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Spatial Probabilistic Model of Block Failure Capacity of Piles in Clay

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Abstract: A probability based model of block failure capacity of pile foundation in clay soil under axial load is developed. The model was based on the first order second moment method. Instead of using point variability, the soil inherent variability is modelled as random field model. Based on this model, a reliability based factor of safety for designing pile group foundation, taking into account bock failure mechanism, is proposed. Furthermore, using simplified lognormal model, the relationship between the factor of safety used in design practice and target reliability may be derived explicitly.

Key words: Block failure, soil variability, random field, model error, reliability index.

1. Introduction

When a pile group is not very large, both theory and experience have shown that a pile group may fail as one unit by breaking into the ground before the load for each individual pile reaches its allowable design load. [1, 2]. Sowers et al. [1] have shown that the minimum spacing to prevent group failure ranges from 1.75 diameters to 2.5 diameters, depending on the number of piles in the group. Nevertheless, block failure may be encountered for pile spacing at even 3 diameters to 6 diameters [3, 4], where the pile and the confined mass of soil work like a rigid unit. It is therefore necessary to investigate this group (or block) failure as an additional failure mode.

2. Block Failure Formulation

The ultimate bearing capacity Q_{gu} of a pile group for the undrained condition is sufficiently modelled by superposition of the friction group capacity, Q_{gf}, and the base group capacity, Q_{gb} (Fig. 1), as:

$$Q_{gu} = Q_{gf} + Q_{gb} \tag{1}$$

where,

$$Q_{gf} = 2\int_0^{D_f}\int_0^{B} fs(x,y)\,\mathrm{d}x\mathrm{d}y + 2\int_0^{D_f}\int_0^{L} fs(x,y)\,\mathrm{d}x\mathrm{d}y \tag{2}$$

and,

$$Q_{gb} = \int_0^{B}\int_0^{L} q_d(x,y)\,\mathrm{d}x\mathrm{d}y \tag{3}$$

where, D_f is the depth of foundation; B is the width of group piles; L is the length of pile group; $f_s(x,y)$ is the inherent variability of shear resistance of soil per unit area. For the case of homogeneous soil with negligible inherent spatial variability, i.e., $f_s(x,y)$ will reduce to f_s and q_d, and Eq. (3) becomes:

$$Q_{gu} = D_f(2B + 2L)\cdot f_s + q_d BL \tag{4}$$

The shear resistance f_s may be assumed equal to undrained shear strength, and q_d may be evaluated using the equation suggested by Terzaghi and Peck [2], that is

$$q_d = 1.2c\cdot N_c + \gamma D_f N_q + 0.4\gamma B N_\gamma \tag{5}$$

where, N_c, N_q and N_γ are Terzaghi bearing capacity factors; c is the undrained shear strength of soil; γ is the unit weight of soil. In the case of cohesive clay soil with angle of internal friction ϕ equal to zero, Skempton [5] has proposed the following simple expression for bearing capacity of a rectangular footing, as a function of soil shear strength and the dimension of the foundation itself:

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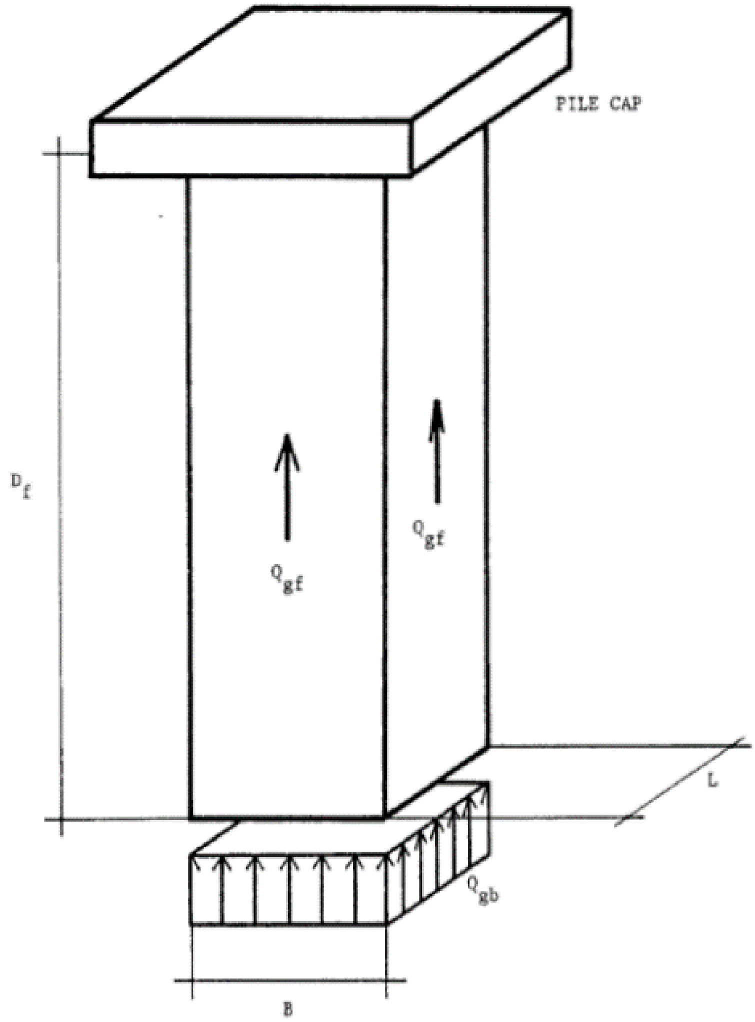

Fig. 1 Block ultimate capacity of a pile group.

$$q_d = 5\,c\left(1 + 0.2\,\frac{D_f}{B}\right)\left(1 + 0.2\,\frac{B}{L}\right) \qquad (6)$$

Furthermore, he showed that the net base resistance q_d becomes practically constant for values D_f/B greater than 2.5, and may be taken equal to $9c$. Both Eqs. (5) and (6) are widely used in the design practice.

3. Probabilistic Model on Block Failure Mechanism

Applying the first order analysis [6, 7] to Eqs. (2) and (3), the expected value of Q_{gu} due to inherent spatial variability of the undrained shear strength may be expressed as [5]:

$$E[Q_{gu}] = D_f[2B + 2L]\cdot E[f_s] + B\cdot L\cdot E\cdot[q_d] \qquad (7)$$

Assuming that Q_{gf} and Q_{gb} are independent random variables, the variance of Q_{gu} becomes:

$$VAR[Q_{gu}] = VAR[Q_{gf}] + VAR[Q_{gb}] \qquad (8)$$

where,

$$VAR[Q_{gf}] = 4\,VAR\left[\int_0^{D_f}\int_0^{B} f_s(x,y)\,\mathrm{d}x\mathrm{d}y\right] + 4$$

$$4VAR\left[\int_0^{D_f}\int_0^{L} f_s(x,y)\,\mathrm{d}x\mathrm{d}y\right] \qquad (9)$$

and,

$$VAR[Q_{gb}] = VAR\left[\int_0^{B}\int_0^{L} q_d(x,y)\,\mathrm{d}x\mathrm{d}y\right] \qquad (10)$$

Consider the first term of Eq. (9), let

$$A = \int_0^{D_f}\int_0^{B} f_s(x,y)\,\mathrm{d}x\mathrm{d}y \qquad (11)$$

The variance of A can be shown to be

$$VAR[A] = \int_0^{D_f}\int_0^{D_f}\int_0^B\int_0^B \rho(\tau_1,\tau_2)$$

$$VAR\,[f_s(x,y)]\,\mathrm{d}x_1\,\mathrm{d}x_2\,\mathrm{d}y_1\,\mathrm{d}y_2 \qquad (12)$$

where, τ_1 and τ_2 are spatial distances in directions X and Y, respectively (Fig. 2) and $\rho(\tau_1,\tau_2)$ = correlation function of the undrained shear strength separated at distances τ_1 and τ_2 in X and Y axis, respectively [8].

