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Constructions with Numbers

Nick Huo Han Huang

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Abstracts: Pascal Triangle is more of a number construction (body) then an array of the binomial coefficients. It is a mathematical body, like the digital code feeds for computer but with 2 dimensions. And there should be bodies with x-dimensions and even abnormal or irregular appearances.

Key Words: Natural Numbers, Polygons, Dots (Vertexes) and Bonds (Edges), Number Placement, Number Process (Sequence), Arithmetic Process, and Repeating (Process).

1. Introduction

A Bond without or with certain same of lengths ends both Dots. 3 bonds sharing 3dots constructs a triangle, the bonds are Edges and dots Vertexes. A Bonds-in-line is one dimension or 2 or more dimensions.

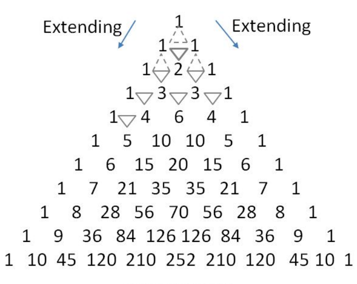
Pascal Triangle is constructed as below.

(1) Name the 1st three dots as 1 and bond them as a triangle.

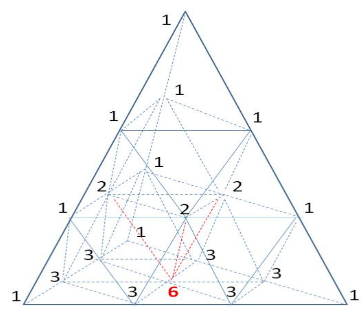
(2) Extend 2 bonds to reach the other 2 dots of the same, and bond the 2 dots with another one in the middle, which is bonded with both neighbor upper 2 dots. Then the 3 new same triangles are constructed, and a larger triangle is formed by the 4 triangles.

(3) Keep extending the 2 bonds to reach the other dots of the same, new dots must be generated to bond with upper neighbor dots to frame new triangles in between. Then triangles jigsaw a larger one. It is a 2-dimension graph.

(4) If the new dots are named as the numbers summed by the numbers for the 2 upper neighbor dots, then the Pascal Triangle[1] is formed, see the below figure.

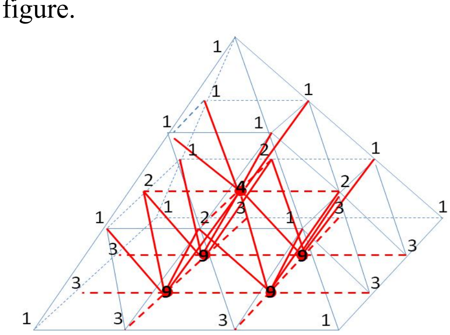


If a triangular pyramid, a 3-dimension object, is extended by 3 bonds (edges) from a dot (vertex), and there must be 3 upper neighbor dots to be bonded by each next new dot, see the red-dot lines in the below figure, i.e. a tetrahedron solidly engaged with octahedrons and triangular pyramids.

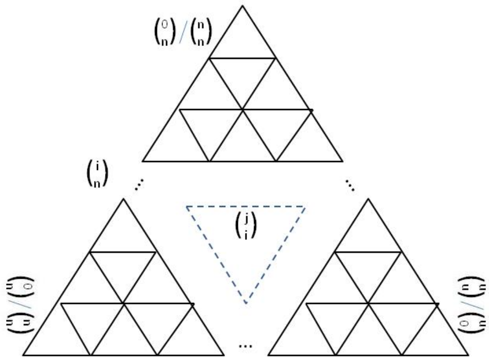


The sum of the dot numbers in n^{th}-row of the Pascal Triangle is 2^n and the sum of the dot numbers on n^{th}-plane of the above tetrahedron is 3^n, n=0 at the vertex.

If it is a rectangular pyramid extended, then the sum of the numbers on n^{th}-plane is 4^n, and the inner new dots are bonded with 4 upper neighbor dots, see the below figure.



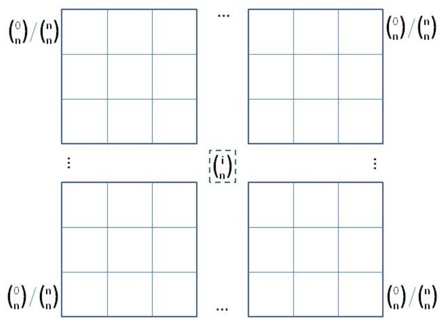
Check the n^{th}-section of the above Triangular Pyramid, the dot numbers layout is as below.



The sum of the numbers on the section plane is as below.

$$S_n=\sum_{i=0}^{n}\binom{i}{n}\left[\sum_{j=0}^{i}\binom{j}{i}\right]=\sum_{i=0}^{n}\binom{i}{n}2^{i}=(1+2)^n=3^n$$

And the same is for the Rectangular Pyramid.



$$S_n=\sum_{i=0}^{n}\binom{i}{n}2^{n}=2^{2n}=4^n$$

No dots and bonds of polygon sections with five or more bonds could be imagined due to the constraints of the below Diophantine of k.

$$\sum_{i=1}^{k}\alpha_i = 180,\ k=1 \text{ to } n \quad (a)$$

$$\sum_{i=1}^{n}\alpha_i = 180*(n-2),\ n>2 \quad (b)$$

While α_i is the i^{th} angle of the polygon in the section plane.

Not loss of generality, take a regular polygon, for example, then the above equations (a) and (b) can be rewritten as below.

$$k=n/(n-2) \quad (c)$$

And then n, $\in \mathrm{N}$, can be the below values only.

[3 (Triangular Pyramid) or 4(Rectangular Pyramid)].

With bonds and dots, what a fantastic world built by numbers instead of molecules or particles! It is natural that for 1-bond extending, i.e. 1-dimension only, there are only dots named 1 in the line, and the sum is $1^n=1$ at any extending steps.

Mathematics is there to be discovered and to be recognized.

Notes:

There are magical bonds among the entries of the number layout in Pascal Triangle. Turn the Pascal Triangle count clock wise, the below table is found. It is observed that the sum formula for the number sequence of k^{th} row is a k-order polynomial of n.

1	1	1	1	1	1	1	1	1	1
1	2	3	4	5	6	7	8	9	10
1	3	6	10	15	21	28	36	45	55
1	4	10	20	35	56	84	120	165	220
1	5	15	35	70	126	210	330	495	715
1	6	21	56	126	252	462	792	1287	2002
1	7	28	84	210	462	924	1716	3003	5005
1	8	36	120	330	792	1716	3432	6435	11440
1	9	45	165	495	1287	3003	6435	12870	24310
1	10	55	220	715	2002	5005	11440	24310	48620

References

[1] "实用数学手册", 叶其孝, 沈永欢, 2nd Edition, Beijing, 2006.

[2] "Teubner Taschenbuch der Mathematik", E. Zeidler.