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A Note on Fermat Equation's Fascination

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Abstract: As former Fermatist, the author tried many times to prove Fermat's Last Theorem in an elementary way. Just few insights of the proposed schemes partially passed the peer-reviewing and they motivated the subsequent fruitful collaboration with Prof. Mario De Paz. Among the author's failures, there is an unpublished proof emblematic of the FLT's charming power for the suggestive circumstances it was formulated. As sometimes happens with similar erroneous attempts, containing out-of-context hints, it provides a germinal approach to power sums yet to be refined.

Key words: Fermat's equation, Fermat's Last Theorem, power sums

1. Introduction

This paper illustrates how a *Fermatist* thinks and acts, an autobiographical argument with a pedagogical intent.

We talk about the low status of Fermatists: people labeled as *amateurs* despite their level of education, discredited as *cranks* and inevitably dismissed for their obstinacy about an insoluble subject.

We rescue an old documentation, explaining that the Fermatists' proofs can be devised in outlandish situations and sometimes contain original hints.

We report an erroneous attempt to show why a Fermat equation at prime indexes should not have positive integer solutions; the self-checking was a sign of healing from the *Fermat's fever*.

We clarify that Fermat's equation is not solely the so-called Fermat's Last Theorem (acronym FLT) but the source of a fecund algebraic research.

2. A Fermatist's Hard Life

It is difficult to explain what the *Fermat's fever* is and how strong it can be, propelling a person to incessantly write hundreds of pages of calculation in order to find a short and elegant way to prove FLT, alternative to the official one [19].

What is the basis for such a compulsive behavior with maniacal traits and an insistence irritating even the number theorists more disposed to examine the submitted works?

Although a minority of the FLT seekers are illiterate adventurers in search of glory, the principal incentive for a foolish and unproductive effort is a burning desire; it goes to the roots of that "art of problem solving" any mathematically-oriented mind is fond of.

The alleged *Mirabilis* (a never found elementary proof of FLT claimed by Pierre de Fermat in 1637 [18]) and the challenge to face an apparently easy issue can exert an irresistible fascination; no matter how much irrational is trying what some of the most formidable scientists of all times failed.

The author once was a *Fermatist* and it meant a complete concentration on the question in the years 2003-2006 with notebooks consumed after exploring several methods (algebraic and geometric) and conceiving strategies in every occasion, from the jolts of a moving bus to the quietness of a seaside beach.

Being a Fermatist meant aprioristic refusals, unpleasant misunderstandings and a frequent redirecting to *«more patient»* reviewers. It meant also receiving diplomatic answers from brilliant researchers and well-referred journals (Figs. $1-3$).

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Egregio Ingegnere,
abbiamo ricevuto la Sua e-mail, con il testo da lei sviluppato sul Teorema di Fermat. Mentre ci congratuliamo con Lei per l'ottimo lavoro, molto apprezzabile, siamo spiacenti di comunicarLe che il nostro Giornale non pubblica articoli scientifici sotto la forma da Lei proposta, in quanto esulano dalla linea editoriale.
Di solito accogliamo argomenti tecnico scientifici, anche a livello di divulgazione elevato, ma non articoli di ricerca da pubblicare su specifici veicoli.
Spiacenti per l'inconveniente, La ringraziamo e la salutiamo cordialmente.

La redazione

Fig. 1 Rejection email dated May 11, 2004.

Dear Dr. Bonacci,
This letter regards your paper "Teorema di Bonacci sull'equazione di Fermat" which you submitted to the Annali.
Unfortunately our journal has a substantial backlog at this time and therefore we cannot consider your paper for publication.
We suggest that you seek another publishing outlet for your paper.
Sincerely yours,
Giorgio Talenti
Editor-in-Chief

Fig. 2 Rejection email dated June 17, 2004.

Dear Dr Bonacci

The Glasgow Mathematical Journal is unable to accept your paper for publication.
The GMJ is a general journal. Your paper is considered more suitable for a specialist journal in number theory.

Yours regretfully,

Alec Mason

Editor-in Chief

Fig. 3 Rejection email dated February 2, 2005.

