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Evaluation of Some Weibull Parameter Estimation Methods for Characterizing Stem Diameter Distribution in a Tropical Mixed Forest of Southern Nigeria

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Stem diameter distribution information is useful in forest management planning. Weibull function is flexible, and has been used in characterising diameter distributions, especially in single-species planted stands, the world over. We evaluated some Weibull parameter estimation methods for stem diameter characterisation in (Oban) multi-species Forest in southern Nigeria. Four study sites (*Aking, Ekang, Erokut* and *Ekuri*) were selected. Four 2 km-long transects situated at 600 m apart were laid in each location. Five 50m × 50m plots were alternately laid along each transect at 400 m apart (20 plots/location) using systematic sampling technique. Tree growth variables: diameter at breast height (Dbh), diameters at the base, middle and merchantable limit, total height, merchantable height, stem straightness, crown length and crown diameter were measured on all trees ≥ 10 cm to compute model response variables such as mean diameters, basal area and stem volume. Weibull parameters estimation methods used were: moment-based, percentile-based, hybrid and maximum-likelihood (ML). Data were analysed using descriptive statistics, regression models and ANOVA at $\alpha_{0.05}$. Percentile-based method was the best for Weibull [location (a), scale (b) and shape (c)] parameters estimations with mLogL = 116.66±21.89, while hybrid method was least-suitable (mLogL = 690.14±128.81) for Weibull parameters estimations. Quadratic mean diameter (D_q) was the only suitable predictor of Weibull parameters in Oban Forest.

Keywords: Diameter distribution, parameter estimation methods, prediction models

Introduction

Diameter distribution is used in most forest management planning packages for predicting stand volume and stand growth (e.g. Sharma and Parton, 2007; Osman *et al*., 2013). The information on diameter distribution can then serve as input for stem biomass and carbon stock estimation by establishing allometric relationships between stem biomass and diameter or stem volume (Beets *et al*., 2012; Özçelik *et al*., 2014). Moreover, forest managers may be interested in estimating the number of trees in different diameter classes in a stand, because the size of the diameter partly determines the industrial use of wood and thus the price of the different products.

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Diameter distributions also provide information about stand stability, and enable planning of silvicultural treatments (Gorgoso-Varela *et al*., 2012; Caetano *et al*., 2014).

Diameter distributions can be used to indicate whether the density of smaller trees in a stand is sufficient to replace the current population of larger trees and to help evaluate potential forest sustainability (Sheykholeslami *et al*., 2011). Since the age of trees is difficult to determine in natural forests, maturity is usually defined by stem diameters. Furthermore, a successful diameter-distribution model requires good prediction or estimation of its parameters. However, this can only be achieved when the most adequate method(s) are adopted. And such confirmation can only be ascertained through a comparative investigation involving all the available estimation methods in a single study and same dataset.

For generally accepted cases considering mixed species stands, Weibull distribution has continuously proven the best of all (McLaughlin, 2014). Similarly, studies abound for Weibull parameter estimation methods in planted stands, and with varied degrees of success (e.g. Adesoye, 2002; Cao, 2007; Podlaski, 2008; Sheykholeslami *et al*., 2011; Ajayi, 2013; Poudel and Cao, 2013). The applicability and suitability of different estimation methods in tropical rainforests, particularly Oban forest, have not been tested. And the most appropriate method(s) for parameter estimations in a tropical rainforest is yet to be ascertained. Therefore, we evaluated four Weibull parameter estimation methods on the same dataset with a view to ascertaining the most appropriate.

Method

The Study Area

This study was carried out in the Oban Forest, which occupies an area of about 251,345 ha in the southern part of Nigeria, within longitudes 8°02' and 8°55'E and latitudes 5°00' and 6°00'N in Akamkpa and Etung Local Government Areas of Cross River State. The forest is bounded by Korup National Park and Ejagham Forest Reserve of Cameroon in the east (Fig. 1). Annual rainfall is generally high throughout the area and decreases from about 3,000 mm in the south to 2,500 mm in the north of the Oban forest. This general trend is affected locally by altitude resulting in higher rainfall in hilly and mountainous areas. The central parts of the forest are estimated to receive about 4,000 mm (Oates *et al*., 2007). Rain falls in one season from March to November with a peak in June and July, and a second peak in September. There is a marked dry season of three to four months between December and February with very few days of rains; the dry season is longer in the north than in the southern part.

The mean annual temperature in the area is 27ºC. The temperatures vary slightly throughout the year with annual range of monthly average temperature of between 3º and 3.5ºC. February/March and November are the hottest months with August being the coolest in the area (Oates *et al*., 2007). Mean monthly relative humidity varies between 78% and 91% with an average of 85%. The prevailing wind is southerly, but during the dry season, the north-east trade winds carry dust-laden air from the Sahara (the Harmattan), as far as Calabar (Oates *et al*., 2007). The Oban Forest is the most extensive area of relatively undisturbed tropical moist forest remaining in Nigeria. It contains approximately 21% of the remaining 1,187,488 ha tropical rainforest in Nigeria (Ojonigu *et al*., 2010), and 52.34% of the standing tropical rainforest (480, 216 ha) in Cross River State (NFIS, 2011).

