# Estimation of Stress in the Crown of Soil-Steel Structures Based on Deformations

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**Abstract:** The paper deals with two stages of life cycle of CSPS (corrugated steel plates structures) buried in soil. The first stage is the construction stage when peaking factor $\omega$ is controlled during backfilling process. The second stage is the serviceability stage where deformations (deflections) of the crown are more difficult to identify due to duration of that stage. The possible range of deformations in serviceability stage is visible at the last stage of construction through deflection factor value $\eta$, where $\eta < 1$. To evaluate the safety of performance of a CSPS soil-steel structure, one should use an increase of deflections generated after completion of construction stage determined by a function $\beta(t)$. The paper concentrates on the second stage of the life cycle. Based on total station measurement of geometrical changes of a shape of CSPS, one can easily convert them into the calculation of a stress level in the steel wall. The papers shows the possibility of estimation of bending moments (main component of the normal stresses) based on deformation of a crown of a structure. A linear relation between deflection and stress level in the crown is found based on the example of the analyzed structure in the paper. That allows to estimate safety level of a structure based on deflection of the crown. The paper deals with CSPS with symmetric deformation during backfilling. In other cases a detailed procedure oriented on specific case is required. The analysis neglects local deformations coming from construction vehicles and equipment used for backfilling.

**Key words:** Corrugated soil-steel structure, stress level, measurements, deformation, deflection, bending moment.

## 1. Introduction

The construction stage is the most critical case in terms of safety for most of the cases of installation of the CSPS (corrugated steel plates structures). Mostly, upward deflection (peaking) occurs in arches and closed type structures [1-4]. They are caused by the backfilling procedure as presented in Fig. 1. The safety of construction of a CSPS is described through Eq. (1):

$$\omega = \frac{w_{\max}}{L} 100\% \qquad (1)$$

where, $w_{\max}$ is the maximum peaking of the crown point and $L$ is the span of a CSPS. On planning stage for a construction of a soil-steel structure, the parameter $\omega$ is estimated and further controlled during the construction stage. Recommended value is $\omega < 2\%$.

In practice, values greater than 2% are obtained, sometimes reaching out $\omega > 3\%$ [4].

In the case of box culverts and low profile arches, the serviceability limit stage is even more important. The downward deflection can be higher than peaking. This case is considered in this paper. The total station method is used to gather data of deflections during both stages.

The change of the chord between chosen points is analyzed if measurements cover several point alongside the periphery [2]. This can be used to evaluate the curvature of the examined CSPS. Changes of the curvature may be used to estimated bending moments and further to calculate the bending stresses in the steel wall [5, 6].

In some experimental structures, also strains are measured to evaluate the internal forces. A special equipment is needed for that [7-9]. The same situation occurs when earth cells are used to measure earth

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pressure around a CSPS [10]. The paper considers evaluation of safety of performance of a CSPS based on deflection. The paper deals with CSPS with symmetric deformation during backfilling. In other cases, a detailed procedure oriented on specific case is required [8]. The analysis neglects local deformations coming from construction vehicles and equipment used for backfilling.

## 2. Change of Deflection of CSPS

The paper presents results of measurements of chosen soil-steel structures as presented in Table 1.

Fig. 2 presents change of deformation in the crown during backfilling and during serviceability stage as a function of time. The highest values of deformation $w_{max}$ is obtained when backfill reaches the crown level $z_g = h$, as presented in Fig. 1. When backfill reaches over the crown, the peaking is reduced and ends up at final post construction value of $w_{min}$. Based on that, the deflection factor is calculated as presented in Eq. (2):

$$\eta = \frac{w_{max}}{w_{max} - w_{min}} \qquad (2)$$

Due to positive and negative values of $w_{min}$, there

Table 1    The set of tested structures.