P.S. la prova dell'esistenza di Dio non può essere logica, in quanto le regole di deduzione si configurano come limitazioni in successione, riduzioni a sottinsiemi veicolate da acclarati vincoli di appartenenza.
Per conoscere il nostro Creatore, se accettiamo l'assunto della sua infinitezza e libertà assoluta, cioè non condizionata, dovremmo seguire un percorso inverso di progressiva emancipazione dagli schemi che si estrinseca, però, come conoscenza non mediata dal raziocinio.
Molti pensano che la matematica avvicini alla divinità invece conduce all'estremità interna, opposta, della spirale.

Fig. 4 Correspondence email dated April 16, 2004.

Any claim about Fermat's Last Theorem, a highly controversial topic, may provoke sarcastic replies like: *«Why don't you try to find an evidence of God's existence too?»*. A message of this kind stimulated unexpected metaphysical reflections (Fig. 4).

3. A Captivating Fermatist's Proof

We present the less awful of a long series of attempts at proving FLT ruled out for publication by the same author and stored among the unpublished files by the Italian Society of Authors and Editors [2-4].

Seeing $c^p = a^p + b^p$ like a sum with restrictions on the positive integer values of the addends imposed by their p-powers, the number of the allowed pairs $(a; b)$ would have an upper bound smaller than one, (i.e., inconsistent) with any prime index $p \geq 3$.

Thus, it would be *impossible* to find a triple of positive integers satisfying the Fermat's equation at prime indexes, because not even a single pair $(a; b)$ would meet the condition $a^p + b^p = c^p$.

The same criterion of counting the pairs of positive integer addends in the equation, would explain why the Pythagorean triples ($p = 2$) are instead allowed.

3.1 The Elementary Sum

Let a, b, c be three positive integers related by

$$a + b = c \tag{1}$$

Let us suppose that $a \leq b$ and denote with n_1 the number of pairs $(a; b)$ satisfying the Eq. (1).

The set of pairs is: $(1; c-1)$, $(2; c-2)$,... with the maximum $\left(\frac{c}{2}; \frac{c}{2}\right)$ reachable only if c is even.

Since we do not know whether c is odd or even, the number of pairs is the unique integer number n_1 between $\frac{c-1}{2}$ and the semi-sum $\frac{c}{2}$:

$$n_1 = \begin{cases} \frac{c-1}{2} & \text{if } c \neq 0 \text{ (mod2)} \\ \frac{c}{2} & \text{if } c = 0 \text{ (mod2)} \end{cases} \tag{2}$$

3.2 The Pythagorean Equation

Let a, b, c be three positive integers related by

$$a^2 + b^2 = c^2 \tag{3}$$

Let us initially suppose that $a \leq b$ and denote with n_2 the number of pairs $(a; b)$ satisfying the Eq. (3).

Differently from the case of n_1, the upper bound of the possible pairs' number n_2 is smaller than the semi-sum $\frac{c^2}{2}$ for two reasons:

$$a < b, \text{otherwise } \frac{c}{a} = \frac{c}{b} = \sqrt{2} \notin \mathbb{Q};$$

the addends are both squared and the repetition of each factor a and b ($aa + bb = c^2$) limits the number of pairs $(a; b)$ to $\frac{c^2}{2ab}$.

Therefore the upper bound is:

$$n_2 < \frac{c^2}{2ab} \tag{4}$$

The Eq. (4) restricts but not excludes the existence of Pythagorean triples. As a classic example, in $3^2 + 4^2 = 5^2$ the limit of possible pairs would be $n_2 < \frac{25}{24}$, i.e., $n_2 = 1$ allowing only the pair $(3; 4)$.

3.3 The Fermat Equation

Let a, b, c, p be four positive integers related by

$$a^p + b^p = c^p \tag{5}$$

with $p > 2$ prime, i.e., the Fermat equation at prime indexes.

Let us initially suppose that $a \leq b$ and denote with n_p the number of pairs $(a; b)$ satisfying the Eq. (5).

Similarly to the case of n_2, the upper bound of the possible pairs' number n_p is sensibly smaller than the semi-sum $\frac{c^p}{2}$ for two reasons:

$$a < b, \text{otherwise } \frac{c}{a} = \frac{c}{b} = \sqrt[p]{2} \notin \mathbb{Q};$$

the addends are both p-powers and the repetition of each factor a and b ($aa^{p-1} + bb^{p-1} = c^p$) limits the number of pairs $(a; b)$ to $\frac{c^p}{2a^{p-1}b^{p-1}}$.