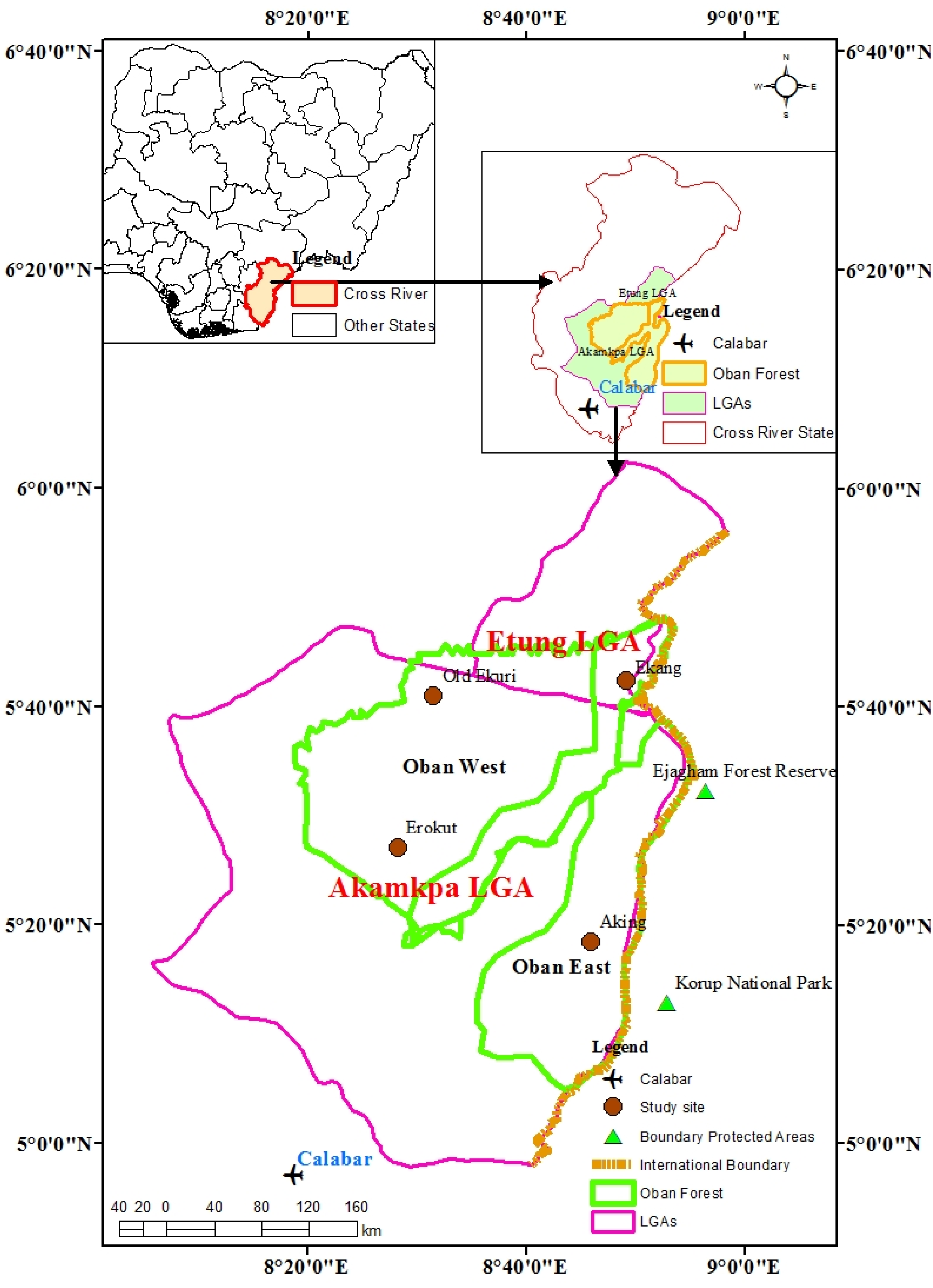

Figure 1. Map of the study area.

Data Collection

Data were collected at four study sites (*Aking*, *Ekang*, *Erokut* and *Ekuri*) within the forest. Systematic (line transect) sampling technique was adopted for plot locations in each of the sites. A distance of 100 m from the forest boundary was measured to locate the first transect in order to avoid edge effect. The starting point of each transect was determined with the aid of prismatic compass and global positioning systems (GPS) receiver (Husch *et al*., 2003). Four transects of 2 km long situated 600 m apart were cut in each of the locations. Five 0.25 ha (i.e. 50 m × 50 m) plots were then alternately laid along each of the transects at 400 m intervals resulting to 20 plots per location. Hence, eighty (80) sample plots were used for the study. The sampling procedure also ensured that all the possible variations within the forest were captured.

The following measurements were collected on all the trees with Dbh ≥ cm within each of the sample plots: diameter at breast height (Dbh); diameter, over bark (cm) at the base (D_b), middle (D_m) and merchantable limit (D_t); crown diameter (CD); total height (H_t); merchantable height (H_m); stem straightness (SQ) and crown length (CL) using Spiegel Relaskop, girth tape and distance measuring tape.

Data Analysis

Computation of Model Variables

Arithmetic mean diameter was computed as:

$$\overline{D} = \frac{1}{n}\sum_{i=1}^{n} D_i \tag{1}$$

Where,

D_i = Dbh of *ith* tree (cm); n = total number of trees per plot.

Quadratic mean diameter was computed using:

$$\overline{D}_q = \sqrt{\frac{1}{n}\sum_{i=1}^{n} D_i^2} \tag{2}$$

Where, D_i and n are as defined in eqn. 1.

Development of Diameter Distribution Models

The Weibull probability density function (pdf) that was used in this study is a three-parameter Weibull distribution function, and is given as:

$$f(x;a,b,c) = \left(\frac{c}{b}\right)\left(\frac{x-a}{b}\right)^{c-1} \exp\left[-\left(\frac{x-a}{b}\right)^{c}\right]; x \geq a \tag{3}$$

Where,

x = tree diameter at breast height (Dbh); a = location parameter (minimum Dbh in the stand) and b = scale parameter, c = shape parameter.

Weibull Parameter Recovery Methods

The following major parameter recovery methods were used: moment-based; percentile-based; hybrid and the maximum likelihood methods. In each of the methods, there were sub-versions and/or modifications.

Moment-based Parameter-recovery Method

For the two approaches, the location parameter was estimated from the predicted minimum diameter in the stand as follows:

$$a = 0.5\hat{\bar{D}}_0 \tag{4}$$

Under the first approach, the predicted mean diameter and diameter variance were used to recover the scale and shape parameters as follows:

$$b = \frac{\hat{\bar{D}} - a}{G_i} \tag{5}$$

$$G_i = \Gamma\left(1 + {}^{i}\!/\!_{c}\right) \tag{6}$$

$$c = b^2\left(G_2 - G_1^2\right) - D_{\text{var}} \tag{7}$$

Where, $\hat{\bar{D}}$ = the predicted mean diameter and Γ (·) is a complete gamma function.

Under the second approach, the Weibull parameters were recovered from quadratic mean diameter and diameter variance as follows:

$$b = -{}^{G_1}\!/\!_{G_2} + \left[\left({}^{a}\!/\!_{G_2}\right)^2\left(G_1^2 - G_2\right) + {}^{\hat{D}_q^2}\!/\!_{G_2}\right]^{0.5} \tag{8}$$

$$c = b^2\left(G_2 - G_1^2\right) - \hat{D}_{\text{var}} \tag{9}$$

Where, D_q = quadratic mean diameter; $\hat{D}_{\text{var}}$ = predicted diameter variance.

