Journal of Mathematics and System Science 6 (2016) 260-261
doi: 10.17265/2159-5291/2016.06.005

Bonds & Dots in Lines on a Plane--about Euler Formula F+V-E=2

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Received: March 16, 2016 / Accepted: April 12, 2016 / Published: June 25, 2016.

Abstracts: There comes the Euler Formula on the way exploring the constraints among bonds and dots by intersected lines on plane.

Key Words: Polygons, Dots (Vertices) and Bonds (Edges), Lattices and Polyhedrons.

1. Introduction

Given n straight lines in a plane, i.e. *2 line divides the plane into 2 parts (areas)*, there are intersections and the partitions are edged with line segments. The intersections are Dots and the Segments Bonds and the partitions either closed or open polygons, i.e. 2 or more-side open areas. See the below figure.

In the figure, there are 6 5-dot lines, and all dots are 2-line intersected, and the closed areas are bonded in polygons, including triangles, quadrangles. For the open areas, the partitions are edged with or without bonds but 2 line-extensions at least.

For the n-line partitions, n>2, it is easy to have the below equations if all dots are 2-line intersected, i.e. *4 areas surround each dot and 2 areas each bond*.

$$N_d = V_2 = \binom{2}{n} \ \& \ N_v = N_d + 2n \quad (1a)$$

$$N_e = n^2 \ \& \ N_b = n(n-2) \quad (1b)$$

$$\sum_{i=2}^{n} O_i = 2n \quad (1c)$$

$$\sum_{i=3}^{n} C_i = \binom{2}{n-1} \quad (1d)$$

$$\sum_{i=2}^{n} [O_i * (i-1)] + \sum_{i=3}^{n} (C_i * i) = 4V_2 \quad (1e)$$

$$\sum_{i=2}^{n} (O_i * i) + \sum_{i=3}^{n} (C_i * i) = 2N_e \quad (1f)$$

While

N_d is the number of dots on the plane, N_v the number of dots as vertices for polygons, including those at far for open areas, and

V_i is the number of the i-line-intersected dots, e.g. V_2 is for 2-line-intersected dots, excluding the far-away ones, and

N_e is the number of sides or edges dividing the plane, N_b the number of edges for closed areas, i.e. the 2-dot ended bonds, and the number of open bonds (with only 1-dot end) is N_e-N_b=2n, and

O_i is the number of the open areas with i-side, and

C_i is the number of polygons with i-bond.

And *the equation (1e) is for the Dot-Conservation, the equation (1f) the Edge-Conservation.* And on each line, the numbers of the dots, edges and bonds are n-1, n and n-2 respectively.

Referred to the below Euler's formula,

F+V-E=2

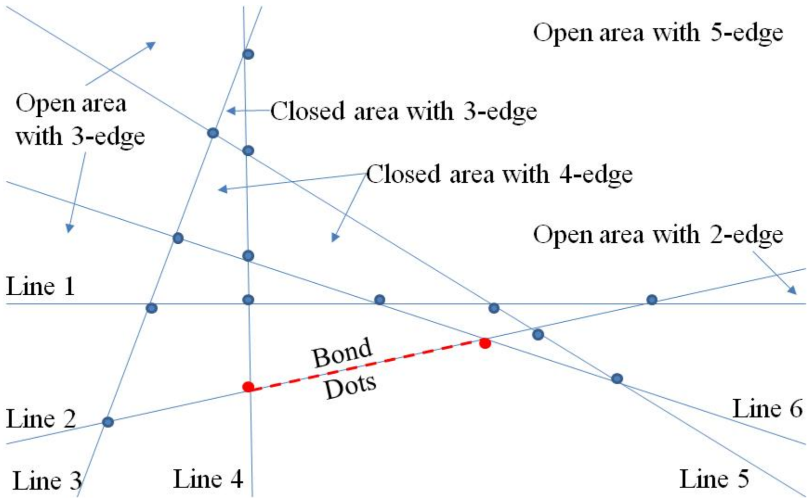
For the above bonds and dots, similarly, the below equation is observed,

$$\sum_{i=3}^{n} C_i + N_d - N_b = 1 \quad (2)$$

Which holds even there are multi-line intersected dots, i.e. the above equation (2) is valid for any bond-dot lattice structures on or mapped on a plane.

It is very interesting that, considering the mapped cases, then, without loss of generality, the out-dots of the structure could be on a plane and recognized as a surface to construct a polyhedron with the other lattices, and *there the equation (2) becomes the famous Euler formula*.

V=Nd,



E=Nb, and

$$F=\sum_{i=3}^{n} C_i + 1$$

It is known that 2 dots are for ending a bond; 2 or more bonds (could be curved) are bonded to surround an area, 2 or more areas could be established by bonds and dots to pave an area seamlessly; and 2 or more surfaces are sealed together to close an empty cubage, and 2 or more small cubic objects can be solidly engaged to fill the empty cubage.

Let D of dots be as the number of vertices, B of bonds the number of edges, S of surfaces the number of Faces, and C of cubage the number of cubic objects, taking 3-dimension connections as considerations, then for any converged objects or bodies, the below equation is observed.

$$D+S-B-C=1 \quad (3)$$

Equation (2) holds while C=0, and the Euler Formula while C=1.

Mathematics is there to be discovered and to be recognized.