Therefore the upper bound is:

$$n_p < \frac{c^p}{2a^{p-1}b^{p-1}} \tag{6}$$

Since $a^p = c^p - b^p$, the Eq. (6) becomes:

$$n_p < \frac{c^p}{2b^{p-1}\left(\sqrt[p]{(c-b)(c^{p-1} + c^{p-2}b + \cdots + b^{p-1})}\right)^{p-1}} \tag{7}$$

Since $c - b \geq 1$, the Eq. (7) becomes:

$$n_p < \frac{c^p}{2b^{p-1}\left(\sqrt[p]{c^{p-1} + c^{p-2}b + \cdots + b^{p-1}}\right)^{p-1}} \tag{8}$$

Since $c > b$, the Eq. (8) becomes:

$$n_p < \frac{c^p}{2b^{p-1}\left(\sqrt[p]{pb^{p-1}}\right)^{p-1}} \tag{9}$$

Since $p^{\frac{p-1}{p}} > 1$, the Eq. (9) becomes:

$$n_p < \frac{c^p}{2b^{p-1}\left(\sqrt[p]{b^{p-1}}\right)^{p-1}} \tag{10}$$

A trivial manipulation of the Eq. (10) leads to:

$$n_p < \frac{c^p}{2b^p \sqrt[p]{b^{p^2-3p+1}}} \tag{11}$$

Since $b^{\frac{p^2-3p+1}{p}} > 1$, the Eq. (11) becomes:

$$n_p < \frac{c^p}{2b^p} \tag{12}$$

Since $c^p < 2b^p$, the Eq. (12) becomes:

$$n_p < 1 \tag{13}$$

The Eq. (13) suffices to exclude the existence of any Fermat triple because the condition $n_p < 1$ means that not even a single pair of integers $(a; b)$ can satisfy the Eq. (5).

3.4 Differences between the Equations

The Eqs. $(6) - (12)$, leads to the condition $n_p < 1$, i.e., not even a single pair of positive integer addends can satisfy the Fermat equation.

If we apply an analogous upper bound search to the Pythagorean equation, we obtain a different result.

In fact, since $a^2 = c^2 - b^2$, the Eq. (4) becomes:

$$n_2 < \frac{c^2}{2b\sqrt{(c-b)(c+b)}} \tag{14}$$

Since $c - b \geq 1$, the Eq. (14) becomes:

$$n_2 < \frac{c^2}{2b\sqrt{(c+b)}} \tag{15}$$

Since $c + b > 2b$, the Eq. (15) becomes:

$$n_2 < \frac{c^2}{2b\sqrt{2b}} \tag{16}$$

A trivial manipulation of the Eq. (16) leads to:

$$n_2 < \frac{c^2}{2\sqrt{2}\, b\sqrt{b}} \tag{17}$$

Whereas the Eq. (12) is narrower than the Eq. (6) and excludes any Fermat triple, the Eq. (17) is wider than the Eq. (4) allowing the Pythagorean triples.

3.5 Analogies Among the Equations

The Eqs. (2), (4) and (6) can be unified by the same upper value:

$$\frac{c^p}{2a^{p-1}b^{p-1}} \tag{18}$$

Whereas for the Eq. (4) and (6) it is evident, the Eq. (2) should be put in the equivalent form:

$$\frac{c}{2a^0b^0} \tag{19}$$

obviously with $a^0 = b^0 = 1$.

3.6 What is Wrong With this Proof?

The Eq. (18) offers the opportunity to reduce the whole proof to the following Eqs. (20) and (21).

In fact, the first logical step would be:

$$a^p + b^p = c^p \Rightarrow \frac{c^p}{2a^{p-1}b^{p-1}} \geq 1 \tag{20}$$

The second logical step would be:

$$\frac{c^p}{2a^{p-1}b^{p-1}} \geq 1 \Rightarrow p < 3 \tag{21}$$

Combining the Eqs. (20) and (21), we get:

$$a^p + b^p = c^p \Rightarrow p < 3 \tag{22}$$

that is our thesis.

The glitch is in the Eq. (20); concisely, counting pairs is not as immediate as it seems [1] when we consider the equation Eq. (3) and it becomes even more complicated if we focus on the Eq. (5).