Percentile-based Parameter-recovery Method

The method involved the use of different combinations of diameter percentiles in recovering the Weibull scale and shape parameters. Under the first approach, 25th and 95th percentiles were used to recover shape and scale parameters as follows:

$$c = \frac{\ln\left[{}^{\ln\left(1-P_{95}\right)}\!/\!_{\ln\left(1-P_{25}\right)}\right]}{\ln\left(\hat{D}_{95} - a\right) - \ln\left(\hat{D}_{25} - a\right)} \tag{10}$$

$$b = \frac{\hat{D}_{95} - a}{\left[-\ln\left(1 - P_{95}\right)\right]^{1/c}} \tag{11}$$

Where, $\hat{D}_{25}$ and $\hat{D}_{95}$ = the predicted diameters at 25th and 95th percentiles; P_{25} and P_{95} = the 25th and 95th percentiles respectively. Under the second approach, 24th and 93rd percentiles were used for shape and scale parameters recovery as follows:

$$c = \frac{\ln\left[\ln\left(1-P_{93}\right)\Big/\ln\left(1-P_{24}\right)\right]}{\ln\left(\hat{D}_{93}-a\right)-\ln\left(\hat{D}_{24}-a\right)} \tag{12}$$

$$b = \frac{\hat{D}_{93}-a}{\left[-\ln\left(1-P_{93}\right)\right]^{1/c}} \tag{13}$$

Where, $\hat{D}_{24}$ and $\hat{D}_{93}$ = predicted diameters at 24th and 93rd percentiles; P_{24} and P_{93} = 24th and 93rd percentiles respectively.

Under the third approach, 31st and 63rd percentiles were used to recover shape and scale parameters as follows:

$$c = \frac{\ln\left[\ln\left(1-P_{63}\right)\Big/\ln\left(1-P_{31}\right)\right]}{\ln\left(\hat{D}_{63}-a\right)-\ln\left(\hat{D}_{31}-a\right)} \tag{14}$$

$$b = \frac{\hat{D}_{63}-a}{\left[-\ln\left(1-P_{63}\right)\right]^{1/c}} \tag{15}$$

Where, $\hat{D}_{31}$ and $\hat{D}_{63}$ = predicted diameters at 31st and 63rd percentiles; P_{31} and P_{63} = 31st and 63rd percentiles respectively.

Under the fourth approach, 24th and 63rd percentiles were used for shape and scale parameters recovery as follows:

$$c = \frac{\ln\left[\ln\left(1-P_{63}\right)\Big/\ln\left(1-P_{24}\right)\right]}{\ln\left(\hat{D}_{63}-a\right)-\ln\left(\hat{D}_{24}-a\right)} \tag{16}$$

$$b = \frac{\hat{D}_{63}-a}{\left[-\ln\left(1-P_{63}\right)\right]^{1/c}} \tag{17}$$

Where, $\hat{D}_{24}$ and $\hat{D}_{63}$ = the predicted diameters at 24th and 63rd percentiles; P_{24} and P_{63} are 24th and 63rd percentiles respectively.

Under the fifth approach, 25th and 50th percentiles were used for shape and scale parameters recovery as follows:

$$c = \frac{\ln\left[\ln\left(1-P_{50}\right)\Big/\ln\left(1-P_{25}\right)\right]}{\ln\left(\hat{D}_{50}-a\right)-\ln\left(\hat{D}_{25}-a\right)} \tag{18}$$

$$b = \frac{\hat{D}_{50} - a}{\left[-\ln\left(1 - P_{50}\right)\right]^{1/c}} \tag{19}$$

Where, $\hat{D}_{25}$ and $\hat{D}_{50}$ = the predicted diameters at 25th and 50th percentiles; P_{25} and P_{50} are 25th and 50th percentiles respectively.

Under the sixth approach, 50th and 95th percentiles were used to recover shape and scale parameters as follows:

$$c = \frac{\ln\left[\ln\left(1 - P_{95}\right) \big/ \ln\left(1 - P_{50}\right)\right]}{\ln\left(\hat{D}_{95} - a\right) - \ln\left(\hat{D}_{50} - a\right)} \tag{20}$$

$$b = \frac{\hat{D}_{95} - a}{\left[-\ln\left(1 - P_{50}\right)\right]^{1/c}} \tag{21}$$

Where, $\hat{D}_{50}$ and $\hat{D}_{95}$ = the predicted diameters at 50th and 95th percentiles; P_{50} and P_{95} = 50th and 95th percentiles respectively.

Hybrid Method

This method involved combinations of the moment-based and percentile-based methods for Weibull parameters recovery. The first approach involved the use of arithmetic mean diameter and the 24th percentile for scale and shape parameters recovery as follows:

$$b = \frac{\hat{D}_{24} - a}{\left[-\ln\left(1 - p_{24}\right)\right]^{1/c}} \tag{22}$$

$$c = a + b\Gamma\left(1 + 1/c\right) - \bar{D} = 0 \tag{23}$$

The second approach involved the use of arithmetic mean diameter and the 31st percentile for scale and shape parameters recovery.

$$b = \frac{\hat{D}_{31} - a}{\left[-\ln\left(1 - p_{31}\right)\right]^{1/c}} \tag{24}$$

The shape parameter, c, was then obtained from:

$$a + b\Gamma\left(1 + 1/c\right) - \bar{D} = 0 \tag{25}$$

The third approach involved the use of arithmetic mean diameter and the 63rd percentile for scale and shape parameters recovery.

$$b = \frac{\hat{D}_{63} - a}{\left[-\ln\left(1 - p_{63}\right)\right]^{1/c}} \tag{26}$$

$$a + b\Gamma\left(1 + 1/c\right) - \bar{D} = 0 \tag{27}$$

The fourth approach involved the use of arithmetic mean and the 95th percentile for scale and shape parameters recovery.

$$b = \frac{\hat{D}_{95} - a}{\left[-\ln\left(1 - p_{95}\right)\right]^{1/c}} \tag{28}$$

$$a + b\Gamma\left(1 + 1/c\right) - \bar{D} = 0 \tag{29}$$

The fifth approach involved the use of quadratic mean diameter and two percentiles for the shape and scale parameters recovery as follows:

$$c = -\frac{\ln\left[\ln(1 - P_{95}) \big/ \ln(1 - P_{25})\right]}{\ln(\hat{D}_{P_{95}} - a) - \ln(\hat{D}_{P_{25}} - a)} \tag{30}$$

$$b = -G_1 \big/ G_2 + \left[\left(a \big/ G_2\right)^2 \left(G_1^2 - G_2\right) + \hat{D}_q^2 \big/ G_2\right]^{0.5} \tag{31}$$

Maximum Likelihood (ML) Method

The ML estimation method, used by Eerikäinen and Maltamo (2003) and Gorgoso-Varela *et al*. (2012), enabled the estimation of the Weibull distribution parameters through iterative procedures as follows:

$$c = \frac{1}{-\frac{1}{n}\sum_{i=1}^{n}\left(x_i - a\right) + \frac{\sum_{i=1}^{n}\left(x_i - a\right)^c \ln\left(x_i - a\right)}{\sum_{i=1}^{n}\left(x_i - a\right)^c}} \tag{32}$$

$$b = \left[\frac{1}{n}\sum_{i=1}^{n}\left(x_i - a\right)^c\right]^{\frac{1}{c}} \tag{33}$$

Where, n = the number of sample observations in a Weibull distribution, x_i = the individual tree Dbh (cm).

