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Qualification of Three Analytical Wake Models

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Abstract: The decrease of wind velocity (wake losses) in downstream area of wind turbine is generally quantified using wake models. The overall estimated power of wind farm varies according to reliability of wake model used, however it's unclear which model is most appropriate and able to give a high performance in predicting wind velocity deficit. In this subject, a qualification of three analytical wake models (Jensen, Ishihara and Frandsen) based on three principal criteria is presented in this paper: (i) the parsimony which characterizes the inverse of model complexity, (ii) the accuracy of estimation in which wake model is compared with the experimental data and (iii) imprecision that is related to assumptions and uncertainty on the value of variables considered in each model. This qualitative analysis shows the inability of wake models to predict wind velocity deficit due to the big uncertainty of variables considered and it sensitivity to wind farm characteristic.

Key words: Wind farm, wind turbine, wake models, parsimony, accuracy, imprecision.

1. Introduction

To cope with the sharp decline in fossil resources, all efforts are recently focuses on alternative and renewable sources of energy among which wind is one of the fastest. The expandability of this type of energy requires a large area and hundred wind turbines, for instance the energy capacity installed in the European Union during 2015 have a steep increase than the previous year by 5.4% [1].

It's noticed that electrical energy, really, produced by the wind farm is less than the summation of the rated power generated by turbines. The power losses can be attributed to the phenomenon named wake effect. This later could be characterized by a reduced stream wise and increased levels of turbulence compared to income airflow. As consequence it impacts the downstream turbines and may even stop some of them. In addition to this a high evolution of turbulence intensity accelerates the fatigue and reduces the lifespan of wind turbines. Considered these two key factors in the wind farm design process can enhance the productivity and minimize the maintenance cost [2].

Different wake models have been developed to characterize the behaviour of far wind wake and evaluate the velocity deficit in downstream region which is strongly depended on down-distance. It could be divided into two principal types of models: analytical and computational [3].

In this regard several studies have been done to compare different engineering wake models in order to analyse the performance of each one in predicting wind velocity deficit. Barthelmie et al. [4] measured free stream and wake wind speed at hub height with variation of the distance between 1.7 and 7.4 rotor diameter. They showed an average absolute error of 15% concerning single wake prediction and also they claimed that due to large uncertainty of measurement it's difficult to make a comparison between models and measurement. Barthelmie concluded that the spread of the wake model predictions is considerable even for these relative simple offshore single wake cases. Recent study also has compared two analytical wake models (Jensen and Frandsen) with CFD (computational fluid dynamics) simulations [5]. It assesses and attempts to link the model performance in the different scenarios (single wind turbine, long row of turbines and infinite wind farm), then provides new calibration of parameters for the engineering models.

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They concluded that the expansion factors calibration for three cases are found to be approximately half of the recommended standard values.

The aim of others studies was increasing output power of wind farm with high wake losses considering the simplest wake model [6]. For example, Behnood et al. propose an algorithm which minimizes the wake effect. The main idea was reducing CP and CT of upstream wind turbines, they could be controlled through pitch control and rotational speed. The influence of wind direction variation in regeneration of new values of CP and CT is shown which increase the overall output power by 1.86%.

The diversity of results obtained by different wake models are strongly depends on various variables [7] shown in Fig. 1. This often requires a comparison of wake models behaviour in different scenarios in purpose to verify, validate and make a choice of the best suitable one that can only give a rough estimation of produced power.

