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Development of a Method for Optimizing Rotating Machines Vibrations Limits

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Abstract: This work consists to a development a new calculation method of rotating machines vibration limits. we will confirmed norms. An evaluation of the absolute vibration parameters of these machines with a comparison of vibration limits of ISO 10816-1 will be also affected.

Key words: Vibration, stiffness, centrifugal force, free movement.

1. Introduction

Rotating machines operate with limits vibrations, an update of these limits maybe required.

From the following calculation we will try to reformulate terms of vibrations machine parameters.

2. Model of the Machines Vibrations [1]:

The rotating machines vibrations will be modeled by the system with one degree on Fig. 1. Rotating machine can be compared with this system (Fig. 2):

- The base is replaced by the bearings.
- The Springs is replaced by the rotor.
- The mass is replaced by the body of the turbine or the pump.
- Damping is ensured by the fluid passing in the turbine or the pump.

3. Basic Equation[1]

The differential equation translating the behavior of the system (displacement of mass x (t) compared to its rest position after release) in a direction can be written:

$$M\ddot{x}(t) + C\dot{x}(t) + kx(t) = F(t) \qquad (2.1)$$

With k is the rigidity of the spring which expresses that the force of recall is proportional to elongation x and F the force applied on the basis.

A viscous damping (or fluid) of coefficient C exerts on the movement of the mass m a force of damping $-C\frac{dx}{dt}$ proportional to the instantaneous speed. The total response of the movement is:

$$x(t) = x_L(t) + x_F(t)$$

where, $x_L(t)$ is the response of the free movement, solution of the equation:

$$M\ddot{x}(t) + C\dot{x}(t) + kx(t) = 0 \qquad (2.2)$$

where, $x_F(t)$ is the response of the movement forced, solution of the equation:

$$M\ddot{x}(t) + C\dot{x}(t) + kx(t) = F(t) \qquad (2.3)$$

The response of the free movement [1] in the case $\xi < 1$ is:

$$x_L(t) = X_0 e^{-\xi\omega_0 t}\sin(\omega_0\sqrt{1-\xi^2}\,t + \phi_0)$$

where, $\xi = \frac{C}{C_C}$ and C_C are critically damping.

The natural frequency of the free movement is $\omega_0 = \sqrt{\frac{k}{M}}$. If $\xi = 1$, then $C = 2\sqrt{kM} = 2M\omega_0 = C_C$. These cases are rare, and when $\xi > 1$, the free response is an exponential function. $x_L(t)$ is represented on the Fig. 3.

By holding account that $F(t) = F_0 e^{j\omega t}$ the response of the forced movement is: $x(t) = Xe^{j(\Omega t - \phi)}$, with:

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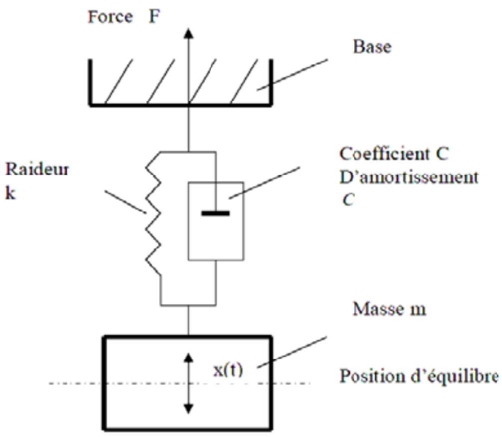

Fig. 1 mechanical system with one degree.

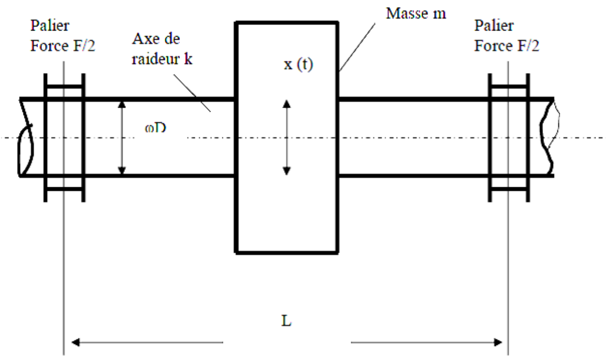

Fig. 2 Rotating machines system.

$$X = \frac{F_0 / k}{\left[(1 - (\frac{\Omega}{\omega_0})^2)^2 + (2\xi \frac{\Omega}{\omega_0})^2 \right]^{1/2}} \tag{2.4}$$

$$tg\,\phi = \frac{2\xi \frac{\Omega}{\omega_0}}{1 - (\frac{\Omega}{\omega_0})^2} \tag{2.5}$$

$$T = \frac{2\pi}{\omega_0 \sqrt{1 - \varepsilon^2}}$$

If $\varepsilon \ll 1$, then $T = \frac{2\pi}{\omega_0}$. With the natural pulsation $\omega_0 = \sqrt{\frac{k}{M}}$. We can have the harmonic response forced to one degree on Fig. 4.