Assessment of Parameter Estimation Methods

The evaluation statistics adopted in this method was negative log-likelihood (*mLogL*) statistics. It produced a more consistent results compared to the other goodness-of-fit statistics tried. The method(s) and/or approaches producing the lowest value(s) of the statistics are the bests. It was computed for each of the methods and/or approaches as using:

$$mLogL = \sum_{j=1}^{n_i}\left[\ln\left(b\right) - \ln\left(c\right) + \left(1 - c\right)\ln\left(\frac{x_{ij} - a}{b}\right) + \left(\frac{x_{ij} - a}{b}\right)^c\right] \tag{34}$$

Where, *mLogL* = negative value of the log-likelihood function of the Weibull distribution;

n_i = the number of trees in the *ith* plot; x_{ij} = the Dbh of tree *j* in the *ith* plot; *a*, *b* and *c* are the Weibull location, scale and shape parameters respectively.

Diameter Distribution Model Fitting

During the model-fitting process, all the tree growth variables measured (i.e. H_t H_m, SQ, CD, Dbh, D_b, D_m, D_t, as well as the computed tree variables at individual tree (i.e. arithmetic mean diameter, quadratic mean diameter) and whole-stand (number of trees/ha, basal area/ha, and stem volume/ha) levels were tried as independent variables. The response (dependent) variables were Weibull parameters. The general form of the models is of the form:

$$f(a,b,c) = b_o + b_1X_1 + b_2X_2 + b_3X_3 + b_nX_n + e_i \qquad (35)$$

Where, *a*, *b*, *c* are the predicted Weibull location, scale and shape parameters, $x_{1,.....,n}$ are the predictors, which represent growth variables at both individual tree and whole-stand levels, $b_o,...,b_n$ are regression parameters, e_i is the standard error term in the regression equations.

Model Evaluation

Model evaluation is an important aspect of model building. It is imperative that some examination of model be made at all stages of model design, fitting and implementation. Therefore, thorough evaluation of models involved two major steps, which were model verification and model validation.

Model Verification

This involved examination of the structure and properties of the models. It implicitly means comparing and evaluating candidate models. The models developed in this study were verified using the following statistics:

Coefficient of Determination (R^2)

This measured the proportion of variation in the dependent variable that has been accounted, or explained by its relationship with the independent variable(s). It was computed as:

$$R^2 = 1 - \left(\frac{RSS}{TSS}\right) \qquad (36)$$

The R^2 value ranges between 0 and 1, and can be expressed in percentage by multiplying the value by 100.

Root Mean Square Error (RMSE)

This was computed using:

$$RMSE = \sqrt{\frac{RSS}{n-p}} \qquad (37)$$

Where, p = the number of parameters in the model, or total number of variables been considered; n = the total number of observations; RSS = the regression sum of squares; TSS = the total sum of squares.

The most suitable models were those with large values of R^2 and least values of $RMSE$.

Significance of Regression (F-ratio)

This was used to test the overall significance of the regression equations. The critical value of F (F-tabulated) at 'α' equals 0.05 was compared with the variance ratio (F-calculated). Where the F-calculated was greater than the critical values (F-tabulated), such equation was considered significant, and therefore accepted for prediction.

Model Validation

Validation involves the testing and comparing of model output with what is observed in the real world. This requires that the predictions of the model be compared with real world data that are independent of the data used in the construction of the models. Models validation requires that some data are set aside, or that new data are obtained for the tests. Model validation was done by dividing the data into two sets. One set for calibrating the models and the other set for validation of the models. The calibrating set was used for model construction while the validating set was used to test the constructed models following Reynolds *et al*. (1988). The models were validate by: (i) testing for the significant differences in mean predicted and observed values of the dependent variables in all cases, using student t-statistics given as:

$$t = \frac{\overline{X}_{obs} - \overline{X}_{pred.}}{\sqrt{S^2 \frac{\left(N_{obs.} + N_{pred.}\right)}{\left(N_{obs.}\right)\left(N_{pred.}\right)}}} \tag{38}$$

Where, $\overline{X}_{obs.}$ = the mean observed value for a particular response variable in the models; $\overline{X}_{pred.}$ = the mean predicted value for that variable; $N_{obs.}$ = the total number of the observed values; $N_{pred.}$ = the total number of the predicted values; S^2 is the pooled within-group variance (for independent samples with equal variance). The t has ($N_{obs.}$-1) + ($N_{pred.}$-1) degrees of freedom.

(ii) The fitting method consistency was evaluated using the bias and mean square error (MSE) values, with the following expressions:

$$Bias = \frac{\sum_{i=1}^{N} Y_{obs.} - \hat{Y}_{pred.}}{N} \tag{39}$$

$$MSE = \frac{\sum_{i=1}^{N} (Y_{obs.} - \hat{Y}_{pred.})^2}{N} \tag{40}$$

Where, $Y_{obs.}$ = observed value; $\hat{Y}_{pred.}$ = predicted value; N = number of data points.

Results

Comparisons of the Weibull Parameter Estimation Methods

Table 1 presents the estimated parameters for the two approaches under moment-based parameter recovery method for the four sites. The first approach, involving the use of arithmetic mean diameter and diameter variance, produced higher values for scale and shape parameters in all the four stands compared to the second approach, which involved the use of quadratic mean diameter and diameter variance. The Weibull parameter estimates under the percentile-based parameter recovery method and the associated approaches for the four sites are presented in Table 2. The estimated mean values for ‘b’ and ‘c’ for the six approaches are shown in Table 2. Table 3 presents the Weibull parameter estimates under Hybrid method and the associated approaches for the four sites. The parameter estimates under the maximum likelihood method for the four study sites are presented in Table 4.

Table 5 presents the result of the evaluation statistics for the Weibull parameter recovery methods. The parameter estimation methods and the associated approaches were ranked in the order of appropriateness, with ranks 1 to 14, indicating the best to the worst methods. Generally, percentile-based method was the best for Weibull parameter estimations (Table 5). Within this method, the first approach, which involved the 25th and 95th percentiles, was the best with hybrid method as the least-appropriate.