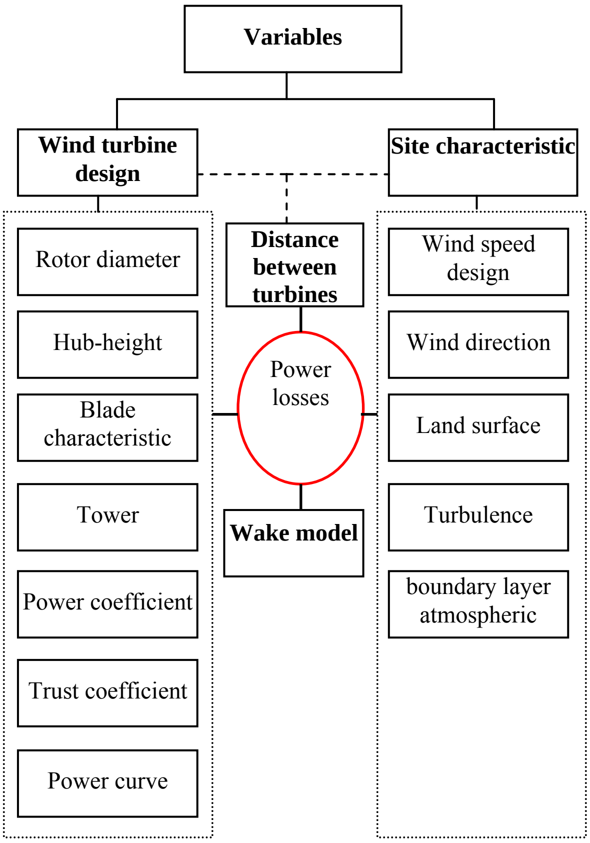

Fig. 1 Variables determining the power losses.

In this work three analytical wake models (Jensen, Ishihara and Frandsen) are qualified in the stage of preliminary design using three criteria: parsimony, accuracy and imprecision.

This paper is organized as follows: In Section 2, three analytical wake models are presented; Section 3 is dedicated to describe briefly the qualification method; The results obtained and discussions are devoted in Section 3 before the conclusions.

2. Analytical Wake Models

2.1 Model 1

The analytical wake model developed by Jensen et al. [8, 9] is a simple far wake model and it is the most model used in optimizing the position of wind turbines. Jensen wake model based on global momentum conservation and on the assumption of a wake with linearly expanding diameter. It is characterized by a uniform velocity profile, which is only dependent on downstream distance from the turbine. Due to the simplification of velocity profile, the model cannot be used to make wake predictions in the near wake region.

$$U_{wake} = U_{In}\,(1 - (\sqrt{1 - C_t})(\frac{D_r}{D_{wake}})) \quad (1)$$

$$D_{wake} = D_r + (2\,\alpha\,\Delta X_{ij}) \quad (2)$$

where,

U_{wake}: Wind velocity in wake area
U_{In}: Incoming wind speed
C_t: Trust coefficient
R_r: Rotor radius
R_{wake}: Wake radius
Δx_{ij}: Distance separate wind turbines
α: Wake decay coefficient.

2.2 Model 2

The analytical wake model developed by Ishihara et al. [10, 11] used wind tunnel data for a model of Mitsubishi wind turbine. The model takes into account the effect of turbulence on the wake recovery. It is not constant and depends on the Atmospheric and rotor generated turbulence, and the downstream distance from the wind turbine. The wake recovery is therefore more dependent on the turbine-generated turbulence. Ishihara is clearly shown that when thrust coefficient C_t (0.31, 082) is large, the rate of wake recovery increases. The same results are found either for ambient turbulence or mechanical generated turbulence.

$$U_{wake} = U_{In} \frac{(C_t)^{0.5}}{32} \left(\frac{1.666}{K_1}\right)^2 \left(\frac{\Delta X_{ij}}{D_r}\right)^{(1-P)} \exp\left(\left(\frac{R_r}{R_{wake}}\right)^2\right) \quad (3)$$

$$R_{wake} = k_1 \frac{(C_t)^{0.25}}{0.833} (D_r)^{(1-(0.5\,P))} (\Delta X_{ij})^{0.5P} \quad (4)$$

$$P = k_2 (I_a + I_w) \quad (5)$$

$$I_w = k_3 \left(\frac{C_t}{\max(I_a, 0.03)}\right)\left(1 - \exp\left(-4\left(\frac{\Delta X_{ij}}{10 D_r}\right)^2\right)\right) \quad (6)$$

where,

I_a: Ambient turbulence

I_w: Mechanical generated turbulence.

2.2 Model 3

The analytical wake model developed by Frandsen et al. [12] is adopted in the SAM (Storpark Analytical Model), the aim of this model is to predict the wind speed deficit in large offshore wind farms using a rectangular site area and straight rows of wind turbines with equidistant spacing between wind turbines and rows. Frandsen considered a cylindrical control volume with constant cross-sectional area equal to the wake region, the shape can presented by a rectangular distribution of the flow speed.

$$U_{wake} = U_{In} \left(0.5 \pm 0.5\left(\sqrt{1 - 2 C_t \left(\frac{A_r}{A_w}\right)}\right)\right) \quad (7)$$

$$D_{wake} = \left(\beta^{(0.5k)} + \alpha\left(\frac{\Delta X_{ij}}{D_r}\right)\right)^{1/k} D_r \quad (8)$$

$$\beta = 0.5 \left(\frac{(1 + \sqrt{1 + C_t})}{\sqrt{1 - C_t}}\right) \quad (9)$$

where,

β: Wake expansion parameter

k: Shape parameter

3. Qualification Method

The overall estimated power of wind farm varies according to wake model used. Hence the need to make a qualification that verifies the adequacy of the model. This study takes into account three criteria of qualification (PAU) developed by vernat [13-15].