3.7 Not a Total Waste of Time

The positive characteristic of the proof is an original criterion for approaching the Fermat equation: counting "correctly" the possible pairs $(a; b)$ in the equation $a^p + b^p = c^p$ at integer variables a, b, c, p, with $a \leq b$ and p prime, we should anyway find decreasing values (even if we do not know whether below unity or not) with the growth of p.

It could be a nice deepening independently of FLT-related ambitions, once the flaws and the mistakes will be fixed by qualified mathematicians.

4. Features of a Fermatist's Activity

Although manifestly simplistic, the previous attempt has been chosen for its curious genesis, underlining the FLT's fascinating power.

The proof was written furiously in a end-of-course ceremony at the Saint-Petersburg State University, where the author had the sudden inspiration of counting pairs in the Fermat equation.

For not risking a text "too large to fit in the margin" as for the legendary Fermat's riddle [18], the author used the biggest sheet available at that moment: a diploma-like card (Fig. 5).

The well-known Latin title *Illuminatio mea* was due to both the enthusiasm of the discovery and the author's habit of wearing Oxford University T-shirts.

An insane belief caused further excitement: Fermat's famous statement *«hanc marginis exiguitas non caperet»* seemed a cryptic indication of a proof establishing that the upper bound for the pairs of addends is very low (below unity).

We may notice how the paper was written in English by an Italian author during his stay in Russia, bizarre circumstances highlighting the cosmopolitan aspect of a robust Fermat's fever.

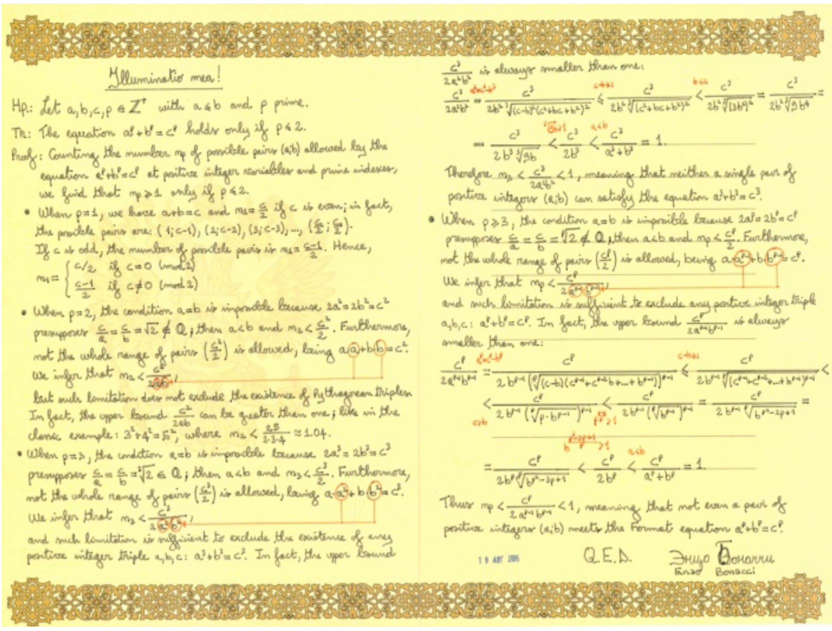
It neither was an isolated episode: two years before in Moscow the author wrote another attempt at proving FLT, named after a Russian woman for sentimental reasons, whose lack of validity was promptly certified by the Einstein Institute of Mathematics (Fig. 6).

5. Recovering from the Fermat's Fever

Strictly speaking, the Fermatist experience ended in 2006, when the author reached the awareness that the search for an elementary proof of FLT was vain and he renounced to anxiety-inducing expectations.

The most promising insights of the huge 2003-2006 work were published in a book [5] which marked a turning point in the author's life. Still under the *Fermat's curse* (Fig. 7), it however elicited the interest of Prof. Mario De Paz (University of Genoa) with whom there began a six-year cooperation aimed at finding the limits of Fermat's equation by employing simple techniques [6-10,13,14].

Beyond the usual hostility towards presumed Fermatists, the Bonacci-De Paz's research raised a constructive criticism [11,12,15-17] and a genuine interest in the mathematical community (Figs. 8 and 9) for the clear invitation to consider the results not like FLT's solutions but as an algebraic investigation.



Illuminatio mea!