| Marked object | Geometry | | | |
| | $L$ (m) | $h$ (m) | $\kappa = h/L$ | Profile of corrugated |
| --- | --- | --- | --- | --- |
| A | 17.594 | 5.459 | 0.310 | SC 380 × 140 × 7 |
| B | 5.22 | 1.85 | 0.354 | Flat $t = 23$ |
| C | 5.00 | 2.50 | 0.500 | MP 150 ×50 × 4 |
| D | 10.36 | 5.50 | 0.531 | MP 150 × 50 × 7 |
| E | 14.960 | 5.349 | 0.358 | SC 380 × 140 × 7 |
| F | 10.36 | 5.50 | 0.531 | MP 200 ×55 × 7 |
| G | 14.96 | 5.25 | 0.351 | SC 380 × 140 × 7 |
| H | 7.625 | 6.73 | 0.883 | MP 200 × 55 × 7 |
| I | 13.46 | 5.004 | 0.372 | SC 380 × 140 × 7 |
| J | 7.625 | 6.73 | 0.883 | MP 200 × 55 × 7 |
| K | 6.495 | 2.38 | 0.366 | SC 380 × 140 × 5.5 |
| L | 10.895 | 2.355 | 0.216 | SC 380 × 140 × 7 |
| M | 11.20 | 6.50 | 0.580 | MP 200 × 55 × 7 |

Note: MP: a corrugated steel structure with corrugation 200 × 55; SC: a corrugated steel structure with corrugation 380 × 140.
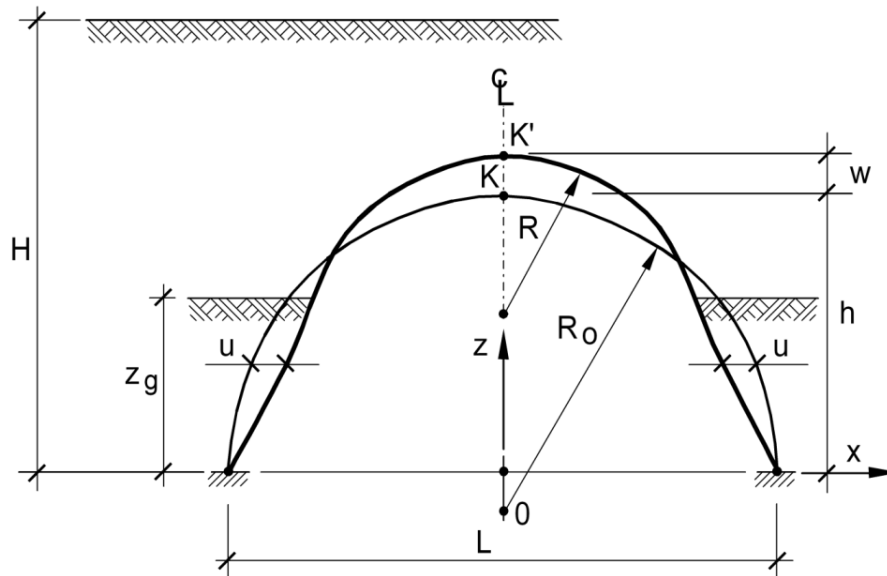


Fig. 1    Schematic deformation of a CSPS during backfilling.

are two types of results: $\eta > 1$ and $\eta < 1$. Based on that, one can divide CSPS structures built as soil-steel structures depending on whether $\eta > 1$ ($\omega$ is checked during construction stage) or $\eta < 1$. Fig. 2 presents examples of two deflection functions in time in both types of situations. Table 1 shows measurements obtained on few selected structures. Measured deflections allow to calculate $\eta$ from Eq. (2). The structures presented in Table 1, marked as C-J were built at the same part of the road. Due to transportation issues, the time for placing the backfill material was extended up to 120 days (Structure I) or 50 days (Structure J), as presented at Fig. 2. As a

result of very different behavior of the Structures I and J reflected through the parameters $\eta$ and $\beta$, given in Table 2, they were used in Fig. 2 for comparison.

During serviceability stage, a reduction of peaking is observed and deflection of the crown progresses. During that time from completion of construction $t_B$ till next measurement in time $t_E$, a downward deflection $w_{exp}$ occurs. To calculate the increase of deflection in time related to 365 days (full year), the following parameter can be used (Eq. (3)):