A 2-dimensional variance function, $\Gamma_2(D_f, B)$, which relates the variance of spatial average within an area and the point variability may be defined for $f_s(x,y)$ as

$$\Gamma_2(D_f, B) = \frac{\int_0^{D_f}\int_0^{D_f}\int_0^B\int_0^B \rho(\tau_1,\tau_2)\,\mathrm{d}x_1\,\mathrm{d}x_2\,\mathrm{d}y_1\,\mathrm{d}y_2}{D_f^2 B^2} \qquad (13)$$

Hence, Eq. (12) becomes

$$VAR[A] = D_f^2\,B^2\,\Gamma^2\,(D_f, B)\,VAR[f_s(x,y)] \qquad (14)$$

For the special case where the correlation structure is separable [9], the 2-dimensional correlation and variance function can be expressed in term of products of the respective one-dimensional function, namely,

$$\rho(\tau_1,\tau_2) = \rho(\tau_1)\rho(\tau_2) \qquad (15)$$

and

$$\Gamma^2(D_f,\mathrm{B}) = \Gamma^2(D_f)\Gamma^2(B) \qquad (16)$$

allowing considerable mathematical simplification. Both triangular and double exponential correlation functions exhibit this "separable" property. Similarly, the second term of Eq. (9) and the variance of Q_{gb} (Eq. (10)) can also be derived.

On the basis of procedures described in the aforementioned paragraphs, the mean and variance of pile block capacity due to inherent spatial variability of the undrained shear strength in a homogeneous soil deposit may be shown as

$$E[Q_{gu}] = \{D_f[2B + 2L) + \gamma_b BL\}\mu_c \qquad (17)$$

And

$$VAR[Q_{gu}] = \{4B^2D_f^2\Gamma^2(B)\Gamma^2(D_f) + 4L^2D_f^2\Gamma^2(L)\Gamma^2(D_f) + \gamma_b^2 B^2 L 2\Gamma^2(B)\Gamma^2(L)\}\delta_c^2\mu_c^2 \qquad (18)$$

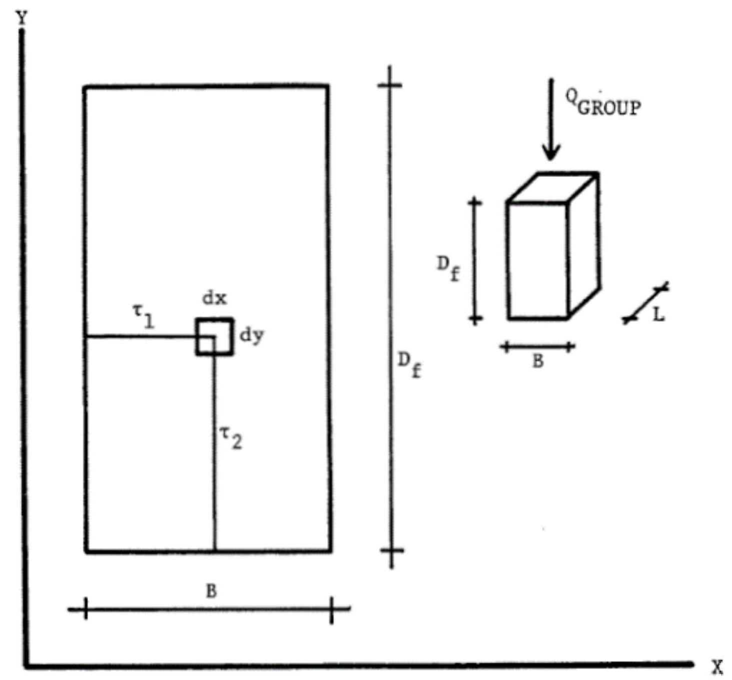

Fig. 2 Spatial distances in 2-dimensional random field.

where, μ_c and δ_c are the mean and c.o.v of the undrained shear strength at a point representing the point variability of the soil shear sterngth, and γ_b is defined as

$$\gamma_b = \left(1+\frac{D_f}{B}\right)\left(1+\frac{B}{L}\right)S \qquad (19)$$

Eqs. (17) and (18) are derived based on the assumption that the correlation structure $\rho(\tau_1, \tau_2)$ of the undrained shear strength can be sufficiently represented by a separable type variance function (e.g., triangular or quadratic exponential type), and the contribution of the base to the block capacity is evaluated based on Skempton's equation given in Eq. (6). For the case typical of most pile foundation, where $B = L$ and $D_f/B \geq 2.5$, the c.o.v of Q_{gu}, is given by

$$\Omega_{Q_{gu}} = \left\{\frac{8\alpha_f^2\Gamma^2(B)\Gamma^2(D_f)+81\Gamma^2(B)\Gamma^2(B)}{(9-4\alpha_f)^2}\right\}^{0.5}\delta_c \qquad (20)$$

in which, α_f is the ratio of D_f to B.