Hp.: Let $a, b, c, p \in \mathbb{Z}^+$ with $a \leq b$ and p prime.

Th: The equation $a^p + b^p = c^p$ holds only if $p \leq 2$.

Proof: Counting the number m_p of possible pairs $(a;b)$ allowed by the equation $a^p+b^p=c^p$ at positive integer variables and prime indexes, we find that $m_p \geq 1$ only if $p \leq 2$.

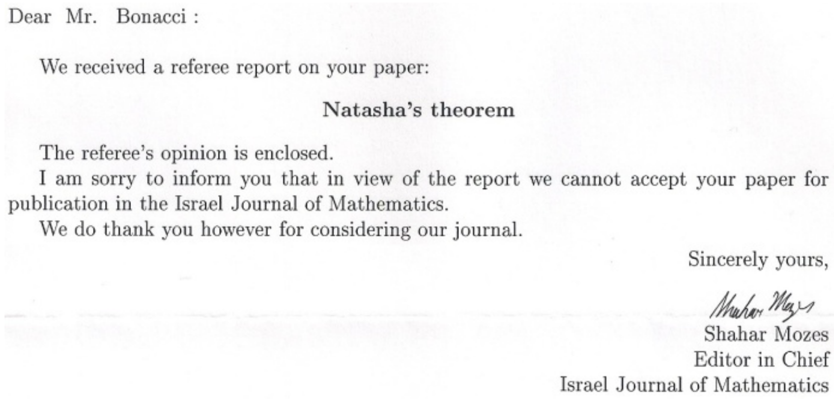
- When $p=1$, we have $a+b=c$ and $m_1 = \frac{c}{2}$ if c is even; in fact, the possible pairs are: $(1;c-1), (2;c-2), (3;c-3), \dots, (\frac{c}{2};\frac{c}{2})$. If c is odd, the number of possible pairs is $m_1 = \frac{c-1}{2}$. Hence,
$$m_1 = \begin{cases} c/2 & \text{if } c \equiv 0 \pmod 2 \\ \frac{c-1}{2} & \text{if } c \not\equiv 0 \pmod 2 \end{cases}$$
- When $p=2$, the condition $a=b$ is impossible because $2a^2=2b^2=c^2$ presupposes $\frac{c}{a}=\frac{c}{b}=\sqrt{2} \notin \mathbb{Q}$; then $a<b$ and $m_2 < \frac{c^2}{2}$. Furthermore, not the whole range of pairs $(\frac{c^2}{2})$ is allowed, being $a \cdot a + b \cdot b = c^2$. We infer that $m_2 < \frac{c^2}{2ab}$, but such limitation does not exclude the existence of Pythagorean triples. In fact, the upper bound $\frac{c^2}{2ab}$ can be greater than one; like in the classic example: $3^2+4^2=5^2$, where $m_2 < \frac{25}{2 \cdot 3 \cdot 4} \cong 1.04$.
- When $p=3$, the condition $a=b$ is impossible because $2a^3=2b^3=c^3$ presupposes $\frac{c}{a}=\frac{c}{b}=\sqrt[3]{2} \notin \mathbb{Q}$; then $a<b$ and $m_3 < \frac{c^3}{2}$. Furthermore, not the whole range of pairs $(\frac{c^3}{2})$ is allowed, being $a \cdot a^2 + b \cdot b^2 = c^3$. We infer that $m_3 < \frac{c^3}{2a^2b^2}$, and such limitation is sufficient to exclude the existence of any positive integer triple a,b,c: $a^3+b^3=c^3$. In fact, the upper bound $\frac{c^3}{2a^2b^2}$ is always smaller than one:
$$\frac{c^3}{2a^2b^2} = \frac{c^3}{2b^2\sqrt[3]{(c-b)^2(c^2+bc+b^2)^2}} \leq \frac{c^3}{2b^2\sqrt[3]{(c^2+bc+b^2)^2}} < \frac{c^3}{2b^2\sqrt[3]{(3b^2)^2}} = \frac{c^3}{2b^2\sqrt[3]{9b^4}} =$$
$$= \frac{c^3}{2b^3\sqrt[3]{9b}} < \frac{c^3}{2b^3} < \frac{c^3}{a^3+b^3} = 1.$$
Therefore $m_3 < \frac{c^3}{2a^2b^2} < 1$, meaning that neither a single pair of positive integers $(a;b)$ can satisfy the equation $a^3+b^3=c^3$.
- When $p \geq 3$, the condition $a=b$ is impossible because $2a^p=2b^p=c^p$ presupposes $\frac{c}{a}=\frac{c}{b}=\sqrt[p]{2} \notin \mathbb{Q}$, then $a<b$ and $m_p < \frac{c^p}{2}$. Furthermore, not the whole range of pairs $(\frac{c^p}{2})$ is allowed, being $a \cdot a^{p-1} + b \cdot b^{p-1} = c^p$. We infer that $m_p < \frac{c^p}{2a^{p-1}b^{p-1}}$, and such limitation is sufficient to exclude any positive integer triple a,b,c: $a^p+b^p=c^p$. In fact, the upper bound $\frac{c^p}{2a^{p-1}b^{p-1}}$ is always smaller than one:
$$\frac{c^p}{2a^{p-1}b^{p-1}} = \frac{c^p}{2b^{p-1}\left(\sqrt[p]{(c-b)(c^{p-1}+c^{p-2}b+\dots+b^{p-1})}\right)^{p-1}} \leq \frac{c^p}{2b^{p-1}\sqrt[p]{(c^{p-1}+c^{p-2}+\dots+b^{p-1})^{p-1}}} <$$
$$< \frac{c^p}{2b^{p-1}\left(\sqrt[p]{p \cdot b^{p-1}}\right)^{p-1}} < \frac{c^p}{2b^{p-1}\left(\sqrt[p]{b^{p-1}}\right)^{p-1}} = \frac{c^p}{2b^{p-1}\sqrt[p]{b^{p^2-2p+1}}} =$$
$$= \frac{c^p}{2b^p\sqrt[p]{b^{p^2-3p+1}}} < \frac{c^p}{2b^p} < \frac{c^p}{a^p+b^p} = 1.$$
Thus $m_p < \frac{c^p}{2a^{p-1}b^{p-1}} < 1$, meaning that not even a pair of positive integers $(a;b)$ meets the Fermat equation $a^p+b^p=c^p$.