$$\beta = 365 \frac{w_{min} - w_{exp}}{t_B - t_E} \left[ \frac{mm}{year} \right] \tag{3}$$
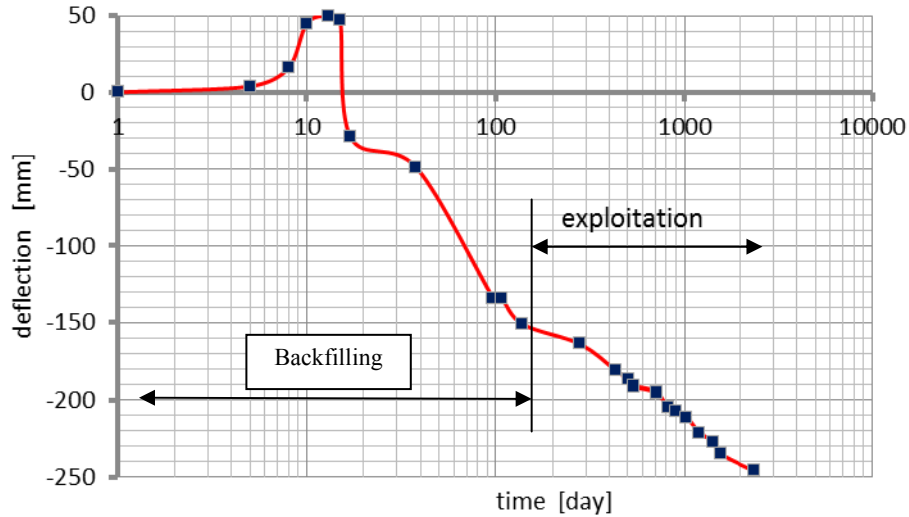


(a)
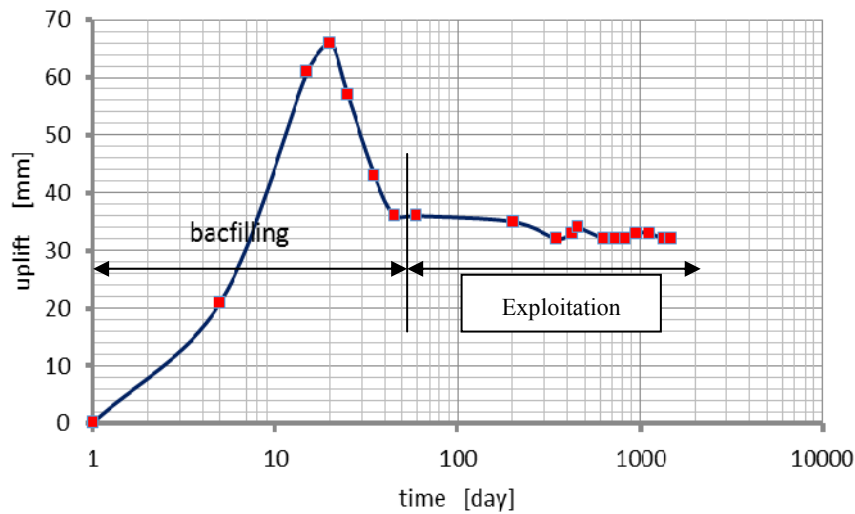


(b)

Fig. 2   Examples of changes of peaking and downward deflection: (a) Structure I ($\eta < 1$); (b) Structure J ($\eta > 1$).

**Table 2   The characteristic features of displacements for chosen structures.**

| Analyzed value | Analyzed object | | | | | |
|---|---|---|---|---|---|---|
| | C | F | G | H | I | J |
| $w_{max}$ (mm) | 50 | 58 | 80 | 60 | 50 | 66 |
| $w_{min}$ (mm) | 38 | 43 | 67 | −10 | −151 | 36 |
| $w_{exp}$ (mm) | 35 | 37 | 46 | −26 | −236 | 32 |
| $\omega$ (%) | 1.000 | 0.560 | 0.535 | 0.787 | 0.371 | 0.866 |
| $\eta$ | 4.167 | 3.867 | 6.154 | 0.857 | 0.249 | 2.200 |
| $\beta$ (mm/year) | 0.692 | 2.323 | 3.452 | 3.429 | 22.17 | 0.960 |
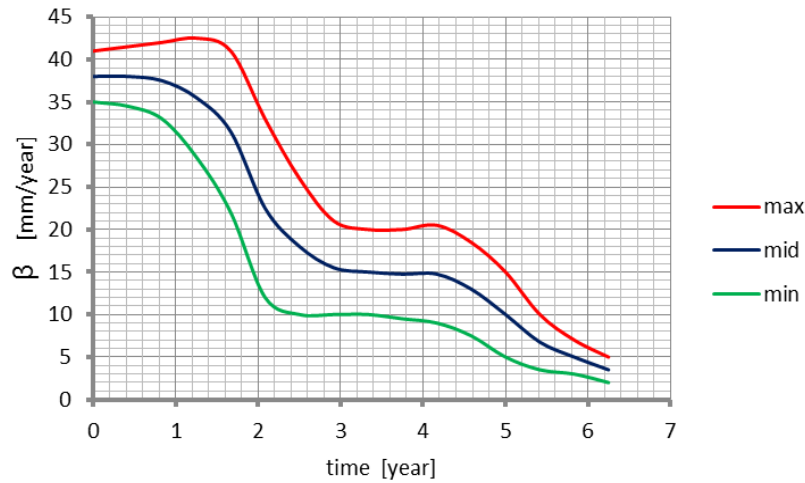