3.1 Parsimony

The Parsimony characterizes the reverse scale of model complexity. A lower numbers of variables (N_{var}) and equations (N_{eq}) make the model more parsimonious.

$$P = \frac{1}{(N_{var} + N_{eq})} \quad (10)$$

3.2 Accuracy

Accurately quantifying analytical wake model is a key aspect of economics in large wind farms. It is a measure of the distance between the space of solutions given by the model and reference behaviour, this tool shows the influence of difference variables considered on power produced. The error between reference data and wake model behaviour can be estimated as following:

$$\text{Error} = \frac{(d_m - d_r)}{d_r} \quad (11)$$

where,

d_r: Data resulting from wake behavior

d_m: Reference data.

3.3 Imprecision

Imprecision in engineering design can be defined as a fuzzy aspect related to the distinction between different values of variable, in which it is not possible to describe with precision the right value when parameters varies stochastically. Generally there are two main sources of imprecision:

Relationships imprecision: This characterizes the ambiguity in relationships between several variables and assumptions take into consideration, however some simplification made in modeling analytical wake model may lead to wrong results.

Data imprecision: the values of some parameters are not known and hardly determined especially for those depended of environment stability.

4. Results and Discussions

4.1 Parsimony

As explained previously the estimation of parsimony is depended on number of equations and variables. Table 1 presents the result obtained according to wake models.

It is clearly shown that Jensen wake model is very parsimonious due to few numbers of equations and coupling variables compared with Ishihara analytical wake model. Frandsen used different equations and various variables to estimate the velocity deficit behind wind turbine for that reason is rather parsimonious.

4.2 Accuracy

We use as a reference power curve measured on Horns Rev offshore wind farm (Fig. 4) which is located in the North sea 14 km the west of Denmark [16]. The wake spreading constant is 0.05, whereas the turbulence intensity is 0.03 and the distance separate turbines is equal to 7D.

It is necessary to point out that this paper is devoted to study single wind wake.

Decision variables take into accounts in formulating a model is an important element that can define result accuracy.

Table 1 Parsimony estimated of three analytical wake models.

	Jensen	Ishihara	Frandsen
Number of equations	2	4	7
Number of variables	7	13	13
Parsimony	**1/9**	**1/17**	**1/20**

Table 2 Characteristic of wind turbine (Vestas V80).

Vestas V80 wind turbine						
D_r (m)	P_n (MW)	$V_{cut\text{-}in}$ (m/s)	$V_{cut\text{-}out}$ (m/s)	H_{hub} (m)	Control type	P
80	2	4	25	70	Pitch	3

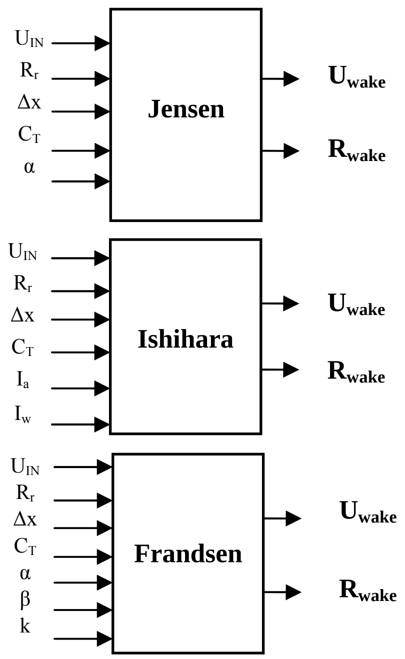

Fig. 2 Decision variables of three wake models.

The following expression is used to calculate the power generated:

$$P_{wt} = 0.3\, U^{3}_{wake}(\theta) \tag{12}$$

Where wind velocity is a function of wind direction variation θ.