19 АВГ 2005

Q.E.D.

Энцо Боначчи
Enzo Bonacci

Fig. 5 Handwritten note dated August 19, 2005.



Dear Mr. Bonacci :

We received a referee report on your paper:

Natasha's theorem

The referee's opinion is enclosed.

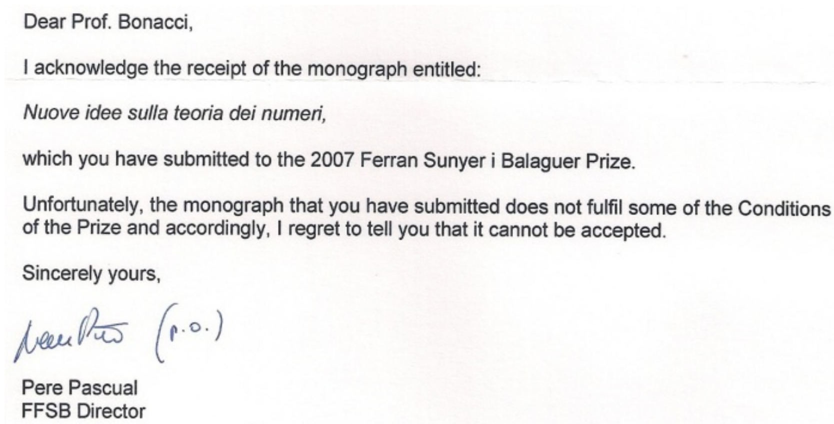
I am sorry to inform you that in view of the report we cannot accept your paper for publication in the Israel Journal of Mathematics.

We do thank you however for considering our journal.

Sincerely yours,

Shahar Mozes
Editor in Chief
Israel Journal of Mathematics

Fig. 6 Rejection letter dated August 21, 2003.



Dear Prof. Bonacci,

I acknowledge the receipt of the monograph entitled:

Nuove idee sulla teoria dei numeri,

which you have submitted to the 2007 Ferran Sunyer i Balaguer Prize.