Table 1

Parameter estimates under moment-based method for the form sites

Site	Statistics	Approach					Site	Statistics	Approach				
		a	b_1	c_1	b_2	c_2			a	b_1	c_1	b_2	c_2
Aking	Mean	6.02	37.77	0.72	1.84	0.07	*Erokut*	Mean	6.25	36.48	0.69	1.75	0.07
	SD	0.80	6.72	0.20	0.32	0.05		SD	1.14	6.33	0.25	0.37	0.04
Ekang	Mean	6.38	33.43	0.82	1.93	0.12	*Ekuri*	Mean	6.82	35.95	0.74	1.88	0.07
	SD	1.15	5.59	0.26	0.35	0.13		SD	1.94	4.22	0.17	0.32	0.06

N.B.: a - location parameter; $b_{1\text{-}2}$ - scale parameter for the first and second approaches; $c_{1\text{-}2}$ - corresponding shape parameter

Table 2

Parameter estimates under percentile-based method for the four sites

Site	Statistics	Approach												
		a	b_1	c_1	b_2	c_2	b_3	c_3	b_4	c_4	b_5	c_5	b_6	c_6
Aking	Mean	12.04	88.31	1.56	78.92	1.53	46.59	1.22	47.29	1.15	45.59	1.26	94.88	1.08
	SD	1.61	25.34	0.25	25.24	0.26	6.80	0.35	7.79	0.37	9.47	0.47	26.49	0.23
Ekang	Mean	12.49	82.54	1.52	75.40	1.48	45.83	1.17	47.18	1.09	46.62	1.21	89.63	1.06
	SD	2.29	24.66	0.27	24.02	0.28	7.70	0.37	8.79	0.38	14.98	0.51	25.82	0.26
Erokut	Mean	12.75	70.40	1.63	62.65	1.60	44.27	1.24	44.92	1.23	43.79	1.36	77.19	1.13
	SD	2.31	20.62	0.25	16.49	0.26	12.81	0.34	13.86	0.34	16.65	0.40	22.13	0.28
Ekuri	Mean	13.65	83.73	1.49	73.84	1.45	51.86	1.09	52.05	1.11	47.99	1.24	92.21	1.00
	SD	3.89	22.34	0.31	15.86	0.31	18.99	0.48	20.48	0.51	14.80	0.51	23.25	0.31

N.B.: a - location parameter; $b_{1\text{-}6}$ - scale parameters for first to sixth approaches; $c_{1\text{-}6}$ - corresponding shape parameters

Table 3

Parameter estimates under hybrid methods for the four sites

Site	Statistics	Approach										
		a	b_1	c_1	b_2	c_2	b_3	c_3	b_4	c_4	b_5	c_5
Aking	Mean	6.02	14.90	0.03	17.26	0.03	29.72	0.03	7.78	0.03	9.45	1.29
	SD	0.80	3.74	0.01	4.47	0.01	6.37	0.01	0.39	0.01	1.33	0.36
Ekang	Mean	6.40	13.05	0.03	16.47	0.03	31.71	0.03	8.06	0.03	11.20	0.91
	SD	1.14	3.29	0.01	3.22	0.01	6.00	0.01	0.41	0.01	1.33	0.37
Erokut	Mean	6.38	14.23	0.04	15.42	0.04	26.50	0.04	7.48	0.04	8.47	1.39
	SD	1.15	2.10	0.01	2.18	0.01	9.65	0.01	0.45	0.01	1.22	0.36
Ekuri	Mean	6.82	14.32	0.04	15.44	0.04	27.08	0.04	7.61	0.04	9.26	1.22
	SD	1.94	2.85	0.02	2.96	0.05	5.44	0.03	0.32	0.02	1.20	0.42

N.B.: a - location parameter; $b_{1\text{-}5}$ - scale parameters for first to fifth approaches; $c_{1\text{-}5}$ - corresponding shape parameters

Table 4

Parameter estimates under maximum likelihood method for the four sites

Site	Statistics	a	b	c	Site	Statistics	a	b	c
Aking	Mean	12.04	26.60	1.08	*Erokut*	Mean	12.49	24.24	1.01
	SD	1.61	7.00	0.24		SD	2.29	6.99	0.26
Ekang	Mean	12.75	21.25	1.03	*Ekuri*	Mean	13.65	22.65	1.02
	SD	2.31	5.68	0.28		SD	3.89	6.17	0.27

N.B.: a - location parameter; b - scale parameter; c - shape parameter

Table 5

Negative Log-likelihood (mLogL) Statistics for parameter recovery methods

Method	Approach	mLogL Statistic	SD	Rank
Moment-based	1	273.4319	13.52	7
	2	1099.7953	71.26	14
Percentile-based	1	53.7073	6.09	1
	2	64.1231	11.70	2
	3	158.7628	29.01	5
	4	164.6199	45.61	6
	5	153.5218	22.19	4
	6	105.2210	16.72	3
Hybrid	1	692.5907	156.99	11
	2	687.6866	93.66	10
	3	677.4845	71.29	9
	4	700.8300	184.11	13
	5	692.1187	137.99	12
ML	1	277.0262	38.03	8

Weibull fits for diameter distributions in the four study sites and the pooled data are shown in Fig. 2. The result for the *Aking*, *Ekang* and *Ekuri* sites revealed that most of the trees were within the Dbh of 20 and 40 cm, while there were fewer trees in the Dbh range of less than 20cm as well as those that were greater than 40 cm, respectively. The result for the *Erokut* stand however showed a different trend, as most of the trees were within the Dbh of less than 50 cm. Beyond the 50 cm Dbh, there were marked dropped in the tree frequencies in the site.

Weibull Parameter Predictions from Stand Attributes

Tables 6 and 7 present the results of the Weibull parameters' predictions under the moment and percentile, and hybrid and maximum likelihood methods respectively. Among the measured tree growth variables tried, the only suitable variable for Weibull parameters' predictions was the quadratic mean diameter (D_q).

The results of validation for the selected models under the four methods and their associated approaches are presented in Tables 8 and 9. For all the methods and approaches, the validation tests were not significant (P >0.05) for scale and shape parameters. This implied that the mean observed and predicted scale and shape parameter values were not significantly different from each other for the four methods and the corresponding approaches adopted. The bias and mean square error (MSE) values for the selected models under the moment-based and the percentile-based methods are presented in Table 10 while Table 11 showed the bias and MSE values for models under the hybrid and maximum likelihood methods. The mean bias and the MSE values were very small and insignificant for virtually all the selected models for the scale and shape parameter under the four methods.