It's clearly proven (Figs. 4 and 5) that analytical wake models under-predict or over-predict the output power of wind turbine depending on wind direction variation. This inaccuracy can be attributed to several variables used by each wake model (Fig. 3). For example wake decay coefficient and thrust coefficient would be the main sources of this inaccuracy in particular using Jensen and Frandsen wake model. β and k are also important parameters which in turn related to thrust coefficient. In other side Ishihara wake model takes into account the effect of turbulence without including the roughness of wind farm, it may be a reason that made inaccuracy on estimation of wind deficit.

Obviously the model accuracy is sensitive to the wind direction especially to low direction. For instance Jensen and Frandsen under-predict the power respectively by -9.39% and -24.34% for 0°, but Ishihara overestimated the output power about 11.32%.

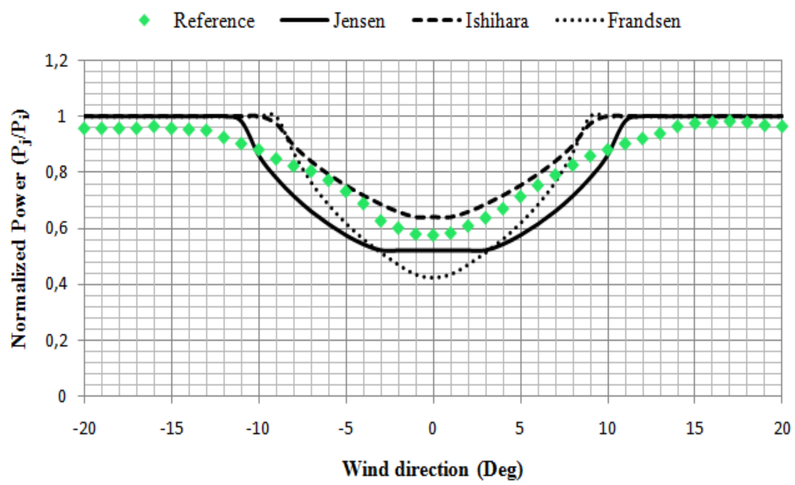

Fig. 3 Normalized power of downstream turbine as a function of wind directions compared wind Horns Rev data.

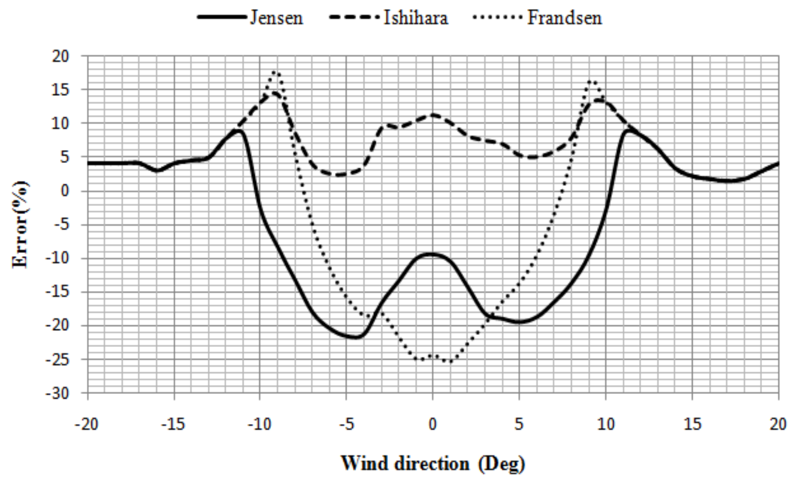

Fig. 4 Accuracy error of Normalized power a function of wind directions.

Concerning high wind direction variation, three analytical wake models over-predict the output power which the error is equal to 4.1%.

In addition to wind direction parameter, it should be noted that is very important to verify the influence of others variables on wind velocity deficit accuracy especially:

Wind turbine design: Induction factor, Power coefficient, blades, pitch control and tower characteristics.

Site: The non-stability of wind speed, and flow atmospheric should be include in modeling in order to verify and validate exactly the ability of predicting wind velocity in wake area and in different types of wind farm included the turbulence, roughness and boundary layer.

4.3 Imprecision

This qualitative study considers that imprecision in wake models is related to assumptions and uncertainty on parameters value.

4.3.1 Assumptions Considered

Three analytical wake models based on resolution averaged Navier-Stokes (RANS or Reynolds Averaged Navier-Stokes models) using mass and moment conservation equations to characterize wake in downstream region. Some assumptions made in modeling wind wake may lead to significant errors.