4. Stiffness Calculation of Rotors

At passage of the critical speed [2]:

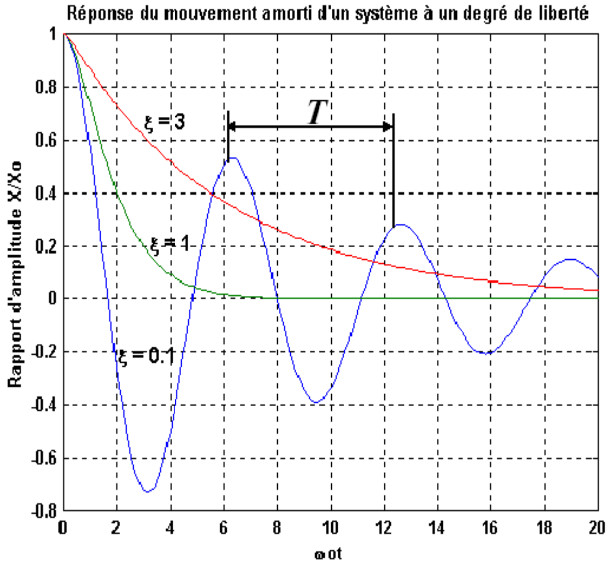

Fig. 3 Mechanical system with one degree.

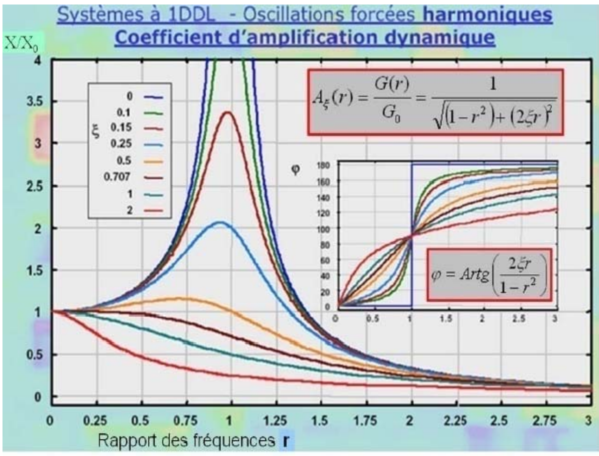

Fig. 4 Harmonic Forced response with one Degree.

$$\Omega = \omega_0 = \sqrt{\frac{k}{M}} \tag{3.1}$$

$$k = M\omega_0^2 \tag{3.2}$$

where, ω_0 is the natural frequency corresponding to the critical speed, k is rotor stiffness, M is rotor mass. The equilibrium is realized. If $F_b = F_c$ (equality of centrifugal forces of unbalance and unbalance correction force with masse m, then:

$$Me\omega_0^2 = mR\omega_0^2$$

With e the excentricity and R rotor radius (distance from the position of the balancing mass to the rotor geometric center).We can write to this effect:

$$m = \frac{Me}{R}$$

On the other hand, the restoring force is given by:

$$F = k \cdot x \tag{3.3}$$

The expression of the stiffness (the restoring force per unit elongation) is given by:

$$k = \frac{mR}{e} \cdot \omega_0^2 \tag{3.4}$$

We suppose that x_1 is vibrations amplitude at critical speed and x_2 is at nominal speed. We pose

$$\tau = \frac{x_2}{x_1} = \frac{x_1}{x_0} \tag{3.5}$$

$$\tau > 1$$

We suppose that τ is vibration ratio of x_2 and x_1 and also the ratio of still admissible X_1 vibration and admissible vibration X_0 according norm 10816-1 (Cf. [1]). We pose :

$$\omega_1 = \rho \cdot \omega_0 \tag{3.6}$$

$$\omega_1 > \omega_0$$

Here, ω_1 and ω_0 are nominal and critical speed.