**Fig. 3   Change of deflection of the crown of the Structure I in time.**

$t_B$ and $t_E$ are set in days. Table 2 summarizes values of $\beta$ based on minimum 4 years measurements during serviceability stage of structures collected in Table 1. For the sake of analysis, structure marked as "I" is considered further on.

In short intervals of time $\Delta t = t_2 − t_1$ (day), temporarily changes of $\Delta w = w_1 − w_2$ (mm) are obtained, thus the following Eq. (4) addresses it:

$$\beta(t) = 365 \frac{\Delta w}{\Delta t} \left[ \frac{mm}{year} \right] \qquad (4)$$

Fig. 3 presents graphs of that function developed for the Structure I. Seasonality of weather conditions may affect the parameter $\beta(t)$. Therefore, there are extreme boundaries of results are presented as well as middle graph representing the trend of changes $\beta(t)$. By comparing values presented in Fig. 3 and Table 2, one can observe stabilization of deflection in time.

## 3. Change of Curvature as a Function of Deflection

Deformation of the CSPS generated during construction stage is determined by two characteristic displacements shown in Fig. 1. These are peaking "$w$" and narrowing "$2u$". The proportions between "$w$" and "$u$" are subject to specific changes [1]. The change of curvature of the steel shell resulted in deformations of a structure is useful to determine the bending moment in the crown [5]. To determine it one can use a procedure based on a radius of a circle based on equilateral triangle as presented in Fig. 4. Based on surveying technique aimed at the points located at the cusps of the triangle with base AB and the top in point K, one can calculate the length of a section $C$ and height $F$ of the triangle. The value of the radius of a curvature is calculated using Eq. (5):
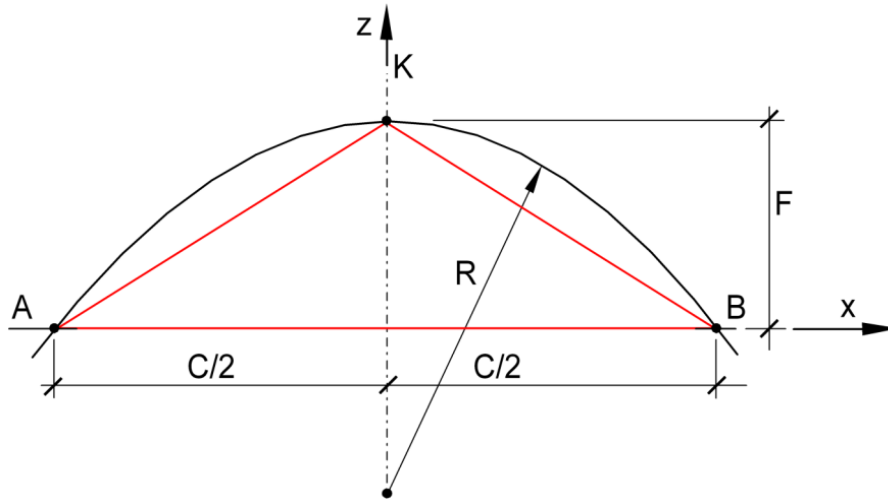
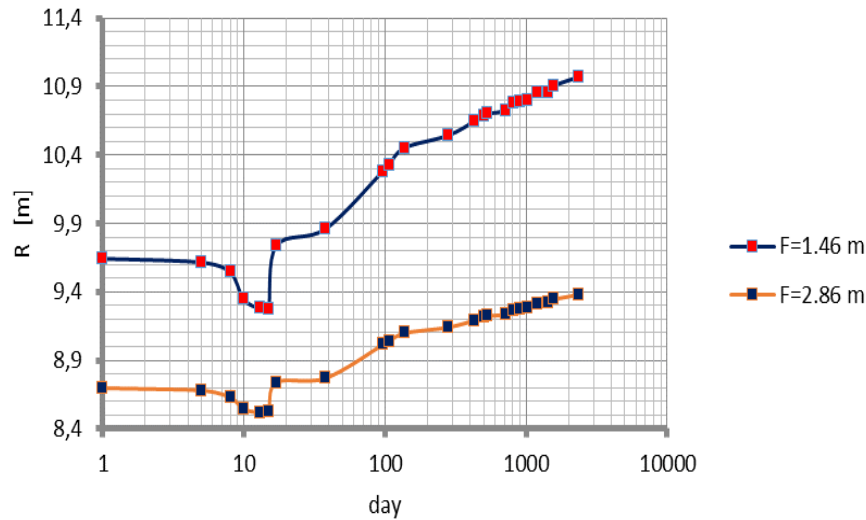**Fig. 4    A scheme used for calculation of the radius of a curvature.**