Jensen considered an ideal rotor, he neglected the wake induced behind wind turbine and assumed that far wake expanded linearly and depended solely on the distance between wind turbines.

Ishihara assumed that wind velocity expanded with gaussian profile. He did not includ the parameter of wake decay constant, and he only introduced the turbulence Value without explaining or showing how it could be estimated.

Frandsen also assumed the wake expanded linear.

We can derive that wind velocity shape approximation and expanding profile in downstream area is an important characteristic which would affect the precision of prediction.

4.3.2 Uncertainty

The trust level associated with various possible values to one variable is a challenge that affects prediction of wind deficit using analytical wake models. Table 3 shows uncertainty variables take into consideration in wake models.

Table 3 Uncertainty variables in wake models.

	Wake models		
	Jensen	Ishihara	Frandsen
Uncertainty Variables	U_{In}	U_{In}	U_{In}
	C_t	C_t	C_t
	α	Ia	α
	-	Iw	β
		k_i	k

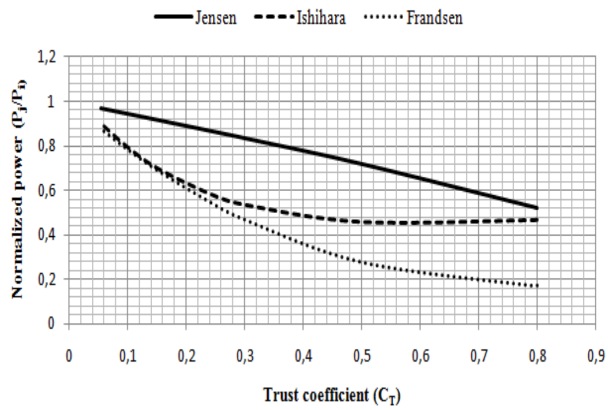

Fig. 4 Trust coefficient influence on power generated.

Wind speed is a crucial parameter of uncertainty on total energy produced of wind farm. It depends on the measure-correlate-predict method and wind shear extrapolation to hub height, moreover there are no standard rules or methods for estimating the uncertainty [17]. Trust coefficient is a key and common parameter between models, it constitutes the biggest challenge for wind turbine. There are different methods which allow to estimated the value of thrust coefficient (induction factor, thrust force, power curve and hub height of wind velocity) may this approximation gives a significant errors. The result (Fig. 6) illustrates that Jensen wake model predicts the power produced approximately when trust coefficient is low (high wind speed), but Frandsen wake model is very sensitive to variation of trust coefficient, it cannot estimate the power correctly.

Likewise the challenge in determining wake decay coefficient can be a source of uncertainty in wake model, it depends on level of turbulence, turbine-induced turbulence, and atmospheric stability, all these parameters are hardly determined [18]. Jensen considered (α = 0.1(ideal rotor) and calibrated it to α = 0.070), than Katic set α = 0.075 for onshore wind farm and α = 0.05 for offshore. However the wake decay constant has influenced by the friction effect at the surface of land.

In the other side there is a big ambiguity in the value of K_i used by Ishihara wake model, the taken values are not known if they are valuable for all types of wind farm. Frandsen takes into account the wake exist behind wind turbine that is characterized by β, this in turn depends on the uncertainty of trust coefficient as well as the shape parameter k in which would take two value 2 or 3.

5. Conclusions

Qualifying analytical wake models is very often crucial step used to verify and validate the ability of wake model to predict wind velocity deficit correctly. This paper proposes a comparison between three analytical wake models (Jensen, Ishihara, and Frandsen) based on three criteria: parsimony, accuracy and imprecision. It pointed out that Jensen wake model was a very parsimonious model compared with two others wake models, several decision variables take into consideration has an important influence on the accuracy of estimating velocity deficit. However the imprecision of wake model was due to the uncertainty on the trust level associated to many variables value, in particular trust coefficient and wake decay coefficient which are related to characteristics and stability of wind farm, in addition we should not neglect the big ambiguity existing on some parameters that would be the main source of errors.

To sum up none of three analytical wake models can estimate the wind velocity deficit approximately but Jensen wake model according to this study still the model that gives a good argument in term of three criteria. There are some discussions about uncertainty on wake models, which deserves more development using for example fuzzy approach that will be the next step of our research work.

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