5. Balancing Quality Vibrations Limits Evaluation

5.1 Vibrations Expressions

The tolerated residual unbalance is defined by norms from the balancing quality. Quality factor G (mm/s) is defined by:

$$G = e \cdot \omega_1$$

where, e is excentricity. ω_1 is maximal angular speed. The stiffness is:

$$k = \frac{mR}{G} \cdot \omega_0^2 \cdot \omega_1 \tag{4.1}$$

$$k = \frac{F_C}{G \cdot \rho} \cdot \omega_0 \tag{4.2}$$

With the centrifugal forces of the mass m given by:

$$F_C = mR\omega_1^2$$

Then:

$$k = C_1 \cdot \omega_0 \tag{4.3}$$

$$C_1 = \frac{F_C}{G \cdot \rho} \tag{4.4}$$

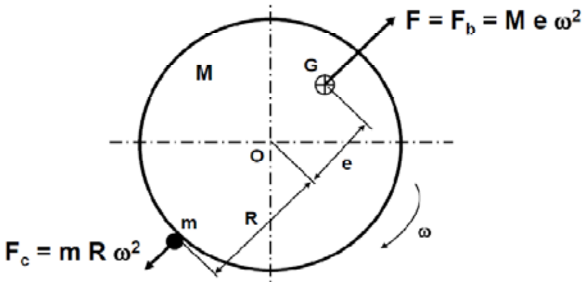

Fig. 5 Disposition on a rotor unbalance [2].

$$k = M\omega_0^2 \tag{4.5}$$

$$C_1 = M \cdot \omega_0$$

The factor C_1 depends to centrifugal force at rotation nominal speed and to the naturalfrequency. For low damping:(cf. [1]) if:

$$\tau = \frac{X_1}{X_0} = \frac{M}{k} \tag{4.6}$$

With Eq. (3.2), we have:

$$k = C_1 \cdot \omega_0 = \frac{M}{\tau} \tag{4.7}$$

$$C_1 = \frac{M}{\tau\omega_0} \tag{4.8}$$

Centrifugal force [3] is:

$$F_C = mR\omega_1^2 = \frac{\rho GM}{\tau\omega_0} \tag{4.9}$$

Unbalance with the mass m is given by:

$$b = mR$$

Then:

$$b = \frac{G}{\tau} \cdot \frac{M}{\omega_0^2} \cdot \frac{1}{\omega_1} \tag{4.10}$$

Unbalance is function of ratio rotor mass and rotating frequency. We have:

$$b = \frac{G}{\tau} \cdot \frac{M}{\omega_0^2} \cdot \frac{T}{2\pi} \tag{4.11}$$

In case of a rotating machines:(case of excitation by a static unbalance, rigid rotor) we have the liaison on Fig. 6. With e like eccentricity, C center of gravity et G is inertia center , z (t) and y(t) are the co-ordinates of the center of gravity along two perpendicular directions. The equations of the movement are:

$$m \frac{d^3}{dt^2}(y(t) + e\cos(\Omega t)) = -ky(t) - c\dot{y}(t)$$

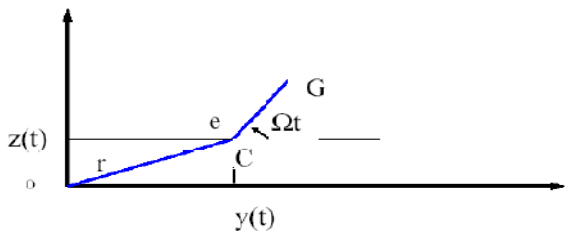

Fig. 6 Rigid liaison.

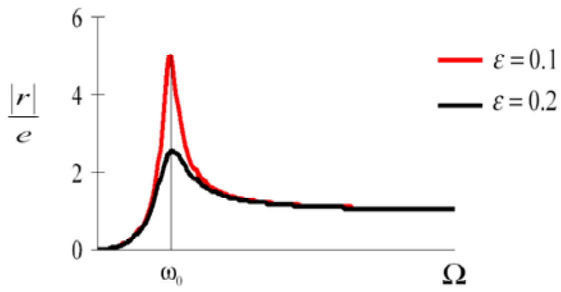

Fig. 7 Critical speed.