**Fig. 5    Change of a curvature of the Structure I.**

$$R = \frac{4F^2 + C^2}{8F} \qquad (5)$$

Fig. 5 presents results of calculations for Structure I. Taking two different levels of measurement Points A and B set by $F$, one will get different functions of $R$. The resulted radii of curvatures is calculated based on Eq. (5) in the crown of the structure for initial stage, i.e., self-weight of a CSPS. When $F = 1.46$ m, thus $R_0 = 9.646$ m and for lower level of measurement $F = 2.86$ m, the value of $R_0 = 8.696$ m.

The analysis of deflection together with change of curvature illustrates the changes of bending moment and bending stress. Fig. 6 presents this relation during construction stage and serviceability stage. Horizontal axis provides change of deflection based in Fig. 2 and change of curvature related to it based on Eq. (5) as shown in Fig. 5. This creates very specific graph for soil-steel structures. In the initial construction phase, the peaking "$w$" increases and curvature as well. When backfill is placed above the crown, a reduction of peaking and curvature occur. One can observe that in the second part of the graph. In the analyzed case, the increase of the curvature $\Delta R$ and change of deflection $\Delta w$ can be express as Eq. (6):

$$\Delta R = 6.13 \cdot \Delta w \qquad (6)$$

Thus change of the curvature radius $R$ is related to

increase of deflection "$w$" of the crown point. That is why surveying is very useful.

The graph presented in Fig. 6 deals with CSPS where $\eta < 1$. Then $w_{min}$ and $w_{exp}$ have negative values. For CSPS where $\eta > 1$, deflection values "$w$" are always positive. Thus lower part of the graph is very limited.

## 4. Change of Curvature and Stress Level in the Steel Wall

The initial radius of curvature is $R_0$. Due to loads applied a bending moment $M$ is generated. For a bar of a small curvature (large radius $R$), the change of the curvature caused by bending moment $M$ related to

bending stiffness $EI$ is shown in Eq. (7):

$$\frac{M}{EI} = \frac{1}{R_0} - \frac{1}{R} \tag{7}$$

Based on the curvature of a bar before and after deformation and taking into account the bending stiffness of the bar, one can arrive at the bending moment as presented in Eq. (8):

$$M = \frac{EI}{R_0} \cdot \frac{R - R_0}{R} \tag{8}$$

To estimated bending stress, one can utilize non-dimensional parameter describing change of the radius $R$ versus its initial value $R_0$ in Eq. (9):