The effect of inherent spatial variability of the undrained shear strength c on block failure mode has been evaluated as functions of the ratio of depth of penetration (D_f) to the vertical scale of fluctuation of c, θ_v, and α_f as presented in Fig. 3 for the typical cases of $B = L$ and $\alpha_f \geq 2.5$. The horizontal scale of fluctuation θ_h is much larger than the vertical scale of fluctuation θ_v.

In this study, the ratio of θ_h/θ_v equal to 9 [9] is used in addition to the c.o.v of 0.4 for δ_c, similar to the case of a single pile, as the depth D_f increases relative to B and θ_v, the averaging area increases, and hence the c.o.v of Q_{gu} (in this case $\Omega_{Q_{gu}}$) decreases as shown in Fig. 3. Moreover, for a relatively short pile (e.g., $\alpha_f = 5$), the percentage contribution of the base capacity Q_{gb} to block capacity Q_{gu} given by Eq. (1) is higher than those of a deep pile foundation (e.g., $\alpha_f = 20$); this fact also contributes to the relatively higher $\Omega_{Q_{gu}}$ values at smaller α_f compared to those of higher α_f. Fig. 3 shows that designing pile group using point variability data will lead to a very conservative design, as the variability of soil parameter would decrease in the case of pile foundation due the averaging effect.

4. Model Error in Group Capacity

Many assumption and simplification has been made

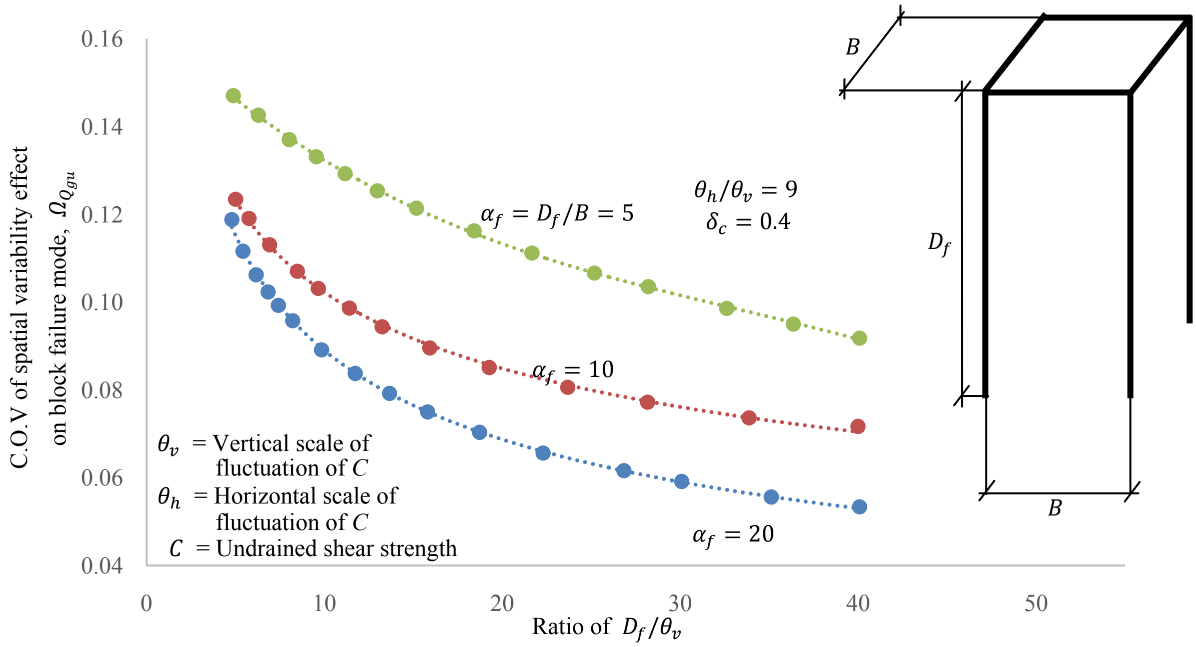

Fig. 3 The effect of spatial variability of undrained shear strength, *c*.