$$m \frac{d^3}{dt^2}(z(t) + e\sin(\Omega t)) = -kz(t) - c\dot{z}(t)$$

With $r(t) = y(t) + iz(t)$, thus $m\ddot{r}(t) + c\dot{r}(t) + kr(t) = me\Omega^2 e^{j\Omega t}$. We have the solution:

$$r(t) = \frac{e(\frac{\Omega}{\omega_0})^2}{(1 - (\frac{\Omega}{\omega_2})^2) + j(2\varepsilon\frac{\Omega}{\omega_0})} e^{j(\Omega t - \varphi)} \tag{4.12}$$

For $\Omega = \omega_0$, that corresponds to critical speed (Fig. 7). The expression of the vibrations in the case of rotating machines and for nominal speed of rotation Ω_1, we can write (cf. [1]):

$$\frac{r_1}{e_1} = \frac{(\frac{\Omega_1}{\omega_0})^2}{\left[(1 - (\frac{\Omega_1}{\omega_0})^2)^2 + (2\varepsilon\frac{\Omega_1}{\omega_0})^2\right]^{1/2}}$$

In the case of a constant speed Ω_1, we will have:

$$\frac{r_1}{e_1} = \frac{c}{k} \tag{4.13}$$

With low damping ($\varepsilon \ll 1$):

$$\frac{r_1}{e_1} = \frac{M}{k}$$

With Eq. (4.11) and for one rotation:

$$b = M \cdot e \tag{4.14}$$

or

$$e = \frac{1}{2\pi} \cdot \frac{G}{\tau} \cdot \frac{1}{\omega_0^2} \tag{4.15}$$

We have (cf. [1]) for rotating machines the ratio:

$$\tau = \frac{r_1}{e} \tag{4.16}$$

then:

$$r = \frac{G}{2\pi} \cdot \frac{1}{\omega_0^2} \tag{4.17}$$

with

$$r(t) = x(t) + iz(t)$$

We have:

$$x(t) = \frac{G}{2\pi} \cdot \frac{1}{\omega_0^2} \tag{4.18}$$

and (Cf.[1]):

$$x_1 = B_2(\omega).\frac{M}{k} = B_2(\omega).\tau \tag{4.19}$$

$$x_2 = B_2(\omega).\tau^2 \tag{4.20}$$

When we pose:

$$\tau = \frac{M}{k} \tag{4.21}$$

the value of x_1 is:

$$x_1 = \frac{G}{2\pi} \cdot \frac{1}{\omega_0^2} \tag{4.22}$$

then

$$B_2(\omega) = \frac{1}{\tau} \cdot \frac{G}{2\pi} \cdot \frac{1}{\omega_0^2} \tag{4.23}$$

$$x_2 = \tau \cdot \frac{G}{2\pi} \cdot \frac{1}{\omega_0^2} \quad (4.24)$$

If we want to change x_2 we increases τ. These values are constants and depends to critical speed. The ratio of vibration at speed $\Omega_1 = \omega_1$ is :(cf. [1]):

$$\frac{x_2}{x_1} = \frac{1}{1-(\frac{\Omega_1}{\omega_0})^2} \quad (4.25)$$

We can write :

$$x_2 = \frac{G}{2\pi} \cdot \frac{1}{\omega_0^2} \cdot \frac{1}{1-(\frac{\Omega_1}{\omega_0})^2} \quad (4.26)$$

If $\Omega_1 = \omega_0$, then x_2 tends to the infinite,

$$x_2 = \frac{G}{2\pi} \times \frac{1}{\omega_0^2 - \Omega_1^2} \quad (4.27)$$

vibrations amplitudes is positive quantity. The expressions obtained are for condition:

$$\Omega_1 < \omega_0 \quad (4.28)$$

If

$$\Omega_1 > \omega_0 \quad (4.29)$$

Then:

$$x_2 = \frac{G}{2\pi} \cdot \frac{1}{\omega_0^2 + j\Omega_1^2} \quad (4.30)$$

We have:

$$2\pi(\omega_0^2 + j\Omega_1^2) \cdot x_2 = G$$

$$x_2 = \frac{G}{2\pi\omega_0^2} = x_1 \quad (4.31)$$

Or $x_2 = 0$, in case where (for example a generator or gas turbine):

$$\Omega_1 > \omega_0 \quad (4.31.a)$$

At nominal speed, we have: $x_2 = x_2 = \frac{G}{2\pi\omega_0^2}$ and $\tau = 1$

5.2 Vibrations Limits

For a movement applied to the base of the spring mass system (Fig. 1), with the form:

$$Y(t) = Y_0 \sin(2\pi ft) \quad (4.32)$$

We have (Cf. [4]):

$$Y(t) = \frac{X}{Y} \sin(2\pi ft + \Phi) \quad (4.33)$$

and with the differential equation of movement: (Cf. [4]).