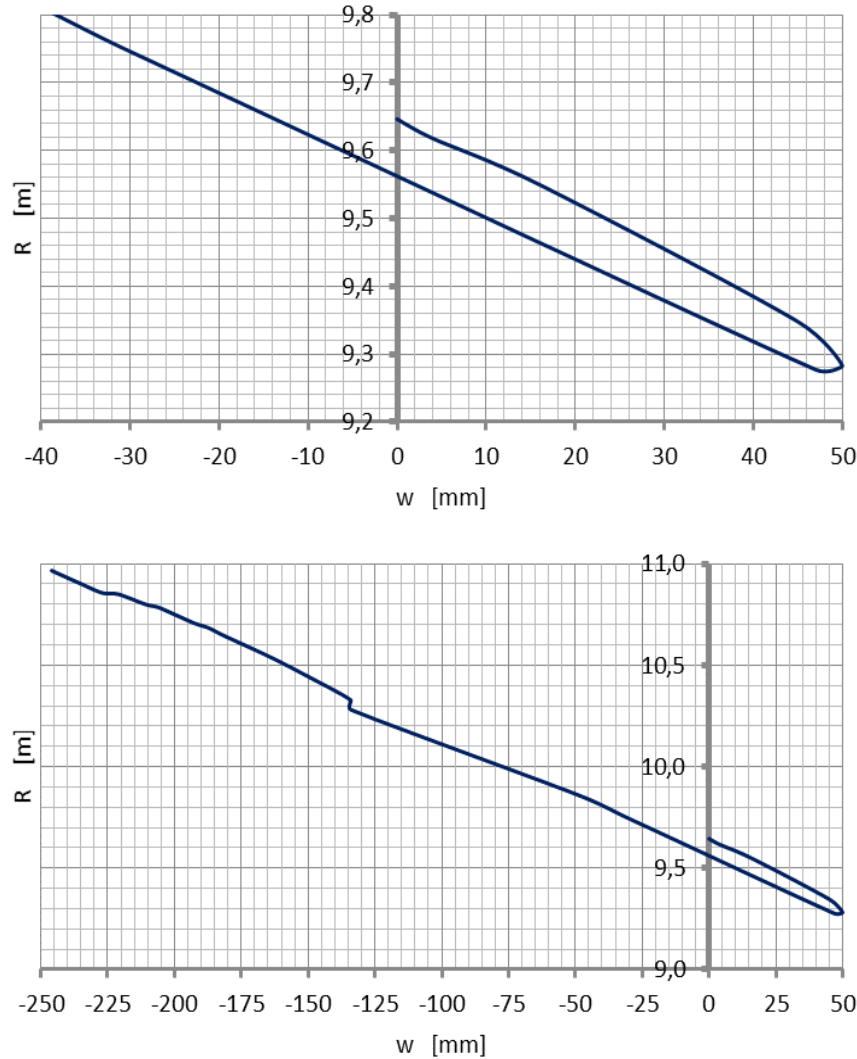$$\rho = \frac{R - R_0}{R} \tag{9}$$



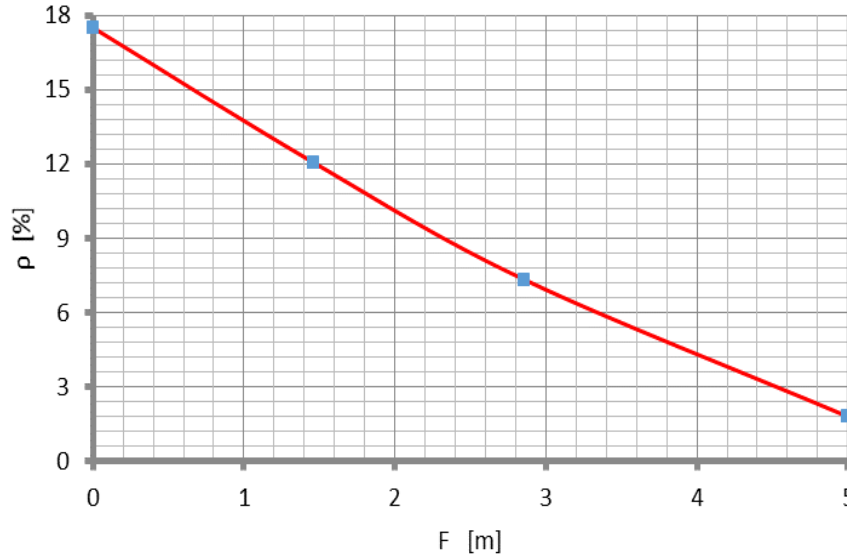Fig. 6   Graphs of the function $R(w)$ based on the Structure I.