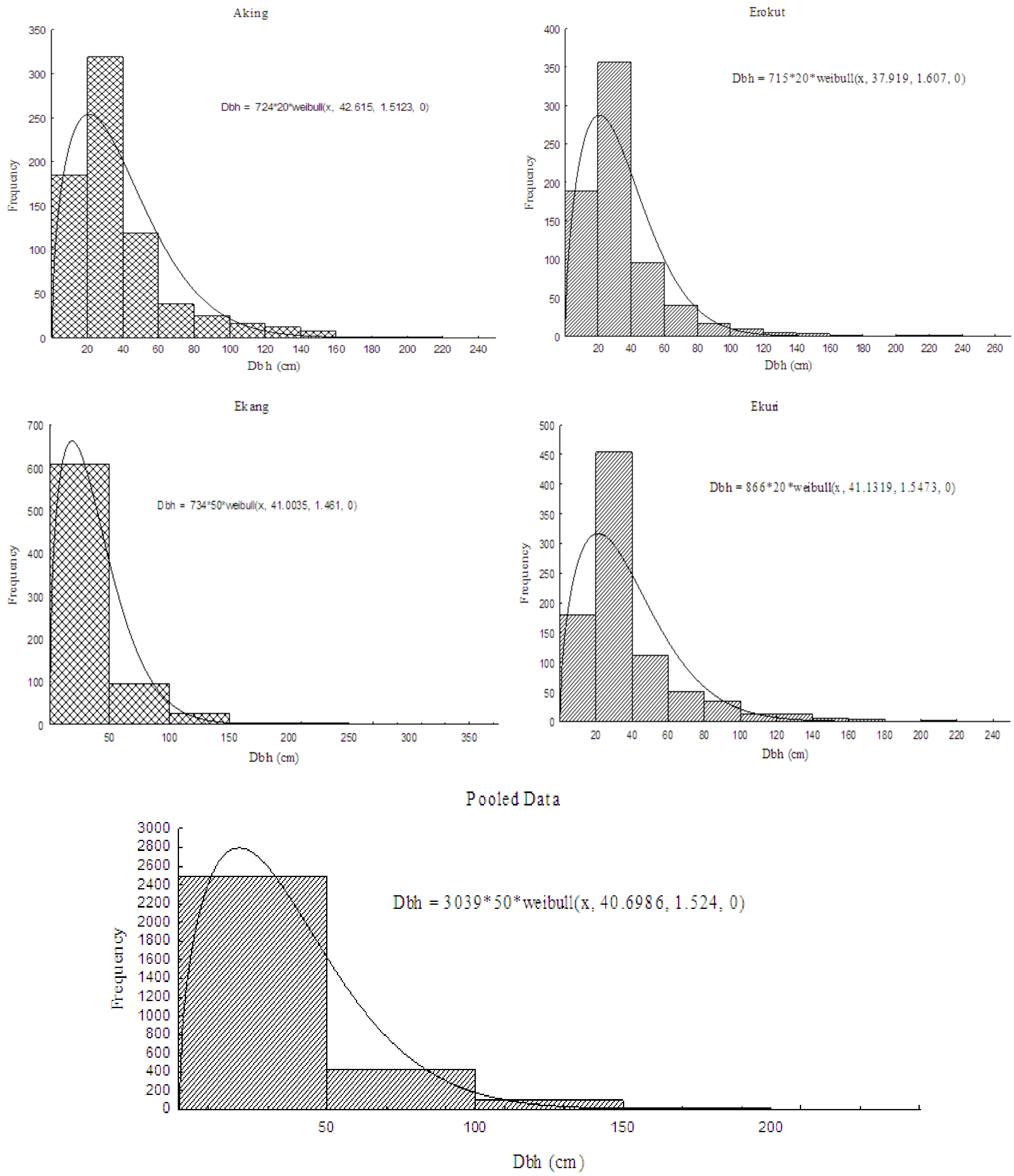

Figure 2. Weibull fits for tree Dbh under the four sites and the pooled data in Oban Forest.

Table 6

Weibull parameters' prediction models under moment and percentile methods

Method	Approach	Model	R^2	RMSE	F	P
Moment-based	1	$b = 9.2045 + 0.6053D_q$	0.8703	2.1368	523.520*	0.000
		$c = 1.6659 - 0.0210D_q$	0.7173	0.1203	198.330*	0.000
		$(a + b) = 21.6069 + 0.6127D_q$	0.6823	3.8267	167.530*	0.000
	2	$b = 3.0011 - 0.0261D_q$	0.4826	0.2475	72.743*	0.000
		$c = 0.4104 - 0.075D_q$	0.5002	0.0682	78.070*	0.000
		$(a + b) = 9.20 - 0.0223D_q$	0.0200	1.420	1.620^{ns}	0.207
Percentile-based	1	$b = -14.8417 + 2.1777D_q$	0.6915	0.3160	174.81*	0.000
		$\ln (b + c) = 3.2138 + 0.0264D_q$	0.7240	0.1490	204.64*	0.000
		$\ln (a + b) = 3.3282 + 0.0251D_q$	0.7151	0.1452	195.74*	0.000
	2	$\ln b = 3.0627 + 0.0268D_q$	0.7056	0.1585	186.92*	0.000
		$\ln (b + c) = 3.1206 + 0.0260D_q$	0.7074	0.1531	188.58*	0.000
	3	$b = 23.8147 + 0.5286D_q$	0.1446	11.768	13.19*	0.0005
		$c = 1.8733 – 0.0157D_q$	0.1374	0.3607	12.43*	0.0007
	4	$b = 26.1782 + 0.4914D_q$	0.1068	13.0090	9.326*	0.0031
		$c = 1.8556 – 0.0160D_q$	0.1346	0.3724	12.13*	0.0008
	5	$b = 19.0073 + 0.6118D_q$	0.1570	12.977	14.52*	0.0003
		$c = 2.2578 – 0.0225D_q$	0.1897	0.4250	18.26*	0.0001
	6	$\ln (b + c) = 3.3456 + 0.0252D_q$	0.6921	0.1538	175.36*	0.000
		$c = 3.3132 + 0.0256D_q$	0.6869	0.1584	171.11*	0.000

N.B: overall significance of the regression models were tested at $\alpha = 0.05$; *significant; ^{ns}not significant