$$\frac{X}{Y}(\omega) = \frac{1}{1 + j(\frac{1}{Q}) \cdot (\frac{\omega}{\omega_0}) - (\frac{\omega}{\omega_0})^2} \quad (4.34)$$

where, $Q = \frac{1}{c}\sqrt{m \cdot k}$. The module is:

$$\left|\frac{X}{Y}\right| = \frac{1}{\sqrt{1 + (\frac{f}{f_0})^4 - (2 - \frac{1}{Q^2})(\frac{f}{f_0})^2}} \quad (4.35)$$

the vibration (velocity) is:

$$v(t) = \frac{dx}{dt} = 2\pi f \left|\frac{X}{Y}\right| \cos(2\pi f + \Phi) \quad (4.36)$$

and the transfer functions:(Cf. [4])

$$\frac{V}{Y}(\omega) = \frac{j\omega}{1 + j\,(\frac{1}{Q}) \cdot (\frac{\omega}{\omega_0}) - (\frac{\omega}{\omega_0})^2} \quad (4.37)$$

For low damping, the module is:

$$\left|\frac{V}{Y}\right| = \frac{\Omega_1}{\sqrt{1 + (\frac{\Omega_1}{\omega_0})^4 - 2(\frac{\Omega_1}{\omega_0})^2}} \quad (4.38)$$

where, f is vibration frequency and f_0 is natural frequency.

We have:

$$\frac{V}{X} = j\omega \quad (4.39)$$

where, V is velocity vibration and X is vibration of the mass of spring mass system. The module is:

$$|V| = |X|(2\pi f) = x_1(2\pi f) \quad (4.40)$$

$$V = \frac{(\frac{G}{2\pi} \times \frac{\Omega_1}{\omega_0^2})}{\sqrt{1 + (\frac{\Omega_1}{\omega_0})^4 - 2(\frac{\Omega_1}{\omega_0})^2}} \quad (4.41)$$

For our machine:

$$V = \frac{\frac{G}{2\pi} \cdot \frac{\rho}{\omega_0}}{\sqrt{1 + \rho^4 - 2\rho^2}} \quad (4.42)$$

Vibration is function of frequency ratio.

Table N°01 Admissibles vibrations according norm ISO 10816-1.

Group	Group N°01	Group N°02	Group N°03	Group N°04
Admissible vibration (mm/s)	0.71	1.12	1.8	2.8
Still admissible Vibration (mm/s)	4.5	7.1	11.2	18
Coefficient τ	6.33	6.33	6.33	6.33

6. Case of Multiple Degrees of Freedom

Free response is :

$$x_1 = X_0 = B_2(\omega)\cdot\frac{M}{k} \tag{5.1}$$

where, $X_0 = \frac{1}{\tau}\cdot\frac{G}{2\pi}\cdot\frac{1}{\omega_0^2}\cdot\frac{M}{k}$. We pose:

$$r = \frac{1}{\tau}\cdot\frac{G}{2\pi} = Constante 1 + \left(\frac{\Omega_1}{\omega_0}\right)^4 - 2\left(\frac{\Omega_1}{\omega_0}\right)^2$$

$$X_0 = \frac{r}{\omega_0^2}\cdot[M]\cdot[k]^{-1} \tag{5.2}$$

for multiple degrees of freedom:

$$\{X_0\} = \frac{r}{\omega_0^2}\cdot[M]\cdot[k]^{-1} \tag{5.3}$$

7. Applications

The following application is concerning a generator of IV group of machines which produces electrical energy. The generator is driven by a gas turbine.

Critical speed: at 50% of nominale speed.

Nominal speed: 3,000 rpm.

Speed ratio : $\rho = 2$

For gas turbine: G = 2.5 mm/s.

Maximale vibration is (see Eq. (4.42)):

$$V = 1.69\, mm/s \tag{6.1}$$

RMS value is:

$$V_{RMS} = \frac{V}{\sqrt{2}} = 1.20\, mm/s \tag{6.1.a}$$

According norm ISO 10816-1 the ratio τ is for all groups machines :

$$\tau = \frac{Vibrations}{Admissibles\ vibrations}$$

where, for admissibles vibrations: $\tau = 1$; For still admissibles vibrations: $\tau = 6.33$; For dangerous vibrations: $\tau > 6.33$. Absolute vibrations (Cf. [1]) depend on τ .