**Fig. 7   Extrapolation of change of the radius of a curvature in the crown.**

It is used to estimate the relative bending moment [5], as per following Eq. (10):

$$M = \frac{EI}{R_0}\rho \qquad (10)$$

where, $EI$ = stiffness of the corrugated steel profile. The value of normal bending stress $\sigma$, developed by bending moment $M$ can be calculated based on Eq. (10) after introducing bending section modulus $W$, as presented in Eq. (11):

$$\sigma = \frac{M}{W} = E\,\frac{f+t}{2R_0}\rho \qquad (11)$$

Bending section modulus $W$ is a specific value for given corrugation type. For corrugation $380 \times 140 \times 7$ and a design radius $R_d = 9{,}930$ mm, (as in the Structure I), the value of the stress is:

$$E\frac{f+t}{2R_0} = 205{,}000\frac{140+7}{2 \cdot 9{,}930} = 1{,}517\ \text{MPa} \quad (12)$$

Based on graphs given in Fig. 5, one can notice that biggest negative change of radius of the curvature was generated during the backfilling process until it reached the crown of the structure. In the given case, the minimal value of factor was calculated as:

$$\rho = \frac{9.282 - 9.646}{9.282}100\% = -3.92\% \qquad (13)$$

After 6 years of service of the Structure I, the change of curvature (positive) calculated based on graphs given in Fig. 5 is:
when $F = 1.46$ m,

$$\rho = \frac{10.969 - 9.646}{10.969}100\% = 12.06\% \quad (14)$$

when $F = 2.86$ m,

$$\rho = \frac{9.383 - 8.696}{9.383}100\% = 7.32\% \qquad (15)$$

As presented in Ref. [5], the procedure to determine the change of the radius of curvature in the crown was based on results obtained from various measurements levels set by $F$, as given in Fig. 4. To arrive at value of $\rho$ in the crown, one must consider case when $F \rightarrow 0$. This result can be obtained only through approximation given in Fig. 7. For this purpose, values $u$ are being used. They are measured at various measurement levels specified by $F$ as presented in Figs. 1, 4 and 7. Based on that a result $\rho_k = 17.\%$ was reached. The value of $\rho$ which is calculated based on displacement $u$ and $w$ was matched against strain gages measurements on Structure A and a satisfactory match of result was achieved [5].

In the case of maximum change of the radius of curvature one obtains the normal stress as:

$$\sigma = 1517 \times 0.175 = 265 \, \text{MPa} \qquad (16)$$

The above calculated value (Eq. (16)) shows that the bending stress is high and by considering local effects and normal force [2], the stress levels are close to the nominal yield strength of the steel used for production of the structure.

## 5. Conclusions

The paper analyzes both important stages of the life cycle for soil-steel structures. The second stage is the serviceability stage where deformations (deflections) of the crown are more difficult to identify due to duration of that stage. The possible range of deformations in serviceability stage is visible at the last stage of construction through deflection factor value $\eta$, where $\eta < 1$. To determine the safety of the soil-steel structure in the serviceability stage, one can check an increase of deflection set as parameter $\beta$.

The factors referred to are determined based on deformations of the crown of a structure. The presented case shows good applicability of surveying to estimate the behavior of a soil-steel structure. Based on deformation of CSSP the bending moments can be evaluated and consequently the bending stress can be calculated. The presented case of the Structure I shows a linear relation between deflection and stress in the crown.

The methodology presented can be used to evaluate existing structure. Based on measurements at given intervals of time the function $\beta(t)$ can be derived. Based on that a change of $\rho$ as per Eq. (9) can be calculated however related to design value $R_0 = 9.96$ m. In the analyzed case the change of $\rho$ will be equal to:

$$\rho = \frac{9.96 - 8.696}{9.96} \times 100\% = 2.65\% \qquad (17)$$

This estimate can be sufficient to evaluate the safety of the soil-steel structure.

The paper deals with CSPS with symmetric deformation during backfilling. In other cases a detailed procedure oriented on specific case is required [8]. The analysis neglects local deformations coming from construction vehicles and equipment used for backfilling.

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