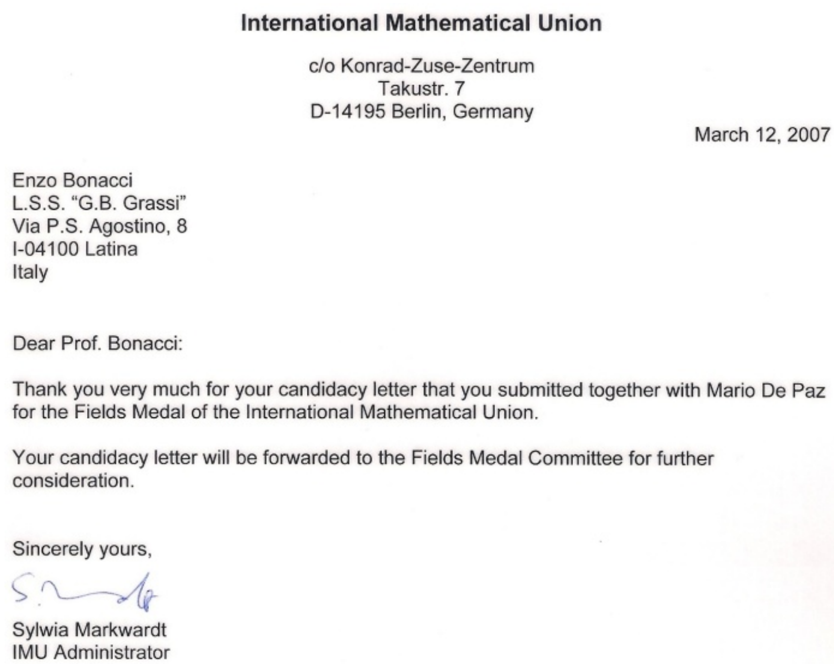
Unfortunately, the monograph that you have submitted does not fulfil some of the Conditions of the Prize and accordingly, I regret to tell you that it cannot be accepted.

Sincerely yours,

(p.o.)

Pere Pascual
FFSB Director

Fig. 7 Rejection letter dated December 5, 2006.



International Mathematical Union

c/o Konrad-Zuse-Zentrum
Takustr. 7
D-14195 Berlin, Germany

March 12, 2007

Enzo Bonacci
L.S.S. "G.B. Grassi"
Via P.S. Agostino, 8
I-04100 Latina
Italy

Dear Prof. Bonacci:

Thank you very much for your candidacy letter that you submitted together with Mario De Paz for the Fields Medal of the International Mathematical Union.

Your candidacy letter will be forwarded to the Fields Medal Committee for further consideration.

Sincerely yours,

Sylwia Markwardt
IMU Administrator

Fig. 8 Acknowledgment letter from the IMU.

INSTITUT D'ESTUDIS CATALANS

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Fax 932 701 180

LXXVII CARTELL DE PREMIS I DE BORSES D'ESTUDI

Premi Ferran Sunyer i Balaguer de Matemàtiques	
Any d'adjudicació:	2008
TÍTOL DEL TREBALL:	*Six moves to checkmate Fermat?*

Hem rebut a la seu de l'Institut d'Estudis Catalans, carrer del Carme, 47, 08001 Barcelona, la tramesa de 1 exemplars de l'obra aspirant al Premi Ferran Sunyer i Balaguer de Matemàtiques presentada per Enzo Bonacci.

Barcelona, dimarts 13 de novembre de 2007

Segell de l'Institut

Fig. 9 Acknowledgment letter from the IEC.

6. Conclusions

Frustrating Fermatist's memories are recalled to demonstrate the enchanting effects of FLT on mathematically-oriented minds.

A 2005 attempt at FLT consisted of two steps: limiting to $\frac{c^p}{2a^{p-1}b^{p-1}}$ the number of the positive integers' pairs $(a;b)$ which satisfy the equation $a^p + b^p = c^p$ and verifying that such upper bound decreases below one for primes $p > 2$.

Discarded as most likely wrong by the author, the proof is now presented for its exotic genesis (a shining example of manuscript written in a Fermat's fever) and in the perspective of future developments.

Namely, the concept of progressive restriction for the number of addends in a p-power sum is suggested for FLT-free improvements.

Acknowledgments

Sincere thanks to the professional interlocutors who took time to study the contributions supplied during the Fermatist period, for their worthy feedback.

Special thanks to Professor Mario De Paz, for his skilled support and caring guidance.

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