The optimal choice of this coefficient can change the thresholds and which takes into account machines design. If $\tau = \frac{X_1}{X_0}$, and considering that admissibles vibrations correspond to the balance, so we add them to the still admissible value. If V_1 correspond to the norm and V_2 correspond to searched vibration, we have:

$$V_2 = V_1 - V_1\cdot\frac{\Delta V}{V} \tag{6.2}$$

We calculate the relative difference $\frac{\Delta V}{V}$ between the value of the ISO 10816-1 norm and the calculated value of the machines group No. 04:

Table N°02 1st Evaluation.

Group	Group N°01	Group N°02	Group N°03	Group N°04
Admissibles vibrations ($\tau = 1$)	0.31	0.48	0.77	1.20
Stilladmissibles vibrations ($\tau = 6.33$)	1.93	3.05	4.9	7.62
Inadmissibles vibrations $\tau > 6.33$	> 1.93	> 3.05	> 4.9	> 7.62

Table N° 03 2nd Evaluation.

Groupe	Group N°01	Group N°02	Group N°03	Group N°04
Admissibles vibrations	0.61	0.96	1.54	2.41
Stilladmissibles vibrations	2.54	4.12	6.45	10.03
Inadmissibles vibrations	> 2.54	> 4.12	> 6.45	> 10.03

Table N° 04 3rd Evaluation.

Group	Group N°01	Group N°02	Group N°03	Group N°04
Admissibles vibrations	0.61	0.96	1.54	2.41
Stilladmissibles vibrations	3.15	4.97	7.99	12.43
Inadmissibles vibrations	> 3.15	> 4.97	> 7.99	> 12.43

Table N° 05 4 th Evaluation.

Group	Group N°01	Group N°02	Group N°03	Group N°04
Admissibles vibrations	0.61	0.96	1.54	2.41
Stilladmissibles vibrations	3.76	5.94	9.54	14.85
Inadmissibles vibrations	> 3.76	> 5.94	> 9.54	> 14.85

Table N° 6 5th Evaluation (corresponding to norm ISO 10816-1).

Groupe	Group N°01	Group N°02	Group N°03	Group N°04
Stilladmissibles vibrations	5.21	8.22	13	20.8

Table N° 07 6 th Evaluation.

Group	Group N°01	Group N°02	Group N°03	Group N°04
Admissibles vibrations	0.61	0.96	1.54	2.41
Stilladmissibles vibrations	4.37	6.90	11.10	17.25
Inadmissibles vibrations	> 4.37	> 6.90	> 11.10	> 17.25

Table N° 08 Vibrations limitsfromoptmization (Cf. [1]).

Group	Group N°01	Group N°02	Group N°03	Group N°04
Admissibles vibrations	0.61	0.96	1.54	2.41
Stilladmissibles vibrations	4.98	7.86	12.63	19.66
Inadmissibles vibrations	> 4.98	> 7.86	> 12.63	> 19.66

$$\frac{\triangle V}{V} = \frac{2.8 - 1.2}{2.8} = 0.57 \tag{6.3}$$

We construct the Table N°02. We suppose also that admissibles vibrations corresponds to equilibirium, they can be added to still admissibles values. We have the five (05) evaluations to obtain vibrations values of norms (Cf. [1]). We also do the same operation on admissible and on still admissible values (Cf. Annexe). Vibrations from optmization (Cf.[1]) are identical to that calculated in the present document.

7.1. Evaluations Number

Considering V_{EA-n} is still admissible vibration after n evaluations, V_{EA-1} is still admissible vibrations of the first evaluation and V_A is admissibles vibrations, we have:

$$V_{EA-n} = V_{EA-1} + nV_A$$

the evaluations number is:

$$n = \frac{V_{EA-n} - V_{EA-1}}{V_A}$$

$$n = (18 - (\frac{18}{6.33}))/2.8 \approx 5$$

8. Conclusions

Our work is based on the optimization study of rotating machines vibration limits (see [1]). Depending on the coefficient as the ratio of vibrations and admissible vibrations according norm ISO 10816-1, absolutes vibrations limits origin is elaborated. We hope that this document will be the object of scientific research in the field of rotating machines.

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