in the formulation of block failure capacity of pile foundation due to the complexity of mechanic of soil response. The engineer purposely simplified the equation to guarantee a direct approach in designing a block of foundation and introduce a factor safety to account for imperfection. Hence, a difference between calculated and measured capacity cannot be avoided. Sidi [8] introduced random correction factor N_g with mean value of one and coefficient variation of 0.06 may be used in the reliability formulation of block failure capacity to account for the imperfection. Taking into account the model error N_g, the true group capacity Q_{gt} can then be written as:

$$Q_{gt} = N_g \cdot [Q_{gf} + Q_{gb}] \quad (21)$$

And the mean value of Q_{gt} may be given by

$$E\cdot[Q_{gt}] = E\cdot[N_g]\cdot D_f[2B + 2L]\, E\cdot[f_s] + B\cdot L\cdot E\cdot[q_d] \quad (22)$$

And the coefficient variation of Q_{gt} may be given by

$$\Omega_{gt} = \sqrt{\Omega_{gu}^2 + \Omega_{N_g}^2} \quad (23)$$

where, Ω_{gu} = coefficient of Q_{gu} given by Eq. (20), and Ω_{gt} = the coefficient variation of the statistics of true capacity Q_{gt} taking into account both the spatial inherent variability of soil parameter and the systematic model error of the block failure capacity, and may readily be used in the reliability formulation.

5. Factor of Safety Based on Lognormal Model

By assuming the load acting on the pile and the capacity block failure mode follow independent lognormal distribution, the safety index β may be derived as

$$\beta = \frac{\lambda_{gt} - \lambda_L}{\sqrt{\xi_{gt}^2 + \xi_L^2}} \quad (24)$$

where, λ_{gt} and ξ_{gt} are parameters of lognormal distribution of resistance R whereas λ_L and ξ_L are parameters of lognormal distribution of resistance L, respectively. By introducing α_{gt} as the ratio of nominal value of resistance used in design (R_n) and the mean

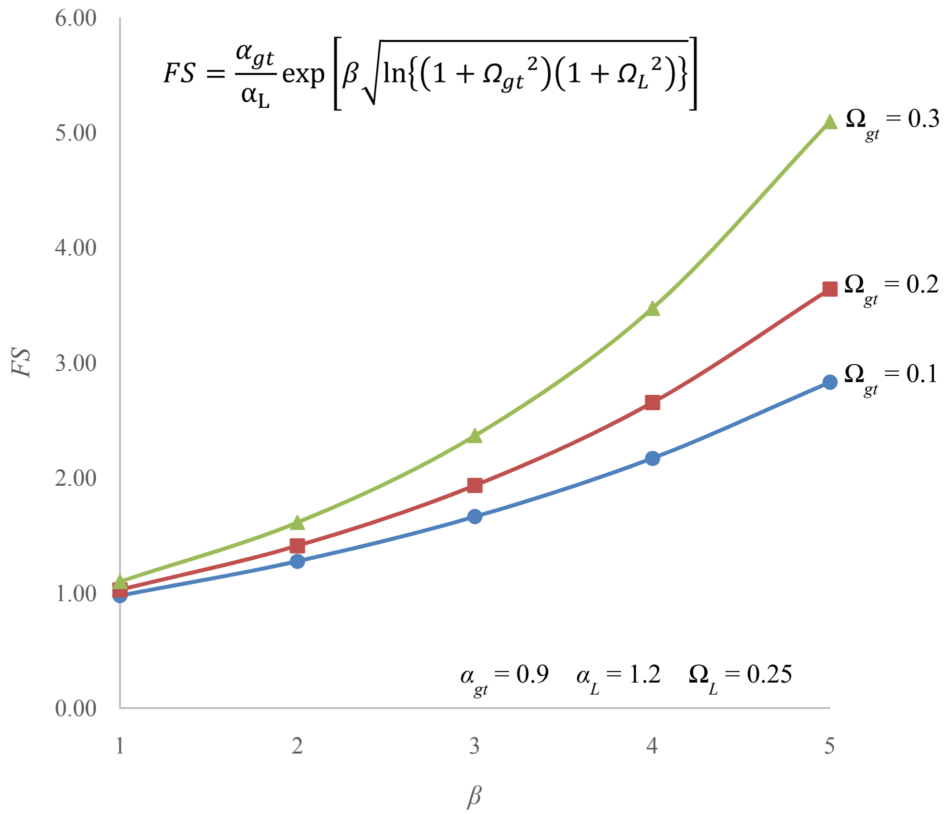

Fig. 4 Variation of factor of safety *FS* with safety index *β*.