Table 7

Weibull parameters' prediction models under hybrid and ML methods

Method	Approach	Model	R^2	RMSE	F	P
Hybrid	1	$b = 13.0 + 0.0301D_q$	0.008	3.026	0.65^{ns}	0.424
		$a + b = 19.2 + 0.0338D_q$	0.009	3.161	0.75^{ns}	0.390
		$b + c = 13.1 + 0.0295D_q$	0.008	3.024	0.62^{ns}	0.433
	2	$b = 12.7 + 0.0752D_q$	0.042	3.282	3.43^{ns}	0.068
		$a + b = 22.7 + 0.0648D_q$	0.027	3.576	2.15^{ns}	0.147
		$b + c = 12.8 + 0.0745D_q$	0.041	3.280	3.37^{ns}	0.070
	3	$b = 11.2 + 0.376D_q$	0.237	6.180	24.25*	0.000
		$\ln (a + b) = 3.15 + 0.0102D_q$	0.291	0.1463	32.01*	0.000
		$\ln (b + c) = 2.69 + 0.0138D_q$	0.290	0.1977	31.91*	0.000
	4	$\ln b = 2.97 + 0.0294D_q$	0.729	0.1642	210.17*	0.000
		$\ln (a + b) = 3.16 + 0.0271D_q$	0.723	0.1536	203.85*	0.000
		$\ln (b + c) = 2.97 + 0.0294D_q$	0.729	0.1642	210.15*	0.000
	5	$\ln b = 1.61 + 0.0133D_q$	0.715	0.0767	195.70*	0.000
		$\ln (a + b) = 2.64 + 0.0060D_q$	0.068	0.2027	5.73*	0.019
		$\ln b + c = 1.90 + 0.0098D_q$	0.752	0.0517	236.57*	0.000
ML	1	$\ln b = 2.2019 + 0.0210D_q$	0.482	0.1991	72.55*	0.000
		$\ln (b +c) = 2.3063 + 0.0196D_q$	0.447	0.1999	63.04*	0.000
		$\ln (a + b) = 2.9703 + 0.0139D_q$	0.595	0.1056	112.46*	0.000

N.B: overall significance of the regression models were tested at $\alpha = 0.05$; *significant; ^{ns}not significant

Table 8

Model validations under moment- and percentile-based methods

Method	Approach	Model	Mean obs.	Mean pred.	t_{cal}	P
Moment	1	$b = 9.2045 + 0.6053D_q$	35.910	35.907	0.00^{ns}	1.00
		$c = 1.6659 - 0.0210D_q$	42.280	48.640	0.06^{ns}	0.95
		$(a + b) = 21.6069 + 0.6127D_q$	0.741	0.740	0.07^{ns}	0.76
	2	$b = 3.0011 - 0.0261D_q$	1.849	1.849	0.01^{ns}	0.99
		$c = 0.4104 - 0.075D_q$	8.214	8.219	0.38^{ns}	0.84
		$(a + b) = 9.20 - 0.0223D_q$	0.082	-0.899	0.02^{ns}	0.98
Percentile	1	$b = -14.8417 + 2.1777D_q$	81.240	81.240	0.00^{ns}	1.00
		$\ln (b + c) = 3.2138 + 0.0264D_q$	4.437	4.435	0.03^{ns}	0.98
		$\ln (a + b) = 3.3282 + 0.0251D_q$	4.377	4.378	0.04^{ns}	0.96
	2	$\ln b = 3.0627 + 0.0268D_q$	4.266	4.266	0.00^{ns}	1.00
		$\ln (b + c) = 3.1206 + 0.0260D_q$	4.267	1.979	0.15^{ns}	0.00
	3	$b = 23.8147 + 0.5286D_q$	47.140	47.137	0.00^{ns}	1.00
		$c = 1.8733 – 0.0157D_q$	1.179	1.181	0.03^{ns}	0.98
	4	$b = 26.1782 + 0.4914D_q$	47.860	47.880	0.00^{ns}	1.00
		$c = 1.8556 – 0.0160D_q$	1.148	1.150	0.03^{ns}	0.97
	5	$b = 19.0073 + 0.6118D_q$	46.000	46.001	0.00^{ns}	1.00
		$c = 2.2578 – 0.0225D_q$	1.267	1.266	0.02^{ns}	0.98
	6	$\ln (b + c) = 3.3456 + 0.0252D_q$	1.068	4.442	0.00^{ns}	1.00
		$c = 3.3132 + 0.0256D_q$	4.457	4.457	0.08^{ns}	0.91

α = 0.05; ns: not significant; obs.: observed value; pred.: predicted value; t: t-calculated; P: probability value

Table 9

Model validation under hybrid and ML methods

Method	Approach	Model	Mean obs.	Mean pred.	t_{cal}	P
Hybrid	1	$b = 13.0 + 0.0301D_q$	14.331	14.180	0.01^{ns}	0.99
		$a + b = 19.2 + 0.0338D_q$	20.696	20.691	0.01^{ns}	0.99
		$b + c = 13.1 + 0.0295D_q$	14.367	14.402	0.10^{ns}	0.92
	2	$b = 12.7 + 0.0752D_q$	16.005	16.008	0.04^{ns}	0.97
		$a + b = 22.7 + 0.0648D_q$	25.547	25.559	0.03^{ns}	0.98
		$b + c = 12.8 + 0.0745D_q$	16.039	16.087	0.13^{ns}	0.90
	3	$b = 11.2 + 0.376D_q$	27.776	27.790	0.02^{ns}	0.99
		$\ln (a + b) = 3.15 + 0.0102D_q$	3.605	3.600	0.21^{ns}	0.83
		$\ln (b + c) = 2.69 + 0.0138D_q$	3.298	3.398	0.02^{ns}	0.98
	4	$\ln b = 2.97 + 0.0294D_q$	4.268	4.267	0.01^{ns}	0.99
		$\ln (a + b) = 3.16 + 0.0271D_q$	4.357	4.356	0.03^{ns}	0.98
		$\ln (b + c) = 2.97 + 0.0294D_q$	4.288	4.267	0.02^{ns}	0.98
	5	$\ln b = 1.61 + 0.0133D_q$	2.196	2.197	0.04^{ns}	0.97
		$\ln (a + b) = 2.64 + 0.0060D_q$	2.903	2.904	0.06^{ns}	0.95
		$\ln b + c = 1.90 + 0.0098D_q$	2.333	2.332	0.07^{ns}	0.95
ML	1	$\ln b = 2.2019 + 0.0210D_q$	3.127	3.125	0.03^{ns}	0.97
		$\ln (b +c) = 2.3063 + 0.0196D_q$	3.582	3.584	0.03^{ns}	0.97
		$\ln (a + b) = 2.9703 + 0.0139D_q$	3.172	3.171	0.09^{ns}	0.93

α = 0.05; ns: not significant; obs.: observed value; pred.: predicted value; t: t-calculated; P: probability value

