec{k}=(k_1,k_2,k_3)$ are the wave frequency and wave vector. In general orthogonal curvilinear coordinates a vector $\vec{A}$ defined as follow:

$$\vec{A}=\vec{e}_1h_1+\vec{e}_2h_2+\vec{e}_3h_3 \tag{60}$$

Let's suppose that (B_{01}, B_{02}, B_{03}) is the unit vector then equate Eq. (59) with Eq. (60), we find $h_1=e^{i(k_1x_1-\omega_1x_4)}, h_2=e^{i(k_2x_2-\omega_2x_4)}$ and $h_3=e^{i(k_3x_3-\omega_3x_4)}$. Using these three coefficients to construct the antisymmetric metric tensor ($g_{\nu\sigma}$); this tensor is in the same form of electromagnetic field strength tensor $F_{\nu\sigma}$ and with the same signs.

$$g_{\nu\sigma}=\begin{bmatrix} 0 & h_1 & h_2 & h_3 \\ -h_1 & 0 & -h_3 & h_2 \\ -h_2 & h_3 & 0 & -h_1 \\ -h_3 & -h_2 & h_1 & 0 \end{bmatrix}=\begin{bmatrix} 0 & e^{i(k_1x_1-\omega_1x_4)} & e^{i(k_2x_2-\omega_2x_4)} & e^{i(k_3x_3-\omega_3x_4)} \\ -e^{i(k_1x_1-\omega_1x_4)} & 0 & -e^{i(k_3x_3-\omega_3x_4)} & e^{i(k_2x_2-\omega_2x_4)} \\ -e^{i(k_2x_2-\omega_2x_4)} & e^{i(k_3x_3-\omega_3x_4)} & 0 & -e^{i(k_1x_1-\omega_1x_4)} \\ -e^{i(k_3x_3-\omega_3x_4)} & -e^{i(k_2x_2-\omega_2x_4)} & e^{i(k_1x_1-\omega_1x_4)} & 0 \end{bmatrix} \tag{61}$$

And now, we will return to the cosmological constant; the cosmological constant splits up into two parts where $F_{\delta\lambda}F^{\delta\lambda}=-2E^2+2B^2$

$$\Lambda=-\frac{4\pi G}{c^4}E^2+\frac{4\pi G}{c^4}B^2 \tag{62}$$

The first term of cosmological constant can be written as:

$$\Lambda_1=\frac{8\pi G}{c^4}\rho_1 \tag{63}$$

$$\rho_1=-\frac{1}{2}E^2=\frac{B.E.}{A} \tag{64}$$

First term is proportional to density of vacuum electric energy; density of vacuum electric energy is equivalent to binding energy per nucleon $\left(\frac{B.E.}{A}\right)$;

$\frac{B.E.}{A}=-\frac{\Delta m}{A}c^2$ where $\Delta m=Zm_p+(A-Z)m_n-M_N$, A is atomic mass number, Z is atomic number, M_N is a nucleus mass, m_p is proton mass and m_n is neutron mass [8]; first term is a function of atomic mass number (A) and it is a continuous quantity. The second term of cosmological constant can be written as:

$$\Lambda_2=\frac{8\pi G}{c^4}\rho_2 \tag{65}$$

$$\rho_2=\frac{1}{2}B^2=\left|\frac{B.E.}{A}\right| \tag{66}$$

Second term is proportional to density of vacuum magnetic energy; density of vacuum magnetic energy equals to the absolute value of binding energy per nucleon; second term is represented by a curve and it is the image of the first term by reflection on the A-axis in $A\Lambda$ -plane; all expected values for the cosmological constant (Λ) lie on the area between the two curves. Symmetric gravitational field equation in empty space is

$$R_{\mu\nu}-\frac{1}{2}Rg_{\mu\nu}=-g_{\mu\nu}\Lambda \tag{67}$$

$$R_{\mu\nu}=\left(\frac{1}{2}R-\Lambda\right)g_{\mu\nu} \tag{68}$$

Equating Eq. (36) with Eq. (68), we find

$$-\frac{8\pi G}{c^2}j_\nu A_\mu-\frac{8\pi G}{c^2}F_{\nu\mu}F_\sigma{}^\mu=\left(\frac{1}{2}R-\Lambda\right)g_{\mu\nu} \tag{69}$$

Antisymmetric gravitational field equation in empty space by analogy to symmetric gravitational field equation is

$$R_{\nu\sigma} - \frac{1}{2} R g_{\nu\sigma} = -g_{\nu\sigma} \Lambda \tag{70}$$

$$R_{\nu\sigma} = \left(\frac{1}{2} R - \Lambda \right) g_{\nu\sigma} \tag{71}$$

Equating Eq. (71) with Eq. (33), we have

$$\frac{8\pi G}{c^4} j_\nu A_\mu = \left(\frac{1}{2} R - \Lambda \right) g_{\nu\sigma} \tag{72}$$

Eqs. (69) and (72) are two states of energy; gravitational object transits between them and changes its state from fermion to boson or vice versa; this transition followed by emitting or absorbing gravitational field; if we added Eq. (69) into Eq. (72) we have

$$-\frac{8\pi G}{c^2} F_{\nu\mu} F_\sigma{}^\mu = \left(\frac{1}{2} R - \Lambda \right) g_{\nu\sigma} + \left(\frac{1}{2} R - \Lambda \right) g_{\mu\nu} \tag{73}$$

If we equate Eq. (58) by Eq. (73), the first term of Eq. (58) has not comparable one in Eq. (73) and equals to zero.

$$\frac{1}{4} \partial_\mu \partial_\nu g^\lambda{}_\lambda = 0 \tag{74}$$

Equating second term of Eq. (58) by the first term of Eq. (73), we find

$$\frac{1}{2} g^{\lambda\sigma} \partial_\mu \partial_\lambda \log \sqrt{-g} = \frac{1}{2} R - \Lambda \tag{75}$$

Equating third term of Eq. (58) by second term of Eq. (73), we find

$$\frac{1}{2} g^{\lambda\sigma} \partial_\sigma \partial_\lambda \log \sqrt{-g} = \frac{1}{2} R - \Lambda \tag{76}$$

Equating Eq. (75) by Eq. (76), we find

$$\frac{1}{2} g^{\lambda\sigma} \partial_\sigma \partial_\lambda \log \sqrt{-g} = \frac{1}{2} g^{\lambda\sigma} \partial_\mu \partial_\lambda \log \sqrt{-g} \tag{77}$$

$$\partial_\sigma = \partial_\mu \tag{78}$$

6. Conclusion

General relativity is very successful theory; differential geometry has been extended by new tensors and operators. These tensors are $g_{\mu\nu}$, $g_{\nu\sigma}$, $g_{\varepsilon\delta}$, $g_{\delta\lambda}$; the four dimensional gradient operators became six operators, these operators are $\partial_\mu, \partial_\nu, \partial_\sigma$, $\partial_\varepsilon, \partial_\delta, \partial_\lambda$.

This study introduced new relations in differential geometry and created new differential geometry analysis undertaken.

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