value of R, μ_{gt}, and α_L as the ratio of nominal value of load used in design (L_n) and the mean value of L, μ_L, and by defining factor of safety FS as the ratio of R_n and L_n given by:

$$FS = \frac{R_n}{L_n} \qquad (25)$$

The factor of safety may be deriving as function safety index β and the related coefficient variation of R and L, as

$$FS = \frac{\alpha_{gt}}{\alpha_L}\exp\left[\beta\sqrt{\ln\left(1 + \Omega_{gt}^2\right)\left(1 + \Omega_L^2\right)}\right] \qquad (26)$$

Eq. (26) shows that the factor of safety depends of the target reliability β and the variation of Q_{gt} and L represented by its coefficient variation. The bigger the coefficient variation the bigger the factor of safety needed to achieve a certain targeted reliability. Of course if the designer is already taking conservative values in determining nominal design values of Q_{gtn} and L_n representing by α_{gt} of less than one and α_L factor of more than one, one will get a smaller FS for a certain target reliability index β. Fig. 4 shows the variation of factor of safety with respect of targeted reliability index. The higher the coefficient variation of the block failure, the higher factor of safety needed for achieving a certain targeted reliability index β. Eq. (26) shows that the factor of safety is a function of both inherent variability of soil shear strength and systematic model error of block equation representing by Ω_{gt}, and the variability of load given by its Ω_L which is readily to use for design purposes.

6. Conclusions

Based on a random field theory, the spatial variability of soil shear strength is modelled probabilistically taking into account horizontal and vertical correlation representing by its auto correlation function. Due to the averaging effect with respect of the area of block capacity, the point coefficient variation of inherent variability of soil shear strength would decrease significantly with the size (depth and width) of the foundation, and hence would lead to a smaller factor of safety needed to achieve a certain target of reliability index. By combining with the model error of block capacity, one could calculate the necessary traditional factor of safety needed to achieve a certain target reliability based on first order second moment method. The model takes into account variability of resistance governed by inherent variability, systematic model error, and the variability of the load itself. A simple lognormal reliability model has been introduced enabling one to determine the required factor of safety as a function of safety index or a certain acceptable risk, variability of soil, and load.

References

[1] Sowers, G. F., Martin, C. B., Wilson, L. L., and Fausold, M. 1961. "The Bearing Capacity of Friction Pile Groups in Homogeneous Clay from Model Studies." In *Proceedings of 5th International Conference of Soil Mechanics and Foundation Engineering*, 155-9.

[2] Terzaghi, K., and Peck, R. B. 1967. *Soil Mechanics in Engineering Practice.* New York: John Wiley.

[3] Peck, R. B., Hanson, W. E., and Thornburn, T. H. 1974. *Foundation Engineering.* 2nd ed. New York: John Wiley.

[4] Zeevaert, L. 1972. *Foundation Engineering for Difficult Subsoil Conditions.* New York: Van Nostrand.

[5] Skempton, A. W. 1951. "The Bearing Capacity of Clays."In *Proceedings of British Building Research Congress,* 180-9.

[6] Ang, A. H-S., and Ellingwood, B. R. 1971. "Critical Analysis of Reliability Principles Relative to Design." In *Proceedings of* the *1st International Conference Application of Statistics and Probability to Soil and Structural Engineering,* 1755-69.

[7] Ang, A. H-S., and Tang, W. H. 1975. *Probability Concepts in Engineering Planning and Design.* Vol. I. New York: Wiley.

[8] Sidi, I. D. 1986. "Probabilistic Prediction of Friction Pile Capacities." Thesis of doctor of philosophy, Department of Civil Engineering, University of Illinois, Urbana.

[9] Vanmarcke, E. H. 1977. "Probabilistic Modeling of Soil Profiles," *Journal of the Geotechnical Engineering Division* 103 (GT11): 1227-46.