Table 10

Bias and MSE values for the models under moment and percentile methods

Method	Approach	Model	Bias	MSE
Moment	1	$b = 9.2045 + 0.6053D_q$	0.0043	0.0036
		$c = 1.6659 - 0.0210D_q$	-0.1715	0.0356
		$(a + b) = 21.6069 + 0.6127D_q$	0.0001	0.000012
	2	$b = 3.0011 - 0.0261D_q$	0.0001	0.00005
		$c = 0.4104 - 0.075D_q$	0.0022	0.0019
		$(a + b) = 9.20 - 0.0223D_q$	0.0833	0.0077
Percentile	1	$b = -14.8417 + 2.1777D_q$	-0.0218	0.1200
		$\ln (b + c) = 3.2138 + 0.0264D_q$	-0.0001	0.00015
		$\ln (a + b) = 3.3282 + 0.0251D_q$	0.2444	0.0637
	2	$\ln b = 3.0627 + 0.0268D_q$	-0.0001	0.000019
		$\ln (b + c) = 3.1206 + 0.0260D_q$	0.0638	0.0045
	3	$b = 23.8147 + 0.5286D_q$	0.0193	0.1302
		$c = 1.8733 – 0.0157D_q$	-0.0002	0.00012
	4	$b = 26.1782 + 0.4914D_q$	0.0223	0.1678
		$c = 1.8556 – 0.0160D_q$	-0.0003	0.00012
	5	$b = 19.0073 + 0.6118D_q$	0.0282	0.1696
		$c = 2.2578 – 0.0225D_q$	-0.0003	0.0002
	6	$\ln (b + c) = 3.3456 + 0.0252D_q$	0.2489	0.0661
		$c = 3.3132 + 0.0256D_q$	-0.0940	0.0095

Table 11

Bias and MSE values for the models under the hybrid and ML methods

Method	Approach	Model	Bias	MSE
Hybrid	1	$b = 13.0 + 0.0301D_q$	0.0025	0.0072
		$a + b = 19.2 + 0.0338D_q$	0.0046	0.0076
		$b + c = 13.1 + 0.0295D_q$	0.0014	0.0072
	2	$b = 12.7 + 0.0752D_q$	0.0025	0.0090
		$a + b = 22.7 + 0.0648D_q$	0.0046	0.0110
		$b + c = 12.8 + 0.0745D_q$	0.0016	0.0090
	3	$b = 11.2 + 0.376D_q$	0.0069	0.0333
		$\ln (a + b) = 3.15 + 0.0102D_q$	0.0003	0.000018
		$\ln (b + c) = 2.69 + 0.0138D_q$	0.0002	0.000033
	4	$\ln b = 2.97 + 0.0294D_q$	-0.0002	0.000018
		$\ln (a + b) = 3.16 + 0.0271D_q$	-0.0001	0.000017
		$\ln (b + c) = 2.97 + 0.0294D_q$	-0.0002	0.000019
	5	$\ln b = 1.61 + 0.0133D_q$	-0.0001	0.000004
		$\ln (a + b) = 2.64 + 0.0060D_q$	-0.0001	0.000038
		$\ln b + c = 1.90 + 0.0098D_q$	-0.00004	0.000002
ML	1	$\ln b = 2.2019 + 0.0210D_q$	0.0001	0.000033
		$\ln (b +c) = 2.3063 + 0.0196D_q$	0.00001	0.000008
		$\ln (a + b) = 2.9703 + 0.0139D_q$	0.0001	0.000033

Discussion

After several modelling trials, percentile-based method was found to be the most suitable compared to the moment-based, hybrid and the maximum likelihood methods, going by the results of the evaluation statistics with 25th and 95th percentile pair as the most appropriate for Weibull parameters recovery and predictions. This is in line with the finding of Gorgoso-Varela *et al*. (2007), who noted that percentile approach was the most accurate in comparisons to the methods of moment and maximum likelihood. This is, however, contrary to the work of Zerda (2012), who reported that percentile method provided poor approximation of true distribution parameters after considering only 25th and 75th percentiles. The result also disagrees with the report by Akbar *et al*. (2014) and George (2014), who tried only 17th and 97th percentiles for shape parameter estimation and 40th and 80th percentile for scale parameter, and concluded that, percentile estimator performed poorer than maximum likelihood and moment-based estimators. The difference in the findings of previous workers compared to the current study may have resulted from the choices of percentiles adopted, and their inability to try and compared other percentile combinations considered in the current work. It could also be that the differences in sample sizes used by the previous workers impacted the results obtained. This corroborates the report by Marks (2005), who noted that sample size affects the successes of Weibull parameter estimation methods. The parameters' estimates may have also been influenced by the species diversity in the Oban Forest.

The result of this study was not also in consonance with Liu *et al*. (2004) and Oyebade *et al*. (2013), who tried only maximum likelihood method for predictions in mono-species planted forests and reported a good prediction of Weibull parameters without testing the appropriateness of other methods, especially the percentile-based. The result also disagrees with the works of Al-Fawzan (2000) and Lei (2008), who independently reported that moment-based method was superior for estimating Weibull scale and shape parameters compared to maximum likelihood method for a mono-species stand of *Pinus tabulaeformis*. Similarly, the result is not in consonance with the report by Sheykholeslami *et al*. (2011) that moment method is the most appropriate for Weibull parameter estimation in a mixed stand. The difference in findings may have resulted from species and structural diversities as well as age composition of the stands in question. Generally, the hybrid method gave poor, and the least reliable estimates of Weibull parameters. This is in line with the finding of Poudel and Cao (2013), who reported hybrid method to be the worst in terms of Weibull parameter estimations.

Only quadratic mean diameter (D_q) was found to be a good predictor of Weibull parameters in Oban Forest. This corroborates the work of Navar (2014), where quadratic mean diameter was found the only suitable predictor variable for Weibull parameter predictions with very high coefficients of determination (R^2) and small RMSE values. When other variables were included in the models, there were virtually no meaningful contributions. The suitability of quadratic mean diameter was confirmed by Gorgoso-Varela *et al*. (2007) with very high (0.99) adjusted R^2. The result is however in sharp contrast with the report by Ajayi (2013), who noted that age is the most-relevant variable for predicting Weibull parameter. This is probably due to the scope of his study, which focused on a monoculture stands using maximum likelihood estimator. The percentile-based method and associated approaches gave good predictions. Better prediction equations were obtained using a combination of 25th and 95th percentiles as well as a combination of 24th and 93rd percentiles compared to other percentile pairs.

Conclusion

The study showed that percentile-based method was the best for Weibull parameter recovery and prediction in Oban Forest. Among all the approaches adopted under this method, the approach involving the 25th and 95th percentiles was the most appropriate. Although moment and maximum likelihood methods had some predictive ability for Weibull parameters, the model selection criteria revealed inadequacies for subsequent adoptions in the study area. The results of this study indicated that a successful parameter recovery or prediction methods or models, as the case may be, for a given forest, or situation, might not be so for others in different areas. Therefore, it is safer to be conscious of the peculiarities of different ecosystems considered as well as the range of data included before generalizations are made about best or worst methods or approaches. It is evident that percentile method and approaches yielded better results, a slight modification in site condition may influence result in other forests. Hence, caution is emphasized before recommendations are made as to which method(s) are appropriate or suitable. For most of the models presented in this study, the validation results showed that the observed and the predicted values were not significantly different from each other. The bias and MSE values were also very small, especially for the 25^{th} and 95^{th} percentile pair, which further justified the suitability of the selected models for prediction studies in Oban Forest.

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